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THE INVARIANTS OF LINEAR DIFFERENTIAL EXPRESSIONS.

BY FRANK IRWIN.

THE INVARIANTS OF LINEAR DIFFERENTIAL EXPRESSIONS.[1]

By Frank Irwin.

Presented by Maxime Bôcher, April 8, 1908. Received June 9, 1908.

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[1] This paper was accepted in June, 1908, by the Faculty of Arts and Sciences of Harvard University in fulfilment of the requirement of a thesis for the degree of Doctor of Philosophy.

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THE following paper deals with linear differential expressions, both ordinary and partial, and of all orders. The term "differential expression," as used in these pages, refers, then, always to linear expressions. After an introduction devoted to the theory of the adjoint differential expression, the invariants and covariants of a differential expression under the three transformations which leave its general form unchanged are considered.

The presentation of the introductory matter (I) is, in the main, a reproduction of the substance of lectures by Professor Bôcher in Harvard University, or an extension to expressions of the nth order of matters discussed in those lectures for the second order. The same remark applies to a good part of §§ 4, 5, 7. Acknowledgment of other indebtedness is made in the text. References to Wilczynski are to his Projective Differential Geometry. The name of Lie might be expected to occur more often in a paper on such a subject; it is, however, in obtaining the results recorded in §8 only that I have made use of his methods.

For permission to use the matter referred to above, as well as for most helpful guidance and suggestion in the preparation of this paper throughout, my warmest thanks are due to Professor Bôcher.

I. THE ADJOINT DIFFERENTIAL EXPRESSION.

§ 1. *Ordinary Differential Expressions.*

The first part of this paper deals with the theory of the adjoint differential expression. Let us begin by recalling briefly the facts in the case of an *ordinary* linear differential expression of the nth order. For details, reference may be made to Darboux, Surfaces, book iv, chapter

5, a treatment here followed, or to Wilczynski, who devotes a chapter to the subject. Further, the ordinary differential expression may be looked upon as a special case of the partial differential expression discussed below.

Let, then, our differential expression be

$$L(u) = a_n \frac{d^n u}{dx^n} + a_{n-1} \frac{d^{n-1}u}{dx^{n-1}} + a_{n-2} \frac{d^{n-2}u}{dx^{n-2}} + \ldots + a_0 u. \tag{1}$$

We *define* as its adjoint the expression

$$M(v) = (-1)^n \frac{d^n(a_n v)}{dx^n} + (-1)^{n-1} \frac{d^{n-1}(a_{n-1}v)}{dx^{n-1}} + (-1)^{n-2} \frac{d^{n-2}(a_{n-2}v)}{dx^{n-2}} + \ldots + a_0 v. \tag{2}$$

If we write $M(v)$ also as

$$M(v) = b_n \frac{d^n v}{dx^n} + b_{n-1} \frac{d^{n-1}v}{dx^{n-1}} + \ldots + b_0 v,$$

the b's will be given by the following formula:

$$b_{n-k} = (-1)^n \sum_{l=0}^{k} (-1)^l \frac{(n-l)\,!}{(n-k)\,!\,(k-l)\,!} \frac{d^{k-l}a_{n-l}}{dx^{k-l}}. \tag{3}$$

We may establish next, for any two functions, u, v, *Lagrange's Identity,*

$$vL(u) - uM(v) = \frac{dS}{dx},$$

where S is bilinear in u, v, and their first n–1 derivatives. From this by integration would be obtained a Green's Theorem for the particular differential expression in question. Further, if a relation of the form of Lagrange's Identity,

$$vL(u) - uN(v) = \frac{dT}{dx},$$

exists between two expressions of the nth order, $L(u)$ and $N(v)$, then $N(v)$ is the adjoint of $L(u)$. For we shall have

$$u[N(v) - M(v)] = \frac{d(S-T)}{dx},$$

and therefore $N(v) = M(v)$. This follows from the proposition, the truth of which is obvious:

Lemma. If $N(v)$ be a linear differential expression, and T an expression bilinear in u, v, and their derivatives, and if

$$uN(v) = \frac{dT}{dx},$$

then
$$N(v) = 0.$$

Since Lagrange's Identity may be written

$$uM(v) - vL(u) = \frac{d(-S)}{dx},$$

we infer that $L(u)$ is the adjoint of $M(v)$: the relation between an expression and its adjoint is reciprocal.

A *multiplier* of $L(u)$ is defined to be a function, $v(x)$, such that $vL(u)$ is a derivative of a differential expression of the $(n-1)$st order,

$$vL(u) = \frac{dP}{dx}.$$

The condition that v should be a multiplier of $L(u)$ is that v should satisfy the differential equation $M(v) = 0$. The sufficiency of the condition is obvious from Lagrange's Identity; its necessity follows from an application of the lemma to

$$uM(v) = \frac{d(P-S)}{dx}.$$

For conditions that $L(u)$ should be self-adjoint, when n is even, the negative of its adjoint, when n is odd, that is, $L(u) = (-1)^n M(u)$, see below, page 15. The problem of making $L(u)$ equal to $(-1)^n$ times its adjoint by multiplying it by a suitable function of x will occupy us later.

§ 2. *Partial Differential Expressions of the Second Order.*

We take up next the theory of the adjoint for partial differential expressions, and here a somewhat different order of presentation will be found advantageous. We consider first expressions of the second order.

Let $L(u)$ be such an expression,

$$L(u) = \sum_{i,j=1}^{m} a_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{m} a_i \frac{\partial u}{\partial x_i} + au. \tag{4}$$

Here we make once for all the convention $a_{ij} = a_{ji}$. Let us inquire as to the condition that a function of the x's should be a multiplier of $L(u)$, the term being defined as follows:

Definition. By a *multiplier* of $L(u)$ is meant a function,

$$v(x_1, \ . \ . \ . \ x_m),$$

such that

$$vL(u) = \sum_i \frac{\partial P_i}{\partial x_i}, \tag{5}$$

where the P's are linear differential expressions of the first order.

First suppose that v is such a multiplier. Writing

$$P_i = \sum_j p_{ij} \frac{\partial u}{\partial x_j} + p_i u,$$

we see that we must have

$$2va_{ij} = p_{ij} + p_{ji}, \tag{6a}$$

$$va_i = \sum_j \frac{\partial p_{ji}}{\partial x_j} + p_i, \tag{6b}$$

$$va = \sum_i \frac{\partial p_i}{\partial x_i}. \tag{6c}$$

Operating on the first of these equations with $\frac{\partial^2}{\partial x_i \partial x_j}$, on the second with $-\frac{\partial}{\partial x_i}$, summing and adding to the last equation, the right side cancels out and we have left

$$\sum_{i,j} \frac{\partial^2 (a_{ij} v)}{\partial x_i \partial x_j} - \sum_i \frac{\partial (a_i v)}{\partial x_i} + av = 0.$$

Our assumptions here are that the second derivatives of the a_{ij}'s, the first of the a_i's, that come in question, exist, and, if we desire that property in the coefficients of the equation last written, are continuous. The left side of that equation is, like $L(u)$, a linear differential expression of the second order; we define it to be the adjoint of $L(u)$.

Definition. By the *adjoint* of $L(u)$ we mean the expression

$$M(v) = \sum_{i,j} \frac{\partial^2 (a_{ij} v)}{\partial x_i \partial x_j} - \sum_i \frac{\partial (a_i v)}{\partial x_i} + av. \tag{7}$$

We have proved, then, that a necessary condition that v should be a multiplier of $L(u)$ is that it should satisfy the differential equation $M(v) = 0$.

The condition is also sufficient. For let v be any solution of $M(v) = 0$. Then choose, for instance, the p_{ij}'s for which $i > j$ at pleasure; then the rest of the p_{ij}'s and the p_i's may be determined to satisfy equations (6a) and (6b). Equation (6c) will thereby be satisfied also, and we shall have

$$vL(u) = \sum_i \frac{\partial P_i}{\partial x_i}.$$

For if (6a) and (6b) are satisfied,

$$\sum_{i,j} \frac{\partial^2 (a_{ij} v)}{\partial x_i \partial x_j} - \sum_i \frac{\partial (a_i v)}{\partial x_i} = - \sum_i \frac{\partial p_i}{\partial x_i}.$$

Now since $M(v) = 0$, the left side is equal to $-av$; that is, equation (6c) is satisfied too, as asserted. These considerations show us that the quantities P_i on the right side of (5) are not uniquely determined by v being given. We may state the result just obtained by saying:

Proposition 1. A necessary and sufficient condition that v should be a multiplier of $L(u)$ is that v should satisfy the differential equation $M(v) = 0$.

If we write $M(v)$ in expanded form,

$$M(v) = \sum_{i,j} b_{ij} \frac{\partial^2 v}{\partial x_i \partial x_j} + \sum_i b_i \frac{\partial v}{\partial x_i} + bv,$$

then the b's, the coefficients of the adjoint, will be given by the formulas

$$\left.\begin{aligned} b_{ij} &= a_{ij}, \\ b_i &= 2 \sum_j \frac{\partial a_{ij}}{\partial x_j} - a_i, \\ b &= \sum_{i,j} \frac{\partial^2 a_{ij}}{\partial x_i \partial x_j} - \sum_i \frac{\partial a_i}{\partial x_i} + a. \end{aligned}\right\} \tag{8}$$

These equations may also be written in symmetrical form,

$$\left.\begin{aligned} \sum_j \frac{\partial b_{ij}}{\partial x_j} - b_i &= - \sum_j \frac{\partial a_{ij}}{\partial x_j} + a_i, \\ \sum_i \frac{\partial b_i}{\partial x_i} - 2b &= \sum_i \frac{\partial a_i}{\partial x_i} - 2a. \end{aligned}\right\} \tag{9}$$

We see thus that if $M(v)$ be the adjoint of $L(u)$, then $L(u)$ is the adjoint of $M(v)$.

Analogous to Lagrange's Identity for ordinary differential expressions we have here too an identity to which we may likewise give that name, holding for any two functions u, v.

Lagrange's Identity. $vL(u) - uM(v) = \sum_i \frac{\partial S_i}{\partial x_i}$,

$$S_i = \sum_j a_{ij}\left(v\frac{\partial u}{\partial x_j} - u\frac{\partial v}{\partial x_j}\right) + \left(a_i - \sum_j \frac{\partial a_{ij}}{\partial x_j}\right) uv.$$

This we readily verify by direct calculation. This identity furnishes, as for ordinary differential expressions, a simple proof of the sufficiency of the condition $M(v) = 0$ for v being a multiplier of $L(u)$. Furthermore we have, here as there, the proposition:

Proposition 2. If between any two differential expressions of the second order, $L(u)$ and $N(v)$, we have an identity of the form of Lagrange's Identity,

$$vL(u) - uN(v) = \sum_i \frac{\partial T_i}{\partial x_i},$$

the *T's* being bilinear expressions in u, v, and their first derivatives, then $N(v)$ is the adjoint of $L(u)$.

For we get with the help of Lagrange's Identity,

$$u[N(v) - M(v)] = \sum_i \frac{\partial(S_i - T_i)}{\partial x_i};$$

so that u is a multiplier of the differential expression $N(v) - M(v)$, and therefore satisfies the differential equation

$$\text{Adjoint of } [N(v) - M(v)] = 0.$$

But u is any function whatever. Therefore the adjoint of $N(v) - M(v)$, and so $N(v) - M(v)$ itself, is identically zero.

Integration of Lagrange's Identity supplies, as noted for ordinary differential expressions, a Green's Theorem for the expression $L(u)$.

Necessary and sufficient conditions that $L(u)$ should be *self-adjoint* are

$$a_i = \sum_j \frac{\partial a_{ij}}{\partial x_j}, \qquad i = 1, \ldots m. \tag{10}$$

For these are, by (8), the conditions that b_i should equal a_i, and from them follows $b = a$. For the cases, so common in mathematical physics, where the coefficients of the second derivatives in $L(u)$ are *con-*

stants, these conditions reduce to $a_i = 0$. Thus Laplace's equation is self-adjoint.

For self-adjoint differential expressions, the S_i's in Lagrange's Identity reduce to

$$S_i = \sum_j a_{ij} \left(v \frac{\partial u}{\partial x_j} - u \frac{\partial v}{\partial x_j} \right),$$

and that identity may be thrown into the form

$$vL(u) - \sum_i \frac{\partial P_i}{\partial x_i} = uL(v) - \sum_i \frac{\partial Q_i}{\partial x_i},$$

$$P_i = v \sum_j a_{ij} \frac{\partial u}{\partial x_j}, \qquad Q_i = u \sum_j a_{ij} \frac{\partial v}{\partial x_j}. \tag{11}$$

On the other hand we have for $L(u)$, if self-adjoint,

$$L(u) = \sum_i \frac{\partial}{\partial x_i} \left[\sum_j a_{ij} \frac{\partial u}{\partial x_j} \right] + au.$$

On inserting this value of $L(u)$ in Lagrange's Identity above, the left side goes over into

$$v \sum_i \frac{\partial}{\partial x_i} \left[\sum_j a_{ij} \frac{\partial u}{\partial x_j} \right] + auv - \sum_i \frac{\partial}{\partial x_i} \left[v \sum_j a_{ij} \frac{\partial u}{\partial x_j} \right],$$

that is,

$$- \sum_{i,j} a_{ij} \frac{\partial u}{\partial x_i} \frac{\partial v}{\partial x_j} + auv.$$

Proposition 3. For self-adjoint differential expressions we get a three-term form of Lagrange's Identity,

$$vL(u) - \sum_i \frac{\partial P_i}{\partial x_i} = uL(v) - \sum_i \frac{\partial Q_i}{\partial x_i} = - \sum_{i,j} a_{ij} \frac{\partial u}{\partial x_i} \frac{\partial v}{\partial x_j} + auv,$$

the P's and Q's being given by (11).

Integration would give a corresponding three-term form of Green's Theorem.

In conclusion, attention may be called to the fact that most of the above can be made to apply directly (1) to ordinary differential expressions of the second order, (2) to differential expressions of the first order, by simply putting the proper coefficients in $L(u)$ equal to zero. A similar remark is in order for the developments of the next paragraph. We note that an expression of the first order can never be self-adjoint, but may be the negative of its adjoint.

§ 3. *Partial Differential Expressions of the nth Order.*[2]

For the general case, partial differential expressions of the nth order, we shall content ourselves with considering differential expressions in two independent variables. The formulas themselves suggest what the extension to the case of a greater number of variables will be, and this suggestion leads throughout to the correct formulas for the latter case. We emphasize once for all this remark, which applies to the whole of the rest of this paper.

We make use here of the following notation:

$$L(u) = \sum_{k=0}^{n} \sum_{p=0}^{n-k} \frac{(n-k)\,!}{p\,!\, q\,!} a_{pq} \frac{\partial^{n-k} u}{\partial x^p \partial y^q}, \tag{12}$$

q being defined by $p + q = n - k$; while the subscripts of any a denote respectively the number of differentiations with regard to x, y in the derivative of u to which that coefficient is attached. We may pass from this notation to that employed for the second order by writing, as subscripts, p ones and q twos.

We inquire first, as for expressions of the second order, as to the existence of *multipliers* of $L(u)$, that is of functions, v, such that

$$vL(u) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}, \tag{13}$$

where P, Q are linear differential expressions of the $(n-1)$st order,

$$P = \sum_{k=0}^{n-1} \sum_{p=0}^{n-1-k} P_{pq} \frac{\partial^{n-1-k} u}{\partial x^p \partial y^q}, \qquad p + q = n - 1 - k,$$

with a similar expression for Q. If v is to be such a multiplier we must have

$$\frac{n\,!}{p\,!\, q\,!} v a_{pq} = P_{p-1,\,q} + \quad \text{a term coming from } \frac{\partial Q}{\partial y}, \quad p + q = n, \quad p = 1, 2, \ldots n,$$

$$v a_{0n} = \quad \text{a term coming from } \frac{\partial Q}{\partial y};$$

---

[2] See Darboux, Surfaces, book iv, chapter 4, and, for the second order, chapter 2 of an article by du Bois-Reymond in Crelle, vol. 104 (1889). Darboux makes use, to obtain the condition for a multiplier, of a very general formula, of which we here deduce the special case we require.

$$\frac{(n-k)\,!}{p\,!\,q\,!}va_{pq}=P_{p-1,\,q}+\frac{\partial P_{pq}}{\partial x}+\text{terms coming from }\frac{\partial Q}{\partial y},\quad \begin{array}{l}p+q=n-k,\\ p=1,2,\ldots(n-k),\end{array}$$

$$va_{0,\,n-k}=\qquad\frac{\partial P_{0,\,n-k}}{\partial x}+\text{terms coming from }\frac{\partial Q}{\partial y},$$

$$k=1,\,2,\,\ldots\,(n-1)\,;$$

$$va_{00}=\qquad\frac{\partial P_{00}}{\partial x}+\text{a term coming from }\frac{\partial Q}{\partial y}.$$

Operate on each of these equations with

$$(-1)^{n-k}\frac{\partial^{n-k}}{\partial x^p\partial y^q},\qquad k=0,\,1,\,\ldots\,n,$$

and add. On the left we get the expression

$$M(v)=$$

$$\sum_{k=0}^{n}\sum_{p=0}^{n-k}(-1)^{n-k}\frac{(n-k)\,!}{p\,!\,q\,!}\frac{\partial^{n-k}(a_{pq}v)}{\partial x^p\partial y^q},\qquad p+q=n-k.\qquad(14)$$

This we define as the *adjoint* of $L(u)$. On the right we get zero. For consider the terms coming from $\frac{\partial P}{\partial x}$. These give

$$\sum_{k=0}^{n-1}\sum_{p=1}^{n-k}(-1)^{n-k}\frac{\partial^{n-k}P_{p-1,\,q}}{\partial x^p\partial y^q}+\sum_{k=1}^{n}\sum_{p=0}^{n-k}(-1)^{n-k}\frac{\partial^{n-k+1}P_{pq}}{\partial x^{p+1}\partial y^q}.$$

If, in the second sum, we put $p=p'-1\;\;k=k'+1$, it goes over into the negative of the first, and the two cancel each other. Similarly for the terms coming from $\frac{\partial Q}{\partial y}$. A necessary condition, then, that v should be a multiplier of $L(u)$, is that it should be a solution of the differential equation $M(v)=0$. That the condition is also sufficient, as well as that P and Q in (13) are not uniquely determined when v is given, follows just as for expressions of the second order. As to the former point, we need merely notice that each of the P's itself occurs in one only of the equations above connecting the a's with the P's and the coefficients of Q, in an equation containing the derivative of a P the sum of whose subscripts is greater, that is of a P which may be supposed to have been already determined from the preceding equations.

Writing the adjoint as

$$M(v) = \sum_{k=0}^{n} \sum_{p=0}^{n-k} \frac{(n-k)\,!}{p\,!\,q\,!}\, b_{pq} \frac{\partial^{n-k} v}{\partial x^p \partial y^q}, \qquad p + q = n-k,$$

we may, from (14), calculate the b's in terms of the a's.

Formulas for the coefficients of $M(v)$ in terms of the coefficients of $L(u)$.

$$\left.\begin{aligned}
&p + q = n: \qquad b_{pq} = (-1)^n a_{pq}. \\
&p + q = n - 1: \\
&\quad b_{pq} = (-1)^n \left[n \left(\frac{\partial a_{p+1,\,q}}{\partial x} + \frac{\partial a_{p,\,q+1}}{\partial y} \right) - a_{pq} \right]. \\
&p + q = n - 2: \\
&\quad b_{pq} = (-1)^n \left[\frac{n(n-1)}{2\,!} \left(\frac{\partial^2 a_{p+2,\,q}}{\partial x^2} + 2\, \frac{\partial^2 a_{p+1,\,q+1}}{\partial x \partial y} + \frac{\partial^2 a_{p,\,q+2}}{\partial y^2} \right) \right. \\
&\qquad\qquad \left. - (n-1) \left(\frac{\partial a_{p+1,\,q}}{\partial x} + \frac{\partial a_{p,\,q+1}}{\partial y} \right) + a_{pq} \right]. \\
&p + q = n - k: \\
&\quad b_{pq} = (-1)^n \sum_{l=0}^{k} \sum_{i=0}^{k-l} (-1)^l \frac{(n-l)\,!}{(n-k)\,!\,i\,!\,(k-l-i)\,!} \frac{\partial^{k-l} a_{p+i,\,q+k-l-i}}{\partial x^i \partial y^{k-l-i}}.
\end{aligned}\right\} \quad (15)$$

Assuming for the moment the fact, which will be proved presently, that $L(u)$ is the adjoint of $M(v)$, we may obtain symmetrical formulas connecting the a's and b's. For the formulas expressing the a's in terms of the b's may be written down from those just given by simply interchanging the letters a and b throughout. If now, from these two sets of formulas we replace, in the identity

$$(-1)^n\, a_{pq} + (-1)^k\, b_{pq} = (-1)^k\, b_{pq} + (-1)^n\, a_{pq}, \qquad p + q = n - k,$$

on the left side a_{pq}, on the right b_{pq}, by their values in terms of the b's, a's respectively, we obtain the desired symmetrical formula,

$$\sum_{l=0}^{k-1} \sum_{i=0}^{k-l} (-1)^l \frac{(n-l)\,!}{(n-k)\,!\,i\,!\,(k-l-i)\,!} \frac{\partial^{k-l}\, b_{p+i,\,q+k-l-i}}{\partial x^i \partial\, y^{k-l-i}} + (-1)^k\, 2 b_{pq}$$

$= (-1)^{n+k}$ times the same function of the a's and their derivatives,

$$p + q = n - k.\text{[3]} \qquad (16)$$

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[3] It should be pointed out that these formulas are not precisely analogous to those obtained for the second order. For, if we put here $n = 2$, $k = 2$, we get, using the notation employed for the second order,

The first equation of (15) shows that a differential expression of odd order cannot be self-adjoint, nor one of even order equal to the negative of its adjoint. Let us call a differential expression that is the negative of its adjoint, $L(u) = -M(u)$, anti-self-adjoint. Then we are led to inquire under what conditions a differential expression will be self-adjoint or anti-self-adjoint, $L(u) = (-1)^n M(u)$. Such conditions may be readily deduced from the symmetrical formulas (16). For let

$$b_{pq} = (-1)^n a_{pq}, \qquad p + q = n - l,$$

for $p = 0, 1, \ldots (n - l)$, and for all values of $l < k$, k being a given even integer. Then, on substituting these values in the left member of (16), all the terms but the last on each side cancel, and we have left

$$b_{pq} = (-1)^n a_{pq}, \qquad p + q = n - k,$$

$p = 0, 1, \ldots (n - k)$. Hence, by mathematical induction, we obtain the conditions (which are, of course, necessary):

Proposition 4. Necessary and sufficient conditions that a differential expression should be self-adjoint or anti-self-adjoint, as the case may be, $L(u) = (-1)^n M(u)$, are that the coefficients of the $(n - k)$th derivatives in $L(u)$, should be $(-1)^n$ times the corresponding coefficients of $M(u)$ for all *odd* values of k.

This proposition has already, in effect, been deduced for expressions of the second order; cf. (10), obtained from the second equation of (8) by putting $b_i = a_i$.

Lagrange's Identity. We may deduce for any differential expression a formula similar to what we have called Lagrange's Identity, or rather a great number of such formulas, by the following process:

Take any term of $vL(u)$, $va\,\dfrac{\partial^k u}{\partial x^p \partial y^q}$, where we now write a simply for the coefficient. We have, to start with,

$$va\,\frac{\partial^k u}{\partial x^p \partial y^q} = \frac{\partial}{\partial x}\left(va\,\frac{\partial^{k-1} u}{\partial x^{p-1} \partial y^q}\right) - \frac{\partial (va)}{\partial x}\,\frac{\partial^{k-1} u}{\partial x^{p-1} \partial y^q}.$$

$$\frac{\partial^2 b_{11}}{\partial x^2} + 2\frac{\partial^2 b_{12}}{\partial x \partial y} + \frac{\partial^2 b_{22}}{\partial y^2} - \frac{\partial b_1}{\partial x} - \frac{\partial b_2}{\partial y} + 2b = \text{the same function of the } a\text{'s},$$

an equation which differs from the last equation of (9), written for the case of two independent variables, by the presence of the terms in the second derivatives; terms that cancel each other, indeed, on the two sides of the equation just written. The remaining equations, $n = 2$, $k = 1$, given by (16) agree with those of (9).

Treating the second term on the right in the same way, and so on as long as we can, we get finally

$$\begin{aligned} va\,\frac{\partial^k u}{\partial x^p\partial y^q} &= \frac{\partial}{\partial x}\left(va\,\frac{\partial^{k-1}u}{\partial x^{p-1}\partial y^q}\right) - \frac{\partial}{\partial x}\left(\frac{\partial(va)}{\partial x}\,\frac{\partial^{k-2}u}{\partial x^{p-2}\partial y^q}\right) + \dots \\ &\pm \frac{\partial}{\partial x}\left(\frac{\partial^{p-1}(va)}{\partial x^{p-1}}\,\frac{\partial^q u}{\partial y^q}\right) \mp \frac{\partial}{\partial y}\left(\frac{\partial^p(va)}{\partial x^p}\,\frac{\partial^{q-1}u}{\partial y^{q-1}}\right) + \dots \\ &+(-1)^{k-1}\frac{\partial}{\partial y}\left(\frac{\partial^{k-1}(va)}{\partial x^p\partial y^{q-1}}\,u\right) + (-1)^k\,\frac{\partial^k(va)}{\partial x^p\partial y^q}\,u. \end{aligned}$$

The last term on the right is the term of $uM(v)$ corresponding to the term of $vL(u)$ chosen. The other terms on the right are derivatives with regard to x or y of expressions bilinear in u, v and their derivatives of order less than n. Applying the same process to all the terms of $vL(u)$, we reach the result:

Lagrange's Identity. For any two functions u, v of x and y,

$$vL(u) - uM(v) = \frac{\partial S}{\partial x} + \frac{\partial T}{\partial y},$$

where S, T are expressions bilinear in u, v and their derivatives of orders up to the $(n-1)$st.

In the process sketched above, there is evidently much that is arbitrary. Thus we might equally well have written

$$va\,\frac{\partial^k u}{\partial x^p\partial y^q} = \frac{\partial}{\partial y}\left(va\,\frac{\partial^{k-1}u}{\partial x^p\partial y^{q-1}}\right) - \frac{\partial}{\partial x}\left(\frac{\partial(va)}{\partial y}\,\frac{\partial^{k-2}u}{\partial x^{p-1}\partial y^{q-1}}\right) + \dots$$

a choice being offered at each, or at least at many of the steps of the process, of what the next term to be written down shall be; the last term, in any case, being evidently as above $(-1)^k\,\frac{\partial^k(va)}{\partial x^p\partial y^q}\,u$. So that the S and T in Lagrange's Identity are far from being uniquely determined.[4]

Corresponding to proposition 2, page 10, we have here also that if between any two differential expressions there holds an identity of the form of Lagrange's Identity, then each is the adjoint of the other. This justifies the assumption made on page 14 above, that $L(u)$ was the adjoint of $M(v)$.

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[4] The process employed first above is that suggested by Darboux, Surfaces, **2**, 73, note. His identity numbered (7) on page 72 is derived by some other of the many possible processes.

II. CHANGE OF DEPENDENT VARIABLE; INVARIANTS AND COVARIANTS. INVARIANTS OF A DIFFERENTIAL EQUATION.

§ 4. *General Properties of Invariants and Covariants.*

We take up next the subject of the transformation of a differential expression by change of dependent variable and of the invariants and covariants of such a transformation.

Taking our differential expression in the form (12), let it go over under change of variable, $u = \psi\,(x, y(\cdot \eta$, into a differential expression $\Lambda(\eta)$, with coefficients α. $\Lambda(\eta)$ will be of the nth order, and its coefficients, the α's, may be readily calculated.

Formulas for the coefficients of the transformed differential expression.

$$\left.\begin{aligned}
&p + q = n: \quad \alpha_{pq} = a_{pq}\psi. \\
&p + q = n - 1: \quad \alpha_{pq} = n\left(a_{p+1,\,q}\frac{\partial\psi}{\partial x} + a_{p,\,q+1}\frac{\partial\psi}{\partial y}\right) + a_{pq}\psi. \\
&p + q = n - 2: \\
&\alpha_{pq} = \frac{n(n-1)}{2\,!}\left(a_{p+2,\,q}\frac{\partial^2\psi}{\partial x^2} + 2\,a_{p+1,\,q+1}\frac{\partial^2\psi}{\partial x\partial y} + a_{p,q+2}\frac{\partial^2\psi}{\partial y^2}\right) \\
&\qquad\qquad + (n-1)\left(a_{p+1,\,q}\frac{\partial\psi}{\partial x} + a_{p,q+1}\frac{\partial\psi}{\partial y}\right) + a_{pq}\psi. \\
&p + q = n - k: \\
&\alpha_{pq} = \sum_{l=0}^{k}\sum_{i=0}^{k-l}\frac{(n-l)\,!}{(n-k)\,!\,i\,!\,(k-l-i)\,!}\,a_{p+i,\,q+k-l-i}\,\frac{\partial^{k-l}\psi}{\partial x^i\partial y^{k-l-i}}.
\end{aligned}\right\}(17)$$

For *ordinary* differential expressions these reduce to

$$\alpha_{n-k} = \sum_{l=0}^{k}\frac{(n-l)\,!}{(n-k)!\,(k-l)\,!}\,a_{n-l}\frac{d^{k-l}\psi}{dx^{k-l}}, \tag{18}$$

while for expressions of the second order,

$$L(u) = \sum_{i,j} a_{ij}\frac{\partial^2 u}{\partial x_i\partial x_j} + \sum_i a_i\frac{\partial u}{\partial x_i} + au,$$

we should get

$$\left.\begin{aligned} \alpha_{ij} &= a_{ij}\psi, \\ \alpha_i &= 2\sum_j a_{ij}\frac{\partial\psi}{\partial x_j} + a_i\psi, \\ \alpha &= \sum_{i,j} a_{ij}\frac{\partial^2\psi}{\partial x_i \partial x_j} + \sum_i a_i\frac{\partial\psi}{\partial x_i} + a\psi = L(\psi). \end{aligned}\right\} \qquad (19)$$

It is in the invariants and covariants of this transformation that we shall interest ourselves. These terms we define as follows:

Definition. By an *invariant* of $L(u)$ under the transformation $u = \psi \cdot \eta$ is meant a function, I, of the a's and their derivatives such that the same function of the coefficients of the transformed differential expression is equal, by virtue of the formulas (17), to the original function multiplied by a power of ψ.

$$I \text{ (α's and derivatives)} = \psi^\mu I \text{ (a's and derivatives)},$$

or, in a convenient abbreviated notation,

$$I(\alpha) = \psi^\mu I(a).$$

If $\mu = 0$, we have an absolute, otherwise a relative invariant.

By a *covariant* is meant a function, not only of the a's and their derivatives, but also of u and its derivatives, having an invariant property defined in a manner similar to the above.

We shall concern ourselves wholly with rational, and principally with rational, integral invariants and covariants, and shall always be speaking of the latter, wherever the contrary is not stated or evident from the context. It will be noticed, however, that certain propositions are true for invariants in general.

We begin with some generalities. Every rational invariant is homogeneous. For make the transformation $u = c \cdot \eta$, c being any constant other than zero. The coefficients of the transformed differential expression are each c times the corresponding coefficient of the original expression, $\alpha_{pq} = ca_{pq}$, and the same is true of their derivatives; so that we have: $I(ca) = c^\mu I(a)$, which shows that I is homogeneous. We shall, in accordance with the usage in vogue for homogeneous functions in general, speak of μ as the degree of the invariant, even when it is not a polynomial. The corresponding proposition for polynomial covariants is that the degree of any term in the a's and their derivatives minus its degree in u and its derivatives is constant and equal to μ.

We proceed now to attach a *weight* to each of the a's and its derivatives.

Definition. The *weight* or *total weight* of the coefficient a_{pq}, $p+q=n-k$, shall be $n-k$, and the weight of $\frac{\partial^{i+j}a_{pq}}{\partial x^i \partial y^j}$, $n-k-(i+j)$; the *partial weights* with respect to x, y of a_{pq}, $p+q=n-k$, shall be p, q respectively, and of $\frac{\partial^{i+j}a_{pq}}{\partial x^i \partial y^j}$, $p-i$ and $q-j$ respectively. The weight of a product shall be the sum of the weights of its factors; and a polynomial will be said to be isobaric, totally or partially, if all its terms are of the same weight, total or partial. With this definition of weight we have the following proposition:

Proposition 5. An invariant may or may not be isobaric; but if not, it is a mere sum of invariants which are isobaric. This statement may be interpreted with respect either to the total or any one of the partial weights.

We give the proof for the former case. Consider the identity, $I(\alpha) = \psi^{\mu} I(a)$. Let the terms of any given weight, w, in $I(\alpha)$ be represented by $G_w(\alpha)$; and let us fix our attention on the corresponding terms, $G_w(a)$, in $I(a)$. Suppose we attribute, for our immediate purposes, to ψ the weight zero, to its first derivatives the weight minus one, and so on. Then a comparison of the formulas (17) shows that α_{pq}, $p+q=n-k$, is of weight $n-k$ in this system of weights, while any of its derivatives is of weight equal to $n-k$ minus the number of differentiations; that is, the weights of the α's and their derivatives are the same as those of the corresponding a's and their derivatives. Thus $G_w(a)$ is of weight w, while all the other terms of $I(a)$ are of some other weight; and consequently there can be no cancelling, whole or in part, between those two sets of terms. Therefore, in $I(\alpha) = \psi^{\mu} I(a)$, $G_w(\alpha)$ must be equal to the terms of weight w on the right side of the equation: i. e., $G_w(\alpha) = \psi^{\mu} G_w(a)$, as was to be proved.

This proposition is of service when we are inquiring as to what invariants of a particular degree exist; in which case we may limit the inquiry to isobaric invariants, since all others can be built up from them by addition.

§ 5. PARTICULAR INVARIANTS.

A simple set of invariants is furnished by the coefficients of the adjoint differential expression. That these are invariants follows at once from the proposition:

Proposition 6. The adjoint of the transformed differential expression, $\Lambda(\eta)$, is ψ times the adjoint of the original expression, $L(u)$.

For make, in Lagrange's Identity,

$$vL(u) - uM(v) = \frac{\partial S}{\partial x} + \frac{\partial T}{\partial y},$$

the change of variable, $u = \psi \cdot \eta$. S, T go over into expressions $\bar{S}$, $\bar{T}$ bilinear in η, v and their derivatives of orders up to the $(n-1)$st. This gives us,

$$v \cdot \Lambda(\eta) - \eta \cdot \psi M(v) = \frac{\partial \bar{S}}{\partial x} + \frac{\partial \bar{T}}{\partial y}.$$

But the existence of an identity of this form between the two expressions $\Lambda(\eta)$ and $\psi M(v)$ shows that they are mutually adjoint.

The coefficients of the adjoint, the b's, are then invariants. They are linear in the a's and their derivatives; cf. the formulas for them (15). Moreover, it may be shown that they are essentially the only linear invariants (see below, page 26). In terms of these invariants and their derivatives — which latter, however, are not invariants — every invariant may be expressed rationally and integrally, simply because the a's and their derivatives can be so expressed.

Further, the b's form a complete system of invariants. This phrase we use in the following sense. Two configurations are said to be equivalent with regard to a given set of transformations if it is possible to find a transformation of the set that takes the first over into the second, and another that takes the second over into the first. A complete set of *absolute* invariants is a set such that if two configurations have the invariants in question equal, each to each, then the two are equivalent. In the case before us we have to do with relative invariants.

Proposition 7. The linear invariants, the b's, constitute a *complete system* of invariants; that is to say, if the linear invariants of two differential expressions are proportional, the expressions are *equivalent*.

Let $L(u)$, $\Lambda(\eta)$ be the two differential expressions, $M(v)$, $M_1(v)$ their adjoints. By hypothesis the coefficients of these latter are proportional; that is, each coefficient of $M_1(v)$ is, say, $\theta(x, y)$ times the corresponding coefficient of $M(v)$. Therefore $M_1(v) = \theta \cdot M(v)$. Now make in $L(u)$ the change of variable $u = \theta \cdot \eta$, and let it go over thereby into $\Lambda_1(\eta)$. The adjoint of $\Lambda_1(\eta)$ is, by proposition 6, θ times the adjoint of $L(u)$, that is $\theta \cdot M(v)$, that is, $M_1(v)$. But $M_1(v)$ was the adjoint of $\Lambda(\eta)$; so that $\Lambda_1(\eta)$ and $\Lambda(\eta)$, being each the adjoint of $M_1(v)$, must be identical; $L(u)$ then goes over, under $u = \theta \cdot \eta$ into $\Lambda(\eta)$. *Q. E. D.*

It is of interest to inquire after processes for deriving, from given invariants, other invariants. One such process is differentiation: the derivative, with regard to any one of the independent variables, of an *absolute* invariant is, in its turn, an absolute invariant; for from $I(a) = I(\alpha)$, $\frac{\partial I(\alpha)}{\partial x_i} = \frac{\partial I(a)}{\partial x_i}$ follows. Since the quotient of any two relative invariants of the same degree is an absolute invariant, this process supplies us with a means of deriving, from two such invariants, a third; a result which, since the denominator, and therefore also the numerator, of the derived invariant are themselves invariants, we may state in another form as follows: If I_1, I_2 be any two relative invariants *of the same degree*, μ, then the Wronskian

$$\begin{vmatrix} I_1 & \frac{\partial I_1}{\partial x} \\ I_2 & \frac{\partial I_2}{\partial x} \end{vmatrix}$$

is also an invariant, and is of degree 2μ. We note that this Wronskian process admits of extensions. If, for instance, I_1, I_2, I_3 be three invariants of the same degree, μ, then both

$$\begin{vmatrix} I_1 & \frac{\partial I_1}{\partial x} & \frac{\partial^2 I_1}{\partial x^2} \\ I_2 & \frac{\partial I_2}{\partial x} & \frac{\partial^2 I_2}{\partial x^2} \\ I_3 & \frac{\partial I_3}{\partial x} & \frac{\partial^2 I_3}{\partial x^2} \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} I_1 & \frac{\partial I_1}{\partial x} & \frac{\partial I_1}{\partial y} \\ I_2 & \frac{\partial I_2}{\partial x} & \frac{\partial I_2}{\partial y} \\ I_3 & \frac{\partial I_3}{\partial x} & \frac{\partial I_3}{\partial y} \end{vmatrix}$$

are invariants. And in general the following precept may be laid down for deriving invariants. Write down, as the first column of an m-rowed determinant, m invariants of the same degree, μ. Take for the elements of any other column the derivatives, with regard to any given one of the independent variables, of the elements of some preceding column. This independent variable may be different for different columns. The determinant so constructed will be an invariant of degree $m\mu$. The proof consists in writing down the transformed Wronskian, when everything except $\psi^{m\mu}$ times the original Wronskian will be seen to vanish.

In particular we may derive invariants by this Wronskian process from our linear invariants, the b's. For instance, let b stand for any given one of the b's. Then $\frac{\partial b_{pq}}{\partial x} b - b_{pq} \frac{\partial b}{\partial x}$ is an invariant of the second degree. But so also is b^2. Therefore

$$\frac{\partial}{\partial y}\left(\frac{\partial b_{pq}}{\partial x} b - b_{pq}\frac{\partial b}{\partial x}\right)b - 2\left(\frac{\partial b_{pq}}{\partial x} b - b_{pq}\frac{\partial b}{\partial x}\right)\frac{\partial b}{\partial y}$$

is an invariant of the third degree; and so on. These invariants are evidently merely the numerators of the various derivatives of $\frac{b_{pq}}{b}$. With regard to them we have the following proposition:

Proposition 8. Let b be any chosen one of the b's. Then every invariant can be expressed rationally, and save for the possible presence of a power of b as a denominator, integrally, in terms of b and the numerators of $\frac{b_{pq}}{b}, \frac{b_{p'q'}}{b}, \ldots$ and the numerators of the derivatives of $\frac{b_{pq}}{b}, \frac{b_{p'q'}}{b}, \ldots$ all these numerators being themselves rational, integral invariants. The notation chosen for the enunciation refers to the case of partial differential expressions in two independent variables: the proposition is valid in every case.

Let the invariant $I(a)$ be expressed in terms of the b's: $I(a) = J(b)$. Put $u = \frac{1}{b}\cdot\eta$, and let $L(u)$ go over into $\Lambda(\eta)$. Since the adjoint of $\Lambda(\eta)$ is $\frac{1}{b}$ times the adjoint of $L(u)$ we shall get $I(\alpha)$ by substituting in $J(b)$, for $b_{pq}, b_{p'q'}, \ldots$ and their derivatives, $\frac{b_{pq}}{b}, \frac{b_{p'q'}}{b}, \ldots$ and their derivatives. But $I(\alpha) = \frac{1}{b^\mu} I(a)$. This gives us

$$J\left(b, b_{pq}, b_{p'q'}, \ldots; \frac{\partial b}{\partial x}, \frac{\partial b_{pq}}{\partial x}, \ldots; \frac{\partial b}{\partial y}, \frac{\partial b_{pq}}{\partial y}, \ldots\right)$$
$$= b^\mu J\left(1, \frac{b_{pq}}{b}, \frac{b_{p'q'}}{b}, \ldots; 0, \frac{\partial}{\partial x}\left(\frac{b_{pq}}{b}\right), \ldots; 0, \frac{\partial}{\partial y}\left(\frac{b_{pq}}{b}\right), \ldots\right).$$

As to a determination of all invariants of the second degree, see below, page 26.

§ 6. *Particular Covariants.*

The simple set of covariants which we now go on to deduce will be, apart from such interest as they may possess in themselves, of use to us later in another connection. For *ordinary* differential expressions the $n + 1$ expressions

$$\sum_{l=0}^{k} \frac{(n-l)!}{(n-k)!\,(k-l)!}\, a_{n-l}\, \frac{d^{k-l}u}{dx^{k-l}}, \qquad k = 0, 1, \ldots n,$$

are absolute covariants. Note that for $k = n$ the expression reduces to $L(u)$. This result is simply a translation into terms of differential expressions of the corresponding facts in the case of ordinary differential equations given by Wilczynski, Chapter II, § 2.[5] And what follows is a mere extension to the case of *partial* differential expressions.

The formulas, (17), expressing the $\boldsymbol{a}$'s in terms of the a's may be given a form more advantageous for some purposes by introducing, in the coefficients of $L(u)$, $\Lambda(\eta)$, further binomial coefficients. Let us put $a_{pq} = \frac{n!}{(n-k)!\,k!}\, c_{pq}$, $p + q = n - k$, and, correspondingly, $\boldsymbol{a}_{pq} = \frac{n!}{(n-k)!\,k!}\, \gamma_{pq}$. $L(u)$ thus becomes,

$$L(u) = \sum_{k=0}^{n} \sum_{p=0}^{n-k} \frac{n!}{k!\,p!\,q!}\, c_{pq}\, \frac{\partial^{n-k}u}{\partial x^p \partial y^q}, \qquad p + q = n - k,$$

while the formulas of transformation are

$$\gamma_{pq} = \sum_{l=0}^{k} \sum_{i=0}^{k-l} \frac{k!}{l!\,i!\,(k-l-i)!}\, c_{p+i,\, q+k-l-i}\, \frac{\partial^{k-l}\psi}{\partial x^i \partial y^{k-l-i}}, \qquad p + q = n - k, \qquad (20)$$

formulas in which everything except the subscripts of the c's is *independent of n*. Now let $L_j(u)$ be an expression of the jth order, $j \leqq n$,

$$L_j(u) = \sum_{k=0}^{j} \sum_{p=0}^{j-k} \frac{j!}{k!\,p!\,q!}\, d_{pq} \frac{\partial^{j-k}u}{\partial x^p \partial y^q}, \qquad p + q = j - k.$$

If we make the change of variable $u = \psi \cdot \eta$, the coefficients of the transformed expression will, by (20), be given by

$$\delta_{pq} = \sum_{l=0}^{k} \sum_{i=0}^{k-l} \frac{k!}{l!\,i!\,(k-l-i)!}\, d_{p+i,\, q+k-l-i}\, \frac{\partial^{k-l}\psi}{\partial x^i \partial y^{k-l-i}}, \qquad p + q = j - k.$$

Now take any two numbers $\bar{p}$, $\bar{q}$ such that $\bar{p} + \bar{q} = n - j$. If we put $d_{pq} = c_{p+\bar{p},\, q+\bar{q}}$ for all values of p, q such that $p + q \leqq j$, the expression just written for δ_{pq} goes over into

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[5] These covariants were first given by Cockle, Phil. Mag., **30** (1865); see Bouton's paper in the Amer. Jour. of Math., **21** (1899).

$$\cdot \sum_{l=0}^{k} \sum_{i=0}^{k-l} \frac{k!}{l!\, i!\, (k-l-i)!} c_{p+\bar{p}+i,\, q+\bar{q}+k-l-i} \frac{\partial^{k-l}\psi}{\partial x^i \partial y^{k-l-i}},$$

that is, by (20), since $p + \bar{p} + q + \bar{q} = n - k$, into $\gamma_{p+\bar{p},\, q+\bar{q}}$. We have then, $\delta_{pq} = \gamma_{p+\bar{p},\, q+\bar{q}}$. Comparing this with the formulas connecting the d's and c's, $d_{pq} = c_{p+\bar{p},\, q+\bar{q}}$, we see that for these values of the d's $L_j(u)$ is an expression that goes over under $u = \psi \cdot \eta$ into the same function of the γ's that $L_j(u)$ itself is of the c's; in other words, it is an absolute covariant. Inserting then these values of the d's in $L_j(u)$, replacing the c's by the a's, and multiplying through by $\frac{n!}{j!}$, we get the proposition:

Proposition 9. The expressions

$$\sum_{k=0}^{j} \sum_{p=0}^{j-k} \frac{(n-k)!}{p!\, q!} a_{p+\bar{p},\, q+\bar{q}} \frac{\partial^{j-k} u}{\partial x^p \partial y^q}, \qquad p + q = j - k,$$
$$j = 0, 1, \ldots n,$$

are absolute covariants for $u = \psi \cdot \eta$. Here $\bar{p}$, $\bar{q}$ are *any* given positive integers (or zeros) subject to the condition: $\bar{p} + \bar{q} = n - j$.

For $j = n$, we get $L(u)$ itself.

For $j = 0$: $\qquad a_{pq}u, \qquad p + q = n.$

For $j = 1$: $\quad n\left(a_{p+1,q}\frac{\partial u}{\partial x} + a_{p,q+1}\frac{\partial u}{\partial y}\right) + a_{pq}u, \qquad p + q = n - 1.$

We note that these covariants are what might, in accordance with a nomenclature we are about to introduce, be called covariants of the differential *equation*.

§ 7. *Multiplication of $L(u)$ by ϕ; Invariants of a Differential Equation.*

Let us now consider briefly a second transformation to which a differential expression may be subjected, namely, that of multiplying it through by a function ϕ of the independent variable or variables. Represent the coefficients of $\phi \cdot L(u)$ by $\bar{a}$'s. Then we define as an invariant of this transformation an expression $I(a)$, such that $I(\bar{a}) = \phi^{\mu} I(a)$.

Between the invariants of $L(u)$ and those of $M(v)$ a simple relation exists.

Proposition 10. An invariant of a differential expression for a multiplication by ϕ is an invariant of its adjoint for change of dependent variable; an invariant for change of dependent variable is an invariant of the adjoint for multiplication by ϕ.

We prove the first part of the proposition. Let $I(a)$ be an invariant of $L(u)$ for multiplication by ϕ; and let $I(a)$, expressed in terms of the b's, be $J(b)$; $I(a) = J(b)$. Let $M(v)$ go over under $v = \phi \cdot v_1$ into $M_1(v_1)$ with coefficients β. Then by proposition 6, page 19, $\phi L(u)$ and $M_1(v_1)$ are mutually adjoint. Therefore $I(\alpha) = J(\beta)$. But

$$I(\bar{a}) = \phi^\mu I(a) = \phi^\mu J(b).$$

Therefore $$J(\beta) = \phi^\mu J(b). \qquad Q.E.D.$$

Expressions $I(a)$ that are invariant not only for change of dependent variable but also for multiplication of $L(u)$ by ϕ it will be natural to speak of as *invariants of the differential equation* $L(u) = 0$.

Now let $I(a) = J(b)$ be any such invariant. By the proposition just proved $J(b)$ is also an invariant of the differential equation $M(v) = 0$. Therefore $J(a)$ is an invariant of $L(u) = 0$; and, the relation between $L(u)$ and $M(v)$ being reciprocal, $J(a) = I(b)$.

Proposition 11. If $I(a) = J(b)$ be an invariant of a differential equation, then so also is $J(a) = I(b)$. We shall call either of two such invariants the *adjoint* of the other.

It is evident that proposition 5, page 19, may be extended to invariants of a differential equation: if not itself isobaric, such an invariant is nothing more than the sum of invariants which are.

As to a complete system of invariants of a differential equation see below, page 29.

§ 8. *Invariants of the First and Second Degree of Differential Expressions and Equations.*

A problem of interest with regard to the invariants of a differential expression or those of a differential equation is that of determining all the invariants of a given degree. The results which I have been able to obtain concern invariants of the first and second degree.

The methods I have employed are as follows. In the first place, as we have seen, we need merely consider invariants isobaric with respect to each independent variable. Next, in the case of invariants of a differential *expression*, we may confine ourselves to such as are homogeneous in *each* b and its derivatives. For if we call the coefficients of $\psi M(v)$ $\bar{b}$'s, we have

$$I(\bar{b}) = \psi^\mu I(b).$$

Now consider the terms of $I(b)$ homogeneous of any given degree in any given one of the b's and its derivatives. The corresponding terms of $I(\bar{b})$ will, since the $\bar{b}$'s are simply the b's multiplied by ψ, be homogeneous of the same degree in the given b and its derivatives; whence it follows that the terms of $I(b)$ in question will themselves constitute an invariant.

The result just obtained enables us to determine at once all linear invariants of a differential expression. For such an invariant may now be taken as containing *one* of the b's and its derivatives only. Then if we consider any of the derivatives of the highest order of that b occurring in $I(b)$, $I(\bar{b})$ will evidently contain uncancelled the same derivative of ψ; so that, if we are to have $I(\bar{b}) = \psi^{\mu} I(b)$, I can contain no derivatives of b at all. (Similar considerations would show that an invariant of any degree, involving one of the b's and its derivatives only, is essentially nothing more than a power of the b.)

Proposition 12. Essentially the only linear invariants of a differential expression are the b's themselves, all others being linear combinations of these invariants.

The general problem, apart from this simple case, may be attacked by the use of Lie's methods, as illustrated in Bouton's paper in the American Journal of Mathematics, vol. 21. The complete system thus obtained of linear partial differential equations, whose solutions are the invariants sought for, takes on, in the case of invariants of a differential expression, a particularly simple form if everything is expressed in terms, not of the a's, but, as above, of the b's and their derivatives. I bring together here the results I have obtained by the use of these and such other methods as suggested themselves, in each particular case, as appropriate.

Proposition 13. Essentially the only invariants of the *second* degree of an *ordinary* differential *expression* are, besides powers and products of the b's themselves, those of the form

$$\frac{db_i}{dx} b_j - b_i \frac{db_j}{dx};$$

of a *partial* differential *expression* in *two* independent variables, those of the form

$$\frac{\partial b_i}{\partial x} b_j - b_i \frac{\partial b_j}{\partial x}, \quad \frac{\partial b_i}{\partial y} b_j - b_i \frac{\partial b_j}{\partial y},$$

b_i, b_j being any two of the b's.

Essentially the only invariant of the *first* degree of an *ordinary* differential *equation* is b_n and essentially the only invariants of the *second* degree are b_n^2 and

$$n b_n b_{n-2} + \frac{n(n-1)}{2}\left(\frac{db_n}{dx} b_{n-1} - b_n \frac{db_{n-1}}{dx}\right) - \frac{n-1}{2} b_{n-1}^2$$

$$= n a_n a_{n-2} + \frac{n(n-1)}{2}\left(\frac{da_n}{dx} a_{n-1} - a_n \frac{da_{n-1}}{dx}\right) - \frac{n-1}{2} a_{n-1}^2,$$

which is na_n times the invariant called I_{n-2}, (23) below.

Essentially the only invariants of the *first* degree of a *partial* differential *equation* of the *second* order in *two* independent variables are b_{11}, b_{12}, b_{22}, and of the *second* degree, besides powers and products of the b_{ij}'s, those of the form

$$\frac{\partial b_{ij}}{\partial x} b_{kl} - b_{ij} \frac{\partial b_{kl}}{\partial x}, \quad \frac{\partial b_{ij}}{\partial y} b_{kl} - b_{ij} \frac{\partial b_{kl}}{\partial y},$$

invariants which involve the b's with two subscripts only.

III. Reduction to Canonical Form.

§ 9. *Ordinary Differential Expressions.*

Another method of treating the problem of invariants, a method that applies to the case of an *ordinary* differential expression, is to reduce that expression by a suitable change of dependent variable, to what we shall call its *canonical* form, namely, a form in which the coefficient of the $(n-1)$st derivative is zero.

The corresponding investigation for the case of ordinary differential *equations* will be found in Wilczynski, Chapter II, § 2.[6] The treatment of the two cases is, to a large extent, identical; so that what follows is given not so much for its own sake as because a number of the results admit of extension to *partial* differential expressions.

Suppose then we have an ordinary differential expression

$$L(u) = a_n u^{(n)} + \ldots . + a_0 u,$$

accents denoting differentiation. Let it be reduced, by the change of variable $u = \theta \cdot \eta$, to canonical form

$$\Lambda(\eta) = A_n \eta^{(n)} + \qquad\qquad A_{n-2} \eta^{(n-2)} + \ldots . + A_0 \eta \cdot$$

---

[6] We note, to avoid confusion, that Wilczynski calls our canonical form semi-canonical. The method is due to Cockle, Philosophical Magazine, **39** (1870); see Bouton's paper in the Amer. Jour. of Math., **21** (1899).

We see, from (18), that to have $A_{n-1} = 0$, θ must satisfy the equation

$$n a_n \theta' + a_{n-1}\theta = 0,$$

or

$$\theta' = -\frac{1}{n}\frac{a_{n-1}}{a_n}\theta. \tag{21}$$

The other coefficients are given by the formula

$$A_{n-k} = \sum_{l=0}^{k} \frac{(n-l)\,!}{(n-k)\,!\,(k-l)\,!}\,\theta^{(k-l)} a_{n-l}. \tag{22}$$

Substituting herein the values of the derivatives of θ obtained from (21) by differentiation, we find that A_{n-k} is θ times a rational function of the a's and their derivatives, say $A_{n-k} = I_{n-k}(a)\,\theta$. Here we use the letter I because the expressions in question are, in fact, invariants. For let $L(u)$ go over under $u = \psi \cdot u_1$ into $L_1(u_1)$ with coefficients a. Then $L_1(u_1)$ will go over by $u_1 = \frac{\theta}{\psi}\eta$ into $\Lambda(\eta)$ above. Since this is a canonical form for $L_1(u_1)$, as well as for $L(u)$, we shall have

$$A_{n-k} = I_{n-k}(\alpha)\frac{\theta}{\psi}.$$

Comparing this with $A_{n-k} = I_{n-k}(a)\theta$, we get

$$I_{n-k}(\alpha) = \psi I_{n-k}(a).$$

The expressions $I_{n-k} = A_{n-k}/\theta$, $k = 0, 2, 3, \ldots n$, are then rational invariants, of the first degree, of the differential expression. Moreover, they are invariants of the differential equation.

For it will be seen from (21) that θ'/θ is the same for $\phi L(u)$ as for $L(u)$ itself, and the same, it is clear, will be true of $\theta^{(k-l)}/\theta$. We see, then, from (22), that I_{n-k}, that is A_{n-k}/θ, formed for $\phi L(u)$ is ϕ times I_{n-k} formed for $L(u)$; or I_{n-k} is an invariant for multiplication by ϕ.

Now, further, suppose that two differential equations, $L(u) = 0$ and $L_1(u_1) = 0$, have these invariants proportional; that is to say, if $L(u)$, $L_1(u_1)$ go over by $u = \theta \cdot \eta$, $u_1 = \theta_1 \cdot \eta$ into canonical forms with coefficients A and $\bar{A}$ respectively, then $A_{n-k}/\theta = \rho(x)\bar{A}_{n-k}/\theta_1$. If now we multiply the former of these canonical forms by $\frac{\theta_1}{\rho\theta}$, it goes over into the latter. We have thus the proposition:

Proposition 14. The expressions

$$I_{n-k} = \frac{A_{n-k}}{\theta}, \quad k = 0, 2, 3, \ldots n,$$

where the A's are the coefficients of the canonical form into which $L(u)$ goes over by $u = \theta \cdot \eta$, form a complete system of invariants of an ordinary differential equation; a complete system, that is, in the sense of equivalence, as explained on page 20.

Next let I be any rational invariant, of degree μ, of the differential expression.

$$\theta^{\mu} I(a_n, a_n', \ldots a_{n-1}, a_{n-1}', \ldots a_{n-k}, a_{n-k}^{(l)}, \ldots)$$
$$= I\ (A_n, A_n', \ldots 0, 0, \ldots A_{n-k}, \ldots A_{n-k}^{(l)}, \ldots)$$

which, since I is homogeneous, is equal to

$$\theta^{\mu} I \left(\frac{A_n}{\theta}, \frac{A_n'}{\theta}, \ldots 0, 0, \ldots \frac{A_{n-k}}{\theta}, \ldots \frac{A_{n-k}^{(l)}}{\theta}, \ldots \right)$$
$$= \theta^{\mu} I\ (I_n, I_{n1}, \ldots 0, 0, \ldots I_{n-k}, I_{n-k,l}, \ldots),$$

if we put $I_{n-k,\,l} = A_{n-k}^{(l)}/\theta$. The expressions $I_{n-k,\,l}$ are, like I_{n-k}, rational invariants of the first degree. This we shall prove in a moment, and thus get the proposition:

Proposition 15. Every rational invariant of a differential expression under change of dependent variable is a rational function of the rational invariants of the first degree

$$I_{n-k} = \frac{A_{n-k}}{\theta}, \qquad I_{n-k,\,l} = \frac{A_{n-k}^{(l)}}{\theta},$$

where the A's are the coefficients of the canonical form into which $L(u)$ goes over under $u = \theta \cdot \eta$, and θ satisfies (21).

$$I(a_n, a_n', \ldots a_{n-1}, a_{n-1}', \ldots a_{n-k}, \ldots a_{n-k}^{(l)}, \ldots)$$
$$= I(I_n, I_{n1}, \ldots 0, 0, \ldots I_{n-k}, \ldots I_{n-k,\,l}, \ldots).$$

In particular, if I be a polynomial, it is a polynomial in these invariants as well.

We note that

$$I_{n-2} = \frac{n a_n a_{n-2} + \frac{n(n-1)}{2}(a_n' a_{n-1} - a_n a_{n-1}') - \frac{n-1}{2} a_{n-1}^2}{n a_n}. \tag{23}$$

This is the invariant of proposition 13, page 26.

It remains to prove that $I_{n-k,\,l}$ is a rational invariant of the first degree. This may be done by mathematical induction. For I_{n-k} is such an invariant. Suppose, then, that $I_{n-k,\,l}$ is.

$$I_{n-k,\,l+1} = \frac{1}{\theta} \overset{(l+1)}{A}_{n-k} = \frac{1}{\theta} \frac{d}{dx} (\overset{(l)}{A}_{n-k}) = \frac{1}{\theta} \frac{d}{dx} (I_{n-k,\,l}\,\theta)$$
$$= I_{n-k,\,l}{}' - \frac{1}{n} \frac{a_{n-1}}{a_n} I_{n-k,\,l}\,.$$

So that $I_{n-k,\,l+1}$ is rational, and will be an invariant of the first degree by the following proposition:

Proposition 16. If I be an invariant of degree k, then so also is

$$I' - \frac{k}{n} \frac{a_{n-1}}{a_n} I.$$

For it is equal to

$$\frac{1}{a_n} \left[\frac{k}{n} (n a_n{}' - a_{n-1})\, I + a_n I' - k a_n{}' I \right]$$
$$= \frac{1}{a_n} \left[\frac{k}{n} (n a_n{}' - a_{n-1})\, I + a_n{}^{k+1} \frac{d}{dx} \left(\frac{I}{a_n{}^k} \right) \right]$$

Here a_n and $n a_n{}' - a_{n-1}$, which is simply $(-1)^n b_{n-1}$, are invariants of the first degree, while $I / a_n{}^k$, and therefore its derivative, too, is an absolute invariant. It is apparent that the whole expression is an invariant of degree k.

§ 10. *Partial Differential Expressions: Conditions for the Possibility of Reduction to Canonical Form.*

We pass now to *partial* differential expressions. Here it is not in general possible, as will appear, to reduce the expression, by a change of dependent variable, to canonical form, where now by a canonical form we mean an expression in which the coefficients of *all* the $(n-1)$st derivatives are zero. Let us ask ourselves under what conditions this will be possible. The problem is of interest, not only in itself, but because it will suggest to us certain expressions analogous to the invariants A_{n-k}/θ, to which we were led, in the case of ordinary differential expressions, by the reduction to canonical form; and these expressions will turn out to be, like their prototypes, invariants of the differential equation $L(u) = 0$. We shall also find something analogous to the invariants $\overset{(l)}{A}_{n-k}/\theta$ of the differential expression $L(u)$.

Let us notice first that the *property*, the conditions for whose existence we are seeking, is an invariant property. It is evidently so for a change of dependent variable; and it is so also for a multiplication of

$L(u)$ by ϕ. For if $L(u)$ go over under $u = \theta \cdot \eta$ into a canonical form $\Lambda(\eta)$, $\phi L(u)$ will go over under the *same* transformation into $\phi\Lambda(\eta)$, which is likewise canonical. We shall be inclined, then, to expect that the conditions in question will consist in the vanishing of expressions which are "invariants of the differential equation." And such proves to be the case.

We examine the question first for an expression of the *second* order. Let

$$L(u) = \sum_{i,j} a_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_i a_i \frac{\partial u}{\partial x_i} + au$$

go over, by $u = \theta \cdot \eta$, into

$$\Lambda(\eta) = \sum_{i,j} a_{ij} \frac{\partial^2 \eta}{\partial x_i \partial x_j} + \sum_i a_i \frac{\partial \eta}{\partial x_i} + a\eta.$$

If this is to be canonical, we must, by (19), page 18, have

$$2 \sum_j a_{ij} \frac{\partial \, log \, \theta}{\partial x_j} + a_i = 0, \qquad i = 1, 2, \ldots m.$$

If

$$A \equiv \begin{vmatrix} a_{11} & \cdot & \cdot & \cdot & a_{1m} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{m1} & \cdot & \cdot & \cdot & a_{mm} \end{vmatrix} \neq 0,$$

these equations may be solved for $\dfrac{\partial \log \theta}{\partial x_i}$, $i = 1, 2, \ldots m$. Note here that A is an invariant of the differential equation. The solution in question will be

$$\frac{\partial \log \theta}{\partial x_i} = - \frac{\begin{vmatrix} a_{11} & \cdots & a_{1,\,i-1} & a_1 & a_{1,\,i+1} & \cdots & a_{1m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{m1} & \cdots & a_{m,\,i-1} & a_m & a_{m,\,i+1} & \cdots & a_{mm} \end{vmatrix}}{2A} = \kappa_i, \qquad (24)$$

let us say. Necessary and sufficient conditions that these equations possess a solution $\log \theta$ are

$$\frac{\partial \kappa_i}{\partial x_j} - \frac{\partial \kappa_j}{\partial x_i} = 0, \qquad i, j = 1, 2, \ldots m. \qquad (25)$$

The expression on the left of this last equation is an absolute invariant of the differential equation. For, first, κ_i, κ_j themselves are absolute invariants for a multiplication of $L(u)$ by ϕ. Next, if $L(u)$ go over by any transformation $u = \psi \cdot \eta$ into an expression with coefficients $\bar{a}$, and $\bar{\kappa}_i$ be the same function of the $\bar{a}$'s that κ_i is of the a's, we see without difficulty from (19), page 18, that

$$\bar{\kappa}_i = \kappa_i - \frac{\partial \log \psi}{\partial x_i}; \tag{26}$$

so that

$$\frac{\partial \bar{\kappa}_i}{\partial x_j} - \frac{\partial \bar{\kappa}_j}{\partial x_i} = \frac{\partial \kappa_i}{\partial x_j} - \frac{\partial \kappa_j}{\partial x_i}.$$

The invariant of the differential equation adjoint (proposition 11, page 25) to the invariant just found is $\frac{\partial \lambda_i}{\partial x_j} - \frac{\partial \lambda_j}{\partial x_i}$, if λ_i be the same function of the b's that κ_i is of the a's, that is, by (8), page 9, if

$$\lambda_i = -\frac{\begin{vmatrix} a_{11} \ldots a_{1,i-1} & 2\sum\limits_j \frac{\partial a_{1j}}{\partial x_j} - a_1 & a_{1,i+1} \ldots a_{1m} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ a_{m1} \ldots a_{m,i-1} & 2\sum\limits_j \frac{\partial a_{mj}}{\partial x_j} - a_m & a_{m,i+1} \ldots a_{mm} \end{vmatrix}}{2A}. \tag{27}$$

The difference, $\left(\frac{\partial \lambda_i}{\partial x_j} - \frac{\partial \lambda_j}{\partial x_i}\right) - \left(\frac{\partial \kappa_i}{\partial x_j} - \frac{\partial \kappa_j}{\partial x_i}\right)$, of these two invariants of the differential equation is an invariant that we shall come across later.

Consider next a differential expression of the nth order. If $u = \theta \cdot \eta$ carry it over into a canonical form, we must have

$$n\left(a_{p+1,q}\frac{\partial \log \theta}{\partial x} + a_{p,q+1}\frac{\partial \log \theta}{\partial y}\right) + a_{pq} = 0,$$
$$p + q = n - 1, \qquad p = 0,\ 1,\ \ldots\ (n-1).$$

Conditions necessary and, in general, sufficient for these equations being *algebraically* solvable are that all three-rowed determinants of the matrix

$$\left\| \begin{matrix} a_{1,n-1} & a_{0n} & a_{0,n-1} \\ \cdots & \cdots & \cdots \\ a_{p+1,q} & a_{p,q+1} & a_{pq} \\ \cdots & \cdots & \cdots \\ a_{n0} & a_{n-1,1} & a_{n-1,0} \end{matrix} \right\|$$

should vanish. Any one of these three-rowed determinants

$$I = \begin{vmatrix} a_{p_1+1,\,q_1} & a_{p_1,\,q_1+1} & a_{p_1q_1} \\ a_{p_2+1,\,q_2} & a_{p_2,\,q_2+1} & a_{p_2q_2} \\ a_{p_3+1,\,q_3} & a_{p_3,\,q_3+1} & a_{p_3q_3} \end{vmatrix}, \quad p_i + q_i = n - 1, \tag{28}$$

is an invariant of the differential equation. That it is invariant for $u = \psi \cdot \eta$ is seen at once from the formulas of transformation (17), page 17. The adjoint invariant is

$$J = (-1)^n. \begin{vmatrix} a_{p_1+1,q_1} & a_{p_1,q_1+1} & n\left(\dfrac{\partial a_{p_1+1,q_1}}{\partial x} + \dfrac{\partial a_{p_1,q_1+1}}{\partial y}\right) - a_{p_1q_1} \\ a_{p_2+1,q_2} & a_{p_2,q_2+1} & n\left(\dfrac{\partial a_{p_2+1,q_2}}{\partial x} + \dfrac{\partial a_{p_2,q_2+1}}{\partial y}\right) - a_{p_2q_2} \\ a_{p_3+1,q_3} & a_{p_3,q_3+1} & n\left(\dfrac{\partial a_{p_3+1,q_3}}{\partial x} + \dfrac{\partial a_{p_3,q_3+1}}{\partial y}\right) - a_{p_3q_3} \end{vmatrix}. \tag{29}$$

And $I + (-1)^{n-1}J$ is an invariant of the differential equation that we shall come across later.

The remainder of the treatment is like that of the second order.

$$\frac{\partial \log \theta}{\partial x} = -\frac{\begin{vmatrix} a_{p_1q_1} & a_{p_1,q_1+1} \\ a_{p_2q_2} & a_{p_2,q_2+1} \end{vmatrix}}{n\,A} = \kappa_1, \quad \frac{\partial \log \theta}{\partial y} = -\frac{\begin{vmatrix} a_{p_1+1,q_1} & a_{p_1q_1} \\ a_{p_2+1,q_2} & a_{p_2q_2} \end{vmatrix}}{n\,A} = \kappa_2, \tag{30}$$

$$A = \begin{vmatrix} a_{p_1+1,q_1} & a_{p_1,q_1+1} \\ a_{p_2+1,q_2} & a_{p_2,q_2+1} \end{vmatrix},$$

p_i, q_i being any positive integers such that $p_i + q_i = n - 1$. The condition for a solution is:

$$\frac{\partial \kappa_1}{\partial y} - \frac{\partial \kappa_2}{\partial x} = 0, \tag{31}$$

where the expression on the left is an invariant of the differential equation. If $\bar{\kappa}_1$, $\bar{\kappa}_2$ refer to $\Lambda(\eta)$ into which $L(u)$ goes over under $u = \psi \cdot \eta$,

$$\bar{\kappa}_1 = \kappa_1 - \frac{\partial \log \psi}{\partial x}, \quad \bar{\kappa}_2 = \kappa_2 - \frac{\partial \log \psi}{\partial y}. \tag{32}$$

The invariant adjoint to (31) is $\dfrac{\partial \lambda_1}{\partial y} - \dfrac{\partial \lambda_2}{\partial x}$, where

$$\lambda_1 = -\frac{\begin{vmatrix} n\left(\dfrac{\partial a_{p_1+1,q_1}}{\partial x} + \dfrac{\partial a_{p_1,q_1+1}}{\partial y}\right) - a_{p_1q_1} & a_{p_1,q_1+1} \\ n\left(\dfrac{\partial a_{p_2+1,q_2}}{\partial x} + \dfrac{\partial a_{p_2,q_2+1}}{\partial y}\right) - a_{p_2q_2} & a_{p_2,q_2+1} \end{vmatrix}}{n\,A}, \tag{32a}$$

with a similar expression for λ_2.

§ 11. *Partial Differential Expressions: Invariants suggested by the Reduction to Canonical Form.*

In the case of ordinary differential expressions we have seen (proposition 15, page 29) that A_{n-k}/θ, $k = 0, 2, 3, \ldots n$, are invariants of the equation $L(u) = 0$, the A's being the coefficients of the canonical form derived from $L(u)$ by putting $u = \theta \cdot \eta$, where θ is defined by (21), page 28. Are there any corresponding phenomena in the case of partial differential expressions? In the first place it is clear, and might be proved in the same way, that for such partial differential expressions as can be reduced to a canonical form by $u = \theta \cdot \eta$, where θ is defined by (30), the coefficients of that form divided by θ are invariants of the equation, to use the term in such a sense, for that particular class of differential expressions. But for other differential expressions the proof that these same functions of the $\boldsymbol{a}$'s and their derivatives were invariants of the equation would no longer hold. It turns out, nevertheless, that they are in fact invariants of the differential equation, as we now go on to show.

Let us see just what it is that we wish to prove. Consider the formulas for the $\boldsymbol{a}$'s in terms of the a's,

$$ {}_{pq} = \sum_{l=0}^{k} \sum_{i=0}^{k-l} \frac{(n-l)\,!}{(n-k)\,!\,i\,!\,(k-l-i)\,!}\, a_{p+i,\, q+k-l-i} \frac{\partial^{k-l} \psi}{\partial x^i \, \partial y^{k-l-i}}, \qquad (17)$$

$$p + q = n - k.$$

Now suppose that we substitute in this formula for ψ and its derivatives θ and its derivatives, $\frac{\partial \theta}{\partial x}, \frac{\partial \theta}{\partial y}$ being given by (30), that is,

$$\frac{\partial \theta}{\partial x} = \kappa_1 \theta, \qquad \frac{\partial \theta}{\partial y} = \kappa_2 \theta,$$

and the higher derivatives of θ being determined from these formulas by differentiation and the substitution, at each step of the process of differentiation, of $\kappa_1 \theta$, $\kappa_2 \theta$ for $\frac{\partial \theta}{\partial x}, \frac{\partial \theta}{\partial y}$ respectively. This rule, it will be noticed, does not completely determine the expressions to be substituted; for we may, to take an instance, in accordance with its directions, substitute for $\frac{\partial^2 \psi}{\partial x \partial y}$ either $\frac{\partial \kappa_1}{\partial y} \theta + \kappa_1 \frac{\partial \theta}{\partial y}$, that is $\left(\frac{\partial \kappa_1}{\partial y} + \kappa_1 \kappa_2 \right) \theta$, or else $\frac{\partial \kappa_2}{\partial x} \theta + \kappa_2 \frac{\partial \theta}{\partial x}$, that is $\left(\frac{\partial \kappa_2}{\partial x} + \kappa_1 \kappa_2 \right) \theta$. But this does not matter. We suppose the expressions to be substituted for any given derivative of ψ to be calculated *in any way whatever* in accordance only with the rule above. These expressions, as we see, will be θ times polynomials in κ_1, κ_2 and their derivatives. If, finally, we divide the whole by θ, we get a rational function of the a's and their derivatives, and it is this latter expression that we wish to prove an invariant of the differential equation. What we have to prove, then, may be stated in the proposition:

Proposition 17. The expression

$$\frac{1}{\theta}\sum_{l=0}^{k}\sum_{i=0}^{k-l}\frac{(n-l)!}{(n-k)!\,i!\,(k-l-i)!}\,a_{p+i,\,q+k-l-i}\,\frac{\partial^{k-l}\theta}{\partial x^i\partial y^{k-l-i}},\qquad (33)$$
$$p+q=n-k,$$

is an invariant of the differential equation $L(u)=0$, where the derivatives of θ are obtained from

$$\frac{\partial\theta}{\partial x}=\kappa_1\theta,\qquad \frac{\partial\theta}{\partial y}=\kappa_2\theta \qquad (34)$$

by the rule above, and κ_1, κ_2 are defined by (30). It is an invariant of degree one.

First, it is an invariant for a multiplication of $L(u)$ by ϕ. For κ_1, κ_2, and therefore their derivatives also, are absolute invariants for this transformation. So too, then, is any derivative of θ divided by θ; while finally each of these latter expressions is multiplied by an a.

Next we have to prove that our expression (33) is an invariant for $u=\psi\cdot\eta$. To this end let us turn back to the absolute covariants of proposition 9, page 24. If we divide any one of these by $(n-j)!\,u$, we get a covariant of the first degree, which, by a change of notation, we may write

$$\frac{1}{u}\sum_{l=0}^{k}\sum_{i=0}^{k-l}\frac{(n-l)!}{(n-k)!\,i!\,(k-l-i)!}\,a_{p+i,\,q+k-l-i}\,\frac{\partial^{k-l}u}{\partial x^i\partial y^{k-l-i}},\qquad (35)$$
$$p+q=n-k.$$

Here we note the close analogy in form with (33). In fact, this covariant may be obtained from the formula (17) for a_{pq}, reproduced on page 34 above, by the substitution for the derivatives of ψ of the corresponding derivatives of u divided by u, just as (35) is obtained from the same formula by the substitution of certain polynomials in κ_1, κ_2, and their derivatives.

Now, since we have parallel with each other

$$\frac{\partial\theta}{\partial x} = \kappa_1\theta \qquad\qquad \frac{\partial u}{\partial x} = \frac{\frac{\partial u}{\partial x}}{u}\,u$$

$$\frac{\partial\theta}{\partial y} = \kappa_2\theta \qquad\qquad \frac{\partial u}{\partial y} = \frac{\frac{\partial u}{\partial y}}{u}\,u,$$

it is evident that, however, from the formulas on the left, we may calculate the value of any derivative of θ, that value, divided by θ, will be the same function of κ_1, κ_2, and their derivatives, as is the corresponding derivative of u divided by u of $\frac{\partial u}{\partial x}\Big/u$, $\frac{\partial u}{\partial y}\Big/u$, and their derivatives. And thus we reach the result that our expression (33) is the same function of κ_1, κ_2, and their derivatives, that the covariant (35) is of $\frac{\partial u}{\partial x}\Big/u$, $\frac{\partial u}{\partial y}\Big/u$, and their derivatives.

From this it follows at once that the former, like the latter, is invariant of degree one. For the two sets of arguments in question are cogredient with each other, since we have seen, (32), page 33, that if $\bar{\kappa}_1$, $\bar{\kappa}_2$ stand for the same functions of the $\bar{a}$'s that κ_1, κ_2 are of the a's, then

$$\bar{\kappa}_1 = \kappa_1 - \frac{1}{\psi}\frac{\partial\psi}{\partial x}, \qquad \bar{\kappa}_2 = \kappa_2 - \frac{1}{\psi}\frac{\partial\psi}{\partial y},$$

while parallel with this we have

$$\frac{\frac{\partial\eta}{\partial x}}{\eta} = \frac{\frac{\partial u}{\partial x}}{u} - \frac{1}{\psi}\frac{\partial\psi}{\partial x}, \qquad \frac{\frac{\partial\eta}{\partial y}}{\eta} = \frac{\frac{\partial u}{\partial y}}{u} - \frac{1}{\psi}\frac{\partial\psi}{\partial y};$$

and this parallelism, of course, extends to the derivatives of the quantities in question. Thus the proof of our proposition is complete.

We see from the formulas for κ_1, κ_2, (30), page 33, that our invariants, if reduced to a common denominator, will be polynomials in the a's, and their derivatives divided by a power of A. These polynomials will then themselves be invariants of the differential equation.

The simplest of our invariants are those derived from a_{pq}, where $p + q = n - 2$. Here we have two invariants,

$$I_1 = \frac{n(n-1)}{2}\left[a_{p+2,q}\left(\frac{\partial\kappa_1}{\partial x} + \kappa_1^2\right) + 2a_{p+1,q+1}\left(\frac{\partial\kappa_1}{\partial y} + \kappa_1\kappa_2\right)\right.$$
$$\left. + a_{p,q+2}\left(\frac{\partial\kappa_2}{\partial y} + \kappa_2^2\right)\right] + (n-1)\left[a_{p+1,q}\kappa_1 + a_{p,q+1}\kappa_2\right] + a_{pq},$$

and I_2, which differs from I_1 in that it replaces $\frac{\partial\kappa_1}{\partial y} + \kappa_1\kappa_2$ in the coefficient of $a_{p+1,q+1}$ by $\frac{\partial\kappa_2}{\partial x} + \kappa_1\kappa_2$. Thus

$$I_1 - I_2 = n(n-1)a_{p+1,q+1}\left(\frac{\partial\kappa_1}{\partial y} - \frac{\partial\kappa_2}{\partial x}\right).$$

Here, since $p + q = n - 2$, $a_{p+1,q+1}$ is an invariant of the differential equation, and the other factor we already know to be such, (31).

In (30) p_i, q_i are subject only to the condition $p_i + q_i = n - 1$. We may, by a special choice of these numbers, considerably simplify I_1 and I_2, or rather their sum. For putting $p_1 = p + 1$, $q_1 = q$, $p_2 = p$, $q_2 = q + 1$, we get

$$\kappa_1 = -\frac{\begin{vmatrix} a_{p+1,q} & a_{p+1,q+1} \\ a_{p,q+1} & a_{p,q+2} \end{vmatrix}}{nA}, \quad \kappa_2 = -\frac{\begin{vmatrix} a_{p+2,q} & a_{p+1,q} \\ a_{p+1,q+1} & a_{p,q+1} \end{vmatrix}}{nA},$$

$$A = \begin{vmatrix} a_{p+2,q} & a_{p+1,q+1} \\ a_{p+1,q+1} & a_{p,q+2} \end{vmatrix}.$$

This would mean that κ_1, κ_2 had been obtained as solutions of the equations

$$n(a_{p+2,q}\kappa_1 + a_{p+1,q+1}\kappa_2) + a_{p+1,q} = 0,$$
$$n(a_{p+1,q+1}\kappa_1 + a_{p,q+2}\kappa_2) + a_{p,q+1} = 0;$$

from which, by differentiation, we get

$$n\left(a_{p+2,q}\frac{\partial\kappa_1}{\partial x} + a_{p+1,q+1}\frac{\partial\kappa_2}{\partial x}\right)$$
$$= -n\left(\frac{\partial a_{p+2,q}}{\partial x}\kappa_1 + \frac{\partial a_{p+1,q+1}}{\partial x}\kappa_2\right) - \frac{\partial a_{p+1,q}}{\partial x},$$

$$n\left(a_{p+1,q+1}\frac{\partial\kappa_1}{\partial y} + a_{p,q+2}\frac{\partial\kappa_2}{\partial y}\right)$$
$$= -n\left(\frac{\partial a_{p+1,q+1}}{\partial y}\kappa_1 + \frac{\partial a_{p,q+2}}{\partial y}\kappa_2\right) - \frac{\partial a_{p,q+1}}{\partial y}.$$

With the help of these four equations, $I = \frac{1}{2}(I_1 + I_2)$ reduces to

$$I = -\frac{n(n-1)}{2}\left[\kappa_1\left(\frac{\partial a_{p+2,q}}{\partial x} + \frac{\partial a_{p+1,q+1}}{\partial y}\right) + \kappa_2\left(\frac{\partial a_{p+1,q+1}}{\partial x} + \frac{\partial a_{p,q+2}}{\partial y}\right)\right]$$

$$+ \frac{n-1}{2}\left[\kappa_1 a_{p+1,q} + \kappa_2 a_{p,q+1}\right] - \frac{n-1}{2}\left[\frac{\partial a_{p+1,q}}{\partial x} + \frac{\partial a_{p,q+1}}{\partial y}\right] + a_{pq},$$

$$p + q = n - 2.$$

For ordinary differential expressions this reduces, as it should, to the invariant of the differential equation which we have called, (23), page 29, A_{n-2}/θ or I_{n-2}, if we put, as proper,

$$\kappa_1 = -\frac{a_{n-1}}{na_n}, \qquad \kappa_2 = 0.$$

For the second order, $n = 2$, m variables, the corresponding invariant is:

$$I = \frac{1}{4A}\left[4\,aA - \sum_{i,j} a_i a_j A_{ij} + 2\sum_{i,j,k} a_i A_{ij}\frac{\partial a_{jk}}{\partial x_k} - 2A\sum_i \frac{\partial a_i}{\partial x_i}\right],$$

where A_{ij} is the cofactor of a_{ij} in

$$A = \begin{vmatrix} a_{11} & . & . & . & a_{1m} \\ . & . & . & . & . \\ . & . & . & . & . \\ a_{m1} & . & . & . & a_{mm} \end{vmatrix}.$$

This becomes for two variables, $m = 2$,

$$I = \frac{1}{4A}\Bigg[4\,aA - (a_1^2 a_{22} - 2\,a_1 a_2 a_{12} + a_2^2 a_{11})$$

$$+ 2\,(a_1 a_{22} - a_2 a_{12})\left(\frac{\partial a_{11}}{\partial x} + \frac{\partial a_{12}}{\partial y}\right)$$

$$+ 2\,(a_2 a_{11} - a_1 a_{12})\left(\frac{\partial a_{12}}{\partial x} + \frac{\partial a_{22}}{\partial y}\right) - 2A\left(\frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y}\right)\Bigg],$$

$$A = a_{11}a_{22} - a_{12}^2.$$

Invariants of a partial differential expression analogous to $\overset{(l)}{A}_{n-k}/\theta$. We have found now invariants of a partial differential equation analogous to the invariants A_{n-k}/θ of an ordinary differential equation. It remains to discover the analogue of $\overset{(l)}{A}_{n-k}/\theta$, which, we remember, was an invariant of the differential *expression*. This merely amounts (see proposition 16, page 30) to an inquiry after a process analogous to the process by which from an invariant I of degree one, or, more generally, of degree k, of an ordinary differential expression, we derived a second, $I' - \frac{k}{n}\frac{a_{n-1}}{a_n} I$. The inquiry is answered by the following proposition:

Proposition 18. If I be an invariant of the kth degree of a partial differential expression, then so also are

$$\frac{\partial I}{\partial x} + k\kappa_1 I,$$

$$\frac{\partial I}{\partial y} + k\kappa_2 I,$$

κ_1, κ_2 being defined by (30), page 33. Further, if $p + q = n - 1$, then

$$a_{p+1,q}\frac{\partial I}{\partial x} + a_{p,q+1}\frac{\partial I}{\partial y} - \frac{k}{n}\,a_{pq} I$$

is an invariant of degree $k + 1$.

We notice that the first two of these invariants may, with the notation of (30), be written as $\frac{1}{\theta^k}\frac{\partial}{\partial x}(\theta^k I)$, $\frac{1}{\theta^k}\frac{\partial}{\partial y}(\theta^k I)$, just as for ordinary differential expressions the derived invariant may, with the notation of (21), page 28, which corresponds to (30), be written $\frac{1}{\theta^k}\frac{d}{dx}(\theta^k I)$.

Proof. The first of the invariants above, formed for the transformed differential expression, is

$$\begin{aligned}\frac{\partial}{\partial x}(\psi^k I) + k\bar{\kappa}_1\psi^k I &= k\psi^{k-1}\frac{\partial \psi}{\partial x} I + \psi^k \frac{\partial I}{\partial x} + k\left(\kappa_1 - \frac{1}{\psi}\frac{\partial \psi}{\partial x}\right)\psi^k I \\ &= \psi^k\left(\frac{\partial I}{\partial x} + k\kappa_1 I\right).\end{aligned}$$

So for the second invariant. To get the third of the above invariants, multiply the first by a_{p_1+1,q_1}, the second by a_{p_1,q_1+1}, and add. This will give us, since each of these multipliers is itself an invariant — for $p_1 + q_1 = n - 1$, (30) — an invariant of degree $k + 1$; and by (30) that invariant will be the third of the expressions above.

IV. CHANGE OF INDEPENDENT VARIABLES; INVARIANTS AND COVARIANTS.

§ 12. *General Properties.*

We come now to change of independent variables and the invariants and covariants of this transformation. A differential expression in the independent variables $x_1, \ldots x_m$ goes over, under the change of variables

$$\xi_i = \xi_i(x_1, \ldots x_m), \qquad i = 1, 2, \ldots m,$$

into another of the same order. With regard to the coefficients of the latter, which we may call $\bar{a}$, let us note, in the general case, certain facts, sufficient for our purposes.

Any derivative of order k of u with respect to the x's is a polynomial in the derivatives, of order k and less, of u with respect to the ξ's, and in the derivatives of the ξ's with respect to the x's, and is linear in the former set of arguments. These facts follow at once, directly for the first derivatives, by mathematical induction for the higher derivatives, from the formula

$$\frac{\partial}{\partial x_i} = \sum_j \frac{\partial \xi_j}{\partial x_i} \frac{\partial}{\partial \xi_j}.$$

Hence the $\bar{a}$'s are polynomials in the a's and in the derivatives of the ξ's with respect to the x's, linear in the a's. The derivatives of the $\bar{a}$'s, on the other hand, with respect to the ξ's, are linear polynomials in the a's and their derivatives with respect to the x's, with coefficients polynomials in the derivatives of the ξ's with respect to the x's, the whole divided by a power of the functional determinant of the ξ's with respect to the x's,

$$J \equiv \begin{vmatrix} \frac{\partial \xi_1}{\partial x_1} & \cdots & \frac{\partial \xi_1}{\partial x_m} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \frac{\partial \xi_m}{\partial x_1} & \cdots & \frac{\partial \xi_m}{\partial x_m} \end{vmatrix}.$$

This follows from the formula

$$\frac{\partial}{\partial \xi_i} = \sum_j \frac{\partial x_j}{\partial \xi_i} \frac{\partial}{\partial x_j} = \sum_j \frac{J_{ij}}{J} \frac{\partial}{\partial x_j},$$

J_{ij} being the cofactor in J of $\dfrac{\partial \xi_i}{\partial x_j}$. For the *second* order, the formulas of transformation run as follows:

$$\left.\begin{aligned} \bar{a}_{ij} &= \sum_{k,l} a_{kl} \frac{\partial \xi_i}{\partial x_k} \frac{\partial \xi_j}{\partial x_l}, \\ \bar{a}_i &= \sum_{k,l} a_{kl} \frac{\partial^2 \xi_i}{\partial x_k \partial x_l} + \sum_k a_k \frac{\partial \xi_i}{\partial x_k}, \\ \bar{a} &= a. \end{aligned}\right\} \tag{36}$$

We next define what we mean by invariants for this transformation.

Definition. By an invariant for a change of independent variables is meant a function of the a's and their derivatives with respect to the x's such that the same function of the $\bar{a}$'s and their corresponding derivatives with respect to the ξ's is equal, by virtue of the formulas of transformation, to the original function multiplied by a power of J, the functional determinant of the ξ's with respect to the x's:

$$I(\bar{a}) = J^w I(a).$$

What we shall have to say about invariants will, in general, as hitherto, refer to polynomial invariants.

As to covariants, besides such as we have already made acquaintance with in the case of change of dependent variable, involving u and its derivatives, we have here a second kind, involving $dx_1, \ldots dx_m$. These two kinds we may distinguish as *covariant differential expressions* and *covariant differential forms* respectively. If we replace u in a covariant differential expression by an *absolute* invariant, it is clear that we shall get an invariant; thus this sort of covariant may be regarded as an operator for deriving invariants; from this point of view it is what is known as a *differential parameter*.

As to the general properties of invariants, we begin with the proposition:

Proposition 19. If we define the weights of the a's and their derivatives as in the case of change of dependent variable, page 19, every invariant is isobaric, of weight w, with respect to any one of the independent variables. Its partial weight, then, with respect to any one of the variables is the same as with respect to any other.

Take the case of two independent variables, x and y. Make the change of variables: $\xi = cx$, $\eta = y$, c being any constant. Then

$$\bar{a}_{pq} = c^p a_{pq}, \qquad \frac{\partial^{i+j} \bar{a}_{pq}}{\partial \xi^i \partial \eta^j} = c^{p-i} \frac{\partial^{i+j} a_{pq}}{\partial x^i \partial y^j}, \qquad J = c,$$

so that we have

$$I\left(\dots c^{p-i}\frac{\partial^{i+j}a_{pq}}{\partial x^i \partial y^j},\dots\right) = c^w I\left(\dots \frac{\partial^{i+j}a_{pq}}{\partial x^i \partial y^j},\dots\right),$$

an equation which not only shows that I, if it be a polynomial, is isobaric, but in other cases is commonly used to define what is meant by isobaric with the given system of weights. We shall speak of w as the weight of the invariant even when it is not a polynomial.

The proposition holds also for covariants if, in the case of covariant differential expressions, we attribute to $\frac{\partial^{a+\cdots}u}{\partial x_i^{a+\cdots}}$ the weight, with respect to x_i, $-a$, and if, in the case of covariant differential forms, we attribute to dx_i the weight one, to dx_j, $j \neq i$, the weight zero, with respect to x_i.

Proposition 20. An invariant may or may not be homogeneous; but if not, it is a mere sum of invariants which are homogeneous.

This is the counterpart of proposition 5, page 19, and the proof is similar in the two cases; for, as noted above, page 40, the $\bar{a}$'s and their derivatives are linear in the a's and their derivatives. So that if we represent by $G_n(a)$ the terms of $I(a)$ of degree n, the corresponding part of $I(\bar{a})$, namely $G_n(\bar{a})$, will be of degree n in the a's and their derivatives.

This proposition may be extended to both kinds of covariants, for the $d\xi$'s are linear in the dx's; and again, as also noted above, the derivatives of u with respect to the x's are linear in the derivatives of u with respect to the ξ's; and this statement may evidently be reversed.

§ 13. *Particular Invariants and Covariants.*

For a differential expression of the *second* order,

$$L(u) = \sum_{i,j} a_{ij}\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_i a_i \frac{\partial u}{\partial x_i} + au,$$

certain simple invariants and covariants may be deduced by the following considerations.

$$\frac{\partial}{\partial x_i} = \sum_k \frac{\partial \xi_k}{\partial x_i}\frac{\partial}{\partial \xi_k}, \qquad \frac{\partial}{\partial x_j} = \sum_k \frac{\partial \xi_k}{\partial x_j}\frac{\partial}{\partial \xi_k},$$

so that

$$\frac{\partial^2 u}{\partial x_i \partial x_j} = \sum_k \frac{\partial^2 \xi_k}{\partial x_i \partial x_j}\frac{\partial u}{\partial \xi_k} + \sum_{k,l}\frac{\partial \xi_k}{\partial x_i}\frac{\partial \xi_l}{\partial x_j}\frac{\partial^2 u}{\partial \xi_k \partial \xi_l}.$$

On the other hand,

$$\frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j}=\sum_{k,l}\frac{\partial \xi_k}{\partial x_i}\frac{\partial \xi_l}{\partial x_j}\frac{\partial u}{\partial \xi_k}\frac{\partial u}{\partial \xi_l}.$$

It appears thus that the coefficient of $\frac{\partial^2 u}{\partial \xi_k \partial \xi_l}$ in $\frac{\partial^2 u}{\partial x_i \partial x_j}$ is the same as the coefficient of $\frac{\partial u}{\partial \xi_k}\frac{\partial u}{\partial \xi_l}$ in $\frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j}$. Now in calculating the $\bar{a}$'s with *two* subscripts, $\bar{a}_{ij}$, we are not concerned with the first derivatives of u with respect to the x's or the ξ's; so that the $\bar{a}_{ij}$'s are expressed in terms of the a_{ij}'s, in the case of $L(u)$ under a change of independent variables, by the same formulas as for the expression $\sum_{i,j} a_{ij}\frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j}$ under the same change of variables, that is, as for the quadratic algebraic form $\sum_{i,j} a_{ij}z_i z_j$ under the linear transformation

$$z_i=\sum_k \frac{\partial \xi_k}{\partial x_i} z_k{}',$$

a linear transformation whose determinant, as we note, is J.

Now the discriminant

$$A\equiv\begin{vmatrix} a_{11} & \cdots & a_{1m} \\ \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot\ \cdot\ \cdot & \cdot \\ a_{m1} & \cdots & a_{mm}\end{vmatrix}$$

is a relative invariant of weight two of the algebraic form. A is therefore also a relative invariant of weight two of the differential expression, $L(u)$. We note that A is also an invariant, for change of dependent variable, of the differential equation.

Again, if $v_1, \ldots v_m, w_1, \ldots w_m$ be two sets of variables contragredient to the $\frac{\partial u}{\partial x}$'s, then

$$-\begin{vmatrix} a_{11} & \cdots & a_{1m} & v_1 \\ \cdot & \cdot\ \cdot\ \cdot & \cdot & \cdot \\ \cdot & \cdot\ \cdot\ \cdot & \cdot & \cdot \\ a_{m1} & \cdots & a_{mm} & v_m \\ w_1 & \cdots & w_m & 0\end{vmatrix}=\sum_{i,j} A_{ij}v_i w_j, \qquad (37)$$

A_{ij} being the cofactor in A of a_{ij}, is invariant of weight two of the algebraic form, and therefore of $L(u)$ also. Now the differentials of the x's are such contragredient variables; so that, if $dx_1, \ldots dx_m$, $\delta x_1, \ldots \delta x_m$ be two sets of differentials, the expressions

$$\sum_{i,j} A_{ij}\, dx_i\, dx_j, \tag{38}$$

$$\sum_{i,j} A_{ij}\, dx_i\, \delta x_j \tag{39}$$

are covariants of weight two. Their coefficients, the A_{ij}'s, are invariants, for change of dependent variable, of the differential equation. Similarly the $(m+p)$-rowed determinant formed by bordering A with p rows and p columns, each of which consists of a set of differentials, is a covariant of weight two.

In the course of the work above we have proved, though we did not at the moment note the fact, that

$$\sum_{i,j} a_{ij} \frac{\partial u}{\partial x_i} \frac{\partial u}{\partial x_j} \tag{40}$$

is an absolute covariant. The analogous covariant exists for differential expressions of the nth order. For take the terms of $L(u)$ involving derivatives of the nth order, and form an expression $C(u)$ by substituting, for $\dfrac{\partial^n u}{\partial x_1^{\beta_1} \ldots \partial x_m^{\beta_m}}$, $\left(\dfrac{\partial u}{\partial x_1}\right)^{\beta_1} \ldots \left(\dfrac{\partial u}{\partial x_m}\right)^{\beta_m}$; then $C(u)$ is the covariant in question. For it is easily established by mathematical induction, that the coefficient of $\dfrac{\partial^{\gamma_1 + \ldots + \gamma_m} u}{\partial \xi_1^{\gamma_1} \ldots \partial \xi_m^{\gamma_m}}$ in $\dfrac{\partial^{\beta_1 + \ldots + \beta_m} u}{\partial x_1^{\beta_1} \ldots \partial x_m^{\beta_m}}$ is the same as the coefficient of $\left(\dfrac{\partial u}{\partial \xi_1}\right)^{\gamma_1} \ldots \left(\dfrac{\partial u}{\partial \xi_m}\right)^{\gamma_m}$ in $\left(\dfrac{\partial u}{\partial x_1}\right)^{\beta_1} \ldots \left(\dfrac{\partial u}{\partial x_m}\right)^{\beta_m}$. Whence it follows, just as for the second order, that $C(u)$ is an absolute covariant. $C[f(x_1, \ldots x_m)]$ is also invariant for change of dependent variable, as well as for multiplication of $L(u)$ by ϕ. For $m = 3$, if f satisfy $C(f) = 0$, $f =$ constant is the equation of the characteristic surfaces of $L(u) = 0$. See Sommerfeld in the Encyklopädie der Mathematischen Wissenschaften, II A7c, Nr. 15. The substitution in $C(u)$ for u of an absolute invariant yields an absolute invariant. Since the coefficient of u in $L(u)$, say a, is an absolute invariant, so then also is $C(a)$. For an ordinary differential expression $C(a)$ reduces to $a_n \left(\dfrac{da}{dx}\right)^n$; so that $C(a)$ is, in a certain sense, the analogue for a partial differential expression of the obvious invariant $\frac{da}{dx}$ of an ordinary differential expression. For $n = 2$,

$$C(a) = \sum_{i,j} a_{ij} \frac{\partial a}{\partial x_i} \frac{\partial a}{\partial x_j}.$$

§ 14. *Reduction to Canonical Form of an Ordinary Differential Expression.*

We may obtain, in the case of an ordinary differential expression, a system of rational invariants in terms of which all others may be rationally expressed, by the same device as that employed, § 9, for change of dependent variable. For let the change of variable, $\xi = \chi(x)$, reduce the ordinary differential expression

$$L(u) = a_n u^{(n)} + \ldots + a_0 u$$

to a canonical form with coefficients $\bar{A}$. We are to have

$$0 = \bar{A}_{n-1} = (\chi')^{n-2} \left[\frac{n\ (n-1)}{2}\ \chi''\, a_n + \chi'\, a_{n-1} \right],$$

or

$$\chi'' = - \frac{2}{n\ (n-1)} \frac{a_{n-1}}{a_n} \chi'. \tag{41}$$

Hence any derivative of χ is χ' times a rational function of the a's and their derivatives, and it follows that $\bar{A}_{n-k}$ is $(\chi')^{n-k}$ times such a function. For let $L(u)$ go over under any transformation, $\xi = \phi(x)$, into an expression with coefficients $\bar{a}$. Then the formula

$$\frac{d^k u}{dx^k} = \sum_{l=1}^{k} f_l \frac{d^l u}{d\xi^l},$$

f_l being a polynomial, homogeneous of degree l, in the derivatives of ϕ, may be established by mathematical induction. Hence $\bar{a}_l$ is not only linear in the a's, but homogeneous of degree l in the derivatives of ξ. We have, then, that $\bar{A}_{n-k} = (\chi')^{n-k} J_{n-k}\ (a)$, J_{n-k} being a rational function. It follows, just as in the similar case of § 9, that J_{n-k} is an invariant of weight $n-k$.

Now let I be any invariant of weight w. Then

$$(\chi')^w\ I\ (a_n, a_n', \ldots a_{n-1}, a_{n-1}', \ldots a_{n-k}, \ldots \overset{(l)}{a}_{n-k}, \ldots)$$

$$= I \left(\bar{A}_n, \frac{d\bar{A}_n}{d\xi}, \ldots 0, 0, \ldots \bar{A}_{n-k}, \ldots \frac{d^l \bar{A}_{n-k}}{d\xi^l}, \ldots \right),$$

which, since I is isobaric, is equal to

$$(\chi')^w I\left(\frac{\bar{A}_n}{(\chi')^n}, \frac{1}{(\chi')^{n-1}}\frac{d\bar{A}_n}{d\xi}, \ldots 0,\ 0, \ldots \frac{\bar{A}_{n-k}}{(\chi')^{n-k}}, \ldots \frac{1}{(\chi')^{n-k-l}}\frac{d^l\bar{A}_{n-k}}{d\xi^l}, \ldots\right)$$

$$= (\chi')^w I\ (J_n, J_{n1}, \ldots 0,\ 0, \ldots J_{n-k}, \ldots J_{n-k,\,l}, \ldots),$$

if we put

$$\frac{1}{(\chi')^{n-k-l}}\frac{d^l\bar{A}_{n-k}}{d\xi^l} = J_{n-k,\,l}.$$

When we have proved that $J_{n-k,\,l}$ is, like J_{n-k}, a rational invariant, and that it is of weight $n-k-l$, we shall, then, have the proposition:

Proposition 21. Every invariant is a function of the rational invariants

$$J_{n-k} = \frac{\bar{A}_{n-k}}{(\chi')^{n-k}}, \qquad J_{n-k,\,l} = \frac{1}{(\chi')^{n-k-l}}\frac{d^l\bar{A}_{n-k}}{d\xi^l},$$

of weights $n-k$, $n-k-l$ respectively. Here the $\bar{A}$'s are the coefficients of the canonical form into which $L(u)$ goes over, if χ satisfy (41), under $\xi = \chi(x)$.

$$I\ (a_n, a_n', \ldots a_{n-1}, a_{n-1}', \ldots a_{n-k}, \ldots \overset{(l)}{a}_{n-k}, \ldots)$$

$$= I(J_n, J_{n1}, \ldots 0,\ 0, \ldots J_{n-k}, \ldots J_{n-k,\,l}, \ldots).$$

In particular, if I be a polynomial, it is a polynominal in these invariants as well.

The simplest of the invariants in question are:

$$J_n = a_n.$$

$$J_{n1} = a_n' - \frac{2}{n-1}\, a_{n-1}.$$

$$J_{n2} = \frac{n(n-1)\ a_n\, a_n'' - 2n\ a_n\, a_{n-1}' - 2(n-1)\ a_n'\, a_{n-1} + 4a_{n-1}^2}{n(n-1)\ a_n}.$$

$$J_{n-2} =$$

$$\frac{6n(n-1)a_n a_{n-2} + 2n(n-1)(n-2)(a_n' a_{n-1} - a_n a_{n-1}') - (n-2)(3n-1)a_{n-1}^2}{6n(n-1)a_n}$$

We shall find later invariants of a partial differential expression of the second order analogous to J_{n1} and J_{n2}.

It remains to prove that $J_{n-k,l}$ is a rational invariant of weight $n-k-l$. Since

$$J_{n-k,l+1} = \frac{1}{(\chi')^{n-k-l-1}} \frac{d[(\chi')^{n-k-l} J_{n-k,l}]}{d\xi}$$
$$= J_{n-k,l}' - \frac{2(n-k-l)}{n(n-1)} \frac{a_{n-1}}{a_n} J_{n-k,l},$$

the case in hand comes under the proposition:

Proposition 22. If I be an invariant of weight w, then

$$\frac{1}{a_n}\left[a_n I' - \frac{2w}{n(n-1)} a_{n-1} I\right]$$

is an invariant of weight $w-1$.

This proposition may be proved as follows. The expression in question is equal to

$$\frac{1}{a_n}\left[\frac{n a_n I' - w a_n' I}{n} + \frac{w}{n}\left(a_n' - \frac{2}{n-1} a_{n-1}\right) I\right].$$

Here $a_n' - \frac{2}{n-1} a_{n-1} = J_{n1}$, and is shown, by direct calculation, with the help of the formulas

$$\bar{a}_n = (\phi')^n a_n,$$
$$\bar{a}_{n-1} = (\phi')^{n-2}\left(\frac{n(n-1)}{2} \phi'' a_n + \phi' a_{n-1}\right),$$

to be an invariant of weight $n-1$. On the other hand, since I^n/a_n^w is an absolute invariant, its derivative is an invariant of weight -1, that is, $n a_n I' - w a_n' I$ is an invariant of weight $w+n-1$.

§ 15. *The Adjoint of the Transformed Differential Expression.*

Proposition 6, page 19, gives us, for a change of dependent variable or a multiplication of $L(u)$ by ϕ, a simple relation between the adjoints of the transformed and the original differential expressions. For a change of independent variables we have the following relation:

Proposition 23. If $L(u)$ and its adjoint $M(v)$ go over, under a change of independent variables, into $\overline{L}(u)$ and $\overline{M}(v)$ respectively, then $\frac{\overline{L}(u)}{J}$ and $\frac{\overline{M}(v)}{J}$ are adjoint. To obtain the adjoint of the transformed differential expression we have, then, to subject $M(v)$ to the following transformations:

$$\xi_i = \xi_i(x_1, \ldots x_m), \qquad i = 1, 2, \ldots m;$$

multiplication by $\frac{1}{J}$;

$$v = J \cdot v_1.$$

Proof.[7] Make the change of variables in question in Lagrange's Identity,

$$vL(u) - uM(v) = \sum_i \frac{\partial S_i}{\partial x_i},$$

where, as we remember, the S's are bilinear in u, v, and their derivatives of orders up to the $(n-1)$st. Then the S's go over into expressions $\bar{S}$ bilinear in u, v, and their derivatives, with regard to the ξ's, of orders up to the $(n-1)$st, and we have

$$v\bar{L}(u) - u\bar{M}(v) = \sum_i \frac{\partial \bar{S}_i}{\partial x_i} = \sum_{i,j} \frac{\partial \xi_j}{\partial x_i} \frac{\partial \bar{S}_i}{\partial \xi_j}.$$

If we divide this equation through by J, we shall find that we may, without altering the value of the right side, put everything on that side under the signs of differentiation with regard to the ξ's, thus getting

$$v\frac{\bar{L}(u)}{J} - u\frac{\bar{M}(v)}{J} = \sum_j \frac{\partial}{\partial \xi_j}\left(\frac{1}{J}\sum_i \frac{\partial \xi_j}{\partial x_i}\bar{S}_i\right).$$

Here we have, between $\frac{\bar{L}(u)}{J}$ and $\frac{\bar{M}(v)}{J}$, an identity of the form of

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[7] The proposition in the text is given by du Bois-Reymond in the article Crelle, vol. **104**, already referred to in the note on page 12. His proof, which is based on the Green's Theorem, or integral form of Lagrange's Identity, runs essentially as follows:

$$\int \cdots \int [vL(u) - uM(v)]dx_1 \ldots dx_m = U,$$

U consisting of terms with less than m integrations. But

$$\int \cdots \int [vL(u) - uM(v)]dx_1 \ldots dx_m = \int \cdots \int [v\bar{L}(u) - u\bar{M}(v)]\frac{1}{J}\,d\xi_1 \ldots d\xi_m,$$

and so also we may transform U to, say, $\bar{U}$. This gives us

$$\int \cdots \int \left[v\frac{\bar{L}(u)}{J} - u\frac{\bar{M}(v)}{J}\right] d\xi_1 \ldots d\xi_m = \bar{U},$$

from which relation, of the form of a Green's Theorem, we infer, just as from a relation of the form of Lagrange's Identity, that $\frac{\bar{L}(u)}{J}$ and $\frac{\bar{M}(v)}{J}$ are adjoint. I have preferred to base my proof on Lagrange's Identity itself.

Lagrange's Identity, and these two expressions are, therefore, in accordance with the proposition noted on page 16, mutually adjoint.

It remains, then, to prove that

$$\sum_{i,j} \frac{1}{J} \frac{\partial \xi_j}{\partial x_i} \frac{\partial \overline{S}_i}{\partial \xi_j} = \sum_{i,j} \frac{\partial}{\partial \xi_j} \left(\frac{1}{J} \frac{\partial \xi_j}{\partial x_i} \overline{S}_i \right),$$

or, what is the same thing, that

$$\sum_{i,j} \overline{S}_i \frac{\partial}{\partial \xi_j} \left(\frac{1}{J} \frac{\partial \xi_j}{\partial x_i} \right) = 0.$$

Now the coefficient of $\overline{S}_i$ in this equation vanishes. For $\frac{1}{J} \frac{\partial \xi_j}{\partial x_i}$ is equal to ι_{ij}, the cofactor in

$$\frac{1}{J} \equiv \begin{vmatrix} \frac{\partial x_1}{\partial \xi_1} & \cdots & \frac{\partial x_1}{\partial \xi_m} \\ \cdot & \cdot \; \cdot \; \cdot & \cdot \\ \cdot & \cdot \; \cdot \; \cdot & \cdot \\ \frac{\partial x_m}{\partial \xi_1} & \cdots & \frac{\partial x_m}{\partial \xi_m} \end{vmatrix}$$

of $\frac{\partial x_i}{\partial \xi_j}$. So that the coefficient in question, viz., $\sum_j \frac{\partial}{\partial \xi_j} \left(\frac{1}{J} \frac{\partial \xi_j}{\partial x_i} \right)$, is equal to $\sum_j \frac{\partial \iota_{ij}}{\partial \xi_j}$, and this expression vanishes. For $\frac{\partial \iota_{ij}}{\partial \xi_j}$ is the sum of the $m-1$ determinants obtained by substituting in ι_{ij} for the elements of each of its columns in turn the derivatives, with regard to ξ_j, of the elements of that column. Consider any one of these $m-1$ determinants,

$$(-1)^{i+j} \begin{vmatrix} \frac{\partial x_1}{\partial \xi_1} & \cdots & \frac{\partial x_1}{\partial \xi_{k-1}} & \frac{\partial^2 x_1}{\partial \xi_j \partial \xi_k} & \frac{\partial x_1}{\partial \xi_{k+1}} & \cdots & \frac{\partial x_1}{\partial \xi_{j-1}} & \frac{\partial x_1}{\partial \xi_{j+1}} & \cdots & \frac{\partial x_1}{\partial \xi_m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial x_{i-1}}{\partial \xi_1} & \cdots & \frac{\partial x_{i-1}}{\partial \xi_{k-1}} & \frac{\partial^2 x_{i-1}}{\partial \xi_j \partial \xi_k} & \frac{\partial x_{i-1}}{\partial \xi_{k+1}} & \cdots & \frac{\partial x_{i-1}}{\partial \xi_{j-1}} & \frac{\partial x_{i-1}}{\partial \xi_{j+1}} & \cdots & \frac{\partial x_{i-1}}{\partial \xi_m} \\ \frac{\partial x_{i+1}}{\partial \xi_1} & \cdots & \frac{\partial x_{i+1}}{\partial \xi_{k-1}} & \frac{\partial^2 x_{i+1}}{\partial \xi_j \partial \xi_k} & \frac{\partial x_{i+1}}{\partial \xi_{k+1}} & \cdots & \frac{\partial x_{i+1}}{\partial \xi_{j-1}} & \frac{\partial x_{i+1}}{\partial \xi_{j+1}} & \cdots & \frac{x_{i+1}}{\partial \xi_m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial x_m}{\partial \xi_1} & \cdots & \frac{\partial x_m}{\partial \xi_{k-1}} & \frac{\partial^2 x_m}{\partial \xi_j \partial \xi_k} & \frac{\partial x_m}{\partial \xi_{k+1}} & \cdots & \frac{\partial x_m}{\partial \xi_{j-1}} & \frac{\partial x_m}{\partial \xi_{j+1}} & \cdots & \frac{\partial x_m}{\partial \xi_m} \end{vmatrix}$$

The same determinant occurs a second time, and a second time only, in $\sum_j \frac{\partial \iota_{ij}}{\partial \xi_j}$. It comes, namely, from ι_{ik} also, if we replace therein the elements of the jth column by their derivatives with regard to ξ_k, — the same determinant, that is, except perhaps as to sign; and it is easily seen that the signs in the two cases are opposite, so that the two determinants cancel each other. Thus the $m\ (m-1)$ determinants, as the sum of which $\sum_j \frac{\partial \iota_{ij}}{\partial \xi_j}$ may be written, cancel each other in pairs; and the latter expression is, as asserted, zero.

V. CONDITIONS FOR $\phi \cdot L(u)$ BEING $(-1)^n$ TIMES ITS ADJOINT.

§ 16. *The Conditions.*

The remainder of this paper will be devoted to a study of the problem: What are the conditions that a differential expression should possess the property of its being possible, by multiplying it by a suitable function, ϕ, of the independent variable or variables, to make $\phi \cdot L(u)$ equal to $(-1)^n$ times its adjoint?[8] After a discussion of ordinary differential expressions I shall give a complete solution of the problem for partial differential expressions of the second order, obtaining also certain results for those of higher order.

Before attacking the problem, let us notice that the property in question is an invariant property. It is, of course, invariant for a multiplication of $L(u)$ by a function of the independent variables. It is invariant for a change of independent variables. For let $L(u)$ go over, under such a change of variables into $\bar{L}(u)$. Now $\phi \cdot L(u)$ and $(-1)^n \phi \cdot L(v)$ are adjoint. Therefore, by the proposition last proved, $\frac{1}{J}$ times the transformed of $\phi \cdot L(u)$ and $\frac{1}{J}$ times the transformed of $(-1)^n \phi \cdot L(v)$ are adjoint. That is, $\frac{1}{J} \phi \cdot \bar{L}(u)$ and $(-1)^n \frac{1}{J} \phi \cdot \bar{L}(v)$ are adjoint. That is, $\bar{L}(u)$ can be made equal to $(-1)^n$ times its adjoint by multiplying it by $\frac{1}{J}\phi$. In the same way, by making use of proposition 6, page 19, we may show that the property in question is invariant for a change of dependent variable; that is, that if $L(u)$ go over, under $u = \psi \cdot \eta$, into $\Lambda(\eta)$, then $\Lambda(\eta)$ may be made equal to $(-1)^n$ times its adjoint by multiplying it by $\phi\psi$. That property, then, persists under all these transformations. In parallelism with this fact, the conditions we shall obtain for its existence are the vanishing of expressions invariant under all these transformations.

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[8] This problem is solved, in the case of partial differential expressions of the second order, in *two* independent variables, by du Bois-Reymond in the article referred to in the last note. The fact that the expression, whose vanishing forms the condition for the possibility of a solution, is an invariant, is not, however, noticed.

Taking first the case of *ordinary* differential expressions, let us begin with those of the second order. The condition that

$$L(u) = a_{11}u'' + a_1u' + au$$

should be self-adjoint is, by (10), page 10, $a_1 = a_{11}'$. The condition that $\phi \cdot L(u)$ should be self-adjoint is, therefore,

$$\phi a_1 = \frac{d}{dx}(\phi a_{11}),$$

or

$$a_{11}\phi + (a_{11}' - a_1)\,\phi = 0.$$

It is always possible, then, to make an ordinary differential expression of the second order self-adjoint by multiplying it by a function of x; the latter function has merely to be a solution of the differential equation last written.

We note that, since $\frac{d}{dx}(\phi a_{11}) = \phi a_1$, $\phi \cdot L(u)$ may be written in the form

$$\phi \cdot L(u) = \frac{d}{dx}(Ku') + Gu,$$

where $K = \phi a_{11}$, $G = \phi a$, and ϕ is determined as above. A differential *equation*, then,

$$u'' + pu' + qu = 0,$$

may be thrown into the form

$$\frac{d}{dx}(Ku') + G(u) = 0,$$

where $K = \phi$, $G = \phi q = Kq$, and ϕ is a solution of $\phi' = p\phi$, or, say, $\phi = e^{\int p dx}$. This is Sturm's Normal Form for such an equation.

For ordinary differential equations of the nth order, the solution of our problem will be found in Wilczynski, page 46. The conditions there obtained consist in the vanishing of the so-called linear invariants of odd weight, that is, in Wilczynski's notation, of Θ_3, Θ_5, etc.

To translate into terms of differential expressions we must substitute $\frac{a_{n-k}}{a_n}$ in the Θ's for the coefficient p_{n-k} of the differential equation

$$u^{(n)} + p_{n-1}u^{(n-1)} + \ldots = 0.$$

The expressions so obtained are evidently, like the Θ's, invariants, both for change of dependent and independent variable, of the differential equation $L(u) = 0$.

Next let $L(u)$ be a *partial* differential expression and of the second order. $\phi \cdot L(u)$ is to be self-adjoint. Necessary and sufficient conditions thereto are, by (10), page 10,

$$\phi a_i = \sum_j \frac{\partial(\phi a_{ij})}{\partial x_j},$$

or

$$\sum_j a_{ij} \frac{\partial \log \phi}{\partial x_j} = a_i - \sum_j \frac{\partial a_{ij}}{\partial x_j}, \qquad i = 1, 2, \ldots m.$$

If

$$A \equiv \begin{vmatrix} a_{11} & \ldots & a_{1m} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ a_{m1} & \ldots & a_{mm} \end{vmatrix}$$

is not zero, we may solve these equations, and get

$$\frac{\partial \log \phi}{\partial x_i} = \frac{\begin{vmatrix} a_{11} & \ldots & a_{1,i-1} & a_1 - \sum_j \frac{\partial a_{1j}}{\partial x_j} & a_{1,i+1} & \ldots & a_{1m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{m1} & \ldots & a_{m,i-1} & a_m - \sum_j \frac{\partial a_{mj}}{\partial x_j} & a_{m,i+1} & \ldots & a_{mm} \end{vmatrix}}{A} = L_i, \tag{42}$$

let us say. Necessary and sufficient conditions that these equations should have a solution, $\log \phi$, are

$$\frac{\partial L_i}{\partial x_j} - \frac{\partial L_j}{\partial x_i} = 0, \qquad i = 1, 2, \ldots m.$$

The expressions on the left are absolute invariants, for change of dependent variable, of the equation $L(u) = 0$. For if we refer to (24) and (27), pages 31–32, we shall find that $L_i = \lambda_i - \kappa_i$; so that

$$\frac{\partial L_i}{\partial x_j} - \frac{\partial L_j}{\partial x_i} = \frac{\partial(\lambda_i - \kappa_i)}{\partial x_j} - \frac{\partial(\lambda_j - \kappa_j)}{\partial x_i} = \left(\frac{\partial \lambda_i}{\partial x_j} - \frac{\partial \lambda_j}{\partial x_i}\right) - \left(\frac{\partial \kappa_i}{\partial x_j} - \frac{\partial \kappa_j}{\partial x_i}\right),$$

that is, is equal, page 32, to the difference of an absolute invariant and the adjoint invariant.

The expressions $\frac{\partial L_i}{\partial x_j} - \frac{\partial L_j}{\partial x_i}$ are *not*, on the other hand, except in the case of *two* independent variables, invariants for a change of independent variables. They are, however, the coefficients of what, to extend somewhat the definition of that term, we may call a covariant, viz., $\sum_{i,j}\left(\frac{\partial L_i}{\partial x_j} - \frac{\partial L_j}{\partial x_i}\right) dx_i \delta x_j$, where the dx's and δx's are two independent systems of differentials. Reserving for the moment, until we have discussed partial differential expressions of the nth order, the proof that the expression above is a covariant, we may state the solution of our problem, for the case in hand, as follows:

Proposition 24. A necessary and sufficient condition for the possibility of making a differential expression of the second order self-adjoint by multiplying it by a function of the independent variables is, if the invariant A does not vanish, the identical vanishing of the expression

$$\sum_{i,j}\left(\frac{\partial L_i}{\partial x_j} - \frac{\partial L_j}{\partial x_i}\right) dx_i \delta x_j, \tag{43}$$

the L's being defined by (42). The coefficients of this expression, $\frac{\partial L_i}{\partial x_j} - \frac{\partial L_j}{\partial x_i}$, are absolute invariants, for change of dependent variable, of the differential equation, and the expression itself is absolutely invariant for change of independent variables.

Let us look now for a moment at the case of partial differential expressions of the nth order. We take, as usual, for illustration, two independent variables. In order, first, that the coefficients of *the* $(n-1)$*st derivatives* in $\phi \cdot L(u)$ should be $(-1)^n$ times the corresponding coefficients of its adjoint, we must, by (15), page 14, have

$$n a_{p+1,q} \frac{\partial \log \phi}{\partial x} + n a_{p,q+1} \frac{\partial \log \phi}{\partial y} = 2 a_{pq} - n\left(\frac{\partial a_{p+1,q}}{\partial x} + \frac{\partial a_{p,q+1}}{\partial y}\right),$$
$$p + q = n - 1, \qquad p = 0, 1, \ldots (n-1). \tag{44}$$

If these equations are to be solvable algebraically for $\frac{\partial \log \phi}{\partial x}$, $\frac{\partial \log \phi}{\partial y}$, it is necessary that all three-rowed determinants of the matrix

$$\left\| \begin{array}{cccc} a_{n0} & a_{n-1,1} & 2\,a_{n-1,0} - n\left(\dfrac{\partial a_{n0}}{\partial x} + \dfrac{\partial a_{n-1,1}}{\partial y}\right) \\ a_{n-1,1} & a_{n-2,2} & 2\,a_{n-2,1} - n\left(\dfrac{\partial a_{n-1,1}}{\partial x} + \dfrac{\partial a_{n-2,2}}{\partial y}\right) \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ a_{1,n-1} & a_{0n} & 2\,a_{0,n-1} - n\left(\dfrac{\partial a_{1,n-1}}{\partial x} + \dfrac{\partial a_{0n}}{\partial y}\right) \end{array} \right\|$$

should vanish. These three-rowed determinants are invariants, for change of dependent variable, of the differential equation. For any one of them may be written as $I + (-1)^{n-1} J$, where I is an invariant of the form (28), page 33, and J is the adjoint invariant (29).

If these conditions are fulfilled, we may solve for $\dfrac{\partial \log \phi}{\partial x}$, $\dfrac{\partial \log \phi}{\partial y}$ any two of the equations (44):

$$\frac{\partial \log \phi}{\partial x} = \frac{\left| \begin{array}{cc} 2\,a_{p_1 q_1} - n\left(\dfrac{\partial a_{p_1+1,q_1}}{\partial x} + \dfrac{\partial a_{p_1,q_1+1}}{\partial y}\right) & a_{p_1,q_1+1} \\ 2\,a_{p_2 q_2} - n\left(\dfrac{\partial a_{p_2+1,q_2}}{\partial x} + \dfrac{\partial a_{p_2,q_2+1}}{\partial y}\right) & a_{p_2,q_2+1} \end{array} \right|}{n\,A} = L_1,$$

$$\frac{\partial \log \phi}{\partial y} = \frac{\left| \begin{array}{cc} a_{p_1+1,q_1} & 2\,a_{p_1 q_1} - n\left(\dfrac{\partial a_{p_1+1,q_1}}{\partial x} + \dfrac{\partial a_{p_1,q_1+1}}{\partial y}\right) \\ a_{p_2+1,q_2} & 2\,a_{p_2 q_2} - n\left(\dfrac{\partial a_{p_2+1,q_2}}{\partial x} + \dfrac{\partial a_{p_2,q_2+1}}{\partial y}\right) \end{array} \right|}{n\,A} = L_2,$$

$$A = \left| \begin{array}{cc} a_{p_1+1,q_1} & a_{p_1,q_1+1} \\ a_{p_2+1,q_2} & a_{p_2,q_2+1} \end{array} \right|, \qquad p_i + q_i = n - 1.$$

And the necessary and sufficient condition for the existence of a solution, $\log \phi$, is $\dfrac{\partial L_1}{\partial y} - \dfrac{\partial L_2}{\partial x} = 0$. Here the expression on the left is an invariant, for change of dependent variable, of the differential equation. For $L_1 = \lambda_1 - \kappa_1$, $L_2 = \lambda_2 - \kappa_2$, the κ's and λ's being given by (30) and (32 a), page 33; and $\dfrac{\partial L_1}{\partial y} - \dfrac{\partial L_2}{\partial x}$ is, therefore, the difference between an invariant and the adjoint invariant.

We shall carry the solution of the problem no further. To complete that solution we should next have to go on and write down the conditions that the coefficients of the derivatives of the $(n-3)$d, $(n-5)$th, etc., orders in $\phi \cdot L(u)$ should be $(-1)^n$ times the corresponding coefficients of its adjoint. (By proposition 4, page 15, we need merely consider the orders $n-k$, where k is odd.) These conditions would, by (15), page 14, be the vanishing of expressions bilinear in the a's and their derivatives and in ϕ and its derivatives; that is, after substitution for the derivatives of ϕ from the equations $\frac{\partial \phi}{\partial x} = L_1\phi$, $\frac{\partial \phi}{\partial y} = L_2\phi$, and from the equations obtained from these by differentiation, and after division by ϕ, of rational functions of the a's and their derivatives. And the question would suggest itself as to whether these latter were invariants.

§ 17. *The Covariant* $\sum_{i,j}\left(\frac{\partial L_i}{\partial x_j} - \frac{\partial L_j}{\partial x_i}\right) dx_i \delta x_j$.

We return now to the proof that the expression (43), page 53, is a covariant for change of independent variables. A proof of this fact is to be found in a paper by E. Cotton, *Sur les Invariants Différentiels de quelques Equations linéaires aux dérivées partielles du second ordre,* in the Annales de l'Ecole Normale, 3e série, vol. 17 (1900), pages 211–244. Cotton's methods are based on the theory of quadratic differential forms. It is perhaps worth while to obtain the result we are interested in independently of that theory, as may be done with no great difficulty. I shall therefore give such a proof, following in general the steps by which Cotton reaches his result. I retain in part his notation. Further, a dash over an expression shall indicate that it is the same function of the $\bar{a}$'s, the coefficients of the transformed differential expression, that the expression without the dash is of the a's.

First, then, the expression

$$\Delta_2 u = \sqrt{A} \sum_{i,j} \frac{\partial}{\partial x_i}\left(\frac{a_{ij}}{\sqrt{A}} \frac{\partial u}{\partial x_j}\right)$$

is an absolute covariant. Here A stands, as usual, for the determinant of the a_{ij}'s, an invariant, as we know, of weight two. The proof goes as follows. Making use of the formulas (36), page 41, for the $\bar{a}$'s, we get

$$
\begin{aligned}
\overline{\Delta_2 u} &= \sqrt{\bar{A}} \sum_{i,j} \frac{\partial}{\partial \xi_i}\left(\frac{\bar{a}_{ij}}{\sqrt{\bar{A}}} \frac{\partial u}{\partial \xi_j}\right) \\
&= J\sqrt{A} \sum_{i,j} \frac{\partial}{\partial \xi_i}\left[\sum_{k,l} \frac{a_{kl}}{J\sqrt{A}} \frac{\partial \xi_i}{\partial x_k} \frac{\partial \xi_j}{\partial x_l} \frac{\partial u}{\partial \xi_j}\right] \\
&= J\sqrt{A} \sum_{i} \frac{\partial}{\partial \xi_i}\left[\sum_{k,l} \frac{a_{kl}}{J\sqrt{A}} \frac{\partial \xi_i}{\partial x_k} \left(\sum_{j} \frac{\partial u}{\partial \xi_j} \frac{\partial \xi_j}{\partial x_l}\right)\right] \\
&= J\sqrt{A} \sum_{i} \frac{\partial}{\partial \xi_i}\left[\sum_{k,l} \frac{a_{kl}}{J\sqrt{A}} \frac{\partial \xi_i}{\partial x_k} \frac{\partial u}{\partial x_l}\right] \\
&= J\sqrt{A} \sum_{i,k,l} \left[\frac{\partial \xi_i}{\partial x_k} \frac{\partial}{\partial \xi_i}\left(\frac{a_{kl}}{J\sqrt{A}} \frac{\partial u}{\partial x_l}\right)\right] \\
&\qquad + \sum_{k,l}\left[a_{kl} \frac{\partial u}{\partial x_l} \sum_{i,j}\left(\frac{\partial^2 \xi_i}{\partial x_j \partial x_k} \frac{\partial x_j}{\partial \xi_i}\right)\right].
\end{aligned}
$$

The first half of this expression

$$
\begin{aligned}
&= J\sqrt{A} \sum_{k,l} \frac{\partial}{\partial x_k}\left(\frac{a_{kl}}{J\sqrt{A}} \frac{\partial u}{\partial x_l}\right) \\
&= \sqrt{A} \sum_{k,l} \frac{\partial}{\partial x_k}\left(\frac{a_{kl}}{\sqrt{A}} \frac{\partial u}{\partial x_l}\right) - \frac{1}{J} \sum_{k,l} a_{kl} \frac{\partial u}{\partial x_l} \frac{\partial J}{\partial x_k} \\
&= \Delta_2 u - \frac{1}{J} \sum_{k,l} a_{kl} \frac{\partial u}{\partial x_l} \frac{\partial J}{\partial x_k}.
\end{aligned}
$$

Now

$$\frac{\partial J}{\partial x_k} = \sum_{i,j} \frac{\partial^2 \xi_i}{\partial x_j \partial x_k} J_{ij},$$

if J_{ij} be the cofactor, in J, of $\frac{\partial \xi_i}{\partial x_j}$. Further

$$J_{ij} = J \frac{\partial x_j}{\partial \xi_i};$$

so that we have finally

$$\overline{\Delta_2 u} = \Delta_2 u - \sum_{i,j,k,l} a_{kl} \frac{\partial u}{\partial x_l} \frac{\partial^2 \xi_i}{\partial x_j \partial x_k} \frac{\partial x_j}{\partial \xi_i} + \text{the same quadruple sum};$$

that is,

$$\overline{\Delta_2 u} = \Delta_2 u. \qquad Q.\ E.\ D.$$

$\Delta_2 u$, then, or, written in expanded form,

$$\Delta_2 u = \sum_{i,j} a_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i,j} \frac{\partial u}{\partial x_i} \left(\frac{\partial a_{ij}}{\partial x_j} - \frac{1}{2A} a_{ij} \frac{\partial A}{\partial x_j} \right),$$

is an absolute covariant. We notice that the terms involving second derivatives are identical in $\Delta_2 u$ and in $L(u)$, so that the latter may be written

$$L(u) = \Delta_2 u + \sum_i d_i \frac{\partial u}{\partial x_i} \qquad au,$$

where

$$d_i = a_i - \sum_j \frac{\partial a_{ij}}{\partial x_j} + \frac{1}{2A} \sum_j a_{ij} \frac{\partial A}{\partial x_j}. \tag{45}$$

Similarly, the transformed differential expression, $\bar{L}(u)$, may be written

$$\bar{L}(u) = \overline{\Delta_2 u} + \sum_i \bar{d}_i \frac{\partial u}{\partial \xi_i} + \bar{a} u.$$

Now since, when $L(u)$ goes over into $\bar{L}(u)$, $\Delta_2 u$, au go over into $\overline{\Delta_2 u}$, $\bar{a}u$ respectively, it follows that $\sum_i d_i \frac{\partial u}{\partial x_i}$ goes over into $\sum_i \bar{d}_i \frac{\partial u}{\partial \xi_i}$, in other words that it is an absolute covariant.

Hence we conclude that the d's are transformed contragrediently to the $\frac{\partial u}{\partial x}$'s. The expression $\sum_{i,j} A_{ij} d_j dx_i$, then, is of the form of (37), page 43, and is, therefore, a relative covariant of weight two; or

$$\sum_i l_i dx_i \tag{46}$$

is an absolute covariant, if we define l_i by the formula

$$l_i = \frac{1}{A} \sum_j A_{ij} d_j. \tag{47}$$

Since (46) is an absolute covariant, the l's must be transformed contragrediently to the dx's,

$$\bar{l}_i = \sum_k \frac{\partial x_k}{\partial \xi_i} l_k. \tag{48}$$

This being the case, the expression

$$\sum_{i,j} \left(\frac{\partial l_i}{\partial x_j} - \frac{\partial l_j}{\partial x_i} \right) dx_i \delta x_j, \tag{49}$$

where the dx's and δx's are two independent sets of differentials, will be an absolute covariant.

Proof. Consider an expression $\sum_{i,j} c_{ij} dx_i \delta x_j$; and let it go over by our change of variables into $\sum_{i,j} \bar{c}_{ij} d\xi_i \delta\xi_j$. Then for the $\bar{c}$'s we readily calculate the formula,

$$\bar{c}_{pq} = \sum_{i,j} c_{ij} \frac{\partial x_i}{\partial \xi_p} \frac{\partial x_j}{\partial \xi_q}.$$

Now the coefficients, $\frac{\partial l_i}{\partial x_j} - \frac{\partial l_j}{\partial x_i}$, in (49) above are transformed cogrediently with the c's. For we have, from (48),

$$\frac{\partial \bar{l}_p}{\partial \xi_q} = \frac{\partial}{\partial \xi_q}\left(\sum_i \frac{\partial x_i}{\partial \xi_p} l_i\right) = \sum_i \frac{\partial^2 x_i}{\partial \xi_p \, \partial \xi_q} l_i + \sum_{i,j} \frac{\partial x_i}{\partial \xi_p} \frac{\partial x_j}{\partial \xi_q} \frac{\partial l_i}{\partial x_j},$$

$$\frac{\partial \bar{l}_q}{\partial \xi_p} = \sum_i \frac{\partial^2 x_i}{\partial \xi_p \partial \xi_q} l_i + \sum_{i,j} \frac{\partial x_i}{\partial \xi_p} \frac{\partial x_j}{\partial \xi_q} \frac{\partial l_j}{\partial x_i}.$$

Therefore

$$\frac{\partial \bar{l}_p}{\partial \xi_q} - \frac{\partial \bar{l}_q}{\partial \xi_p} = \sum_{i,j} \left(\frac{\partial l_i}{\partial x_j} - \frac{\partial l_j}{\partial x_i}\right) \frac{\partial x_i}{\partial \xi_p} \frac{\partial x_j}{\partial \xi_q},$$

as asserted. Hence it follows that just as we have

$$\sum_{i,j} \bar{c}_{ij} d\xi_i \delta\xi_j = \sum_{i,j} c_{ij} dx_i \delta x_j,$$

so also we have

$$\sum_{i,j} \left(\frac{\partial \bar{l}_i}{\partial \xi_j} - \frac{\partial \bar{l}_j}{\partial \xi_i}\right) d\xi_i \delta\xi_j = \sum_{i,j} \left(\frac{\partial l_i}{\partial x_j} - \frac{\partial l_j}{\partial x_i}\right) dx_i \delta x_j. \qquad Q.E.D.$$

Now the covariant (49) is identical with the expression (43) which we wish to prove a covariant. To establish this identity we need merely to obtain the explicit form of (49). From formulas (47), (45) we get

$$l_i = \frac{1}{A} \sum_j A_{ij} d_j = \frac{1}{A} \sum_j A_{ij} \left(a_j - \sum_k \frac{\partial a_{jk}}{\partial x_k} + \frac{1}{2A} \sum_k a_{jk} \frac{\partial A}{\partial x_k}\right)$$

$$= \frac{1}{A} \sum_j A_{ij} \left(a_j - \sum_k \frac{\partial a_{jk}}{\partial x_k}\right) + \frac{1}{2A^2} \sum_k \left(\frac{\partial A}{\partial x_k} \sum_j a_{kj} A_{ij}\right).$$

The first part of this expression will be seen to be equal to L_i, as defined by (42), page 52. Since, further,

$$\begin{aligned}\sum_j a_{kj}A_{ij} &= 0, \qquad && k \neq i, \\ &= A, && k = i,\end{aligned}$$

we get finally

$$l_i = L_i + \frac{1}{2A}\frac{\partial A}{\partial x_i}.$$

Hence

$$\frac{\partial l_i}{\partial x_j} - \frac{\partial l_j}{\partial x_i} = \frac{\partial L_i}{\partial x_j} - \frac{\partial L_j}{\partial x_i},$$

and our expression, (43), page 53, is identical with (49), and is therefore an absolute covariant. But this is what we set out to prove.

In the case of two independent variables, $m = 2$, our covariant is

$$\left(\frac{\partial L_1}{\partial y} - \frac{\partial L_2}{\partial x}\right)(dx\delta y - dy\delta x).$$

Here the second factor is itself a covariant of weight one; so that, in this case, the condition of proposition 24, page 53, would be the vanishing of $\frac{\partial L_1}{\partial y} - \frac{\partial L_2}{\partial x}$, which is not only an absolute invariant, for change of dependent variable, of the differential equation, but an invariant, of weight minus one, for change of independent variables as well.

I collect here for reference the covariants that we have come across in the course of our work above, adding a couple of invariants from Cotton's paper.[9]

$$\begin{aligned}\Delta_2 u &= \sqrt{A}\sum_{i,j}\frac{\partial}{\partial x_i}\left(\frac{a_{ij}}{\sqrt{A}}\frac{\partial u}{\partial x_j}\right) \\ &= \sum_{i,j} a_{ij}\frac{\partial^2 u}{\partial x_i\,\partial x_j} + \sum_{i,j}\frac{\partial u}{\partial x_i}\left(\frac{\partial a_{ij}}{\partial x_j} - \frac{1}{2A}\,a_{ij}\frac{\partial A}{\partial x_j}\right),\end{aligned}$$

$\sum_i d_i\frac{\partial u}{\partial x_i}$, and $\sum_i l_i dx_i$ are absolute covariants; d_i and l_i are defined by (45) and (47).

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[9] For bibliography, see the note, page 239, of Cotton's article. The invariant, for $m = 2$, $\frac{\partial L_1}{\partial y} - \frac{\partial L_2}{\partial x}$ is also given, in explicit form, by Rivereau in the Bull. de la Soc. Math. de France, **29**, 7 (1901); it is identical, as is easily shown, with what Rivereau calls $2I$.

$$\Delta(l) = \sum_{i,j} a_{ij} l_i l_j,$$

$$\Delta_2(l) = \sum_i \frac{\partial d_i}{\partial x_i} - \frac{1}{2A} \sum_i d_i \frac{\partial A}{\partial x_i},$$

are absolute invariants.

For *one* independent variable, these invariants reduce to $\frac{(2a_1 - a_{11}')^2}{16 a_{11}}$ and $\frac{1}{8} \frac{a_{11}}{2a_1 - a_{11}'} \left(\frac{(2a_1 - a_{11}')^2}{a_{11}} \right)'$ respectively. The first of these is the square of the invariant J_{n1}, page 46, for $n = 2$, divided by $16 a_{11}$; while J_{n2}, for $n = 2$, is $8 \Delta(l) - 4 \Delta_2(l)$. Thus we have found, for the second order, invariants of a partial differential expression analogous to the invariants J_{n1}, J_{n2} of an ordinary differential expression.

We shall accept from Cotton the fact that $\Delta_2(l)$ is an absolute invariant. Next as to $\Delta(l)$. Since $\sum_{i,j} \frac{A_{ij}}{A} dx_i dx_j$ is an absolute covariant, — cf. (38), page 44, — any invariant of this quadratic differential form of weight w will be an invariant of $L(u)$ of weight $-w$. Now since, page 57, the l's are contragredient to the dx's,

$$- \begin{vmatrix} \frac{A_{11}}{A} & \cdots & \frac{A_{1m}}{A} & l_1 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \frac{A_{m1}}{A} & \cdots & \frac{A_{mm}}{A} & l_m \\ l_1 & \cdots & l_m & 0 \end{vmatrix},$$

that is, $\sum_{i,j} \frac{a_{ij}}{A} l_i l_j$, is an invariant of weight two of the differential form. $\sum_{i,j} a_{ij} l_i l_j$ or $\Delta(l)$ is, then, an absolute invariant of $L(u)$.

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THE DAMPING OF THE OSCILLATIONS OF SWINGING BODIES BY THE RESISTANCE OF THE AIR.

By B. Osgood Peirce.

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THE DAMPING OF THE OSCILLATIONS OF SWINGING BODIES BY THE RESISTANCE OF THE AIR.

By B. Osgood Peirce.

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When a body, free to turn about a fixed axis, like a horizontal pendulum, a suspended magnet, or the coil of a d'Arsonval galvanometer, is disturbed from a position of equilibrium, and is then allowed to swing under the action of a righting moment the intensity of which is proportional to the angular deviation of the body from the position of rest which it originally had, the damping effect of the resistance which the air offers to the motion is sooner or later made evident by a reduction in the amplitude of the swings. In many cases the phenomena can be quantitatively explained, with an approximation quite good enough for every practical purpose, if one assumes that the resisting couple has a moment equal at every instant to the product of a constant of the apparatus and the angular velocity which the body then has; and more than seventy years ago Gauss and W. Weber gave an exhaustive mathematical treatment,[1] based upon this hypothesis, of the behavior of such swinging magnets as they employed in their magnetic measurements at Göttingen. It appeared from their analysis, which in simplified form is given in most modern treatises on Physics, that if the resistance follows the law stated above, the ratio of any two successive elongations of the magnet must have a constant value; and they used the natural logarithm (λ) of this ratio, under the name of the "logarithmic decrement" of the motion, in many of their equations.

The resistance which air, under given conditions of temperature, pressure, and confinement, offers to a body of given form and dimensions, moving through it at a *uniform* velocity, v, has been studied by a great number of experimenters under a great variety of physical conditions, and a resumé of the results at which they have arrived can be found in the articles of Finsterwalder on Aërodynamik and of Cranz on Ballistik in the fourth volume of the Encyklopädie der Mathematischen Wissenschaften.[2]

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[1] Gauss, Resultate des Magnetischen Vereins, 1837. W. Weber, Resultate des Magnetischen Vereins, 1837, 1838; Maassbestimmungen, **2**; Math.-phys. Abhandlungen der K. Sächs. Gesellschaft, 1852. Du Bois-Reymond, Monatsberichte der Berl. Akad., 1869, 1870.

That under otherwise given conditions the air resistance, when v is large, is a complicated function of v, is shown by the practical formulas based on experiments made with rotating projectiles of the standard Krupp form. For a projectile of this kind of given size, in free air, the expressions are av^2, bv^3, cv^5, dv^3, ev^2, $fv^{1.7}$, $gv^{1.55}$, according as v, measured in meters per second, lies in one or other of the intervals between the values 50, 240, 295, 375, 419, 550, 800, and 1000. The constants are different for projectiles of different diameters and vary with the temperature of the air, the barometric pressure, and other circumstances.

In order to determine the resistance which the air offers to a given body moving uniformly through it at a comparatively small velocity, v, many different observers have made use of the whirling table in some form. The phenomenon to be studied is in any case a very complex one, since the moving body drags with it, as it moves, a certain mass of air, and the viscosity of the air contributes an uncertain amount to the quantity to be measured. It appears, however, from the experiments of Schellbach, von Loessl, Langley, Recknagel, Hagen, and others,[3] that when proper corrections have been made for the effect of the wind which the table takes with it as it turns, the air resistance varies as the square of the velocity[4] for all values of v between 50 and 0.2. For velocities much less than 20 centimeters per second the viscosity of the air appears to determine the resistance which is approximately proportional to the velocity. It is well to remember that a solid sphere, to take a concrete example, moving in an infinite homogeneous liquid at rest at infinity, in a straight line, with *constant* velocity, would encounter no resistance from the liquid if there were no viscosity; but that even in a homogeneous, perfect liquid, a sphere moving with *changing* velocity would meet with a resistance from the liquid, and the inertia of the sphere would in consequence of this be apparently increased in a manner which could be mathematically accounted for in the equation of motion of the sphere, if the mass of the sphere were increased by half the mass of the displaced liquid.

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[2] Leipzig, B. G. Teubner, 1903.

[3] Schellbach, Ann. d. Phys., **143**, 1871. Recknagel, Zeitschrift d. Ver. deutsch. Ing., **30**, 1886. F. v. Loessl, Die Luftwiderstandsgesetze. Langley, Experiments in Aërodynamics. Cranz, Aeussere Ballistik, Leipzig, 1895. Thiesen, Ann. d. Phys., **26**, 1885. Mach und Salcher, Wiener Berichte, 1887, 1889.

[4] Mohn, Grundzüge der Meteorologie, Zweite Auflage, p. 137: "Durch vergleichende Versuche über Druck und Geschwindigkeit des Windes, hat man gefunden dass der Winddruck dem Quadrate der Geschwindigkeit proportional ist." On page 138, however, the pressure of the wind in kilograms per square meter is given as 0.15, 1.87, 5.96, 15.27, 34.35, 95.4, according as the velocity in meters per second is 0.5, 4, 7, 11, 17, or 28.

If at the point (x, y, z) in a viscous fluid at the time t the components of the velocity are u, v, w, if the applied body forces which urge the fluid have the components X, Y, Z, if ρ is the density, and if μ represents a constant of the fluid which measures its coefficient of viscosity, the equations of motion of the fluid as established by Navier and Poisson[5] are usually written in the forms:

$$\rho\left(\frac{\partial u}{\partial t} + u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z}\right) = \rho X - \frac{\partial p}{\partial x} + \tfrac{1}{3}\mu\cdot\frac{\partial \varpi}{\partial x} + \mu\cdot\nabla^2(u),$$

$$\rho\left(\frac{\partial v}{\partial t} + u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z}\right) = \rho Y - \frac{\partial p}{\partial y} + \tfrac{1}{3}\mu\cdot\frac{\partial \varpi}{\partial y} + \mu\cdot\nabla^2(v),$$

$$\rho\left(\frac{\partial w}{\partial t} + u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z}\right) = \rho Z - \frac{\partial p}{\partial z} + \tfrac{1}{3}\mu\cdot\frac{\partial \varpi}{\partial z} + \mu\cdot\nabla^2(w), \qquad (1)$$

where
$$\varpi = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z},$$

and p represents the arithmetical mean of the normal pressures on any three mutually perpendicular planes through the point (x, y, z).

Using these equations, Stokes, in a paper[6] presented to the Cambridge Philosophical Society in December, 1850, determined the resistance which a sphere making small harmonic oscillations of complete period T, in an infinite viscous liquid, would encounter, and showed that if θ represented the distance of the centre of the sphere from its mean position at the time t, the value of this resistance would be

$$\left(\frac{1}{2} + \frac{9}{4\,af}\right)M'\cdot\frac{d^2\theta}{dt^2} + \frac{9\pi M'}{2\,afT}\left(1 + \frac{1}{af}\right)\cdot\frac{d\theta}{dt}, \qquad (2)$$

or

$$M''\frac{d^2\theta}{dt^2} + 2m\frac{d\theta}{dt}, \qquad (3)$$

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[5] Navier, Mémoire de l'Académie des Sciences, **6**, 1822. Poisson, Journal de l'École Polytechnique, **13**, 1829.

[6] Stokes, Mathematical and Physical Papers, II.

where a is the radius of the sphere, M', the mass of the displaced liquid, and $f^2 = \pi\rho/\mu T$: M_0 is the mass of the sphere.

Such a sphere, oscillating under the action of this resistance and a restoring force $(b^2\theta)$ proportional to the displacement, would have an equation of motion of the form

$$M \cdot \frac{d^2\theta}{dt^2} + 2\,m \cdot \frac{d\theta}{dt} + b^2\theta = 0, \tag{4}$$

where $M = M_0 + M''$: all the coefficients are to be considered contant, since b^2 is fixed, but they would be different for a different period of oscillation.

For an infinitely long cylinder of revolution also, oscillating in a viscous liquid, in a direction perpendicular to the axis of the cylinder, Stokes found an equation of motion of this same familiar form which had long been used to explain the behavior of pendulums, though it had been founded on a theory quite different from his. As early as 1828 Bessel[7] had pointed out the necessity of allowing for the inertia of the air which accompanies a pendulum in its motion, and the work of Sabine, Dubuat, Poisson, Baily, Plana, South, and others, had made it clear that in practical cases the moment of inertia of the swinging system might be twice that of the pendulum bob, and that the "resistance" of the air might be accounted for in many practical cases by assuming it to be proportional to the first power of the angular velocity. This equation had been used by Gauss for determining the motion of swinging bar magnets, as has been already mentioned, and it still forms the foundation of much modern work, as, for instance, that on the properties of damped d'Arsonval galvanometers.[8]

If, however, a swinging magnet presents to the air a relatively large surface, or if the magnet is provided with a large mica damping vane, it often happens that the resistance of the air cannot be satisfactorily explained on the assumption that it is proportional to the angular velocity at every instant, and that at the beginning of the motion it seems to be much more nearly proportional to the square of the angular velocity. It will be convenient, therefore, to consider first the manner in which the amplitude of an oscillating body would decrease if the motion were resisted by a couple of moment proportional to the square of the angular velocity. A roughly approximate solution of this problem was printed by Poisson in 1811, but is not accurate enough for practical purposes. We shall do well to attack it in another way.

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[7] Bessel, Untersuchungen über die Länge des einfachen Secunden Pendels, Berlin, 1828. Bottomley, Phil. Mag., **23**, 1887. Graetz, Reibung, Winkelmann's Handbuch der Physik, I. O. E. Meyer, Pogg. Ann., **113**, 1861; **125**, 1863; **142**, **143**, 1871; **148**, 1873. Wied. Ann., **23**, 1887. Kundt und Warburg, Pogg. Ann., **155**, 1875. Crookes and Stokes, Proceedings Royal Society, 1888.

[8] Dorn, Ann. der Physik, **17**, 1882; **35**, 1888. F. Kohlrausch: Ueber die Inconstanz der Dämpfungsfunction eines Galvanometers und ihren Einfluss auf die Absolute Widerstandsbestimmung mit dem Erdinductor, Ann. der Phys., **26**, 1885. Schering, Ann. der Phys., **9**, 1880. Jaeger, Instrumentenkunde, 1903. Dorn, Ann. der Physik, **17**, 1882.

If θ is the angular deviation in radians of the moving body from the position of equilibrium, and $b^2\theta$ the restoring moment, the moment of the couple due to the resistance of the air is of the form $2\,a(d\theta/dt)^2$; and if K represents the moment of inertia of the swinging system, the equation of motion is

$$K \cdot \frac{d^2\theta}{dt^2} + 2\,a\left(\frac{d\theta}{dt}\right)^2 + b^2\theta = 0, \tag{5}$$

or

$$\frac{d^2\theta}{dt^2} + 2\,a\left(\frac{d\theta}{d\,t}\right)^2 + \beta^2\theta = 0, \tag{6}$$

when the body is swinging in the positive direction.

If for $d\theta/dt$ we write ω, $d^2\theta/dt^2$ is equal to $\omega \cdot d\omega/d\theta$, and the equation becomes

$$\omega \cdot d\omega + (2\,a\omega^2 + \beta^2\theta)d\theta = 0, \tag{7}$$

which will become exact if we multiply through by $e^{4a\theta}$, so that,

$$\omega^2 = 2\,c \cdot e^{-4a\theta} + \frac{\beta^2}{8\,a^2} - \frac{\beta^2\theta}{2\,a}, \tag{8}$$

or

$$\omega^2 = 2\,c\,e^{-k\theta} + m - mk\theta, \tag{9}$$

where c is a constant of integration.

If $-\theta_0$ is the value of the angular deviation at any elongation on the negative side, and if θ_1 is the next elongation on the positive side, then, for the same value of c,

$$2\,c\,e^{k\theta_0} + m + mk\theta_0 = 0, \tag{10}$$

$$2\,c\,e^{-k\theta_1} + m - mk\theta_1 = 0, \tag{11}$$

or

$$(1 + k\theta_0)\,e^{-k\theta_0} = (1 - k\theta_1)e^{+k\theta_1}, \tag{12}$$

where $k = 4\,a$. This equation does not involve β.

For swings of large amplitude, it is easy to find θ_1 graphically, when k and θ_0 are given, by aid of this last equation. When θ_0 is small, however, we may, in any practical case, develop each number of (12) in a very convergent power series of which we need keep only terms of order lower than the fourth.

This procedure gives the equation

$$2\,k(\theta^3{}_0 + \theta^3{}_1) - 3(\theta^2{}_0 - \theta^2{}_1) = 0, \tag{13}$$

which is satisfied when $\theta = -\theta_0$ and from this we may find, by aid of a second development, the very approximate result,

$$\theta_1 = \theta_0 - \tfrac{2}{3}\,k\theta_0{}^2. \tag{14}$$

If terms of the fourth order are kept, we may obtain the expressions

$$\theta_1 = \theta_0 - \tfrac{2}{3}\,k\theta^2{}_0 + \tfrac{4}{9}\,k^2\theta^3{}_0, \tag{15}$$

$$\theta_2 = \theta_0 - \tfrac{4}{3}\,k\theta^2{}_0 + \tfrac{16}{9}\,k^2\theta^3{}_0,$$

but for most practical purposes (14) is quite accurate enough.

After the swinging system has come momentarily to rest at the elongation $-\theta_0$, it moves in the positive direction with an angular velocity which increases to a maximum at a position determined by the constants of the motion, and has the value ω_0 when $\theta = 0$.

It is easy to see from (3) that

$$\omega_0{}^2 = 2\,c + m, \tag{16}$$

and from (9) that

$$2\,c = -\,m(1 + k\theta_0)\,e^{-k\theta_0}, \tag{17}$$

so that

$$\omega_0{}^2 = m - m(1 + k\theta_0)e^{-k\theta_0}; \tag{18}$$

and it is evident that ω_0 is greater, other things being given, the greater the amplitude of the motion; that is, the greater the value of θ_0. Equation (16) shows, however, that the greatest value which ω_0 can have is $\sqrt{m}$, and it is interesting to determine what elongation on the positive side of the zero point corresponds to this angular velocity at $\theta = 0$.

If in (12) we suppose θ_0 to grow large without limit, θ_1 approaches the limit $1/k$, and it appears that however great the angle through which the swinging system may have been turned out of the position of equilibrium at the outset, the amplitude of the next elongation cannot be greater than $1/k$th of a radian, and the next turning point to this (on the same side of the zero as the original disturbance) must come at an angular distance from the position of equilibrium not greater than about $0.594/k$ radians. The subsequent swings decrease regularly in amplitude in such a manner as to make the logarithmic decrement decrease towards zero. At any time during the motion the determination of two successive amplitudes serves to determine k through (12), for it is easy to solve the transcendental equation to any desired accuracy.

If we differentiate (6) with respect to t, and represent $d\omega/dt$ by r, we shall get the equation

$$\frac{rdr}{4\alpha r + \beta^2} + \omega d\omega = 0, \qquad (19)$$

or

$$r - \frac{\beta^2}{4\alpha} \log \left(r + \frac{\beta^2}{4\alpha} \right) = C - 2\alpha\omega^2, \qquad (20)$$

and C, the constant of integration, may be determined from a consideration of the fact that when $\omega = 0$, θ is $-\theta_0$.

If a swinging system oscillates about a position of equilibrium under the action of a righting moment proportional to the deviation and a resisting couple proportional during the whole motion to the *first* power of the instantaneous angular velocity, the equation of motion has the familiar form

$$\frac{d^2\theta}{dt^2} + 2\alpha \frac{d\theta}{dt} + \beta^2\theta = 0. \qquad (21)$$

If $\rho^2 = \beta^2 - \alpha^2$, and if m and n are the roots of the equation

$$x^2 + 2\alpha x + \beta^2 = 0, \qquad (22)$$

$$m = -\alpha + \rho i, \qquad n = -\alpha - \rho i,$$

and we have $$\theta = e^{-\alpha t}(L \cos \rho t + M \sin \rho t), \qquad (23)$$

or $$\theta = A e^{-\alpha t} \sin (\rho t - e), \qquad (24)$$

where A and e are constants of integration. If, using t and θ as coordinates, we plot (24), it is clear that the curve $\theta = Ae^{-\alpha t}$ touches the curve $\theta = Ae^{-\alpha t} \sin (\rho t - e)$ when $\rho t - e = (2k + \frac{1}{2})\pi$, so that if the time be counted from the date of one of these points of tangency, the corresponding solution of (21) may be written in the form

$$\theta = Be^{-\alpha t} \cos \rho t. \qquad (25)$$

The complete period of the oscillation (T) is $2\pi/\rho$. The ratio of the amplitudes at two consecutive elongations is $e^{\alpha\pi/\rho}$ and the logarithmic decrement is $\alpha\pi/\rho$. The ratio of the amplitudes at two consecutive elongations on the same side of the position of equilibrium is $e^{2\alpha\pi/\rho}$, and we have

$$\alpha = 2\lambda/T, \qquad \beta^2 = 4(\pi^2 + \lambda^2)/T^2. \qquad (26)$$

The maxima of the curve (24) occur at times defined by the equation

$$\tan (\rho t - e) = \rho/\alpha; \quad \text{or} \quad \sin (\rho t - e) = \rho/\beta,$$

and the curve $$\theta = \frac{A\rho}{\beta} e^{-at}$$

passes through all these points, which are spaced at equal time intervals T.

If, then, the curve which represents as a function of the time (t) the deviation (θ) of a swinging body from the position of equilibrium be drawn, and if the motion be of the kind defined by the equation (21), the maxima will be spaced at equal time intervals, and it will be possible to pass through all the crests a curve of the family $\theta = Ce^{-at}$ where C and a are constants. It is easy to see whether or not this last condition is satisfied in any given case, if one has measured a series of successive amplitudes on the same side (d_1, d_2, d_3, d_4, d_5, . . . d_r). If we measure t from the date of the first of these elongations, the desired curve must have an equation of the form $\theta = d_1 e^{-at}$, and aT may be determined from any other amplitude (say the kth) for

$$d_k = d_1 e^{-(k-1)aT}.$$

If the value of aT thus found be the same for all values of k, the condition is satisfied. Sometimes when the period of the oscillation is extremely short, the maximum points seem to form a continuous curve, unless the diagram be much drawn out horizontally. In such a case as this one may use, in making the test just described, *not* a series of successive amplitudes on the same side of the position of equilibrium, but points on the curve, taken at convenient values of t equally spaced.

The Damping of the Quick Oscillations of a Light System suspended between two Stretched Wires by the Resistance of the Air and Frictional Forces in the Wire.

It will appear from the observations recorded in this paper that if a small magnetic needle be mounted horizontally with a minute galvanometer mirror upon a short, stiff, vertical piece of wire or glass filament stretched between two vertical pieces of fine wire, and if the needle be turned horizontally out of its position of rest through an angle of say 5° and then allowed to oscillate, the curve drawn through the crests of the oscillations as represented on a photograph record will usually not coincide exactly with any exponential curve of the family mentioned above. If a curve of this family be drawn nearly through a number of crests in the middle of the diagram, it will usually fall somewhat below the observed curve at each end. It will be convenient to instance a few typical cases at the outset.

I. Figure 1 (Plate 1) is a copy of a photographic record obtained from a short-period mirror galvanometer. The one-centimeter-long needle of this instrument, made of watch spring, was mounted on a short, stout, inflexible piece of glass fibre, together with a minute bit of very thin mirror, and the fibre was suspended, like the coil of a marine d'Arsonval galvanometer,[9] between two pieces of extremely fine gimp, under gentle tension. The light from an electric projecting lantern about twenty feet from the galvanometer, shining through a small hole in a brass plate used as a lantern slide, fell upon the galvanometer mirror, and a sharp image of the hole was formed on a piece of sensitive paper on a horizontal revolving drum at a considerable distance from the mirror. The needle was first deflected a little off scale by a steady current sent through the coils of the galvanometer when the light was screened, the screen was then removed and a record of the manner of decay of the amplitude of the excursions of the needle obtained when the galvanometer circuit had been suddenly broken. The moment of the couple due to the mutual action of the magnet and the earth's field was relatively inappreciable. Three different drums and three pieces of chronograph clock work were used in making the records discussed in this paper, but for the fastest speeds an electric motor driving a worm gear accurately cut for the purpose by Mr. G. W. Thompson, the mechanician of the Jefferson Laboratory, was employed, and this left nothing to be desired. The apparatus was put together by Mr. John Coulson, who helped me in all the work and took many of the photographic records. Most of these records, of which I have a very large number, were about 50 cms. long and 20 cms. wide, but much larger ones could be obtained if desirable.

On one of the photographs taken with the apparatus just described a series of measurements of the amplitudes of the oscillations, as depicted on the diagram, were made at times, represented by whole centimeters from the time origin, on the figure. The successive values for the excursions were: 1260, 1006, 791, 646, 521, 420, 349, 280, 231, 190, 159, 131, on a scale of equal parts, and, at the scale distance used, these numbers were accurately proportional to the angular amplitudes of the needle at the times concerned. If, then, the resistance to the oscillations were proportional to the angular velocity, it should be possible to draw a curve of the family $y = A \cdot e^{-at}$ the successive ordinates of which, taken at the proper time interval (T), should have the lengths indicated above.

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[9] See M, Figure 3.

If, however, we assume that when $t = 0$, $y = 1260$, and use the other numbers given above, in succession for determining aT, we get for this product the different values 0.225, 0.238, 0.224, 0.221, 0.220, 0.217, 0.214, 0.212, 0.210, 0.207, and it is evidently impossible to find the curve sought exactly. The differences, while much too great to be accidental, are intrinsically not very large, and a curve of the family $y = A \cdot e^{-at}$ may be drawn which will pass through the fourth, fifth, and sixth points, and the ordinates of which at the ends of the series will be 1231 and 116, instead of 1260 and 131. Corresponding ordinates of the observed and calculated curves are shown in the following table. The calculated curve has, as a whole, a less curvature than the observed curve, and the ratio of any excursion to the next decreases slightly with the time. The period was about 1/100th of a second.

TABLE I.

Observed.	Calculated.	Observed.	Calculated.
1260	1231	349	339
1006	993	280	273
791	801	231	221
646	646	190	178
521	521	159	143
420	420	131	116

In the case of this particular quickly oscillating system, therefore, the first double amplitude of which was not larger than 6°, the motion can be explained during a considerable part of its course with fair approximation on the assumption that the resistance due to the air and to frictional forces in the fibre is proportional to the angular velocity. The deviations from this law, while real, are not greater than one often finds in the motion of a suspended magnet or the coil of a d'Arsonval galvanometer, when swinging slowly over a small arc. Indeed two d'Arsonval galvanometers of the same type and apparently very like each other may depart from the usual law in opposite directions if the periods are long; in one the logarithmic decrement may grow larger as the amplitudes of a long series of swings decay, while in the other it may become smaller. In the case of a galvanometer of this kind in the Jefferson Laboratory, the ratio of one excursion to the next *increased* from 1.063 to 1.086 in an hour and a quarter, while the amplitude decreased to about four tenths of its original value. The complete period of swing was 158 seconds. A similar galvanometer in the same room has a coil the swings of which decay at a *decreasing* rate as the amplitudes grow less.

In his Anleitung zur Bestimmung der Schwingungsdauer einer Magnetnadel (1837), Gauss describes a suspended magnet the logarithmic decrement of the swings of which increased on a certain occasion from 1168×10^{-6} to 1301×10^{-6} in 422 oscillations. The actual value of the logarithmic decrement for this magnet and for a given amplitude varied from day to day, being usually smaller in cloudy weather.

II. After a number of records had been made like that reproduced in Figure 1, a small vertical mica damping vane of about 3 square centimeters area, was fastened symmetrically to the little glass rod which carried the mirror of the swinging system, and a new series of records were obtained. The restoring moment was the same as before, but the moment of inertia had been increased somewhat, as well as the resistance due to the air. Under these circumstances the period was much longer than before, while the manner of decay of the amplitudes was much the same. Figure 2 (Plate 1) represents on a reduced scale one of the smaller photographs. Figure A was plotted from a large record in which the crests of successive oscillations were 4.5 millimeters apart at the beginning of the diagram and nearly 4.9 millimeters at the end. Such a gradual change of period during the motion often accompanies the swinging of a magnet under the torsional forces of a stretched wire.[10]

The values, in ten thousandths of a radian, of a number of successive amplitudes, as obtained from the photograph, were: 597, 556, 518, 481, 448, 419, 390, 367, 341, 320, 300, 280, 262, 246, 230, 217, 203, 190, 179, 168, 159, 148, 140, 132, 124, 117, 111, 104, 98, 92.

These numbers, used as ordinates of points with equally spaced abscissas, give a curve of the form shown in Figure A by the full line WHCDK. The dotted line VCDG shows a curve of the family $y = A \cdot e^{-at}$, which coincides almost exactly with the full line between the points C and D.

The curves HT, CL represent attempts to determine the constants of an equation of the form (6) which should yield a curve of amplitudes like the observed curve. Both HT and CL pass exactly through two adjacent points of the line WHCDK, and the other points were determined by a series of applications of the equation (8). Some of the characteristics of certain of the records which I obtained resemble those of oscillations under a resistance proportional to the square of the angular velocity, but it is evident that in the case here considered the resistance does not nearly follow this law. We may notice that according to Poisson's rather rough approximation, HT and CL would be straight lines.

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[10] Guthe, Physical Review, 1908.

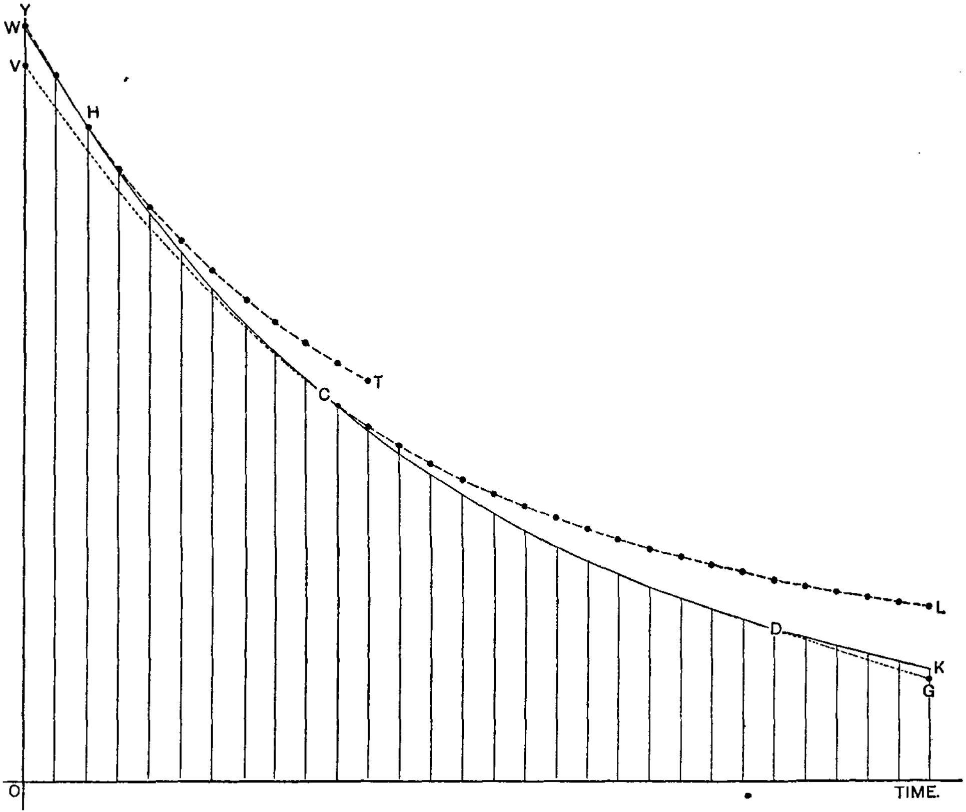

FIGURE A.

III. A new suspended system was then made of two 15 millimeter long magnetic needles mounted horizontally, one over the other (together with a small mirror), on a short bit of glass fibre stretched between two short lengths of No. 36 steel wire. The restoring forces came from the torsional forces in these wires. The mirror and needles together exposed to the air a resisting surface of less than a square centimeter area. The period was about 1/48th of a second. The numbers in the first column of the next table show the lengths of successive ordinates (taken at equal time intervals) of the curve drawn on the photographic record through the crests of the oscillations. The next column gives the lengths of the corresponding ordinates of a curve of the family $A \cdot e^{-at}$ drawn exactly through the fifth and tenth crests. The very tips of the needles at the beginning of the motion passed over about 10 centimeters of path per second.

TABLE II.

1705	1509	418	411
1445	1341	379	365
1230	1141	342	324
1075	1058	313	288
940	940	282	256
828	835	255	227
734	742	230	202
652	658	208	179
582	586	185	159
520	520	174	142
465	462		

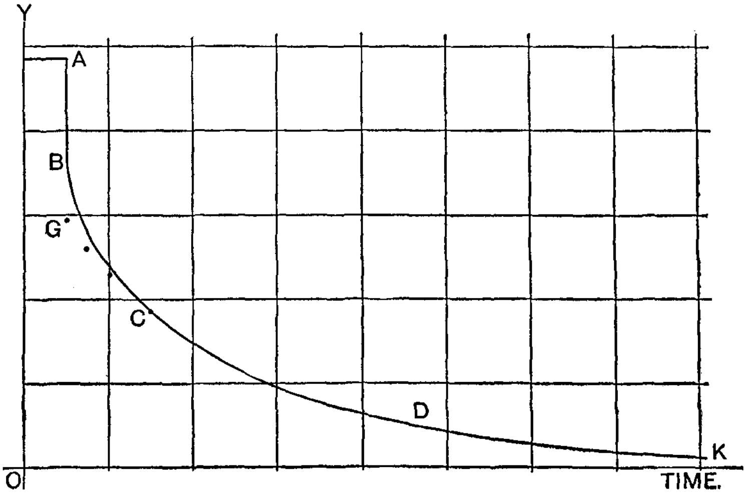

Figure B.

IV. Figure B shows the manner of decay of the oscillations of a light suspended system under the action of very strong restoring forces. A small mirror and two 15 millimeter long watch-spring magnets were mounted on a square vertical mica vane, of about 3 square centimeters area, which was fastened symmetrically on a slender but stiff bit of glass filament. The filament was stretched between two pieces of No. 36 B. & S. steel wire about 2 centimeters long. The righting moment was due partly to the torsional forces in the wire and partly to a strong electromagnetic field about the needles. When the circuit of the magnetic field used to deflect the needle through the initial angle θ_0 was suddenly broken, the vane and its belongings moved quickly (in perhaps 1/250th of a second) through the position of equilibrium and out on the other side to a turning point corresponding to a deviation of about three fourths of θ_0. After this the amplitudes decreased slowly and continuously. The curve drawn through the crests of the oscillations consists at the start of a vertical line, as it would if, for instance, the resistance followed the law of the square of the angular velocity. After a short time, however, the curve, like most of those which I have obtained, follows more nearly a course which corresponds to the equation $y = A \cdot e^{-at}$. The numbers in the next table show well enough what the character of the agreement is. The first column gives ordinates of the photographic record taken at equal time intervals. The second column gives corresponding ordinates of a curve of the family $y = A \cdot e^{-at}$ which falls in very nearly with the first curve for a portion of the middle of its course.

TABLE III.

4840–3750	2950	1100	1095
2950	2695	1012	1001
2560	2463	930	915
2295	2251	862	836
2065	2057	800	764
1880	1880	745	698
1710	1718	695	639
1560	1570	645	583
1430	1435	600	533
1315	1312	. . .	. . .
1200	1200	390	310

V. Figure C represents curves taken with this apparatus when the filament was made of a piece of manganine wire. One curve is here displaced an arbitrary amount with respect to the other, for purposes of comparison. The sudden drop (ST) from the original deflected position to one of much smaller displacement, after which the decrease of amplitude is gradual, is clearly shown. The two curves show different values of the original deflection.

The Damping of the Slow Oscillations of a d'Arsonval Galvanometer Coil, which is wound on a Nonmetallic Core, and is swinging between the Poles of its Magnet.

If the coil of a d'Arsonval galvanometer be wound on a wooden spool, and if its circuit be open, the damping of its oscillations is due principally, unless the copper wire is magnetic, to air resistance, and only slightly to frictional forces within or at the surface of the gimp from which the coil hangs. When, however, the circuit of the coil is closed through an outside resistance x, electromagnetic damping is added, and the damping coefficient of the motion is larger than before, or, if x is small enough, the motion ceases to be periodic. In many instances it is possible and desirable to damp the coil critically, but this is sometimes impracticable, — as, for instance, in such instruments of long period (400 or 500 seconds) as are used in testing massive iron cores, — and there are certain kinds of absolute measurements where a relatively undamped instrument is preferable. The throw of

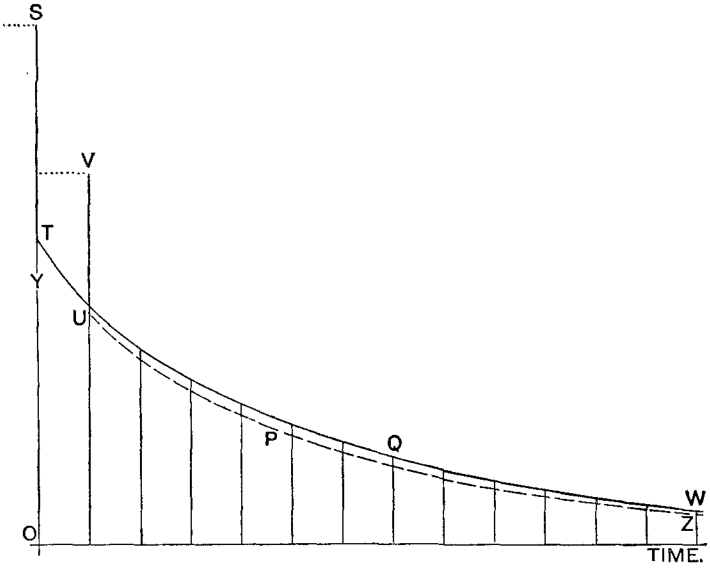

Figure *C*.

a d'Arsonval galvanometer due to a given change of the flux of magnetic induction through its circuit is usually to be quantitatively explained only by attributing to the resistance of the circuit a value much greater than the real one. This apparent resistance [11] may be many times as great as the real resistance; its value depends upon the constants of the motion of the coil, and it not infrequently happens that a knowledge of these "constants" is important, even though the amplitudes do not always decrease exactly according to the assumption that the resistance to the motion is equivalent to a couple of moment proportional to the angular velocity.

If a coil of the ordinary Ayrton-Mather form, without a damping vane, swing between the poles of its magnet with the coil circuit open, the amplitude generally decreases slowly, and if the coil be hung successively by pieces of gimp of different lengths or stiffnesses, the period changes with the restoring moment, and the damping coefficient (a) remains small, though it often changes somewhat with the amplitude. If with a given suspension we determine the quantity a in the equation $y = A \cdot e^{-at}$ from two amplitudes of about 5° near the beginning of the motion, and then from two amplitudes of about 2° after the coil has made twenty or thirty swings, the latter value will usually be sensibly smaller than the other, but the difference is not very great unless the restoring force is weak, as it is in very sensitive instruments.

VI. In the case of a certain galvanometer of the Ayrton-Mather type which I studied at length, the value of a fell from 0.00403 to 0.00356 as the motion progressed, when a piece of very fine steel gimp was used to hang the coil. When stiff gimp was employed, the value of a remained much more nearly constant while the amplitude decreased, and was nearly the same for different lengths of the gimp. The first column in the next table shows the period as determined principally by the stiffness of the gimp, the second column gives the corresponding value of a determined after twenty or thirty swings had been executed and the double amplitude had fallen below 4°.

TABLE IV.

T.	Damping Coefficient.
2.59	0.0029
3.62	0.0028
4.57	0.0031

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[11] Robinson, The Electrician, 1901. White, Physical Review, 1904. Peirce, These Proceedings, 1906.

The resistance of the instrument was about 21 ohms, but a considerable fraction of this was in the gimp.

When the coil circuit was closed by a resistance of 400 ohms, and the coil was hung successively by several different pieces of gimp of different lengths, the damping coefficient (a) slowly decreased as the amplitude decreased, so that the logarithmic decrement was not quite constant during the whole motion in any case, but the value of a for a double amplitude of say 4° was practically the same for widely different periods.

The next two tables show the results of measurements of a good number of photographic records. In the first case, as has been said, the outside resistance of the circuit was 400 ohms, in the second case it was 200 ohms.

TABLE V.

T.	Damping Coefficient.
9.28	0.0113
4.57	0.0113
3.62	0.0113
2.61	0.0114

TABLE VI.

T.	Damping Coefficient.
7.58	0.0193
4.57	0.0192
3.62	0.0192
2.61	0.0191

If the coil was deflected out of its position of equilibrium through an angle of perhaps 10°, and was then suddenly released, the amplitude fell at once to a much smaller value, especially when the coil was closed through a resistance of say 400, and then decreased gradually in much the same manner as the swings represented by Figure 5. The phenomenon is, however, not so marked as when the damping is fairly large and due wholly to air resistance.

When the circuit of the coil of a d'Arsonval galvanometer of the form described is closed through an outside resistance, x, so that the whole resistance is $(g + x)$, the damping coefficient of the motion is theoretically the sum of the corresponding coefficient when the circuit is open and the coefficient which the electromagnetic damping would cause if the air damping were absent, and this last should be proportional reciprocally to the apparent resistance $(g' + x)$ of the circuit, where g' is usually considerably larger than g. A set of five photographic records were obtained with the coil mentioned above when it was suspended by a certain short piece of wire which gave the system a period of about 2.60 seconds. The next table shows (1) the values of x, (2) the corresponding values of a determined by a series of measurements of the diagrams from amplitudes not greater than 4°, (3) the values which the damping coefficient (a') would have if the air damping were absent, as calculated by aid of Table IV, and finally the reciprocal (y) of a'. Since a' should theoretically be of the form

$$\frac{1}{y} = \frac{b}{x + g'}, \tag{29}$$

if the observed values of x and y be plotted, the locus should be a straight line the intercept of which on the axis of abscissas is the value of g'.

TABLE VII.

x.	a.	a'.	y.
400	0.0114	0.0085	11.76
200	0.0191	0.0162	6.17
100	0.0358	0.0315	3.24
50	0.0595	0.0552	1.81
20	0.1120	0.1091	0.92

As a matter of fact, the points indicated by this table lie almost exactly on a line which cuts the x axis at a point the abscissa of which is a little less than forty. The apparent resistance of the galvanometer is, therefore, a trifle less than 40 ohms, while its real resistance with this wire is less than 20 ohms.

THE MOTION OF A SUSPENDED SYSTEM WHICH CARRIES A RELATIVELY LARGE DAMPING VANE UNDER RIGHTING COUPLES OF DIFFERENT STRENGTHS.

In order to study the effects of different restoring moments upon a swinging system furnished with a given damping vane, I used the apparatus represented in Figure 3 (Plate 2). G is a uniformly wound solenoid the horizontal axis of which lies in the meridian at a place where H is known. From a fine fibre in a narrow chimney inserted in the top of the solenoid at the centre hangs a small bar magnet (Q) fastened to a stiff mica vane in the manner shown at N. The axis of the magnet is coincident with the axis of the solenoid. A small mirror on the vertical wire which carries the vane and magnet lies in a vertical plane which makes an angle of 45° with the vertical plane through the axis of the solenoid, and, receiving the light from a small round hole in a brass plate in the slide holder of a distant Schuckert projecting lantern, throws it upon a sheet of bromide paper wound upon the drum D, where a small very sharp image of the hole is formed. The drum may be turned uniformly at very various speeds, either by clockwork or by an alternating motor actuated by a 60 cycle, 110 volt street circuit. The magnetic field about the suspended magnet can be given any desired value within wide limits by sending through G a suitable steady current from a battery of large storage cells. A current from another similar battery sent through the coil K serves to deflect the magnet out of the meridian against the given restoring field. When the current in K is suddenly interrupted, the suspended system oscillates with continually decreasing amplitude about the horizontal meridian and makes a record of its motion upon the photographic paper. In order that the seam in the paper on the drum may not come at an undesirable place in the record, the break in K's circuit is made automatically by the drum when it reaches a given position, but the system of relays by which this is accomplished is not indicated in the figure.

Experience gained with this apparatus shows that if the original deflection caused by a steady current in K is not more than 5° or 6°, and if the intensity of the magnetic field about the magnet is not too great, the record obtained after K's circuit has been suddenly broken is such that it is possible to draw a curve of the family $y = A \cdot e^{at}$ which shall, within the errors of observation, pass through all the crests of the diagram except the first two or three. We may assume that the motion in a case like this could be mathematically explained on the assumption that a body of fixed moment of inertia (I),— quite different, however, from the moment of inertia of the actual suspended system swinging in vacuo, — is oscillating under the action of the restoring moment due to the magnetic field and a retarding moment equal at every instant to the product of a damping coefficient ($2a$) and the angular velocity of the system. If the intensity of the field about the magnet be somewhat changed, I will have nearly its old value, but the damping coefficient, though constant for a given system swinging with a given period, has a new value when the period is changed. The change of the damping coefficient usually follows the direction of the change indicated by Stokes's theoretical treatment of the resistance encountered by a sphere making harmonic oscillations of small amplitude in a viscous liquid. It is usually rather difficult to determine the apparent moment of inertia of the system (I) with accuracy from observations of the period of the oscillations (for there generally is a fixed period), the value of the damping factor, the intensity of the external magnetic field about the magnet, and the moment of the magnet in that field, but such values of *I* as my observations give do not seem to change in any such manner as Stokes's formula for the sphere demands. Of course the two cases are mathematically quite different.

If the magnetic field about the magnet is relatively intense, and if the original deflection is as great as 10°, the system swings through its position of equilibrium, when it is released, to an elongation on the other side only a fraction (perhaps a half or a quarter) of the original deflection. From this time on the amplitude decreases slowly and regularly, much as in the case figured in Diagram C.

If a seasoned magnet placed in G be subjected to a magnetic field of several units' strength, the magnetic moment changes, and it is necessary to determine the amount of this change with some care if one needs to know the restoring couple which acts upon the swinging system. I have used for measurements of this kind a simple induction-coefficient apparatus shown diagrammatically in Figure 4 (Plate 2). P and Q are two similar solenoids which may be set anywhere on a horizontal east-west track *vw*. O is a mirror magnetometer the deflections of the needle of which can be determined by the telescope and scale (T, S). A horizontal scale *ab* in the meridian carries a wooden holder which contains a seasoned magnet (M_0) protected from sudden temperature changes, in Gauss's B Position with respect to the magnetometer needle. P and Q are so connected in series with a storage battery, a rheostat, and a standard centiamperemeter that a current can be sent in opposite directions through the solenoids; it is then easy, when a current stronger than any to be used in the subsequent determinations is passing through the circuit, to arrange the positions of P and Q near O on *vw*, so that the current shall not affect the needle. After this adjustment has been made, the magnet to be tested is placed in P somewhere near the middle of the solenoid and so near the needle that the latter is deflected off scale, and the wooden holder containing M_0 is placed on *ab* at such a distance from the needle that the latter is brought back exactly to its undeflected position. If then a current of suitable, small intensity be sent through the solenoid circuit, the change of the moment of the magnet in P from M to M' causes a scale reading z owing to a deflection (δ) of the needle; and if the current has not been too strong, this deflection disappears when the circuit is broken. If the field to which the magnet has been exposed has been fairly large, however, the moment is permanently changed by a small amount, and it is then necessary to follow the same magnetic journey in the testing which is to be taken in the damping experiments.

If the distance of the centre of the auxiliary magnet from the centre of the needle is d_0 centimeters, and if l_0 is the half magnetic length of this magnet, the moment of the couple which it exerted upon the needle before the latter was deflected, and which just balanced the moment due to the magnet to be tested, was $\frac{M_0 d_0}{(d^2_0+l^2_0)^{\frac{3}{2}}}$.

When the magnet under the test is removed, the needle deflection (δ_0) caused by the auxiliary magnet alone is usually too large to be easily measured by aid of the telescope and scale; but if this magnet be removed on its track to such a distance that the deflection δ' can be determined, and if the distance between the centres of the magnet and needle is then d',

$$\frac{\tan \delta_0}{\tan \delta'} = \frac{d_0}{(d^2_0 + l^2_0)^{\frac{3}{2}}} \cdot \frac{(d'^2 + l^2_0)^{\frac{3}{2}}}{d'}, \tag{30}$$

and M′ can be determined in terms of M by means of the equation

$$\frac{M' - M}{M} = \frac{\tan \delta}{\tan \delta_0}. \tag{31}$$

VII. The first magnet (Q′) used with this apparatus was about 4.0 centimeters long and weighed about 7 grams. The whole suspended system had a moment of inertia in vacuo almost exactly equal to 43.0, and the magnetic moment of the seasoned magnet (Q′) when placed with its axis perpendicular to the meridian was about 29.8 units. Its induction coefficient under these circumstances was about 0.0242; its moment in a field of 10.37 gausses was 38.7. Most of the records were made with the drum revolving very slowly at the rate of a turn in 348 seconds: the normal length of a record was 479 millimeters. The periodic time of the swinging system varied from 50.8 seconds to 1.20 seconds in the fields actually used. The torsion coefficient of the fibre was under all circumstances here considered much too small to be appreciable.

Figures 5 (Plate 2) and 6 (Plate 3) represent oscillations of the suspended system of which the magnet Q' was a part under fields of about 2 and 12 gausses respectively. In the case shown in Figure 7 (Plate 3) the magnet was deflected through an angle of perhaps 10° and then suddenly released. The record begins at the point **O**, where a nearly straight line indicates that the magnet was on its way through the position of equilibrium and out on the other side to a point corresponding to a deflection of about 2.5°, after which the amplitude decreased gradually and regularly. The field here was about 19.3 units. Figures 8, 9 (Plate 4) show the effect of suddenly applying a comparatively strong field (14.3 gausses) when the system is already swinging in a field of about 2 units.

The curious irregularity in the spacing of the record in the last diagram after the strong field was applied came from the fact that the magnet was making oscillations in a vertical plane with an amplitude of about 2′. When the system was at rest, the axis of the magnet and the axis of the solenoid were in the same vertical plane but differed from each other in direction by a small fraction of one degree.

To illustrate the fact that in a weak field where the period of the oscillation is long the amplitude of the motion decreases regularly with a practically constant decrement, and that in somewhat stronger fields the departure from this law is nearly inappreciable, except perhaps at the very beginning of the motion, two or three sets of typical measurements will serve. In very strong fields, when the initial deflections are fairly large, the motion cannot be explained with any good approximation to accuracy on the assumption that the air resistance furnishes a couple proportional to the angular velocity.

TABLE VIII.

Periodic Time, 13.2 Seconds.

Successive Amplitudes.

Measured.	Computed.	Measured.	Computed.
857	857	302	283
763	760	269	249
680	673	239	222
605	600	213	195
539	533	189	172
480	480	169	160
427	423	150	148
380	374	134	136
339	325		

When the periodic time was as short as 1.2 seconds, a curve of the family $A \cdot e^{-at}$ which passed through the crests of the figure at the middle of the diagram fell distinctly below the crests at the beginning.

From measurements of photographic records taken with Q' for eight different values of the current in the solenoid, the period (T), the damping coefficient ($2\,a$), the logarithmic decrement (λ) were determined for every case; the intensity of the magnetic field (H) about the magnet was then found by adding the original strength of the field to

TABLE IX.

PERIODIC TIME, 5.20 SECONDS.

Successive Amplitudes.

Measured.	Computed.	Measured.	Computed.
795	789	390	388
744	740	365	364
696	693	343	341
655	650	320	320
611	610	302	300
574	571	285	281
536	536	266	264
504	502	250	248
474	471	235	232
445	441	220	217
418	417	205	197

that caused by the measured steady current in the solenoid, and a fairly approximate value of the moment of the magnet was computed from the H thus found and the results of measurements made with the magnet in the induction coefficient apparatus described above. When these quantities were known, it was comparatively easy to determine β from the equation $(\pi^2 + \lambda^2)/T^2 = \beta^2$, and then to get an approximate value of the apparent moment of inertia of the swinging system from the formula $I = MHT^2/(\pi^2 + \lambda^2)$. Some of the results obtained by studying many records of the motion of this suspended system are given in the next table.

TABLE X.

Period.	M H.	Damping Coefficient.	Logarithmic Decrement.
15.90	6.38	0.00914	0.0726
13.8	9.86	0.00927	0.0640
9.9	16.64	0.00985	0.0487
8.05	24.2	0.01029	0.0414
7.63	29.9	0.01067	0.0407
2.85	213	0.01467	0.0209
2.18	359	0.01651	0.0180
1.12	1418	0.01907	0.0115

As has been said above, it is possible to obtain from these data values for the apparent moment of inertia of the oscillating system, but since a slight change in any one of several of the quantities measured might introduce a great change in the quantity computed, the results must be considered rough. Such a change in the intensity of the earth's field as might come from a passing train of electric cars at two hundred yards distance would appreciably affect the first value given.

The results of this computation are respectively 161, 188, 163, 157, 174, 173, 171, 178. So far as one may judge from these and from similar sets obtained with other systems there is no very strong evidence that I changes materially with T, unless it be for extreme values. The damping coefficient is by no means constant, for its value increases rapidly with the restoring force but not according to any easily recognizable law.

VIII. In the next series of experiments with the apparatus represented in Figure 6 Q' was displaced by another small bar magnet 6.0 centimeters long which, when placed perpendicular to the earth's field at room temperature, had a magnetic moment of 101.2 units. This new magnet (Q'') had a moment 129.5 in a field of 9.07 gausses, and a moment 140.2 in a field of 19.93 gausses, when the field was slowly increased. The same mica vane (x) was used as in the work with Q'.

The results of measurements made upon photographic records made with fields of seven different strengths appear in the next table.

TABLE XI.

Period.	M H.	Damping Coefficient.	Logarithmic Decrement.
14.53	12.5	0.0094	0.0683
6.31	68.0	0.0120	0.0379
4.47	123	0.0136	0.0304
2.97	270	0.0158	0.0235
1.81	669	0.0196	0.0177
1.23	1808	0.0222	0.0137
0.81	4396	0.0283	0.0114

At another time a long series of observations were made with the same system, under somewhat different initial circumstances of field and perhaps of moisture in the atmosphere, with the results given below.

TABLE XII.

Period.	I.	Period.	I.
12.30	287	1.60	285
10.44	295	1.27	295
7.90	285	1.13	297
3.29	280	0.98	293
2.38	280	0.81	292
1.92	294		

Here again the apparent moment of inertia is nearly constant but the damping coefficient increases rapidly as the field about the magnet becomes more intense.

Many kinds of physical measurements concern themselves with the behavior of oscillating systems, and it is often necessary to determine what the apparent moment of inertia of a system is if the motion is in air, and what the exact value of the damping coefficient is at any time. If this is not constant throughout the whole motion, — as it should be if it follows the Gaussian law, which assumes the existence of a fixed logarithmic decrement, — it is necessary to find out how it varies with period and amplitude. If one uses a d'Arsonval galvanometer to measure changes of magnetic flux in a large mass of iron, and for reasons of sensitiveness at some point of a hysteresis diagram needs to introduce extra resistance into the circuit or to remove some which is there already, one cannot compute the effect of the change unless one knows, not the real, but the apparent, resistance of the galvanometer coil, and this depends upon the "constants" of the motion which must be determined with some care; it would not be difficult to show that such deviations from the Gaussian law as one frequently encounters in practice need to be carefully taken into account in accurate work. The fact that the swinging system comes to rest in a comparatively short time suggests that the law may not be exactly followed at any part of the motion.

If, then, a swinging magnet or galvanometer coil is exposed to a relatively strong air damping, we must expect that unless the amplitude is very small there will be an appreciable departure from the Gaussian law. If the system be turned out of the position of equilibrium through a considerable angle and then released, it moves rapidly through this position and out on the other side to a new elongation corresponding to a displacement much smaller than the one from which it started; and this modifies profoundly the theories of some ballistic instruments, but after this the subsequent decrease of the amplitude takes place slowly and regularly, accompanied usually by a slowly decreasing logarithmic decrement. For any small number of swings after the first few, however, the constancy of the logarithmic decrement can often be assumed with sufficient accuracy for ordinary purposes.

The moment of inertia of the swinging system cannot as a rule be computed with any fair approximation from a knowledge of the masses and the geometrical dimensions of the bodies of which the system seems to be made up, for a comparatively large mass of air accompanies the visible system and materially increases the inertia.

The apparent moment of inertia of the system seems usually to remain practically unaltered when the moment of the restoring couple which dominates the swings is changed within wide limits, but under these circumstances the coefficient of damping generally increases rapidly as the restoring moment is increased, and the period decreases. If the restoring moment is due to an external field the periodic time remains fairly constant as the amplitude decreases; but if the moment comes from the torsional rigidity of a stiff wire, the period frequently lengthens somewhat as the amplitude grows small.

In case of a d'Arsonval galvanometer coil hung by different pieces of gimp or wire successively, the damping coefficient is practically the same for large differences of period if the resistance of the coil circuit is unchanged; but if this resistance is changed, the damping coefficient changes in a manner to be quantitatively explained by assuming that the coil has an apparent resistance larger than its real resistance. This apparent resistance may be considered as a constant of the coil as long as the level of the instrument is unchanged. If the righting moment of a swinging coil or magnet exposed to air damping is weak and comes from the torsional rigidity of a piece of fine gimp or fibre, the motion often seems to be anomalous because it depends upon obscure elastic changes.

The Jefferson Laboratory,
Cambridge, Mass.

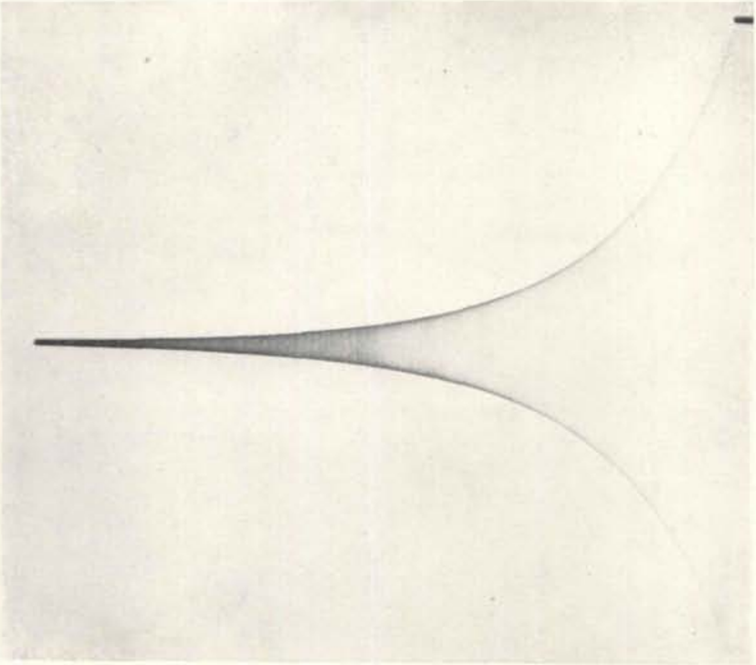


FIGURE 1.

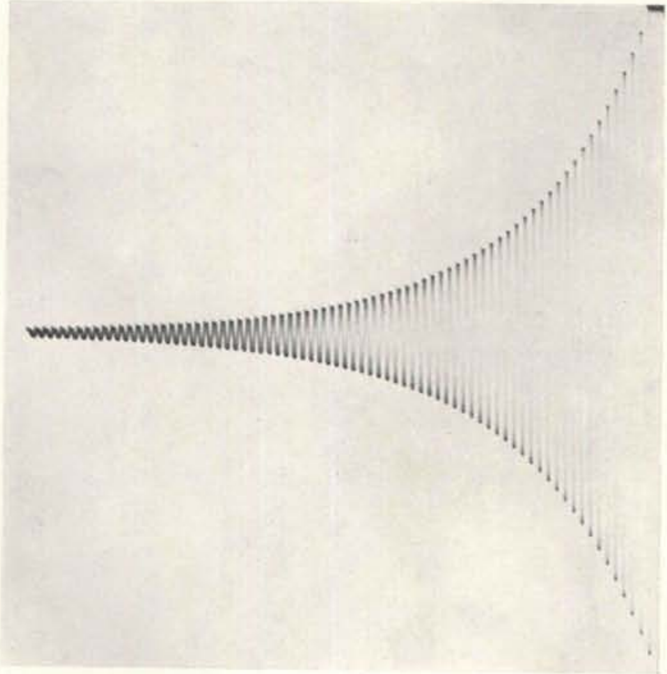


FIGURE 2.

Seite 88 I

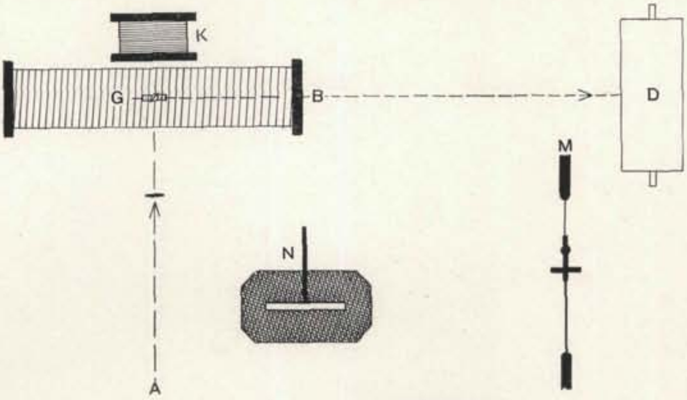
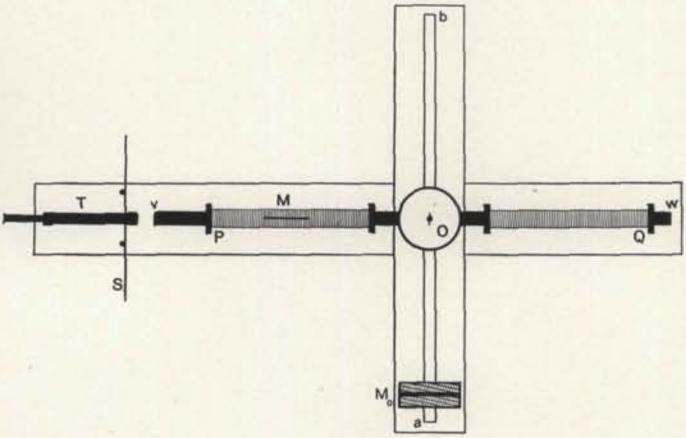


FIGURE 3.


b
T
v
M
w
P
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Q
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M_0
a

FIGURE . 4

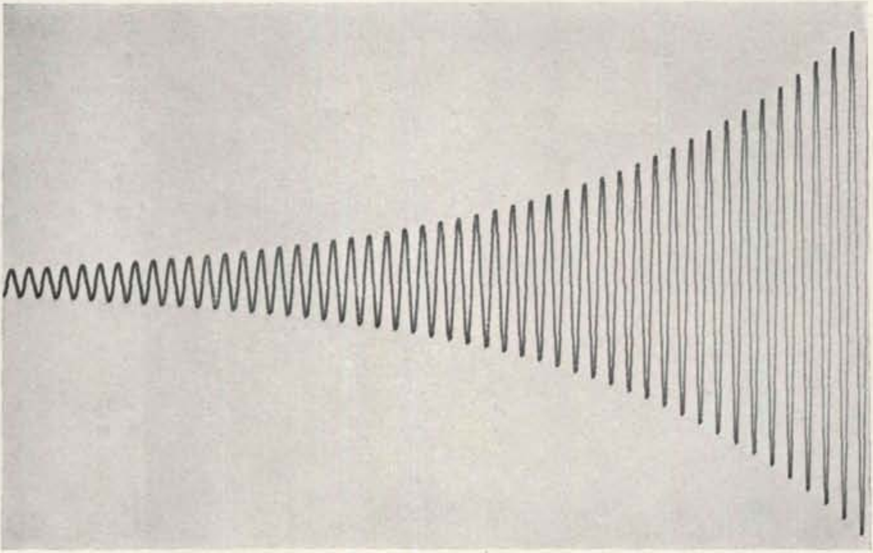

FIGURE 5.

Seite 88 II

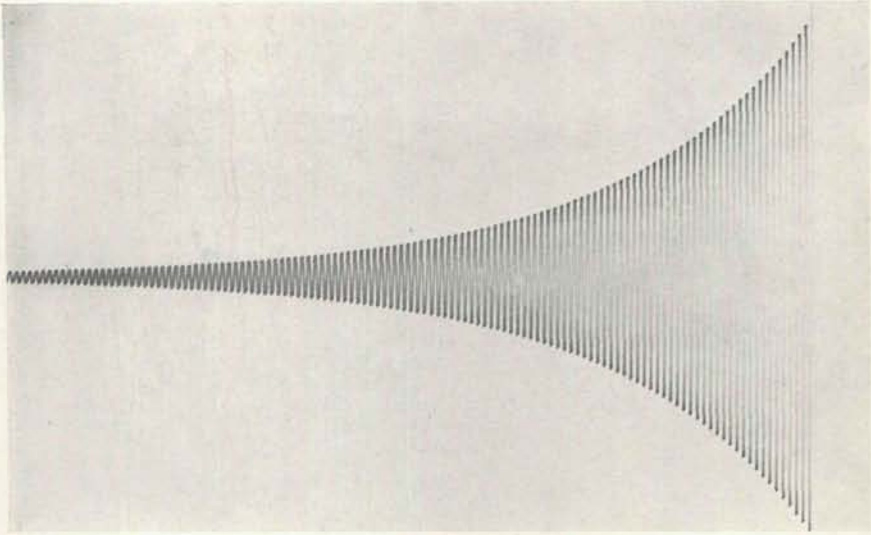

FIGURE 6.

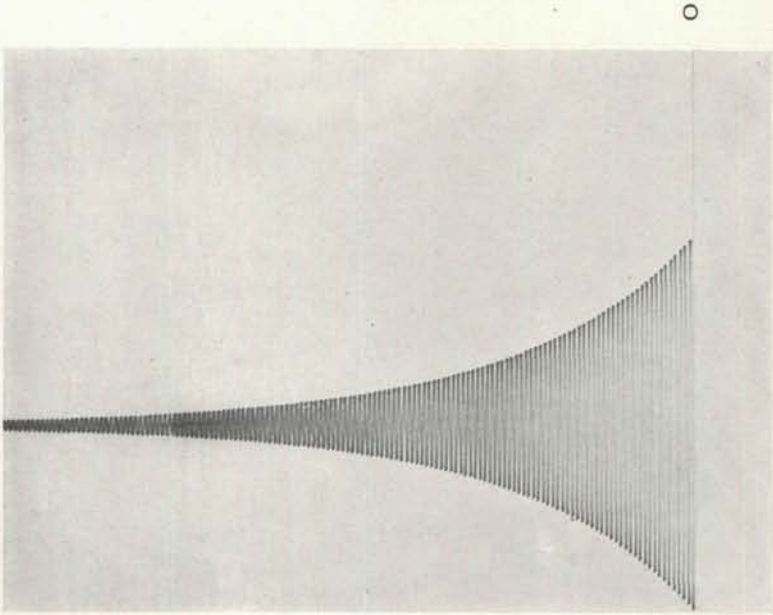

FIGURE 7.

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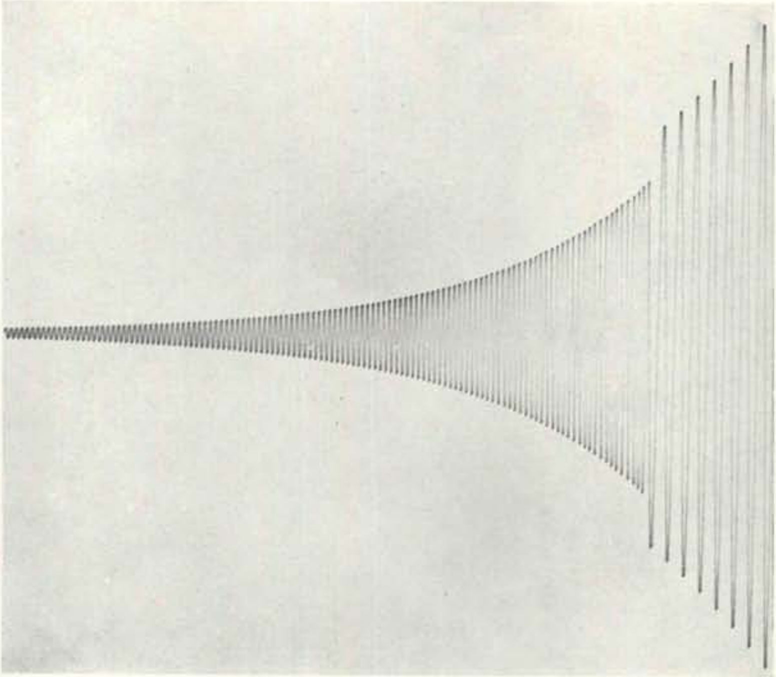

FIGURE 8.

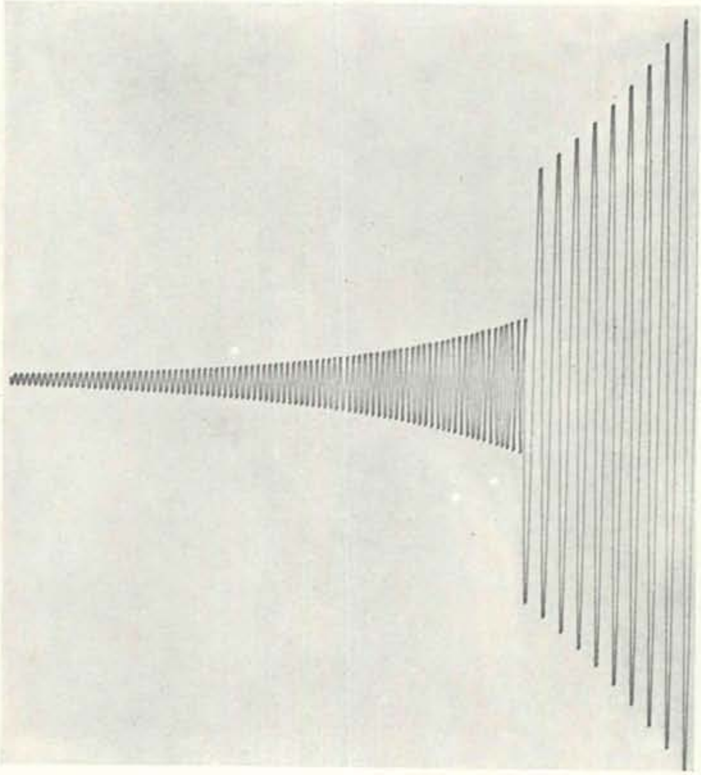

FIGURE 9.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

NOTE CONCERNING THE SILVER COULOMETER.

By Theodore William Richards.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

NOTE CONCERNING THE SILVER COULOMETER.

By Theodore William Richards.

Received August 7, 1908.

In a recent paper Messrs. Smith, Mather and Lowry have recounted their numerous and carefully executed experiments on the silver voltameter, or coulometer.[1] Their results are valuable, for they appear to have shown that it is possible to obtain an accurate result with the silver coulometer without the trouble of interposing either porous cup or siphon between the anode and the cathode.

One of the respects in which their experiments have differed from those of others is their use of a large volume of electrolyte. It is not surprising that this device tends toward accomplishing the desired end; for when the volume of the electrolyte is sufficiently large, the anomalous substances formed at the anode are so much diluted as to have but slight effect on the cathode. Moreover, the chance that these substances will be affected by dissolved air is much greater in the larger volume.

Although in this respect the new English work is of great service, there are one or two points in which exception may be taken to the authors' conclusions, and this note is written to call attention to these points. The coming International Meeting upon Electrical Standards renders it desirable that the matter receive promptly as much discussion as possible.

First among the minor points is the much disputed question as to whether silver crystals deposited in the coulometer contain liquid inclusions. Upon page 570 of their paper Smith and Mather speak of having attempted to test this point by reheating deposits previously dried at 160° to 240° in eight cases and to over 400° on three other occasions. My own experience leads me to believe that these temperatures were not any of them high enough to effect the expulsion of the included mother liquor. The same objection applies in a smaller degree to the work of van Dijk.[2] Silver containing included mother liquor does not give up this mother liquor until the temperature has been raised to so high a point that the metal becomes somewhat softened, and then the mother liquor is set free by a series of small explosions or decrepitations. The temperature needed is probably over 600°, as may be inferred from the statements made in my previous article on this subject.[3] It is probably true that the current density and other conditions at the time of the deposit cause variations in the amount included, but I have never by any process obtained silver which did not include a trace of mother liquor. That the inclusions are not due to extraneous impurities in the silver nitrate, but really occur with the purest salt, is conclusively shown by the recent experiments of Duschak and Hulett.[4] Therefore it is clear that the weight of silver dried at 160° does not give the precise weight deposited by the current, although the amount of included mother liquor may be so constant as not to interfere with the use of the weight obtained in this way as a technical measure of current strength.

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[1] Phil. Trans. Roy. Soc. London., Series A., **207**, 545 (1908).

Messrs. Smith and Lowry have done good service in emphasizing the importance of using really pure silver nitrate — a precaution not always heeded by physicists. One detail of their argument does not seem to be proved, however. They state that nitric acid causes a decrease in the amount of the deposit, — a very probable effect, which might have been predicted beforehand; but this conclusion can hardly be drawn from the results which they give on page 595. When small quantities of nitric acid (corresponding to about 0.1 to 0.2 of a per cent of the amount of silver nitrate present) were added, the average of their four results showed not a decrease but an *increase* in the weight of the deposit by 7 parts in 100,000; and when as much as 1 per cent of nitric acid is present, the average deficiency was only 4 parts in 100,000 as an average of seven experiments showing a rather large probable error. One would therefore be inclined to infer on the basis of their experiments that a small amount of nitric acid has no effect — or at least a much smaller effect than they are inclined to ascribe to it. One finds it difficult to agree with their conclusion on page 596:

---

[2] Van Dijk and Kunst, Ann. der Phys., **14**, 569 (1904); Van Dijk, ibid., **19**, 249 (1906).

[3] These Proceedings, **37**, 435 (1902).

[4] Trans. Am. Electrochem. Soc. (1908).

"We conclude, therefore, that whilst the abnormally low values which are observed from time to time can only be explained by the presence of acid, it may be very difficult in practice to add nitric acid without at the same time introducing other impurities which may more than counterbalance the effects produced by the acid itself." [5]

To this supposed influence of nitric acid they ascribe the fact that on thirteen occasions they found less silver in the experiments where a porous cup was interposed between the anode and the cathode than where the cup was absent. They infer that the porous cup was not adequately washed from nitric acid. This is possible, although it seems more probable that, as they suggest, cyanide, which is notoriously difficult to wash out of porous material, was the real cause of the deficiency, as indeed they suggest on the third line of page 564. They obtained good results when their porous cups were ignited for some time in an electric furnace. This treatment would drive off not only nitric but also hydrocyanic acid, and might oxidize any remaining cyanide.

To sum up the last paragraph: it may be pointed out that there is little evidence presented that nitric acid, if present in traces, would have produced a deficiency in the silver deposited, and some doubt as to whether nitric acid was present in the experiments of Smith, Mather and Lowry with porous cups. Hence the conclusions of those gentlemen concerning the unsatisfactory behavior of their insufficiently washed cups are of doubtful value. Nevertheless it would obviously be well in future work to make sure that nitric acid is wholly absent, and they have done a service by calling attention to the danger of incomplete washing of the porous cell if that is used. It is not probable that this difficulty affected the determinations made at Harvard, because cyanide was not used for washing the cells, and, as is stated in one of the papers,[6] the solution around the cathode in our cases remained wholly neutral. Moreover, in the Harvard experiments the porous cup method was compared with another method free from any possible defect of this kind, and found to give the same result.[7]

One other point may be mentioned in which the results of Messrs. Smith, Mather and Lowry differ from the Harvard results. The English experimenters were unable to find that freshly formed anode liquid deposited silver upon contact with the silver surface. This is a crucial experiment, and the result is purely a question of fact, not of interpretation. Clearly for some reason or other the anode irregularities were less prominent in the experiments of Smith, Mather and Lowry than in other cases, and one is inclined to refer the difference in this respect between the results which they obtain and the Harvard results to other causes as yet unknown. Possibly the fact that they used electrically deposited silver for their anode may not only account for their inability to deposit silver from the heavy anode liquid, but also contribute toward the constancy of their results with Lord Rayleigh's voltameter. Electrically deposited silver, being arranged in definite crystals, may dissolve with less irregularity than a fused lump. Further experiments must decide the uncertainty. For the present, until this question has been settled, it would seem to be advisable to use electrically prepared silver as the anode, if a porous cup is not employed.[8]

---

[5] Professor Smith, in a letter kindly written after he had seen the manuscript of the present note, explains that there was some doubt as to the purity of some of their nitric acid. This doubt may have applied equally to that used in washing their porous cells, however.

[6] These Proceedings, **35**, 141 (1899).

[7] These Proceedings, **37**, 420 (1902).

It is to be hoped in view of these points still remaining unsettled that the International Congress on Electrical Standards will not define too positively the true electro-chemical equivalent. It is equally obvious that with the exception of these disputed points the matter is in a much more definite state than it was twenty-five years ago. There can be no doubt that the final result of Lord Rayleigh and Mrs. Sidgwick was the best of all the early absolute determinations, all things considered, because of their having taken account of the inclusion of mother liquor. In this respect this pioneer work is better even than some of the most recent work. Probably it was not over 0.05 per cent in error — a remarkable degree of accuracy for that time.

In brief, the contents of this note may be summarized as follows. While it is clear that Smith, Mather and Lowry have done good service in showing that large volumes of liquid, taken in connection with the electrically prepared anodes, will give good uniform results with the silver coulometer, and that the results thus obtained are like those obtained with clean porous cups and siphons between the electrodes, there are still a few minor points of detail left to be decided, especially the question as to the amount of included liquid in the silver.

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[8] I am glad to hear from Professor Smith that the National Physical Laboratory of England proposes to test this and other doubtful points in the near future.

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ARTIFICIAL LINES FOR CONTINUOUS CURRENTS IN THE STEADY STATE.

By A. E. Kennelly.

ARTIFICIAL LINES FOR CONTINUOUS CURRENTS IN THE STEADY STATE.

BY A. E. KENNELLY.

Received August 26, 1908.

ARTIFICIAL lines are well known to electrical engineers, in telegraphy and telephony, as devices for electrically imitating actual lines of communication in a compact and convenient manner. They are employed industrially in most duplex or quadruplex systems. They are also employed in the laboratory for testing methods of telegraphing, or of telephoning, under conditions that are electrically akin to those of practice.

Artificial telegraph lines contain associated resistance and capacity. Those used in telephony are sometimes provided with inductance and leakance in addition. These quantities are rarely associated distributively, as in actual lines.[1] They are associated for convenience and economy in lumps or sections. Thus an artificial telegraph line containing resistance and capacity AE, Figure 1, may be composed of say four similar sections of resistance AB, BC, CD, and DE, each representing the resistance of say 50 miles (or kilometers) of line. Each section is provided at its centre with a condenser having a capacity of 50 miles of line. The whole line AE will thus purport to represent 200 miles of line. The imitation must, however, be necessarily imperfect, by reason of the lumpiness of the capacity, which is divided into four blocks, and connected to the line at four points only, instead of being distributed uniformly; i. e., indefinitely subdivided, and connected at an infinite number of points, as in the actual line. The smaller the number of sections in the artificial line, the easier and cheaper it will be to build, but the lumpier and more imperfect the imitation will be. The question arises, therefore, as to what are the comparative electrical behaviors of the artificial line and of the line imitated, under any set of assigned conditions.

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[1] An exception is found, however, in the artificial lines for duplexing long submarine cables, where the proper proportions of resistance and capacity are associated distributively.

It is the object of this paper to present the quantitative laws that, from the engineering standpoint, control continuous-current artificial lines (sections of resistance and leakance) in the steady state. The basis for the construction of these formulas is given in the Appendix. All of the formulas apply equally to simple alternating-current artificial lines (sections of resistance, inductance, capacity, and leakance) when interpreted vectorially, or expanded from one dimension to two, in the well-known way; but in order to keep within reasonable limits of space the explicit discussion of alternating-current lines cannot here be considered.

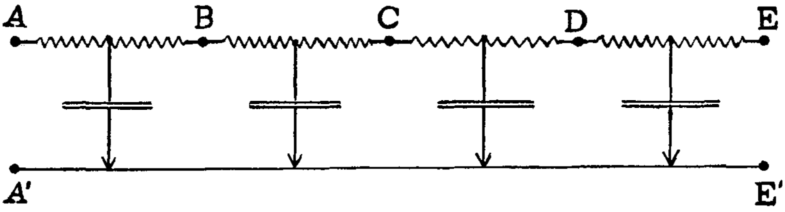

FIGURE 1.— Single-conductor type of artificial line.

TYPES OF ARTIFICIAL LINE.

There are two types of artificial line; namely, the ground-return-circuit line of Figure 1, and the metallic-return-circuit line of Figure 2, which are sometimes respectively defined as the single-conductor and double-conductor artificial lines. The former is characteristic of wire telegraphy, and the latter of wire telephony. In order that two such types of line should be equivalent, ignoring questions of lumpiness, circuit balancing, and circuit symmetry, it is necessary and sufficient that each section AB of Figure 1 should have a resistance $\frac{AB + A'B'}{2}$

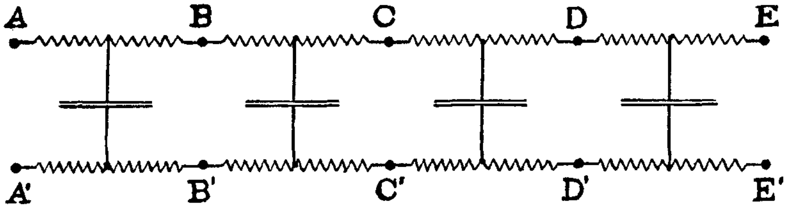

FIGURE 2. — Double-conductor type of artificial line.

of Figure 2, and that the capacity in each section of Figure 1 should be twice the capacity in each section of Figure 2; so that the CR product, i. e. the total resistance R in the line circuit and the total capacity C across the circuit, shall be the same. The double-conductor line of Figure 2 has, therefore, twice the total resistance, and half the total capacity, of the single-conductor line of Figure 1 for the same electrical retardation, and is thus the cheaper type to build, for a given CR. Since, then, to any double-conductor line of Figure 2 there is always a corresponding electrically equivalent single-conductor line of Figure 1, and the latter is, perhaps, the simpler to analyze and discuss, we may confine our attention entirely to the single-conductor or ground-return-circuit artificial line.

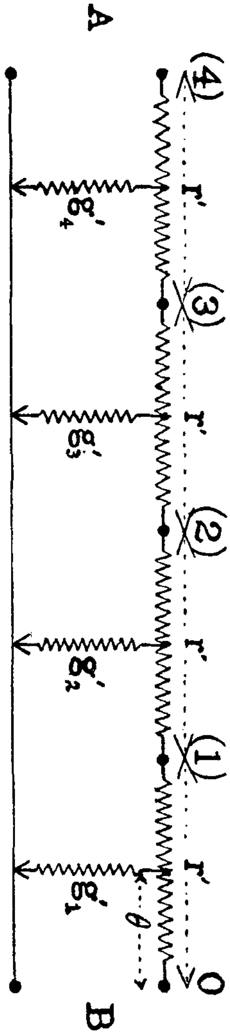

FIGURE 3. — Single-conductor type of artificial line for continuous currents.

FUNDAMENTAL RELATIONS AND NOTATION.

The continuous-current type of single-conductor artificial line is indicated in Figure 3. Let there be m sections. In the case presented, $m = 4$. Let each section represent a nominal length, l, kilometers (or miles) of line, and have a conductor resistance of r' ohms. Let the leak connected to the centre of each section have a conductance of g' mhos and a resistance of $R' = \frac{1}{g'}$ ohms. Let the total nominal length of the line be $L = ml$ kilometers and let $\lambda = \frac{l}{2}$ be the nominal length of a half section in kilometers.

First determine the nominal or apparent attenuation-constant of the artificial line as though the resistance and leakance were distributed as in an actual line:

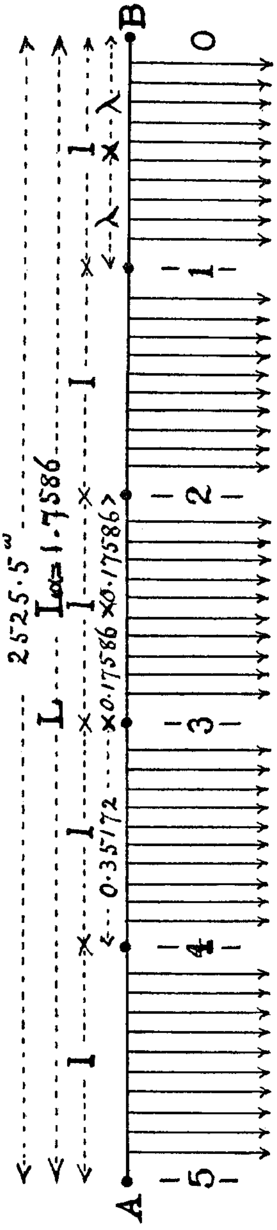

FIGURE 4. — Single-conductor type of real line with distributed leakance.

$$a' = \frac{\sqrt{g'r'}}{l} = \frac{1}{l}\sqrt{\frac{r'}{R'}} \quad \text{hyp. per km.} \quad (1)$$

Call the product la' of the nominal section length l and nominal attenuation-constant the nominal *hyperbolic angle* subtended by the section. Then La' will be the nominal hyperbolic angle of the whole artificial line, and $\lambda a'$ the nominal hyperbolic angle of a half section. These "hyperbolic angles" will be expressed in units of hyperbolic measure corresponding to radians in circular measure, and the unit may be denoted by the abbreviation "hyp."

Find the nominal surge-resistance of the artificial line, as though the resistance and leakance were uniformly distributed.

$$r_o' = \sqrt{\frac{r'}{g'}} = \sqrt{r'R'} \qquad \text{ohms.} \quad (2)$$

The above nominal values of attenuation-constant a', hyperbolic line angles $\lambda a'$, la', and La', as well as the surge-resistance r_o', will then have been obtained as though the resistance r' and leakance g' were presented in an actual uniform line of distributed leakance. They are therefore vitiated by lumpiness. We proceed to correct for lumpiness as follows:

$$\sinh \lambda a = \lambda a' \qquad \text{numeric.} \quad (3)$$

that is, the hyperbolic sine of the true semi-section hyperbolic angle is equal to the nominal semi-section hyperbolic angle; or

$$\lambda a = \sinh^{-1}(\lambda a') = \theta \qquad \text{hyp.} \quad (4)$$

where θ represents the true semi-sectional hyp.-angle. Similarly, the true value of the surge-resistance, corrected for lumpiness, is

$$r_o = r_o' \cosh \lambda a = r_o' \cosh \theta = r_o' \sqrt{1 + (\lambda a')^2} \quad \text{ohms.} \qquad (5)$$

We now obtain from (3), (4) and (5) the true attenuation-constant a of the artificial line, the true surge-resistance r_o, and the true hyperbolic angles λa, la, and La subtended by a half-section, a section, or the whole line, respectively. These various quantities also define the actual line (Figure 4) which the artificial line imitates, after being corrected for lumpiness. The actual line of distributed leakance which is electrically equivalent to an artificial line, after correcting the latter for lumpiness, may be defined as the "*imitated line.*"

As an example, consider an artificial line of $m = 5$ sections, as shown in Figures 5, 6, and 7, with a total nominal length $L = 500$ km.; so that each section has a nominal length $l = 100$ km., and a nominal semi-length $\lambda = 50$ km. The conductor-resistance of each section is $r' = 500$ ohms, corresponding to a nominal linear conductor-resistance of 5 ohms per km. The leak of each section has a resistance $R' = 4000$ ohms or a conductance of 0.00025 mho (0.25 millimho), corresponding to a nominal linear leakance of 2.5 micromhos per km., or a linear insulation-resistance of 400,000 km.-ohms. The nominal attenuation-constant of the artificial line will be, by (1), $a' = 0.0035355$ hyp. per km. The nominal hyperbolic angle subtended by a half-section, a section, and the whole line, will be respectively $\theta = \lambda a' = 0.17678$, $la' = 0.35355$, $La' = 1.7678$ hyps. The nominal surge-resistance will be by (2) $r_o' = 1414.2$ ohms. We must now find the corrected values for these quantities corresponding to the imitated line shown in Figure 4.

With reference to formula (3), we find in tables of hyperbolic functions [2] that the angle whose hyperbolic sine is 0.17678 must be $\lambda a = 0.17586$ hyp.; which is the true angle of a semi-section of the artificial line, corrected for lumpiness. The true angle subtended by a section will be $la = 0.35172$ hyp., and by the whole line 1.7586 hyps. The true attenuation-constant of the artificial line, or the natural attenuation-constant of the imitated line, will be $a = 0.0035172$ hyp. per km. The true surge-resistance by (5) $r_o = 1436.13$ ohms. In other words the artificial line will behave externally in all respects, after the steady state has been attained, as though it were an actual smooth line of distributed leakance with these corrected constants. The correction has in this case diminished the nominal attenuation-constant and hyperbolic line angles by 0.52 per cent, but has increased the surge-resistance by 1.55 per cent. The linear conductor-resistance of the imitated line, Figure 4, will be $ar_o = 5.051$ ohms per km. The linear leakance of the imitated line will be $a/r_o = 2.44989 \times 10^{-6}$ mho per km., corresponding to a linear insulation resistance of 408,320 km.-ohms.

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[2] The best tables probably are "Tafeln der Hyperbelfunctionen und der Kreisfunctionen" by Dr. W. Ligowski, Berlin, Ernst & Korn, 1890.

Figures 5, 6, and 7 are diagrams of the voltage and current distribution over the artificial line above defined, for the respective cases of line grounded, freed, and grounded through 750 ohms, at B, the distant end. The steady impressed emf. at the sending end A is assumed as 100 volts in each case. Conductances are written in millimhos. All of the numerical work on these diagrams was carried out by the ordinary formulas of Ohm's law, and inspection will show that the arithmetical results are consistent. The various formulas given in this paper admit, therefore, of being checked by reference to these diagrams.

ARTIFICIAL LINE FREED AT FAR END. (Figure 6.)

Sending-End Resistance.

The sending-end resistance of an artificial line at the *n*th junction; i. e., the resistance offered to ground by the line, at and beyond the *n*th junction, is

$$R_f = r_o \coth L_2 a = r_o \coth 2n\theta \qquad \text{ohms,} \qquad (6)$$

where L_2 is the length of the line in km. reckoned from the far free end. When the sending-end resistance is measured at A, Figure 6, so as to include the whole line, $L_2 = L$, and $n = m$. As L_2 increases from 0 to ∞, $\coth L_2 a$ diminishes from ∞ to 1. Thus, in Figure 6, with $r_o = 1436.1$ ohms, and $m = 5$; or $L = 500$, $L_2 a = 1.7586 = 2m\theta$, $\coth 2m\theta = 1.0612$, and $R_f = 1436.1 \times 1.0612 = 1523.99$ ohms, as indicated at A. In the case of a smooth actual line, such as is shown in Figure 4, L_2 may be varied continuously between 0 and L kms.; but in an artificial line, L_2 can only be varied in steps of 2θ. That is, formula (6) applies to all points of the imitated line, but only to the junction points of the artificial line.

At the *n*th leak, excluding the same, the sending-end resistance is

$$R'_{n,f} = r_o' \frac{\cosh (2n-1)\theta}{\sinh (2n-2)\theta} = \frac{r_o \cosh (2n-1)\theta}{\cosh \theta \sinh (2n-2)\theta} \qquad \text{ohms.} \qquad (7)$$

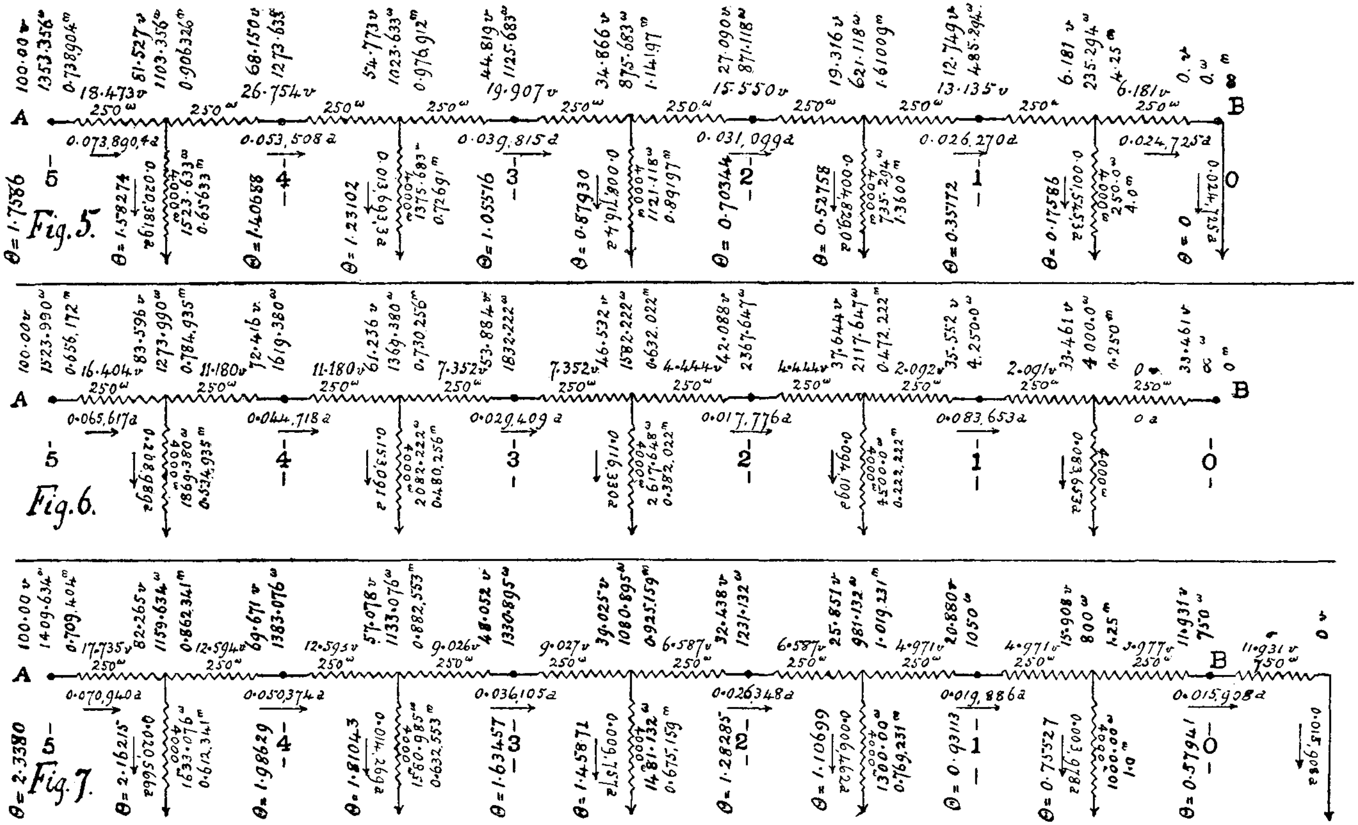


Fig. 5.
A
B
5
4
3
2
1
0
θ = 1.7586
θ = 1.58274
θ = 1.40688
θ = 1.23102
θ = 1.05516
θ = 0.87930
θ = 0.70344
θ = 0.52758
θ = 0.35172
θ = 0.17586
θ = 0
Fig. 6.
A
B
5
4
3
2
1
0
Fig. 7.
A
B
5
4
3
2
1
0
θ = 2.3380
θ = 2.16215
θ = 1.98629
θ = 1.81043
θ = 1.63457
θ = 1.45871
θ = 1.28285
θ = 1.10699
θ = 0.93113
θ = 0.75527
θ = 0.57941
250ω
750ω

FIGURES 5, 6, 7. — Five-section artificial line carrying continuous currents. Sending end connected to 100 volts e.m.f. Distant end grounded, freed, and to ground through resistance. In Figure 6 the indicated leak currents should be divided by ten.

At the nth leak, including the same, the sending-end resistance is

$$R'_{f,n} = r_o' \frac{\cosh (2n - 1)\theta}{\sinh 2n\theta} = \frac{r_o \cosh (2n - 1)\theta}{\cosh \theta \sinh 2n\theta} \quad \text{ohms.} \quad (8)$$

When the number of sections of artificial line becomes indefinitely great, the two immediately preceding expressions respectively become:

$$R'_{\infty, f} = r_o' \epsilon^{\theta} = \frac{r_o \epsilon^{\theta}}{\cosh \theta} \quad \text{ohms,}$$

and

$$R'_{f, \infty} = r_o' \epsilon^{-\theta} = \frac{r_o \epsilon^{-\theta}}{\cosh \theta} \quad \text{ohms,} \quad (9)$$

where ϵ is the base of Napierian logarithms.

The ratio of the sending-end resistance at and excluding the $(n + 1)$th leak to that at and including the nth leak is

$$\frac{R'_{n+1, f}}{R'_{f,n}} = \frac{\cosh (2n + 1)\theta}{\cosh (2n - 1)\theta}. \quad (10)$$

This is the ratio of the extreme sending-end resistances, when ascending from one leak where it is a local minimum, to the next higher leak where it is a local maximum. When the artificial line becomes indefinitely long, this ratio tends to the limit $\epsilon^{2\theta}$.

Voltage. Far End Free.

The voltage e_o at the far free end of the artificial line, Figure 6, will be:

$$e_o = \frac{e_m}{\cosh 2m\theta} = \frac{e_m}{\cosh L_2 \alpha} \quad \text{volts,} \quad (11)$$

where e_m is the voltage impressed on the mth junction, or sending end. If the voltage ϵ_n should be impressed on the line at the nth leak, the formula is

$$e_o = \frac{\epsilon_n \cosh \theta}{\cosh (2n - 1)\theta} \quad \text{volts.} \quad (12)$$

Thus, if $e_m = 100$ volts, and $m = 5$, as in Figure 6, $2m\theta = 1.7586$ hyps. and $\cosh 2m\theta = 2.9883$; so that $e_o = 100/2.9883 = 33.46$ volts.

The voltage at junction (n) is

$$e_n = e_o \cosh 2n\theta = e_m \frac{\cosh 2n\theta}{\cosh 2m\theta} \quad \text{volts.} \quad (13)$$

The voltage at the nth leak is

$$\epsilon_n = e_o \frac{\cosh(2n-1)\theta}{\cosh\theta} = \epsilon_m \frac{\cosh(2n-1)\theta}{\cosh(2m-1)\theta} \quad \text{volts.} \qquad (14)$$

Consequently, the voltages at successive junctions, $e_o, e_1, e_2, \ldots e_n$, are respectively proportional to $\cosh 0$, $\cosh 2\theta$, $\cosh 4\theta, \ldots \cosh 2n\theta$; that is, to the cosine of the hyperbolic angle of the junction, measured from the far free end.

Similarly, the voltages at successive leaks, $\epsilon_1, \epsilon_2, \ldots \epsilon_n$ are respectively proportional to $\cosh\theta$, $\cosh 3\theta, \ldots \cosh(2n-1)\theta$; that is, to the hyperbolic angle of the leak, measured from the far free end.

As we ascend along the line by steps of θ from the far free end, the voltages increase as follows:

Angular Distance from far free end. Hyps.	Point.	Voltage Symbol.	Value. Volts.
0	End	e_o	e_o
θ	Leak 1	ϵ_1	$e_o \frac{\cosh\theta}{\cosh\theta}$
2θ	Junction 1	e_1	$e_o \cosh 2\theta$
3θ	Leak 2	ϵ_2	$e_o \frac{\cosh 3\theta}{\cosh\theta}$
4θ	Junction 2	e_2	$e_o \cosh 4\theta$
. . .	. . . . . .	. . .	. . . .
$(2n-1)\theta$	Leak n	ϵ_n	$e_o \frac{\cosh(2n-1)\theta}{\cosh\theta}$
$2n\theta$	Junction n	e_n	$e_o \cosh 2n\theta$

Current Strength. Far End Free.

The current strength at the sending end is:

$$I_m = \frac{e_m}{r_o \coth L_2 a} = \frac{e_m}{r_o \coth 2m\theta} \quad \text{amperes,} \qquad (15)$$

where e_m is the voltage impressed on the mth junction.

The current strength at the nth junction is:

$$I_n = I_m \frac{\sinh 2n\theta}{\sinh 2m\theta} \quad \text{amperes.} \qquad (16)$$

Thus in Figure 6 the current at the sending end is 0.065617 ampere. The current at junction 3 will be 0.065617 × $\frac{\sinh 1.05516}{\sinh 1.7586}$ = 0.029409 ampere.

At the nth leak, the ratio of ongoing to arriving line current is

$$\frac{I_{n-1}}{I_n} = \frac{\sinh 2\,(n-1)\theta}{\sinh 2\,n\theta}. \qquad (17)$$

The current escaping at the nth leak is:

$$i_n = \epsilon_n g' = e_o g' \frac{\cosh\,(2\,n-1)\,\theta}{\cosh\theta} = \epsilon_m g' \frac{\cosh\,(2\,n-1)\,\theta}{\cosh\,(2\,m-1)\,\theta}$$

$$= e_m g' \frac{\cosh\,(2\,n-1)\,\theta}{\cosh\theta\cosh 2\,m\theta} \quad \text{amperes.} \qquad (18)$$

Line grounded at Far End. (Figure 5.)

Sending-End Resistance.

The sending-end resistance at the nth junction with the far end grounded is:

$$R_g = r_o \tanh L_2 a = r_o \tanh 2\,n\theta \quad \text{ohms.} \qquad (19)$$

In the case represented by Figure 5, for $m = 5$, $R_g = 1436.1 \times 0.94235 = 1353.3$ ohms. As we ascend the line from junction to junction, the resistances are in proportion to the hyp. tangents of the angles of those junctions.

The sending-end resistance at and excluding the nth leak is:

$$R'_{n,g} = r_o' \frac{\sinh\,(2\,n-1)\,\theta}{\cosh\,(2\,n-2)\,\theta} \quad \text{ohms.} \qquad (20)$$

The sending-end resistance at and including the nth leak is:

$$R'_{g,n} = r_o' \frac{\sinh\,(2\,n-1)\,\theta}{\cosh 2\,n\theta} \quad \text{ohms.} \qquad (21)$$

When n is indefinitely increased, (20) becomes:

$$R'_{\infty,g} = r_o' \epsilon^{\theta} \quad \text{ohms,} \qquad (22)$$

and (21) becomes:

$$R'_{g,\infty} = r_o' \epsilon^{-\theta} \quad \text{ohms.} \qquad (23)$$

The ratio of local maximum resistance just before a leak to the local minimum resistance just after the preceding leak is:

$$\frac{R'_{n+1,\,g}}{R'_{g,\,n}} = \frac{\sinh\,(2\,n\,+\,1)\,\theta}{\sinh\,(2\,n\,-\,1)\,\theta}. \qquad (24)$$

When n is increased indefinitely, this ratio becomes:

$$\frac{R'_{\infty,\,g}}{R'_{g,\,\infty}} = \epsilon^{2\theta}. \qquad (25)$$

Receiving-End Resistance. Far End grounded.

The receiving-end resistance, or resistance which the artificial line appears to offer, as judged by an observer at the far end, from the received current to ground and the impressed emf. at the sending end, is:

$$R_l = r_o \sinh L_2\alpha = r_o \sinh 2m\theta \qquad \text{ohms.} \qquad (26)$$

In the case of Figure 5, $R_l = 1436.1 \sinh 1.7586 = 1436.1 \times 2.81602 = 4044.2$ ohms. The received current to ground at the far end will therefore be $100/4044.2 = 0.02472$ ampere.

Voltage. Far End grounded.

The emf. at the nth junction in terms of the emf. e_m impressed on the mth junction is:

$$e_n = e_m \frac{\sinh 2\,n\theta}{\sinh 2\,m\theta} \qquad \text{volts,} \qquad (27)$$

or, in terms of the current i_o to ground at the far end, it is;

$$e_n = I_o r_o \sinh 2\,n\theta \qquad \text{volts.} \qquad (28)$$

Consequently, the voltages at successive ascending junctions are proportional to the hyperbolic sines of the angles of those junctions.

The emf. at the nth leak is:

$$\epsilon_n = \epsilon_m \frac{\sinh\,(2\,n\,-\,1)\,\theta}{\sinh\,(2\,m\,-\,1)\,\theta} \qquad \text{volts,} \qquad (29)$$

in terms of the emf. ϵ_m at the mth leak; or

$$\epsilon_n = I_o r_o \frac{\sinh\,(2\,n\,-\,1)\,\theta}{\cosh\theta} = I_o r_o' \sinh\,(2\,n - 1)\theta \qquad \text{volts,} \qquad (30)$$

in terms of the current to ground and of the surge-resistances, corrected and nominal.

Consequently, the voltages at successive ascending leaks are proportional to the hyperbolic sines of the angles of those leaks.

Current. Far End grounded.

The current at the sending end is:

$$I_m = \frac{e_m}{r_o \tanh L_2 a} = \frac{e_m}{r_o \tanh 2\,m\theta} \qquad \text{amperes.} \qquad (31)$$

The current at junction n is:

$$I_n = I_m \frac{\cosh 2\,n\theta}{\cosh 2\,m\theta} = \frac{e_m}{r_o} \cdot \frac{\cosh 2\,n\theta}{\sinh 2\,m\theta} \qquad \text{amperes.} \qquad (32)$$

The current at junction 0, or the grounded end, is:

$$I_o = \frac{e_m}{r_o \sinh 2\,m\theta} = \frac{e_m}{r_o \sinh L_2 a} = \frac{I_m}{\cosh 2\,m\theta} = \frac{I_m}{\cosh L_2 a} \quad \text{amperes.} \qquad (33)$$

At the nth leak, the ratio of ongoing to arriving line current is:

$$\frac{I_{n-1}}{I_n} = \frac{\cosh 2\,(n-1)\,\theta}{\cosh 2\,n\theta}. \qquad (34)$$

The current escaping at the nth leak is:

$$i_n = \epsilon_n g' = \epsilon_m g' \frac{\sinh\,(2\,n-1)\,\theta}{\sinh\,(2\,m-1)\,\theta} = 2\,I_o \sinh\theta \sinh\,(2\,n-1)\theta \qquad \text{amperes.} \qquad (35)$$

By comparing formulas (6) and (19), (13) and (27), (16) and (32), it will be seen that with the far end free, the sending-end resistances follow the cotangents, voltages the cosines, and currents the sines, of the hyp. angles of the junctions; but that with the far end grounded, the sending-end resistances follow the tangents, voltages the sines, and currents the cosines, of said angles.

LINE GROUNDED AT FAR END THROUGH A RESISTANCE σ. (Figure 7.)

First Case. Let σ be not greater than r_o.

Find the hyperbolic angle ϕ of the terminal load σ from

$$\tanh\phi = \frac{\sigma}{r_o}. \qquad (36)$$

Then treat the artificial line as grounded directly, but with the angles of all its leaks and junctions increased by ϕ. Formulas (19) to (35) will then apply, except where the strength of the current to ground enters into consideration, as in (26), (28), (30), and (33). The surge-resistance r_o must then be replaced by a new surge-resistance

$$r_o'' = \frac{r_o}{\cosh\phi} \qquad \text{ohms.} \qquad (37)$$

Thus, the sending-end resistance becomes, by (19):

$$R_{g\sigma} = r_o \tanh(L_2\alpha + \phi) = r_o \tanh(2m\theta + \phi) \quad \text{ohms.} \qquad (38)$$

The resistance at and excluding the nth leak becomes, by (20):

$$R'_{n,g\sigma} = r_o' \frac{\sinh[(2n-1)\theta + \phi]}{\cosh[(2n-2)\theta + \phi]} \qquad \text{ohms.} \qquad (39)$$

The resistance at and including the nth leak becomes, by (21):

$$R'_{g,n\sigma} = r_o' \frac{\sinh[(2n-1)\theta + \phi]}{\cosh(2n\theta + \phi)} \qquad \text{ohms.} \qquad (40)$$

The ratio of local maximum resistance just before a leak to the local minimum just after the preceding leak is:

$$\frac{R'_{n+1,g\sigma}}{R'_{g,n\sigma}} = \frac{\sinh[(2n+1)\theta + \phi]}{\sinh[(2n-1)\theta + \phi]}. \qquad (41)$$

For example, the sending-end resistance of the line in Figure 7, with $\sigma = 750$ ohms, whose hyperbolic angle is $\tanh^{-1}\frac{750}{1436.1} = 0.57941$ hyp., becomes by (38), $1436.1 \times \tanh 2.338 = 1409.6$ ohms.

The receiving-end resistance is, by (26) and (37):

$$\begin{aligned} R_{l\sigma} &= r_o'' \sinh(L_2\alpha + \phi) = r_o'' \sinh(2m\theta + \phi) \\ &= r_o \sinh 2m\theta + \sigma \cosh 2m\theta \qquad \text{ohms.} \end{aligned} \qquad (42)$$

Thus, in Figure 7, $r_o'' = 1224.7$ ohms by (37), and

$$R_{l\sigma} = 1224.7 \sinh 2.338 = 6285.4 \text{ ohms.}$$

The voltage at the nth junction is, by (27) and (28):

$$e_n = e_m \frac{\sinh(2n\theta + \phi)}{\sinh(2m\theta + \phi)} = I_o r_o'' \sinh(2n\theta + \phi) \qquad \text{volts;} \qquad (43)$$

so that the voltage at the distant end of the line is:

$$e_o = e_m \frac{\sinh \phi}{\sinh (2\,m\theta + \phi)} = I_o r_o'' \sinh \phi \qquad \text{volts.} \qquad (44)$$

Thus, in Figure 7, the voltage at the distant end B is

$$100 \times \frac{\sinh 0.5794}{\sinh 2.338} = 11.931.$$

The voltage at the nth leak is, by (29) and (30):

$$\epsilon_n = \epsilon_m \frac{\sinh [(2\,n - 1)\,\theta + \phi]}{\sinh [(2\,m - 1)\,\theta + \phi]} = I_o r_o'' \frac{\sinh [(2\,n - 1)\theta + \phi]}{\cosh \theta} \quad \text{volts.} \qquad (45)$$

The current at the sending end is, by (31):

$$I_{m\sigma} = \frac{e_m}{r_o \tanh (L_2 a + \phi)} = \frac{e_m}{r_o \tanh (2\,m\theta + \phi)} \quad \text{amperes.} \qquad (46)$$

At junction n it is, by (32):

$$I_{n\sigma} = I_{m\sigma} \frac{\cosh (2\,n\theta + \phi)}{\cosh (2\,m\theta + \phi)} = \frac{e_m}{r_o} \frac{\cosh (2\,n\theta + \phi)}{\sinh (2\,m\theta + \phi)} \quad \text{amperes.} \qquad (47)$$

At the distant end, through σ, it is, by (33):

$$\begin{aligned} I_{o\sigma} &= \frac{e_m}{r_o'' \sinh (2\,m\theta + \phi)} = \frac{e_m}{r_o'' \sinh (L_2 a + \phi)} \\ &= \frac{I_{m\sigma} \cosh \phi}{\cosh (2\,m\theta + \phi)} = \frac{I_{m\sigma} \cosh \phi}{\cosh (L_2 a + \phi)} \quad \text{amperes.} \qquad (48) \end{aligned}$$

At the nth leak, the ratio of ongoing to arriving current is, by (34):

$$\frac{I_{\sigma,\, n-1}}{I_{\sigma,\, n}} = \frac{\cosh [2\,(n - 1)\,\theta + \phi]}{\cosh (2\,n\theta + \phi)}. \qquad (49)$$

For example, the received current to ground through σ is, by (48), $100/6285.4 = 0.01591$ ampere.

Second Case, with σ not less than r_o.

Find the hyperbolic angle of the terminal load σ from the formula:

$$\tanh \phi' = \frac{r_o}{\sigma}. \qquad (50)$$

Then treat the artificial line, actually grounded through σ, as though it were freed at the far end, but with its angular length increased at all points by ϕ' hyps. Formulas (6) to (18) will then apply, except that where the strength of the received current to ground enters into consideration, as in (56) and (62), the surge-resistance r_o must be replaced by a new surge-resistance:

$$r_o''' = \frac{r_o}{\sinh \phi'} \quad \text{ohms.} \qquad (51)$$

Thus, the sending-end resistance at junction n becomes, by (6):

$$R_{f\sigma} = r_o \coth (L_2 a + \phi') = r_o \coth (2\,n\theta + \phi') \quad \text{ohms.} \qquad (52)$$

The resistance at the nth leak, excluding the same, is, by (7):

$$R'_{n,\,f\sigma} = r_o' \frac{\cosh [(2\,n - 1)\,\theta + \phi']}{\sinh [(2\,n - 2)\,\theta + \phi']} \quad \text{ohms.} \qquad (53)$$

The resistance at the nth leak, including the same, is, by (8):

$$R'_{f\sigma,\,n} = r_o' \frac{\cosh [(2\,n - 1)\,\theta + \phi']}{\sinh (2\,n\theta + \phi')} \quad \text{ohms.} \qquad (54)$$

The ratio of resistance at and excluding the $(n + 1)$th leak to that at and including the nth leak is, by (10):

$$\frac{R'_{n+1,\,g\sigma}}{R'_{g\sigma,\,n}} = \frac{\cosh [(2\,n + 1)\,\theta + \phi']}{\cosh [(2\,n - 1)\,\theta + \phi']}. \qquad (55)$$

The receiving-end resistance is, by (26):

$$\begin{aligned} R_{l\sigma} &= r_o''' \cosh (2\,m\theta + \phi') = r_o''' \cosh (L_2 a + \phi') \\ &= r_o \sinh 2\,m\theta + \sigma \cosh 2\,m\theta \quad \text{ohms.} \qquad (56) \end{aligned}$$

The voltage at junction n is, by (13):

$$e_{n\sigma} = e_m \frac{\cosh (2\,n\theta + \phi')}{\cosh (2\,m\theta + \phi')} = e_o \frac{\cosh (2\,n\theta + \phi')}{\cosh \phi'} \quad \text{volts.} \qquad (57)$$

At the distant end, or junction 0, it is:

$$e_{o\sigma} = \frac{e_m \cosh \phi'}{\cosh (2\,m\theta + \phi')} = \frac{e_m \cosh \phi'}{\cosh (L_2 a + \phi')} \quad \text{volts.} \qquad (58)$$

At the nth leak, it is, by (14):

$$\epsilon_{n\sigma} = \epsilon_m \frac{\cosh [(2\,n - 1)\,\theta + \phi']}{\cosh [(2\,m - 1)\,\theta + \phi']} = \frac{e_o \cosh [(2n - 1)\,\theta + \phi']}{\cosh \theta \cosh \phi'} \quad \text{volts.} \qquad (59)$$

The current strength at the sending end or junction m is:

$$I_{m\sigma} = \frac{e_m}{r_o \coth (L_2 a + \phi')} = \frac{e_m}{r_o \coth (2\,m\theta + \phi')} \quad \text{amperes.} \qquad (60)$$

At junction n it is, by (16):

$$I_{n\sigma} = I_{m\sigma} \frac{\sinh (2\,n\theta + \phi')}{\sinh (2\,m\theta + \phi')} \quad \text{amperes.} \qquad (61)$$

At the receiving end, or junction 0, it is:

$$I_{o\sigma} = I_{m\sigma} \frac{\sinh \phi'}{\sinh (2\,m\theta + \phi')} = \frac{e_m}{r_o''' \cosh (2\,m\theta + \phi')} \quad \text{amperes.} \qquad (62)$$

At the nth leak the ratio of ongoing to arriving line current is, by (17):

$$\frac{I_{n-1,\sigma}}{I_{n\sigma}} = \frac{\sinh [2\,(n-1)\,\theta + \phi']}{\sinh (2\,n\theta + \phi')}. \qquad (63)$$

The current escaping at the nth leak is, by (18):

$$i_{n\sigma} = \epsilon_{n\sigma} g' = e_{o\sigma} g' \frac{\cosh [(2\,n-1)\theta + \phi']}{\cosh (\theta + \phi')} = \epsilon_{m\sigma} g' \frac{\cosh [(2\,n-1)\theta + \phi']}{\cosh [(2\,m-1)\theta + \phi']}$$

$$= e_{m\sigma} g' \frac{\cosh [(2\,n-1)\,\theta + \phi']}{\cosh \theta \cosh (2\,m\theta + \phi')} \quad \text{amperes.} \qquad (64)$$

As an example, let $\sigma = 3750$ ohms. Then $\phi' = \tanh^{-1} \frac{1436.13}{3750} =$ 0.403535 hyp. The sending-end resistance at junction 1 is, by (52), 1436.1 coth 0.755255 = 2250 ohms, which by Figure 8 is evidently correct. Again, the received current strength for the same case with $e_1 = 10$ volts, at junction 1, will be,by (62), $I_{o\sigma} = \frac{10}{3464.1 \cosh 0.75526}$ = 0.00222 amperes, which is also easily seen to be correct, from Figure 8.

Third Case $\sigma = r_o$. Exponential Case.

In the particular and intermediate case in which $\sigma = r_o$, either of the preceding sets of formulas applies under limit conditions. We have $\phi = \phi' = \infty$, by (36) and (50). Consequently, the sending-end resistance becomes at any junction:

$$R_{gr_0} = r_o \quad \text{ohms.} \qquad (65)$$

The resistance at any leak, excluding the same, is:

$$R'_{gr_0} = r_o' \, \epsilon^{\theta} \quad \text{ohms.} \qquad (66)$$

ϵ being the Napierian base.

The resistance at any leak, including the same, is:

$$R'_{r_{01}} = r_o' \epsilon^{-\theta} \qquad \text{ohms.} \qquad (67)$$

Thus, in Figure 9, where one section of artificial line is grounded at the distant end through a resistance $\sigma = r_0 = 1436.13$ ohms, the

Fig. 8

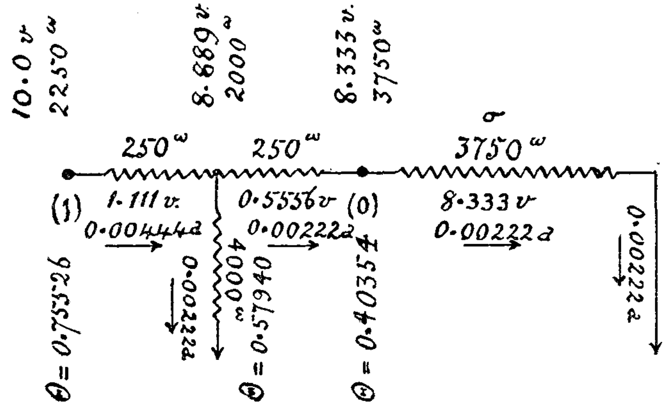
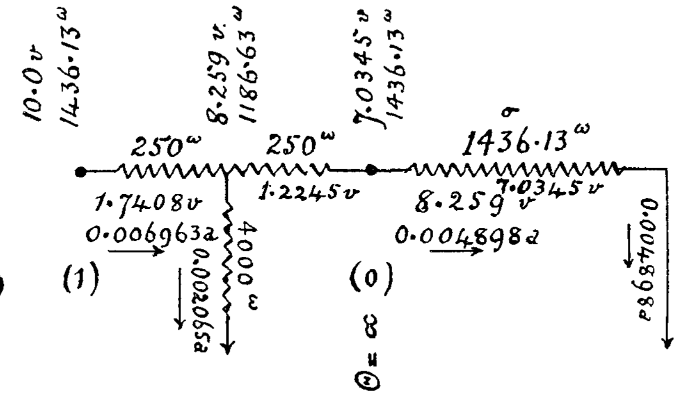

10.0 v
2250 ω
8.889 v
2000 ω
8.333 v
3750 ω
σ
250 ω
250 ω
3750 ω
(1)
1.111 v
0.004444 a
0.5556 v
(0)
0.002222 a
8.333 v
0.00222 a
0.00222 a
Θ = 0.75526
0.002222 a
4000 ω
Θ = 0.57940
Θ = 0.40354

Fig. 9


10.0 v
1436.13 ω
8.259 v
1186.63 ω
7.0345 v
1436.13 ω
σ
250 ω
250 ω
1436.13 ω
1.7408 v
0.006963 a
1.2245 v
8.259 v
7.0345 v
0.004898 a
(1)
0.002065 a
4000 ω
(0)
Θ = ∞
0.004898 a

sepuing-end resistance at junction (1) is r_o, at leak 1 is $1414.2 \times \epsilon^{0.17586} = 1686.1$ ohms, excluding the leak, and $1414.2\ \epsilon^{-0.17586} = 1186.13$ ohms, including the leak.

The receiving end resistance is:

$$R_{lr_0} = r_o\, \epsilon^{L_2 a} = r_o\, \epsilon^{2m\theta} \qquad \text{ohms.} \qquad (68)$$

Thus in Figure 9 the receiving-end resistance is 1436.1 × 1.422 = 2042 ohms.

The voltage at junction n is:

$$e_{nr_0} = e_m \, \epsilon^{2\theta(n-m)} = e_o \, \epsilon^{2n\theta} \qquad \text{volts.} \qquad (69)$$

At the distant end, or junction 0, it is:

$$e_{or_0} = e_m \, \epsilon^{-L_2 a} = e_m \, \epsilon^{-2m\theta} \qquad \text{volts.} \qquad (70)$$

At the nth leak, it is:

$$\epsilon_{nr_0} = \epsilon_m \, \epsilon^{2\theta(n-m)} = e_o \, \frac{\epsilon^{(2n-1)\theta}}{\cosh \theta} \qquad \text{volts.} \qquad (71)$$

The current strength at the sending end, or junction m, is:

$$I_{mr_0} = \frac{e_m}{r_o} \qquad \text{amperes.} \qquad (72)$$

At junction n it becomes:

$$I_{nr_0} = I_m \, \epsilon^{2\theta(n-m)} = I_o \, \epsilon^{2n\theta} \qquad \text{amperes.} \qquad (73)$$

At the receiving end it is:

$$I_{or_0} = \frac{e_m}{r_o} \, \epsilon^{-L_2 a} = \frac{e_m}{r_o} \, \epsilon^{-2m\theta} = I_m \, \epsilon^{-2m\theta} \qquad \text{amperes.} \qquad (74)$$

At any leak the ratio of ongoing to arriving line current is

$$\frac{I_{n-1, r_0}}{I_{n r_0}} = \epsilon^{-2\theta}. \qquad (75)$$

GENERAL PROPOSITIONS.

Equal Increase of Receiving-End Resistance due to Resistance inserted at either End of Line.

When a resistance σ is added to the line at the sending end, the sending-end resistance is obviously increased by σ; but the receiving-end resistance is increased by $\sigma \cosh La = \sigma \cosh 2\,m\theta$ ohms. Comparing this result with formula (42), it is evident that a resistance σ adds $\sigma \cosh 2\,m\theta$ to the resistance of an artificial line, or $\sigma \cosh La$ to that of a smooth line, whether it be added at the sending or receiving end. Thus, if to the sending end of the artificial line in Figure 5, a resistance of 750 ohms be added, the sending-end resistance will be increased to 2103.4 ohms, and the voltage at the end A of the artificial line will thereby be reduced from 100 to 64.343 volts. The effect of this will be to reduce the received current at B to 0.01591 ampere, the same as in Figure 7.

Best Resistance of Receiving Instrument.

Electromagnetic receiving instruments may be divided into two classes; viz. (1) those, as of the movable-coil type, in which the magneto-mechanical force, or torque, is directly proportional to the ampere-turns, and (2) those, like simple non-polarized relays, in which the magneto-mechanical force, or torque, may be nearly proportional to the square of the ampere-turns at low magnetic saturation, but, as saturation increases, to perhaps a lower power of ampere turns than the first. In either case, the magneto-mechanical force may be expressed by:

$$F = a\,(I_o n_1)^p \quad \text{dynes or dyne-perp. cms.,} \qquad (76)$$

where F is the force or torque, a is a constant of the instrument, I_o is the received current in amperes, n_1 the number of turns in the winding, and p some exponent not greater than 2. The received current I_o is expressed by (42) or (56). The number of turns n_1 in a given winding space is well known to be sensibly proportional to $\sqrt{\sigma}$, where σ is the resistance of the winding in ohms, provided that the size of copper wire selected is within the fairly wide range that keeps the ratio of covered diameter to bare diameter sensibly constant. Consequently, we have approximately:

$$F = a'\left(\frac{e_m\sqrt{\sigma}}{r_o \sinh 2\,m\theta + \sigma \cosh 2\,m\theta}\right)^p \quad \text{dynes, or dyne-perp. cms.} \qquad (77)$$

In order to make this force a maximum by varying σ, we differentiate F with respect to σ in the usual way, and equate to zero. We then obtain

$$\sigma = r_o \tanh 2\,m\theta = r_o \tanh La \qquad \text{ohms.} \qquad (78)$$

That is, the best resistance for the electromagnetic winding of the receiver is equal to the sending-end resistance R_g of the line, no matter what the exponent p which expresses the relation between torque and ampere-turns.[3]

---

[3] See Ayrton and Whitehead paper in Bibliography.

IMITATIVE ACCURACY OF ARTIFICIAL LINES.

As the preceding formulas indicate, an artificial line does not correspond electrically to the real smooth line having the same linear constants (resistance and leakance per km.) as its nominal linear constants, but to some other real smooth line having somewhat different linear constants. In other words, an artificial line has an imitation error due to lumpiness. The amount of this error will differ with the degree of lumpiness, and would obviously disappear if the number of line sections were made indefinitely great. In general, the fewer the sections the greater the lumpiness, and the greater the lumpiness error. With any given artificial line, however, the lumpiness error depends upon the particular quantity considered, and is not the same for all quantities. Thus, let a' and r_o' be the nominal attenuation-constant and surge-resistance of the uncorrected artificial line, by (1) (2); while a and r_o are the corresponding constants, corrected for lumpiness, according to (4) and (5). Then the ratio of received ground current over the artificial line to that over the real line of same nominal linear constants will be $\frac{r_o' \sinh La'}{r_o \sinh La}$. Again, the ratio of sending-end resistances with the far end grounded will be $\frac{r_o \tanh La}{r_o' \tanh La'}$, a distinctly different ratio; while in respect to, say, voltage at the free distant end, the ratio will be again different. Consequently there is no single correction factor for the lumpiness of an artificial line, and each particular quantity will have to be corrected, according to the preceding formulas.

EQUIVALENCE BETWEEN SINGLE-SECTION ARTIFICIAL LINES AND UNIFORM SMOOTH LINES.

In Figure 10, let AOB represent a uniform smooth actual line of L kms. in length, with a linear conductor-resistance of r ohms per km. and a linear dielectric conductance of g mhos per km. Its attenuation-constant will then be $a = \sqrt{gr}$ hyps. per km., and its surge-resistance $r_o = \sqrt{\frac{r}{g}}$ ohms. Its hyperbolic angle will be La, and its semi-hyperbolic angle λa hyps. Then let a single section of artificial line be constructed, as in Figure 11, with a total conductor-resistance of r' ohms, a leak at the centre of g' mhos, or $R' = 1/g'$ ohms. This single section of artificial line will be the complete external equivalent of the actual uniform line in Figure 10 in the steady state, if:

Fig. 10

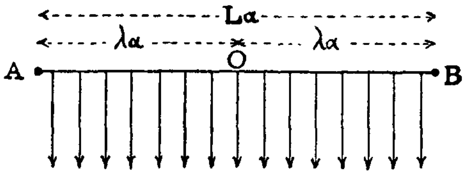


Fig. 11

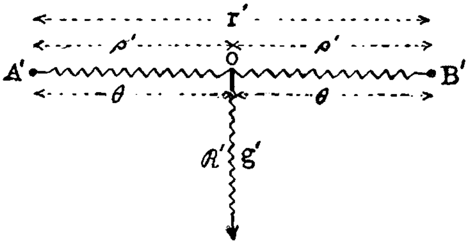
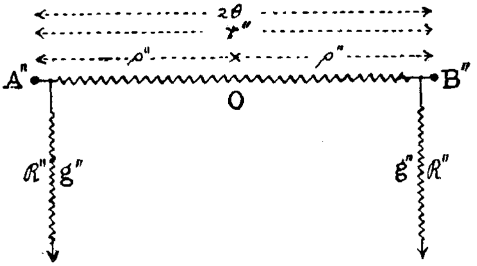


Fig. 12



FIGURES 10, 11, 12. — Section of uniform actual line with distributed leakance, equivalent T and equivalent Π.

$$\rho' = \frac{r'}{2} = r_o \tanh \lambda a \qquad \text{ohms,} \qquad (79)$$

and

$$g' = \frac{r_o \sinh La}{{r_o}^2} = \frac{\sinh La}{r_o} \qquad \text{mhos,} \qquad (80)$$

or

$$R' = \frac{{r_o}^2}{r_o \sinh La} = \frac{r_o}{\sinh La} \qquad \text{ohms.} \qquad (81)$$

That is, the half resistance ρ' is to be equal to the sending-end resistance of each half of the actual line at O, when grounded at A and B; while the resistance of the central leak is to be r_o^2 divided by the receiving-end resistance of the whole line grounded.

Thus, considering the actual smooth line of 500 km. length of which the artificial line represented in Figures 5, 6, and 7 is the external equivalent, we have $r = 5.051$ ohms per km., $g = 2.4499$ micromhos per km., $a = 0.0035172$ hyps. per km., $r_0 = 1436.13$ ohms, $La = 1.7586$ hyps., $\lambda a = 0.8793$ hyp. From (79) we obtain :

Fig. 13

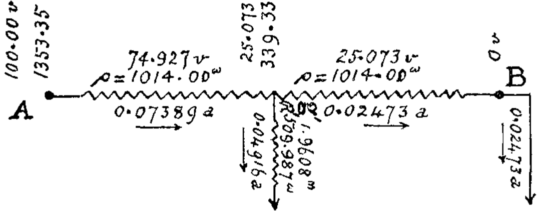

100·00 v
1353·35 ω
25·073 v
339·33 ω
θ = 0·8793
74·927 v
ρ = 1014·00 ω
25·073 v
ρ = 1014·00 ω
0 v
A
B
0·07389 a
0·04916 a
509·987 ω
1·9608 ω
0·02473 a
0·02473 a

Fig. 14

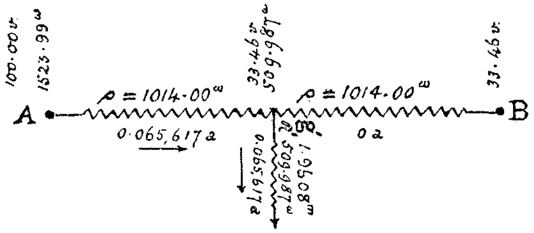
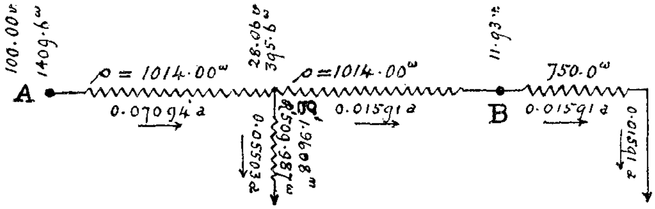

100·00 v
1523·99 ω
33·46 v
509·987 ω
ρ = 1014·00 ω
ρ = 1014·00 ω
33·46 v
A
B
0·065,617 a
0 a
0·065,617 a
509·987 ω
1·9608 ω

Fig. 15


100·00 v
1409·6 ω
28·06 v
395·6 ω
ρ = 1014·00 ω
ρ = 1014·00 ω
11·93 v
750·0 ω
A
B
0·07094 a
0·05503 a
509·987 ω
1·9608 ω
0·01591 a
0·01591 a
0·01591 a

FIGURES 13, 14, 15. — Equivalent T of line imitated in Figures 5, 6, and 7. Grounded, freed, and grounded imperfectly.

$$\rho' = 1436.1 \times 0.70607 = 1014.0 \text{ ohms},$$

and by (81) $R' = 1436.1/2.81602 = 509.987$ ohms.

The above values of ρ' and R' have been employed in Figures 13, 14, and 15 to produce a single-section artificial line. It will be seen by comparing these Figures respectively with Figures 5, 6, and 7, that although the internal distributions of voltage and current differ, the external distributions are identical. That is, the distribution of voltage and current at the ends of, and anywhere external to, the artificial line are identical for the single-section artificial line of Figures 13, 14, and 15; or for the five-section artificial line of Figures 5, 6, and 7; or for the actual smooth uniform line of a = 0.0035172 hyp. per km. and r_o = 1436.1 ohms, there imitated.

For brevity and convenience, let a single-section artificial line, like that of Figure 14, formed of a conductor-resistance r' ohms, with a leak of R' ohms at the centre, be called a T, from the graphical resemblance. Then, any real smooth uniform line may be replaced by its equivalent T, without any change in the electrical system external to the T, after the steady state has been attained. This proposition, like the rest, applies not only to a continuous-current system, but also to any single-frequency alternating-current system.

In duplex and multiplex telegraphy, artificial lines are required to balance real lines, not only in the steady state, but also in the preceding unsteady state; so that it is not possible to employ an equivalent T for such artificial lines. In telephony, however, it is commonly believed that the electrical phenomena in ordinary conversation are substantially steady state single-frequency phenomena, and that the conditions in the unsteady state are so transient that they may be practically ignored. If this is correct, then it follows that, except for purposes of adjustment, and of convenience in altering the length of line, there is nothing to be gained by employing a multisection artificial line for embodying the laboratory equivalent of an actual line. In other words, a single-section artificial line of properly selected constants should be just as good as a multisection artificial line, in regard to carrying on conversation. It is important to have this question settled experimentally. The experiment, if unsuccessful, cannot, however, be competent to determine whether the unsteady state enters appreciably into the phenomena of practical telephonic transmission, owing to the presence of multiple frequencies or harmonics.

Conversely, if we have a given T line, we can determine its hyperbolic angle and surge-resistance; that is, we can determine the actual smooth uniform line to which it corresponds; for in Figures 10 and 11

$$\sinh \theta = \sinh \lambda a = \sqrt{\frac{\rho'}{2R'}} \qquad (82)$$

and

$$r_o = \sqrt{\rho'(\rho' + 2R')} \qquad \text{ohms.} (83)$$

Thus, the T of Figures 13, 14, and 15 has $\rho' = 1014$ ohms and $R' = 509.99$ ohms. Hence by (82), $\sinh \theta = 0.99707$, from which the semi-angle $\theta = 0.8793$ hyp., which is also the semi-angle λa of the equivalent smooth line. Again, $r_0 = 1436.1$ ohms by (83). These are the constants for the line simulated by the T.

Instead of a T, or conductor with a single central leak, we may substitute for any actual smooth uniform line a conductor with two equal terminal leaks, as shown in Figure 12. Such a conductor may be called a Π for convenience and brevity. In Figure 12, the values to be assigned to the conductor-resistances r'' and leak resistances R'' R'' ohms, in order to replace a smooth line of length L kms., semi-length λ kms., attenuation-constant a hyp. per km., and surge-resistance r_0 ohms, are:

$$r'' = r_0 \sinh La \qquad \text{ohms,} \quad (84)$$

$$R'' = \frac{r_0^2}{r_0 \tanh \lambda a} = \frac{r_0}{\tanh \lambda a} \qquad \text{ohms,} \quad (85)$$

or

$$g'' = \frac{r_0 \tanh \lambda a}{r_0^2} = \frac{\tanh \lambda a}{r_0} \qquad \text{mhos.} \quad (86)$$

That is, the conductor resistance r'' must be equal to the receiving-end resistance of the imitated line when grounded, and each leak must be the square of the surge-resistance divided by the sending-end resistance of half the imitated line grounded.

Thus, with $L = 500$ kms., $\lambda = 250$ kms., $a = 0.0035172$ hyp. per km., $r_0 = 1436.13$ ohms, $La = 1.7586$ hyps., $\lambda a = 0.8793$ hyp., we have $r'' = 1436.13 \times 2.81602 = 4044.2$ ohms, and $R'' = 1436.13/0.70607 = 2034.05$ ohms. These values have been used in Figures 16, 17, and 18 to construct the Π there indicated. It will be seen by comparing these Figures with 5, 6, 7, and with 10, 11, 12, respectively, that the external distributions of resistance, conductance, voltage, current, and power are the same for all.

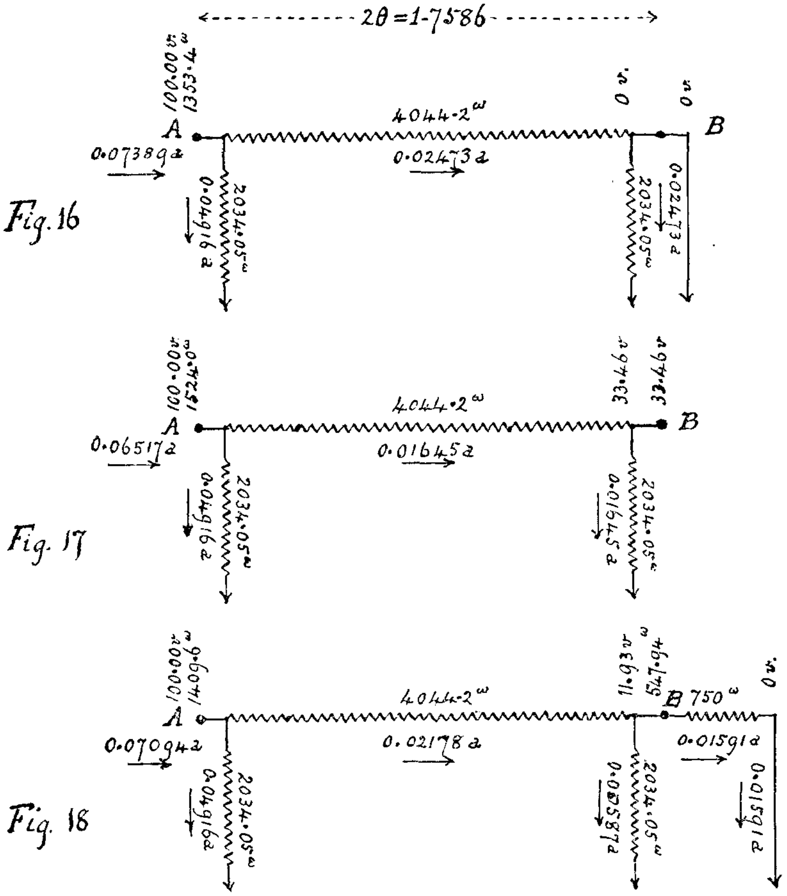
Consequently, any smooth uniform line in the steady state, carrying either continuous or single-frequency alternating currents, may be completely replaced, so far as concerns all external conditions, either by one equivalent T, or by one equivalent Π. Either of these forms of equivalent conductor may be selected for replacing the line, according to convenience.

Conversely, any given Π may have its hyperbolic angle and surge-resistance determined; that is, its equivalent smooth uniform line can be determined by the following formulas:

$$\tanh\theta = \tanh\lambda a = \sqrt{\frac{g''r''}{2+g''r''}} = \sqrt{\frac{r''}{2R''+r''}}, \qquad (87)$$

and

$$r_o \sqrt{\frac{r''}{g''\,(2+g''r'')}} = R''\sqrt{\frac{r''}{2R''+r''}} = R''\tanh\theta \text{ ohms.} \qquad (88)$$



2θ = 1.7586
100.00v
1353.4ω
A
0.07389a
4044.2ω
0.02473a
0v
0v
B
0.04916a
2034.05ω
2034.05ω
0.02473a
Fig. 16
100.00v
1524.0ω
A
0.06517a
4044.2ω
0.01645a
33.46v
33.46v
B
0.04916a
2034.05ω
0.01645a
2034.05ω
Fig. 17
100.00v
1409.6ω
A
0.07094a
4044.2ω
0.02178a
11.93v
547.94ω
B 750ω
0v
0.01591a
0.04916a
2034.05ω
0.00587a
2034.05ω
0.01591a
Fig. 18

FIGURES 16, 17, 18. — Equivalent Π of line imitated in Figures 5, 6, and 7. Grounded, freed, and grounded imperfectly.

Thus, with $r'' = 4044.2$ ohms, and $R'' = 2034.05$ ohms, as in Figures 16, 17, and 18, we have $\tanh\theta = \sqrt{4044.2/8112.3} = 0.70606$, and $r_o = 2034.05 \times 0.70606 = 1436.1$ ohms, as before.

It is possible, by known methods of substitution, to derive combinations of resistance and leakance that shall replace a given T or Π; as, for instance, a combination like that shown in Figure 19. All such conductors must manifestly be either graphically symmetrical about a vertical through their centre 0, or must be reducible to such symmetry. In general, these combinations are unnecessarily complex and have little practical interest. From this standpoint, a multiple-section artificial line like that of Figures 5, 6, and 7 may be regarded as a complex substitute for the simple T of Figure 11, or the simple Π of Figure 12.

It may be observed, however, that the total leakage of current to ground in corresponding Figures is the same for a smooth uniform line, its equivalent T, equivalent Π, or equivalent 5-section artificial line. On reflection, this proposition is almost self-evident.

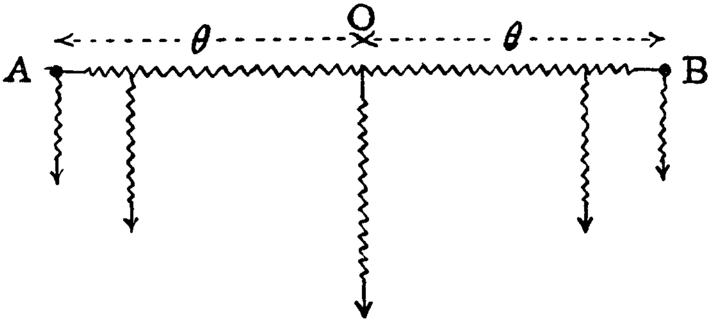


Figure 19. — Complex substitute for an actual line of distributed leakance.

As an instance of the use of substituting equivalent T's for sections of actual line, consider the case represented in Figure 20, of a uniform line of attenuation-constant α, and surge-resistance r_o, loaded with resistances of $\Sigma = 2\,\sigma$ ohms, at uniform intervals of l kms. Required the equivalent smooth line.

First substitute uniform T's for the sections of uniform line, as in Figure 21, by formulas (79), (80), and (81). Then load the T's by adding σ to each end, as in Figure 22. Finally replace the loaded T's by their equivalent lengths of smooth line, as in Figure 23, using formulas (82) and (83). We deduce by this process the following results:

$$\sinh \lambda a' = \sinh \lambda a \sqrt{1 + \frac{\sigma \coth \lambda a}{r_o}}, \tag{89}$$

$$\cosh \lambda a' = \cosh \lambda a \sqrt{1 + \frac{\sigma \tanh \lambda a}{r_o}}, \tag{90}$$

Fig. 20

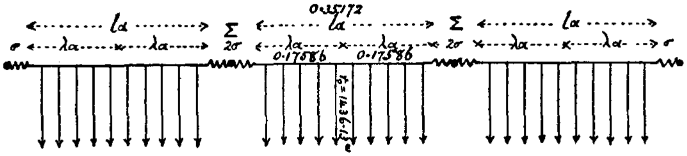

la
σ
λa
Σ
2σ
0·35172
0·17586
$r_0 = 1436 \cdot 12^{\omega}$

Fig. 21

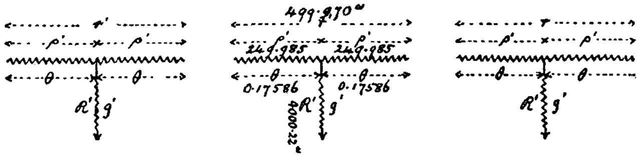

r′
ρ′
θ
R′
g′
499·970ω
249·985
0·17586
4000·22ω

Fig. 22

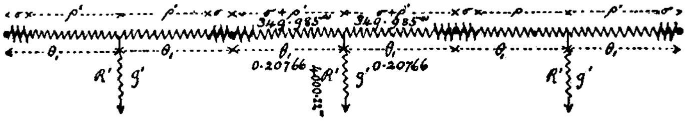
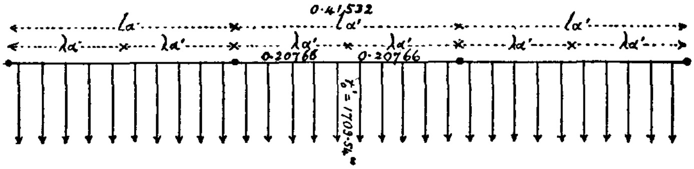

σ
ρ′
σ+ρ′
349·985ω
θ₁
0·20766
R′
g′
4000·22ω

Fig. 23


la
la′
λa
λa′
0·41532
0·20766
$r_0' = 1709 \cdot 54^{\omega}$

Figures 20, 21, 22, 23. — Reduction of a uniform actual line with loads in series to an equivalent unloaded actual line.

$$\tanh \lambda a' = \tanh \lambda a \sqrt{\frac{1 + \dfrac{\sigma \coth \lambda a}{r_0}}{1 + \dfrac{\sigma \tanh \lambda a}{r_0}}} = \sqrt{\tanh \lambda a \tanh (\lambda a + \delta)}, \tag{91}$$

if $\sigma < r_0$ and $\dfrac{\sigma}{r_0} = \tanh \delta$; or

$$\coth \lambda a' = \sqrt{\coth \lambda a \coth (\lambda a + \delta')}, \tag{92}$$

if $\sigma > r_0$ and $\dfrac{\sigma}{r_0} = \coth \delta'$.

Also

$$\sinh la' = \sinh la \sqrt{1 + \frac{2\sigma}{r_0} \coth la + \left(\frac{\sigma}{r_0}\right)^2}; \tag{93}$$

$$\cosh la' = \cosh la + \frac{\sigma}{r_0} \sinh la; \tag{93a}$$ [4]

---

[4] This formula (93a) was first published by Dr. Campbell. See Bibliography.

$$r_o' = r_o\sqrt{\left(\tanh \lambda a + \frac{\sigma}{r_o}\right)\left(\coth \lambda a + \frac{\sigma}{r_o}\right)} \text{ ohms}; \qquad (94)$$

$$= r_o\sqrt{1 + \frac{2\,\sigma}{r_o}\coth la + \left(\frac{\sigma}{r_o}\right)^2} \qquad \text{ohms}; \qquad (95)$$

$$\frac{r_o'}{r_o} = \frac{\sinh la'}{\sinh la}. \qquad (96)$$

Thus, if a uniform line of attenuation-constant a = 0.0035172 hyp./km., and surge-resistance r_o = 1436.13 ohms, has a resistance Σ = 200 ohms, inserted at intervals of 100 kms., required the corresponding constants of the loaded line. Here, as indicated in Figure 21, σ = 100 ohms and λa = 0.17586 hyp. If we compute the equivalent T's of the sections of unloaded line, we find ρ' = 249.985 ohms and R' = 4000.215 ohms. The hyperbolic corrections for these lengths of sections are thus only 0.015 ohm in conductor-resistance and 0.215 ohm in leak-resistance. Adding on the loads to the ends of the T's, we have, as in Figure 22, ρ' = 349.985 ohms and R' = 4000.215 ohms. Using formulas (82) and (83), we obtain for the equivalent smooth line $\lambda a'$ = 0.20766 hyp., la' = 0.41532 hyp., and r_o' = 1709.54 ohms. The apparent conductor-resistance of the loaded line is, therefore, $r_o'la'$ = 710.06 ohms, or 10.06 ohms more than the actual resistance of conductor and loads. The apparent total leak r_o'/la' = 4116.2 ohms, or 116.2 ohms in excess of the actual total leak.

As an example of the use of substituting equivalent Π's for sections of smooth line, consider the case represented in Figure 24 of a uniform line of attenuation-constant a, and surge-resistance r_o, loaded with uniform leakances of Γ mhos at uniform intervals of l kms. Required the constants of the equivalent smooth line.

First divide the leakage conductances into equal parts $\gamma = \Gamma/2$, as in Figure 25. Then substitute for the unloaded line sections their equivalent Π's by formulas (84), (85), and (86), as in Figure 26. Next add on the terminal leakances γ to the pillars of the Π, as in Figures 27. Finally, deduce as in Figure 28, by formulas (87) and (88), the equivalent smooth line.

We also obtain by this process the following relations: —

$$\tanh \theta' = \tanh \lambda a' = \tanh \lambda a \sqrt{\frac{1 + \gamma r_o \coth \lambda a}{1 + \gamma r_o \tanh \lambda a}} \qquad (97)$$

$$r_o' = \frac{r_o}{\sqrt{(1 + \gamma r_o \tanh \lambda a)\,(1 + \gamma r_o \coth \lambda a)}} \qquad \text{ohms}. \qquad (98)$$

Thus, in Figure 24, the load leaks have resistances of 10,312 ohms, or conductances of 0.096971 millimho, the line sections have lengths l = 100 kms., the attenuation-constant 0.0035172 hyp. per km., the hyperbolic angles la = 0.35172, λa = 0.17586, r_o = 1436.13 ohms.

Fig.24

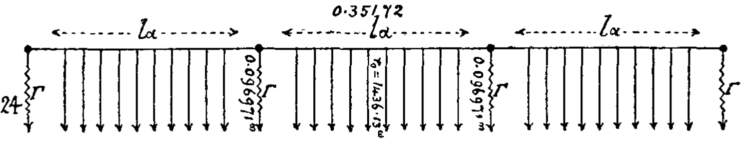

0·35172
la
la
la
r
r
r
r
0·096971ᵐ
r₀ = 1436·13ω
0·096971ᵐ

Fig.25

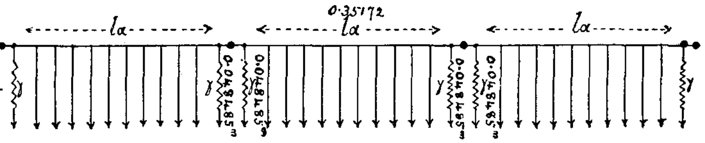

0·35172
la
la
la
γ
γ
0·048485ᵐ
0·048485ᵐ
γ
0·048485ᵐ
0·048485ᵐ
γ

Fig.26

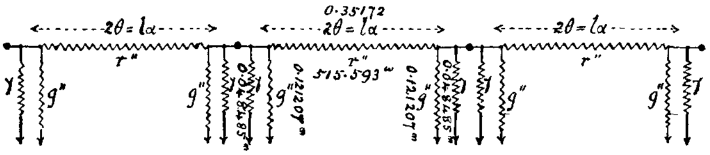

0·35172
2θ = la
2θ = la
2θ = la
r''
r''
r''
γ
g''
g''
0·048485ᵐ
γ
g''
0·121207ᵐ
515·593ω
0·121207ᵐ
g''
0·048485ᵐ
γ
g''
g''
γ

Fig.27

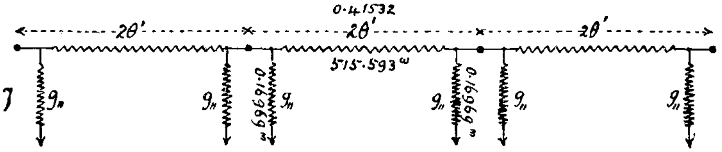
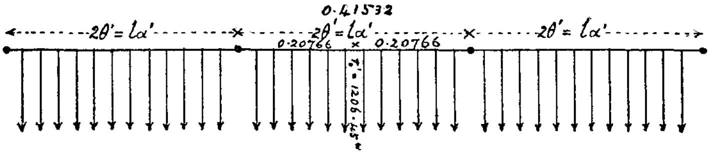

0·41532
2θ'
2θ'
2θ'
515·593ω
g₁₁
g₁₁
0·16969ᵐ
g₁₁
g₁₁
0·16969ᵐ
g₁₁
g₁₁

Fig.28


0·41532
2θ' = la'
2θ' = la'
2θ' = la'
0·20766
0·20766
r₁ = 1206·45ω

FIGURES 24 TO 28. — Reduction of a uniform actual line with loads in derivation to an equivalent unloaded actual line.

Required the corresponding constants of the loaded line. The load leaks are bisected in Figure 25 to 0.048486 millimho each. The equivalent Π of each unloaded line section, as shown in Figure 26, has a resistance of r'' = 515.593 ohms and a leakance g'' of 0.121207 millimho. Adding the γ loads to the pillars of the Π, we have, as in Figure 27, $g_{\shortparallel}$ = 0.16969 millimho. Finally, reducing the loaded Π's to their equivalent smooth-line sections by formulas (87) and (88), we obtain $\lambda a' = 0.20766$ hyp. or $la' = 0.41532$ hyp. and $r_o' = 1206.45$ ohms, as in Figure 28. The apparent conductor resistance of a section of loaded line is $la'r_o' = 501.06$ ohms, or 1.06 ohms in excess of the total actual resistance. The apparent total leakance of a section is 0.34425 millimho, or 0.00272 millimho in excess of the total actual leakance.

It may be observed by comparing Figures 20–23 and 24–28, or formulas (91) and (97), that if loads are applied at assigned uniform distances along a smooth line, a leak load Γ will produce the same equivalent attenuation-constant as a resistance load Σ in the conductor, if $\frac{\sigma}{\gamma} = r_0^2$; that is, if the resistance $1/\gamma$ of a semi-leak be a third proportional to the resistance σ of a semi-conductor-load, and the surge-resistance of the unloaded line. In other words, the attenuation-constant of the loaded line will be the same, whether the loads are inserted in series, or applied in derivation, provided that $\sigma : r_o :: r_o : 1/\gamma$. The surge-resistance of the loaded line will not, however, be the same in these two cases. The surge-resistance will be less with leaks than with series coils. The two values have the unloaded surge-resistance as their geometrical mean.

In all cases of direct-current lines, loads, either in series coils or in leaks, necessarily increase the attenuation-constant of the line. With alternating-current lines, this limitation is removed.

Summary of Conclusions.

Every artificial line composed of similar mid-leak sections, carrying either continuous or alternating currents in the steady state, may be reduced trigonometrically to its equivalent smooth line, and reciprocally. The resistance, current, and voltage at the various junctions and leaks along the line are simple hyperbolic functions of their angles.

Every smooth line in the steady state, carrying either continuous or alternating currents, may be externally completely replaced by one and only one T, or single-section mid-leak artificial line; or by one and only one Π, or single conductor with equal terminal leaks, and reciprocally. This proposition has numerous applications in telegraphy, telephony, power transmission, and distribution.

List of Symbols employed.

a = Attenuation-constant of a smooth line, or of an artificial line after being corrected for lumpiness (hyps. per km.).

a' = The uncorrected attenuation-constant of an artificial line, or the attenuation-constant of a smooth line after being loaded (hyps. per km.).

L = Total length of a line (kms.).

L_2 = A length of line, partial or total, measured from receiving end (kms.).

l = Length of a section of artificial or real line (kms.).

λ = Length of a semi-section of artificial or real line (kms.).

θ, λa = Hyperbolic angles of a semi-section of line (hyps.).

Θ = Total hyperbolic angle from far end (hyps.).

ϕ, ϕ' = Hyperbolic angles of a terminal load (hyps.).

la, La, L_2a = Hyperbolic angles of a section, or length of line (hyps.).

r = Linear conductor-resistance of a line (ohms per km.).

r' = Conductor resistance of a section of artificial line (ohms).

ρ, ρ' = Conductor resistance of a semi-section of artificial line (ohms).

R, R' = Resistance of a central leak in a section of artificial line or T (ohms).

Σ = Resistance of a series load in a line (ohms).

σ = Resistance of a semi-series load or of a single terminal load (ohms).

r'' = Conductor resistance of a Π (ohms).

R'' = Resistance of each leak of a Π (ohms).

r_o' = Nominal or apparent surge-resistance of an artificial line, uncorrected for lumpiness (ohms)

r_o = Surge-resistance of a smooth line, or of an artificial line corrected for lumpiness (ohms).

r_o'', r_o''' = Surge-resistances at receiving ends of terminally loaded lines (ohms).

R_f, R_g = Sending-end resistance of a line respectively freed and grounded at far end (ohms).

$R_{f\sigma}$, $R_{g\sigma}$ = Sending-end resistance of a line grounded at far end through terminal load (ohms).

R'_{nf}, R'_{ng} = Sending-end resistance of a line at nth leak, excluding same (ohms).

R'_{fn}, R'_{gn} = Sending-end resistance of a line at nth leak, including same (ohms).

R_l, $R_{l\sigma}$ = Receiving-end resistance of a line grounded at far end directly, or through terminal load (ohms).

e_m, e_n, e_o = Voltage at mth junction, nth junction and far end (volts).

ϵ_m, ϵ_n = Voltage at *m*th or *n*th leak (volts).
ϵ = Base of Napierian logarithms.
I_m, I_n, I_o = Currents in line at sending-end, *n*th junction, and far end (amperes).
i_m, i_n = Currents in *m*th and *n*th leaks (amperes).
g = Linear leakance of smooth line (mhos per km.).
g' = Conductance of central leak in a T or in a section of artificial line (mhos).
g'', $g_{\prime\prime}$ = Conductance of each leak in a Π (mhos).
m, n = Total number, and reference number, of section junctions in artificial line.
a, a' = Receiving instrument magnetic constants.
n_1 = Number of turns in receiving instrument windings.
F = Force, or torque, exerted by receiving electromagnetic instrument (dynes, or dyne-perp. cms.).
p = Numerical exponent.
Γ, γ = Load leaks, and semi-leaks (mhos).

APPENDIX.

The demonstrations of the various formulas appearing in the foregoing paper have been omitted in order to save space. Nearly all of these formulas are, however, based upon and derived from the following propositions:

(1) Any alternating continued fraction is expressible as a constant continued fraction. Thus to n stages:

$$\cfrac{1}{a+\cfrac{1}{b+\cfrac{1}{a+\cfrac{1}{b+}}}} = \frac{b}{\sqrt{ab}} \times \cfrac{1}{\sqrt{ab}+\cfrac{1}{\sqrt{ab}+\cfrac{1}{\sqrt{ab}+\cfrac{1}{\sqrt{ab}+}}}}$$

(2) Any constant continued fraction is expressible as a simple single fraction or ratio of a hyperbolic sine and cosine. Thus the nth convergent of

$$\cfrac{1}{c+\cfrac{1}{c+\cfrac{1}{c}}} = \frac{\sinh n\theta}{\cosh\,(n+1)\,\theta} \text{ if } n \text{ is even; or}$$

$$= \frac{\cosh n\theta}{\sinh\,(n+1)\,\theta} \quad \text{if } n \text{ is odd,}$$

where

$$\theta = \sinh^{-1}\left(\frac{c}{2}\right).$$

(3) Any terminally loaded constant-continued fraction is expressible as a simple fraction or ratio of hyperbolic sine and cosine. Thus the nth ascending convergent of

$$\cfrac{1}{c+\cfrac{1}{c+\cfrac{1}{c+m}}} = \frac{\sinh\,(n\theta+\phi)}{\cosh\,[(n+1)\,\theta+\phi]} \quad \text{if } n \text{ is even; or}$$

$$= \frac{\cosh\,(n\theta+\phi)}{\sinh\,[(n+1)\,\theta+\phi]} \quad \text{if } n \text{ is odd;}$$

where ϕ is an auxiliary hyperbolic angle.

(4) The sending-end resistance of any artificial line composed of similar sections, whether the leaks are in the middle or not, may always be expressed as a terminally loaded alternating continued fraction.

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CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION
FOR RESEARCH. — No. 14.

THE EFFECT OF ALKALOIDS ON THE EARLY DEVELOPMENT OF TOXOPNEUSTES VARIEGATUS.

By Sergius Morgulis.

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THE EFFECT OF ALKALOIDS ON THE EARLY DEVELOPMENT OF TOXOPNEUSTES VARIEGATUS.[1]

By Sergius Morgulis.

Presented by E. L. Mark. Received October 1, 1908.

It was found by Mathews (:**01**) that upon adding small quantities of pilocarpine hydrochloride to the sea-water the process of development could be hastened and abnormally large embryos produced, while the addition of atropine sulphate resulted in a slowing of the developmental process and in the production of dwarf embryos. The effect, according to this author, is especially well marked on the developing eggs of the star-fish, Asterias Forbesii; Torald Sollman (:**04**), whose work upon the influence of atropine and pilocarpine on the development of star-fish and sea-urchin eggs was done apparently under Mathews' direction, maintains (p. 355) that "the effects [i. e. acceleration or retardation] of the poisons were very similar on both Arbacia and Asterias."

A size above the normal is rather an unusual condition, and it therefore seemed highly desirable to find out in what relation the overgrown lavae stand to the normal ones, especially from a cytological point of view, as such knowledge might contribute something to the understanding of the general problem of growth. The work the results of which are given in the present paper was undertaken originally for the purpose of studying the cellular nature of the larvae, both those larger and those smaller than the normal ones, as well as to test the influence of other alkaloids upon the rate of growth and the size of developing embryos.

Although my experiments have not yielded the anticipated results, a brief statement of the work may not be without interest. The experiments were all made on eggs of Toxopneustes variegatus.[2] Obviously the first point to be determined was the size relations of the larvae developing in various alkaloid solutions. As the outcome has shown no marked influence in the direction of either an increase or a decrease in size, it is clear that no basis for a cytological study presented itself.

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This work was done at the Bermuda Biological Station during the past summer, and it gives me pleasure to express my thankfulness to Dr. E. L. Mark, the Director of the Station, for the many courtesies extended me while there.

EXPERIMENTS WITH ATROPINE SULPHATE AND PILOCARPINE HYDROCHLORIDE.

Bearing in mind the fact that the developing echinoderm eggs are very sensitive to changes in their environment, and are more or less easily affected by external conditions, a few special precautions were taken in carrying out the experiments. The eggs to be fertilized were kept in finger bowls containing sea-water about an inch deep, and were mixed with very small quantities of the spermatic fluid. In this way there was eliminated a possible disturbing factor due to the disintegration of superfluous sperms. The eggs were allowed to settle to the bottom of the dish and then transferred to finger bowls each containing 300 c.c. of the solution to be tried. To insure also an equal distribution of eggs among the several dishes, thus avoiding another possible source of error, the same number of drops of fluid containing the fertilized eggs was added, by means of a pipette, to each dish. It goes without saying that each experiment had its own control, and that the eggs of the same individual were used in both experiment and control.

The eggs were examined at intervals, and outline camera drawings were made of the developing larvae. If any differences in the larvae of a set were observed, drawings were prepared of each type separately. Besides, the drawing of each larva was compared with a few other larvae, so that every drawing was representative of a number of larvae. These drawings served later for reference, and also for a comparison of the sizes attained by embryos in different solutions.

Measurements, wherever such are given, were made on the drawings, the numbers indicating the full length of the drawing in millimeters, and though not giving the actual size of larvae, offer a basis for comparison of the larvae with one another.

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[2] Toxopneustes variegatus is found in great abundance in Bermuda, and its eggs may be easily obtained, according to the writer's observation, from about the middle of May till the middle of August, June and July being the most favorable period.

Solutions of atropine and pilocarpine of a very weak concentration (about 1 : 60000) exert no influence whatever upon the developing eggs, neither during the cleavage stages nor later when the larval stage is reached. But with the increase of concentration of those solutions their effects become pronounced, the necessary strength, however, being different for the two reagents. Definite results may be obtained with atropine by adding ½ c.c. of a 0.5 per cent aqueous solution to 100 c.c. of sea-water (1 : 40000), while pilocarpine in the same concentration does not produce any noticeable influence. In no case, except when the concentration of the atropine or pilocarpine was strong enough to injure the eggs, has there been any influence produced upon the developing eggs during segmentation; the effect was shown only in stages involving the transformation of the gastrula to a pluteus and in those following it.

The larvae developing in atropine solutions of the strength indicated are invariably smaller than the normal ones. The pilocarpinized larvae, when they develop in sea-water to which there has been added from 1 c.c. to 2 c.c. of a 0.5 per cent aqueous solution of pilocarpine per each 100 c.c. of sea-water (1 : 10000 or 20000), are also smaller than normal ones; but in weaker solutions, those containing from 0.5 c.c. to 1 c.c. of the 0.5 per cent pilocarpine solution to every 100 c.c. of sea-water, the larvae may be either quite normal, so far as size is concerned, or they may vary from the normal, being either slightly larger or slightly smaller. The following two Tables (I, II), presenting the notes of two experiments started at the same time, but with eggs of different animals, well illustrate this point.

From these tables it will be seen that cleavage is not in the least affected by any of the three different strengths of atropine and pilocarpine used. But the influence became apparent on the next day, when the surface of the water of control dishes was teaming with plutei, while in atropine the young were still in the gastrula stage, or just beginning to change to plutei, and very few were swimming at the surface. It will also be seen that in one case the pilocarpinized embryos are slightly larger than the normal ones, while in the other set of experiments they are smaller by just as much. In addition to the fact that the differences in the size-relations of the embryos are quite insignificant, the fact that those differences are not of a constant nature indicates that they are chance variations rather than the result of the action of pilocarpine.

TABLE I.

Atropine Sulphate, 0.5% aq. sol. Pilocarpine Hydrochloride, 0.5 % aq. sol.

Eggs fertilized.	Time of observation.	No. of c.c. of sol. added to 100 c.c. of sea-water.			Control.	No. of c.c. of the sol. added to 100 c.c. of sea-water.		
		1.5 c.c.	1 c.c.	0.5 c.c.		1.5 c.c.	1. c.c.	0.5 c.c.
Toxopneustes variegatus. July 16, 10.30 A.M.	July 16, 12.00	4- and 8-cell stages.	4- and 8-cell stages.	4- and 8-cell stages.	4- and 8-cell stages.	4- and 8-cell stages.	4- and 8-cell stages.	4- and 8-cell stages.
	July 17, 9 A. M.	Gastrulae, very few at the surface.	Gastrulae, calcareous rods well developed.	Gastrulae changing to Plutei	Some are changing to Plutei, but most have the arms well developed. Meas. 24.	Gastrulae; some are beginning to change over to Plutei. Small Plutei (meas. 22) are very rare.	Gastrulae changing to Plutei and small Plutei much like those in the control.	
	July 18, 9 A. M.	Dead.	Gastrulae. None are found at the surface.	Small Plutei.	Large Plutei. Meas. 45.	Large Plutei, but smaller than the normal ones. Meas. 41.	Large Plutei. Meas. 45–47.	Large Plutei. Meas. 45–48.

TABLE II.

Atropine Sulphate, 0.5 % aq. sol. | Pilocarpine Hydrochl., 0.5% aq. sol.

Eggs fertilized.	Time of observation.	No. of c.c. of the sol. added to 100 c.c. of sea-water.			Control.	No. of c.c. of the sol. added to 100 c.c. of sea-water.		
		1.5 c.c.	1 c.c.	0.5 c.c.		1.5 c.c.	1 c.c.	0.5 c.c.
Toxopneustes variegatus. July 16, 10.30 A. M.	July 16, 12.30 P. M.	Second and third cleavages.				Second and third cleavages.		
	July 17, 9 A. M.	Gastrulae, free-swimming, but few at the surface.	Free-swimming gastrulae.	Changing to Plutei.	Small Plutei. Meas. 26.	Small Plutei. Meas. 25.	Changing to Plutei and small Plutei.	Small Plutei like the normal ones.
	July 18, 9 A. M.	Dead.	None at surface.	Small Plutei slightly smaller than those of the Control for July 17. Meas. 24.	Large Plutei. Meas. 46.	Plutei. Meas. 40–42.	Large Plutei. Meas. 44.	Large Plutei. Meas. 44.

TABLE III.

Eggs fertilized.	Time of observation.	Control.	Pilocarpine.		Atropine 0.5 c.c. to 100 c.c. sea-water.	Pilo. 0.5 c.c. Atrop. 0.5 c.c. to 100 c.c. sea-water.	Pilo. 1 c.c. Atrop. 0.5 c.c. to 100 c.c. of sea-water.
			1 c.c. to 100 c.c. sea-water.	0.5 c.c. to 100 c.c. sea-water.			
July 27, 5.15 P. M.	July 27, 6.45 P. M.	4- and 8-cell stages in all experiments.					
	July 28, 9 A. M.	Free-swimming gastrulae.					
	July 29, 9 A. M.	Large Plutei. Average meas. length, 44, width, 36.	Large Plutei. Average meas. length, 41, width, 25.	Large Plutei. Average meas. length, 44, width, 12.	Small Plutei. Average meas. length, 28, width, 16.	Small Plutei. Average meas. length, 29, width, 18.	Small Plutei. Average meas. length, 27, width, 16.
	July 30, 9 A. M.				Plutei. Average meas. length, 29, width, 16.	Plutei. Average meas. length, 37, width, 20.	Plutei. Average meas. length, 37, width, 23.

Since my results did not agree with those obtained from the similar investigations of my predecessors, and since they are derived from entirely different species, I repeated the experiments with atropine and pilocarpine a great many times, but always with the same result. Although a small reduction in the size of embryos did occur, there was no increase of size nor acceleration of the development under the influence of pilocarpine.

The suggestion has been made that atropine and pilocarpine respectively inhibit and accelerate the oxidizing processes going on in the cells, thus causing either a decrease or an increase in the size of the embryos. It might be expected, therefore, that a mixture of appropriate quantities of atropine and pilocarpine would neutralize each other's action. In none of my own trials have I succeeded in neutralizing their effects, but, as had been already observed by Sollman, the depressive action predominates, and the embryos show a greater tendency to die out in the mixture than in either atropine or pilocarpine alone. In all of my experiments the larvae developing in the mixture of atropine and pilocarpine were intermediate in size as compared with those developing in either of those solutions alone. This will be seen from Table III.

EXPERIMENTS WITH MORPHINE SULPHATE.

Eggs were placed in sea-water with various quantities of a 0.5 per cent aqueous solution of morphine sulphate soon after they had been fertilized. In none of the experiments, save those where the concentration proved directly injurious, has there been an influence exerted upon the developing eggs during the segmentation stages, the effect becoming apparent after the first day only. In sea-water with but $\frac{1}{10}$ to $\frac{1}{20}$ of 1 c.c. of the standard morphine solution (0.5 per cent) to each 100 c.c. the rate of development as well as the size of the larvae remained absolutely normal, but in concentration of $\frac{1}{6}$ to $\frac{1}{8}$ c.c. of the morphine solution to 100 c.c. of sea-water the size of the developing embryos suffers a slight, though noticeable reduction. The segmentation, however, is perfectly normal. With stronger solutions the effect becomes more pronounced, of course; and when 1 c.c. of the morphine solution is added to 100 c.c. of sea-water the effect is no longer limited to the size of the plutei, but is seen also in a general slowing of the developmental process. In solutions two and three times that strength cleavage is very much retarded and is quite abnormal. In the following Table (IV) are given the records pertaining to the concluding experiment with various strengths of the morphine solution.

TABLE IV.

MORPHINE SULPHATE, 0.5% aqueous solution.

Eggs fertilized.	Time of observation.	Control.	Number of c.c. added to every 100 c.c. of sea-water.						
			3 c.c.	2 c.c.	1 c.c.	½ c.c.	¼ c.c.	⅛ c.c.	$\frac{1}{16}$ c.c.
July 14, 5.15 P. M.	July 15, 9 A. M.	Free swimming gastrulae.	Many-cell stages. Abnormal. Dying out.	Same as 3 c.c.	Blastulae.	Gastrulae, some changing to Plutei.	Free swimming gastrulae.	Gastrulae.	Gastrulae.
	7 P. M.	Small Plutei.	Dead.	Dead.	Changing to Plutei.	Some are changing to Plutei. Small Plutei	Small Plutei.	Small Plutei, as in control.	Same as control.
	July 16, 9 A. M.	Large Plutei, 1⅓ the size of those of last evening.	Dead.	Dead.	Changing to Plutei.	Plutei, ¾ the normal size.	Plutei, slightly smaller than the normal ones.	Plutei, not so large as the control.	Large Plutei, normal size.

EXPERIMENTS WITH COCAINE HYDROCHLORIDE.

I have not performed as many experiments with cocaine as with some of the other alkaloids, but the effect of cocaine upon the size of larvae can be inferred even from the rather insufficient data at my disposal. As was also observed in the experiments with other reagents, the influence of cocaine is not revealed during the first day, segmentation going on normally. Even in sea-water containing 2 c.c. of the standard cocaine solution (0.5 per cent aqueous solution) to each 100 c.c. gastrulae may appear at the same time as in the control, and they are in all essentials normal. In sea-water with $\frac{1}{2}$ to 1 c.c. of the cocaine solution per 100 c.c. the plutei are invariably from $\frac{1}{8}$ to $\frac{1}{4}$ smaller than the normal ones. But in weaker concentrations ($\frac{1}{4}$ to $\frac{1}{8}$ c.c. of a 0.5 per cent solution of cocaine to 100 c.c. of sea-water), though the size of the plutei may be slightly reduced, there is considerable variation in size between the plutei of different lots of eggs. It may be assumed, however, that the limit of toxicity of the cocaine is probably $\frac{1}{8}$ to $\frac{1}{16}$ c.c. of a 0.5 per cent solution to 100 c.c. of sea-water.

EXPERIMENTS WITH STRYCHNINE.

The sulphate of strychnine was used in a 0.5 per cent aqueous solution. As in all foregoing experiments no effect has been observed upon segmenting eggs in sea-water to which from $\frac{1}{32}$ c.c. to 1 c.c. of the strychnine solution was added. The blastula stage is reached at the same time in all the several concentrations. But from this stage on the influence of the poison becomes quite pronounced in the stronger solutions, where fewer larvae come to the surface, and where also the process of gastrulation lags behind that of the control. The limit of toxicity of strychnine differs for eggs of different animals, but $\frac{1}{32}$ c.c. of the standard solution (0.5 per cent) diluted in 100 c.c. of sea-water is invariably ineffective. The plutei developed in various strychnine solutions ($\frac{1}{16}$ c.c. to 1 c.c.) are smaller than normal ones; the differences, however, are not constant, being greater or smaller in different sets of eggs, as was also the case in experiments with all other reagents. Table V contains the records of one of the experiments.

From this table it can be seen that as the strychnine solution reaches an effective concentration, it also causes a reduction of the size of the larvae, although the early stages in the development are not in the least modified.

TABLE V.

Strychnine Sulphate, 0.5% aqueous solution.

Eggs fertilized.	Time of observation.	Control.	Number of c.c. added to every 100 c.c. of sea-water.						
			2 c.c.	1 c.c.	$\frac{1}{2}$ c.c.	$\frac{1}{4}$ c.c.	$\frac{1}{8}$ c.c.	$\frac{1}{16}$ c.c.	$\frac{1}{32}$ c.c.
July 8, 4.45 P. M.	July 8, 8.15 P. M.	Blastulae.	2-cell stage.	Blastulae.	Blastulae.	Blastulae.	Blastulae.	Blastulae.	Normal.
	July 9, 9 A. M.	Gastrulae.	Dead.	Beginning of gastrulation.	Gastrulae, very few at surface.	Gastrulae, few at surface.	Gastrulae.	Gastrulae.	Normal.
	2 P. M.	Small Plutei. Meas. 24.	Dead.	Free-swimming gastrulae.	Small Plutei. Meas. 20.	Gastrulae.	Gastrulae, some changing to Plutei.		Normal.
	6.30 P. M.	Plutei, about one and one-half times the previous size.	Dead.	Gastrulae.	Small Plutei. Meas. 29–30.		Changing to Plutei.	Small Plutei, about $\frac{2}{3}$ of the normal size.	Normal.
	July 10, 9 A. M.	Plutei. Meas. 41.	Dead.	Changing to Plutei; also small Plutei about $\frac{3}{5}$ the normal size.	Plutei, smaller than the control.				Normal.

EXPERIMENTS WITH DIGITALIN AND QUININE.

As in the previously described experiments, five tenths per cent aqueous solutions of digitalin and of quinine sulphate have been used as standard solutions, and of these, various quantities of each were added to the sea-water. Each of these substances proved to be more toxic than the other alkaloids with which I experimented. The addition of from ½ c.c. to 1 c.c. of the digitalin solution to 100 c.c. of sea-water was sufficient to retard cleavage, and to produce various abnormalities in the segmentation process. (The eggs would divide into two unequal portions, from which other cells are budded off quite irregularly, so that after a time the whole egg is broken up into a mass of small and large fragments, of either round, oval, or triangular shape.) In sea-water with half that amount of digitalin (⅛ c.c. to ¼ c.c.) cleavage is also retarded, but no abnormalities are to be observed. In none of these dilutions of digitalin, however, can development proceed very far, rarely beyond the gastrula stage. But in still more dilute concentrations of digitalin, as when only one or two drops of the standard solution is added to 100 c.c. of the sea-water, the eggs develop more or less normally, differing with different lots of eggs, and reach the pluteus stage. But the plutei are as a rule smaller than those developed in pure sea-water. In Table VI are combined the data from two separate experiments to illustrate the above statement.

Quinine is likewise very injurious to the developing eggs in concentrations ranging from 1 c.c. to 2 c.c. of the standard solution in 100 c.c. of sea-water. The effect is shown in a retardation of the segmentation process, which is greater the stronger the solution. But when much smaller quantities of the quinine solution (¼ c.c. to ½ c.c.) are diluted in 100 c.c. of sea-water, the segmentation of the eggs is normal and unchecked. In none of the solutions do the eggs develop very far, but the stage reached in the various solutions is inversely proportional to the concentration; while in a concentration 1 : 80000 (¼ c.c. to 100 c.c. of sea-water) the eggs may develop up to the gastrula stage, they probably never go beyond the 8-cell stage in a concentration 1 : 10000. Table VII reproduces the record of one of the experiments.

Unfortunately, lack of time did not permit me to complete the experiments with quinine and to determine the limit of toxicity of this alkaloid and its effect upon the size of the developing larvae. It does not seem to me improbable, however, that, as in the previous experiments, the size of embryos would have been reduced in quinine solutions in which their development was possible.

TABLE VI.

DIGITALIN, 0.5% aqueous solution.

Eggs fertilized.	Time of observation.	Control.	Volume of solution added to 100 c.c. of sea-water.					
			1 c.c.	½ c.c.	¼ c. c.	⅛ c.c.	2 Drops.[1]	1 Drop.[1]
July 3, 4.45 P. M.	July 3, 7 P. M.	16-cell stage.	Eggs not yet divided. Few in first cleavage.	Cleavage abnormal. 2-cell stages common. 4-and 8-cell stages are occasionally found.	4-cell stages common. Third cleavage in progress.	8-cell stages.	.	
	8.15 P. M.	Blastulae.	. . .	. . .	. . .	. . .	Blastulae (in some experiments not yet formed).	Blastulae.
	July 4, 9 A. M.	Free-swimming gastrulae.	Cleavage very abnormal. No definite stages.	Same.	. . .	. . .	Beginning of gastrulation.	Beginning of gastrulation.
	July 5, 6.45 P. M.	Large Plutei. Aver. meas. 43.	Dead.	Dead.	. . .	. . .	Small Plutei, about ½ the normal size.	Plutei, about ⅛ smaller than the normal.
	July 6, 9 A. M.	Large Plutei. Meas. 49.	Dead.	Dead.	. . .	. . .	Few alive. Large Plutei, meas. 39.	Plutei, the largest meas. 42.

[1] The results of the last two columns are based on a different experiment from that of the preceding columns.

TABLE VII.

QUININE SULPHATE, 0.5% aqueous solution.

Eggs fertilized.	Time of observation.	Number of c.c. of the Quinine solution added to every 100 c.c. of sea-water.					
		Control.	2 c.c.	1.5 c.c.	1 c.c.	0.5 c.c.	0.25 c.c.
Toxopneustes variegatus. July 31, 4.45 P. M.	July 31, 8.30 P. M.	Advanced stages of segmentation. Segmentation cavity present.	Most of the eggs are still unsegmented. Very few 2-cell stages.	2- and 4-cell stages are found. Two-cell stages are more abundant.	2- and 4-cell stages are common. 8-cell stages are also frequently found.	Advanced stages of segmentation. Segmentation cavity is seen.	Segmentation far advanced and normal.
	Aug. 1, 10 A. M.	Numerous free-swimming gastrulae at the surface. Calcareous rods well developed.	None of the developing eggs is beyond the 4-cell stage. Most eggs are dead. None swim at the surface.	None are found at the surface. No further advance in segmentation. Dying.	Cleavage did not probably proceed beyond the 16- or 32-cell stage. None at surface. Dying.	None at surface. Larvae began to die out after the blastula stage was reached.	Few blastulae swimming at the surface.
	8.30 P. M.	Large Plutei. Arms well developed. All alive.	Dead.	Dead.	Dead.	Dead.	Dead.

SUMMARY.

From the facts obtained in the foregoing experiments it may be inferred that alkaloids, such as atropine, pilocarpine, morphine, digitalin, strychnine or quinine, when present in sea-water in very small quantities, produce no influence upon the developing eggs of Toxopneustes variegatus, and become effective only with the approach to a certain concentration, which is different for the different alkaloids and also for different lots of eggs. The length of time from the moment the eggs are subjected to the influence of these alkaloids till the effect becomes noticeable differs, of course, for the various alkaloids, but as a rule the stronger the solution the earlier in the developmental process does its effect become pronounced. In the weaker solutions the effect is seen only in later stages, the earlier stages (segmentation) remaining unaffected. The increasing influence is not due to a gradual concentration of the originally weak solution through evaporation of the water, as was determined by measuring the volume of water in dishes before and after the experiment, but seems rather to be the result of accumulated effects due to a prolonged action of the poison. In solutions which were effective and yet not sufficiently strong to check noticeably the development of the eggs in the earlier stages, the larvae as a rule were smaller than the normal ones.

Pilocarpine does not hasten the development of eggs of Toxopneustes variegatus, and larvae developing in pilocarpine solutions are either of the normal size or else they are smaller than the normal ones, depending upon the strength of the solution and the lot of eggs. Pilocarpine and atropine mixed in various proportions do not, in my experience, neutralize each other's action, but the depressing effect predominates.

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THE PREFACE OF VITRUVIUS.

By Morris H. Morgan.

THE PREFACE OF VITRUVIUS

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THAT the Latin treatise on architecture, extant under the name of Vitruvius in manuscripts of the ninth, tenth, eleventh, twelfth, and fifteenth centuries, is a genuine work, and that it was first published in the earlier half of the Augustan age,[1] are two propositions which ought no longer to be doubted. The theory that it is a forgery of the third, fourth, or even of a later century, — a theory propounded originally by Schultz[2] and supported much later by Ussing,[3] — has never been seriously entertained by many scholars, and it has been recently refuted on the grounds both of subject matter[4] and of language.[5] The ascription of the work to the time of the Emperor Titus is a much older idea. Suggested at first, apparently, in the seventeenth century,[6] it was discussed but rejected by the Spanish translator Ortiz;[7] it was supported by the English translator Newton[8] towards the end of the eighteenth, and it has been revived at the beginning of the twentieth century in a series of learned articles by M. Victor Mortet.[9] But what Degering has said[10] of the arguments of the last of these scholars applies equally well to the arguments of them all; many, taken by themselves, may show that our Vitruvius might possibly have been written in the Flavian period, but not one of them shows that it must have been written at that time, and none of them show that it could not have been written in the Augustan age.

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[1] Cf. Degering, Berl. Phil. Woch., **27**, 1292 ff. (1907), and Morgan, Harvard Stud. in Cl. Philol., **17**, 9 ff. (1906). After the printing of this article had begun, I received L. Sontheimer's dissertation, Vitruvius und seine Zeit., Tübingen, 1908. I have added a few remarks upon it in footnotes 13, 18, 49, and 51.

[2] First in his letter to Goethe in 1829, published in Rhein. Mus., **4**, 329 (1836) ; reprinted by his son, together with a much longer argument in Untersuchung über das Zeitalter des . . . Vitruvius, Leipzig, 1856.

[3] In Danish in 1896; more fully in English: Observations on Vitruvius, published in London by the Royal Institute of British Architects, in 1898.

[4] See especially Degering, Rhein. Mus., **57**, 8 ff. (1902); Krohn, Berl. Phil. Woch., **17**, 773 ff. (1897); and Schmidt, Bursian's Jahresbericht, **108**, 118 ff. (1901).

[5] Morgan, Language of Vitruvius, These Proceedings, **41**, 467 ff. (1906); cf. Hey in Archiv f. Lat. Lex., **15**, 287 ff. (1907); Degering, Berl. Phil. Woch., **27**, 1566 ff. (1907); Nohl, Woch. Kl. Phil., **23**, 1252 ff. (1906).

[6] See Perrault's Vitruve, ed. 1673, note to Vitr., 1 pr. 1.

[7] Madrid, 1787, preface.

[8] London, 1791, Vol. 1, p. ix.

On the other hand, strong evidence is not wanting that this work was produced early in the Augustan age, and that it could not have been produced later. Some of this evidence I have myself offered;[11] more is to be found in the writers whom I have already cited; and some new evidence I may present upon another occasion.

But in spite of it all, the preface which stands at the very opening of the work seems at first thought to contain words and ideas which belong only to a time when the Roman Empire had been established for a considerable period and when more than one emperor had already occupied the throne. In translations into modern languages, as well as in such commentaries as those of Newton, Schultz, Ussing, and Mortet, these words and ideas are so represented or expounded that the difficulty of applying them to an earlier age has seemed well-nigh insuperable to many scholars, and not merely to those who are approaching the critical study of Vitruvius for the first time. If, however, we are convinced that the earlier part of the Augustan age is a date which suits the rest of the work, it is obvious that this difficulty cannot be insuperable. To solve it we must rid ourselves of all those shades of meaning in language and all those novelties of thought which were imperial growths, and we must ask ourselves at every point whether the words and ideas in question are such as might well have been used by one who was brought up under the Republic and who wrote soon after its fall. If they are such, we must explain and translate them accordingly, and so the difficulty will disappear. In the present article, therefore, I propose to comment upon the preface line by line, and then to give an English translation of it. Having been engaged during the past six or seven years upon a translation (still unfinished) of the whole of Vitruvius, I have often had occasion to think of the points in question, and so perhaps I am not unqualified to deal with them. At the same time I am submitting a specimen of my methods to the criticism of scholars, for I do not intend to be so diffuse in my commentary when I come to publish my translation.

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[9] Rev. Archéologique, Ser. III, **41,** 39 ff. (1902); Ser. IV, **3,** 222 ff., 382 ff. (1904); **4,** 265 ff. (1904); **8,** 268 ff. (1906); **9,** 75 ff. (1907); **10,** 277 ff. (1907); **11,** 101 ff. (1908). These articles contain much useful material for the study of Vitruvius.

[10] Berl. Phil. Woch., ib., 1468.

[11] Harvard Studies, **17,** 9 ff. (1906). But M. Mortet (Rev. Phil., **31,** 66 (1907)) has rightly observed that nothing can be proved from Vitr. 243, 18, which I had quoted as evidence that Vitruvius could not have written after 22 B. C. For we do not know that Vitruvius was speaking only of the city of Rome in this passage. In the municipalities, aediles continued to serve as *curatores ludorum* long after praetors superseded them in Rome.

For the convenience of readers of this article, I begin by printing the Latin text from Rose's second edition, setting in the margin the page and line of his first edition, to which commentaries always now refer.

TEXT.

Cum divina tua mens et numen, imperator Caesar, imperio **P.1,** 1
potiretur orbis terrarum invictaque virtute cunctis hostibus
stratis, triumpho victoriaque tua cives gloriarentur et
gentes omnes subactae tuum spectarent nutum populusque
Romanus et senatus liberatus timore amplissimis tuis cogitationibus 5
consiliisque gubernaretur, non audebam, tantis occupationibus,
de architectura scripta et magnis cogitationibus
explicata edere, metuens ne non apto tempore interpellans
subirem tui animi offensionem. cum vero attenderem te non
solum de vita communi omnium curam publicaeque rei constitutione 10
habere sed etiam de opportunitate publicorum aedificiorum,
ut civitas per te non solum provinciis esset aucta,
verum etiam ut maiestas imperii publicorum aedificiorum
egregias haberet auctoritates, non putavi praetermittendum
quin primo quoque tempore de his rebus ea tibi ederem. ideo 15
quod primum parenti tuo de eo fueram notus et eius virtutis
studiosus. cum autem concilium caelestium in sedibus immortalitatis
P.2, 1 eum dedicavisset et imperium parentis in tuam
potestatem transtulisset, idem studium meum in eius memoria
permanens in te contulit favorem. itaque cum M. Aurelio
et P. Minidio et Gn. Cornelio ad apparationem ballistarum
5 et scorpionum reliquorumque tormentorum refectionem fui
praesto et cum eis commoda accepi. quae cum primo mihi
tribuisti, recognitionem per sororis commendationem servasti.
cum ergo eo beneficio essem obligatus ut ad exitum vitae
non haberem inopiae timorem, haec tibi scribere coepi quod

animadverti multa te aedificavisse et nunc aedificare, reliquo quoque tempore et publicorum et privatorum aedificiorum pro amplitudine rerum gestarum ut posteris memoriae tradantur curam habiturum. conscripsi praescriptiones terminatas, ut eas attendens et ante facta et futura qualia sint opera per te posses nota habere. namque his voluminibus aperui omnes disciplinae rationes.

COMMENTARY.

1. *divina tua mens et numen:* "your divine intelligence and will." It may be asked whether a writer of the earlier Augustan period would speak of or to the ruler in such language.[12] But the use of the adjective *divinus* and the substantive *numen* does not necessarily convey imperial ideas of deification or of the "divinity that doth hedge a king." In fact both words are applied to living Romans in republican Latin. Thus Cicero, speaking to Julius Caesar face to face, used the phrase *tua divina virtus* (Marc. 26); of Pompey he has *homo divina quadam mente* (Mil. 21), and *Pompei divino consilio* (Imp. P. 10); he speaks of the ancestors of the Romans as *homines divina mente et consilio praeditos* (L. A. 2, 90), and calls Marius and Africanus each a *divinum hominem* (Sest. 50; Arch. 16; Mur. 75). They were then dead, but to the living Octavian he was still more complimentary; cf. Phil. 5, 43, *hunc divinum adulescentem;* 13, 19, *Caesaris incredibilis ac divina virtus;* 5, 23, *C. Caesar divina animi magnitudine;* 3, 3, *adulescens, paene potius puer, incredibili ac divina quadam mente atque virtute.* And he does not withhold the adjective, with a celestial addition, from the men of certain legions when he says *caelestis divinasque legiones* (Phil. 5, 28). As for *numen,* that it does not necessarily imply actual deification or imperial ideas is clear from Cicero again, as where he is speaking to the Roman people: *numen vestrum aeque mihi grave et sanctum ac deorum immortalium in omni vita futurum* (Post Red. 18, cf. 25, *cum vobis qui apud me deorum immortalium vim et numen tenetis*); and similarly Phil. 3, 32, *magna vis est, magnum numen unum et idem sentientis senatus.* In these passages *numen* implies no more than in Lucretius, 3, 144, *cetera pars animae . . . ad numen mentis momenque movetur.* It means no more than "will," although it is a very strong word to

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[12] See Wölfflin in Archiv. für Lat. Lex., **10**, 301 (1896), where in commenting on Ussing's first article he says: "Beispielweise muss man zu bestimmen suchen ob der Vf., wenn er unter Augustus lebte, der Kaiser in der Vorrede anreden konnte mit der Worte *divina tua mens et numen.*

use in that sense; cf. Paul. Fest. 172, *numen quasi nutus dei ac potestas.* In view of all this a writer of the earlier part of the Augustan age may well have applied *divina mens et numen* to the all-powerful ruler, and we need not here raise the question whether he was already receiving divine worship. In another passage (233, 4) Vitruvius uses the phrase *divina mens* of the intelligence of learned men who could predict changes in the weather; he has it also four times referring to "divine Providence" (138, 10; 184, 17; 218, 19; 231, 18); and the adjective *divinus* is applied to qualities of the gods in two other places (185, 7; 245, 6). He does not use the word *numen* except in our passage.

imperator Caesar: Here two questions come up for consideration: (1) whether Augustus, after he had received that name, was addressed by any other; (2) whether there is any English word by which *imperator* in this passage can be properly translated. As for the first question, it is generally believed that Vitruvius was aware that the name Augustus [13] had been bestowed, and this leads Ussing [14] to assert that an inferior like Vitruvius could not have avoided addressing him by that name. To this it might be rejoined that perhaps the use of the name did not at once become common, and that the absence of it here in Vitruvius points to a date soon after the name was conferred in 27 B. C. But we need not have recourse to this argument; for what are the facts about the use of this name by persons who were speaking or writing to Augustus and employing, as Vitruvius does, the vocative case? The answer is that we know very little about the matter,[15] for we have very little evidence upon which to base a conclusion. We know that Valerius Messala once addressed him in the Senate with the words *Caesar Auguste* (Suet. Aug. 58). We find *Auguste* once in Horace in a formal public ode (4, 14, 3), but *Caesar* in an ode equally formal and public, and published at the same time as the other (4, 15, 4). In view of this, what is to be thought of Ussing's contention that in one of his Epistles (2, 1, 4) Horace as an intimate friend may quite suitably use *Caesar*, his family name? If we turn to Propertius, we find *Auguste* twice (3, 10, 15; 5, 6, 38), and never *Caesar* in the vocative. This might seem to support Ussing's theory. But we must not forget Ovid. In the longest poem of the Tristia he has *Auguste* once (2, 509), but *Caesar* in the vocative five times (27; 209; 323; 551; 560). He uses *Auguste* in only one other passage in his works (M. 1, 204), but he has *Caesar* in the vocative seven times besides those already mentioned in the Tristia (F. 2, 637; Tr. 3, 1, 78; 5, 5, 61, all three in prayers, which are formal things; Tr. 4, 2, 47; 5, 11, 23; P. 2, 7, 67; 4, 9, 128). This is all the evidence that I have been able to find.[16] It is little enough, and it includes only one prose example, but we must remember how small is the amount of Augustan prose that has survived to us. In view of it all, we are not entitled to criticise Vitruvius for using *Caesar* instead of *Auguste*. Elsewhere he addresses his patron six times with the vocative *Caesar* (11, 1; 83, 18; 104, 22; 133, 6; 158, 8; 218, 13), and five times with the vocative *imperator* (32, 22; 64, 16; 83, 13; 103, 1; 243, 19). In our preface he combines the two in *imperator Caesar*. His patron had been an *imperator* ever since 43 or 42 B. C. (cf. Cic. Phil. 14, 28, and 37; CIL. 9, 2142), and long after the name *Augustus* was given to him his inscriptions regularly begin with the words *imperator Caesar*. It seems perfectly natural that he should be addressed in this way by one who had served in the army. But can the word *imperator* as thus used be translated into English? I think not. If we employ "emperor," it carries with it later Roman and modern ideas. And even if it did not, "emperor Caesar" in the vocative is not idiomatic English. Nobody would say "Emperor William" to the Kaiser, though we use the phrase when we speak about him. The word "general" sometimes suits an *imperator* of the republican period, but by no means always, since its scope is too narrow. And to print "General Caesar" here would certainly be an absurdity. The word *imperator,* therefore, cannot be translated here, but must be transliterated like other Roman titles, such as "consul" and "praetor."

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[13] This belief rests on the usual interpretation of 107, 3, *pronai aedis Augusti,* where the name seems to be recognized. But Sontheimer (see above, note 1) holds that we have here merely the adjective *augusti* agreeing with *pronai,* and that consequently the phrase means something like "a majestic temple-pronaos." He thinks that there was no "temple" built at the rear of this pronaos, but that the structure consisted of a pronaos only, containing the tribunal. This theory is attractive, but I have not yet had time fully to weigh it. Some objections, which may not be insuperable, readily suggest themselves. But in this article I need only say that the disappearance of the name *Augusti* would strengthen my arguments in support of this preface as an early production. As for the reading *angusti,* found in cod. S. (in general, as Degering, Berl. Phil. Woch., **20,** 9 ff. (1900), has shown, of the same independent value as H and G), I cannot accept this reading in spite of Krohn (Berl. Phil. Woch., **17,** 781 (1897)). It is improbable that Vitruvius should have spoken of a temple here without naming the divinity to whom it was dedicated. Cod. H, which I have seen, and Cod. G, of which I have a photograph of this page, both have *augusti.* Cod. E does not contain the passage. The reading *angusti* is, however, found in several of the late manuscripts. In Florence I have seen it in Codd. Laur., 30, 11; 12; 13; also in Cod. XVII, 5, of the Bibl. Naz. Centrale (though here the corrector gives *augusti*); and in Venice in Cod. Marc. CCCCLXIII. Of these five manuscripts, the first three belong to the class of H (lacuna in 2, 18) and the other two to the class of G and S. On the other hand, Cod. Laur. 30, 10, which Degering (ibid.) says comes directly from S, has *augusti.* It does indeed belong to the class of G and S. In Rome I observed that Cod. Urb. 293, and also the Vallicellanus (both of the G and S class) have *augusti.*

[14] Observations, 10.

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[15] It has been briefly treated by Friedländer, S. G. **2**, 557 (sixth edition), but he does not include Ovid and Propertius in his examination.

[16] It may be interesting to note that Martial addresses the reigning emperor of his day as *Auguste* nine times and as *Caesar* fifty-one times; cf. Friedländer's edition, **2**, index, p. 371.

2. *imperio orbis terrarum:* "the right to command the world." There is nothing necessarily "imperial" in this expression, any more than in Ad Herenn. 4, 13, cited below on *imperium transtulisset* (2, 1); cf. Vitruvius, 138, 11, cited below on *potiretur.* And the word *imperium,* aside from its technical sense when applied to a high military official (cf. Cic. Phil. 5, 45, *demus imperium Caesari, sine quo res militaris administrari, teneri exercitus, bellum geri non potest*), had also the general meaning of "right to rule," "supreme power," from Plautus down. Cf. Plaut. Men. 1030, *iubeo hercle, siquid imperist in te mihi;* Caes. B. G. 7, 64, 8, *civitati imperium totius provinciae pollicetur;* Cic. Font. 12, *sub populi Romani imperium dicionemque ceciderunt.*

potiretur: "engaged in acquiring." This is a true imperfect in sense, as in 31, 7, *cum Alexander rerum potiretur,* though in 161, 13, *cum Demetrius Phalereus Athenis rerum potiretur,* it has no doubt a completed meaning. With *orbis terrarum imperium* it occurs also in 138, 11, *ita divina mens civitatem populi Romani egregia temperataque regionem conlocavit, uti orbis terrarum imperii potiretur.* True imperfects are also *gloriarentur* (line 3), *spectarent* (4), and *gubernaretur* (6) in our preface, like the main verb *audebam* (6). For such imperfect subjunctives combined with the imperfect indicative, where the *cum* clause, coincident in time, is circumstantial, cf. Vitr. 156, 26; 250, 16; 251, 14 and 21; 283, 9; Cic. D. N. 1, 59, *Zenonem cum Athenis essem, audiebam frequenter;* Fin. 2, 61, *Decius cum se devoveret, . . . cogitabat?* The circumstances to which Vitruvius refers are of course the struggle with Caesar's murderers, and then with Antony, ending with Actium, the conquest of Egypt, the days of formal triumphs in Rome, and the beginning of the rule of Octavian there. This passage shows that Vitruvius's work could not have been published before August 13–15 (the days of the triple triumph) in 29 B. C.

4. *tuum spectarent nutum:* "awaiting your nod," "your beck and call." Vitruvius has *nutus* elsewhere only in its literal sense (33, 22),

but this metaphorical sense is common enough in republican writers; cf. Cic. Parad. 5, 39, *quem nutum locupletis orbi senis non observat;* Q. F. 1, 1, 22, *tot urbes tot civitates unius hominis nutum intuentur.* The verb *specto,* though common in Vitruvius, is found only here in this particular sense but it may be paralleled from Cicero; cf. Verr. 2, 33, *cum iudex . . . voluntatem spectaret eius,* etc.; Q. F. 1, 1, 35, *non legem spectare censoriam;* RA. 22, *omnes in unum spectent.*

populusque Romanus et senatus: for this unusual order cf. Cic. Fam. 15, 2, 4; Sall. Jug. 41, 2, and Weissenborn on Liv. 7, 31, 10. Vitruvius has elsewhere the usual order (20, 17; 176, 17).

cogitationibus: "conceptions," so in Vitr. 34, 9; 103, 1; 161, 3; 216, 24. Somewhat similarly "ideas," 31, 7 and 23; 36, 9; 156, 1; "notions," 103, 20; "devices," 137, 12; 138, 9; 269, 9; other shades of meaning are "consideration," 215, 20; "reflection," 1, 7; 12, 4 and 5; "deliberation," 15, 2; "power of thought," 36, 4; 132, 11; and in the phrase *cogitatio scripturae,* 263, 9, like our "thread of the discourse." On Vitruvius's use of the plural of this and other abstracts I have written elsewhere.[17]

6. *tantis occupationibus:* "in view of your serious employments." The phrase may be either an ablative absolute (so with Rose's punctuation) or a dat. incommodi. With most commentators I take *occupationibus* as referring to Augustus, though Schneider refers it to Vitruvius.

7. *de architectura scripta et magnis cogitationibus explicata:* "my writings and long-considered ideas on architecture," or literally "things written and set forth with long reflection." For *cogitatio* in this sense, cf. 12, 5, *cogitatio est cura, studii plena et industriae vigilantiaeque, effectus propositi cum voluptate.* For *magnis,* "great," in the sense of "much," "long" (not "grand" or "important"), cf. 214, 7, *quod magno labore fabri normam facientes perducere possunt,* "the result which carpenters reach very laboriously with their squares." This is like the vulgar use shown in Bell. Hisp. 12, *magnum tempus consumpserunt;* cf. Justin, 11, 10, 14, *magno post tempore* (see Schmalz, Antibarbarus s. v. *magnus*). Somewhat similar are *magno negotio* in Caes. B. G. 5, 11, 2 (cf. Bell. Alex. 8), and *magna industria,* Sall. Hist. 4, 2 M. The phrase *de architectura . . . explicata* does not necessarily signify that Vitruvius's book was finished before the time indicated in the next sentence, and that it was merely slightly revised before being dedicated to his patron and published.[18] If there is any

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[17] Language of Vitruvius (cited above in Note 5), p. 473.

[18] This is the theory of Krohn, Berl. Phil. Woch., **17**, 773 f. (1897), and Dietrich, Quaestionum Vitr. Specimen, answered by Degering, Berl. Phil.

particular force beyond the natural logic of the Latin language to be attached to the perfect tenses of *scripta* and *explicata,* Vitruvius may refer merely to his preliminary collections and studies, and perhaps especially to what he elsewhere sometimes calls *commentarii,* — the notes and abstracts made by himself and other architects in the course of their professional studies: cf. 3, 17, *litteras architectum scire oportet uti commentariis memoriam firmiorem efficere possit;* 132, 27, *philologis et philotechnis rebus commentariorumque scripturis me delectans.* With regard to *magnis cogitationibus,* Ussing and Mortet [19] are troubled because they take *magnis* in the sense of "grand" or "lofty," and feel that Vitruvius would be presumptuous in applying much the same language to his own thoughts and to those of Augustus (cf. *amplissimis tuis cogitationibus* just above). Mortet therefore proposes to take *magnis cogitationibus* with *edere* in the same construction (presumably dative) as *tantis occupationibus,* and he translates as follows: "Je n'osais pas mettre au jour pour vous mes écrits sur l'architecture à cause de vos si grandes occupations, ni vous soumettre mes commentaires sur cet art, alors que vous avez de grands soucis de gouvernement." But strange as Vitruvius may often be in his methods of expressing himself, I know of no other passage in his whole work that is so distorted in arrangement as this one would be if we accept the explanation of Mortet, who indeed does not pretend to have found any parallel for it. His other explanation, that perhaps *et* before *magnis* means "even," is not happier nor is either explanation necessary.

10. *publicae rei constitutione:* "the establishment of public order"; cf. Cic. Marc. 27, *hic restat actus, in hoc elaborandum est, ut rem publicam constituas.*

11. *de opportunitate publicorum aedificiorum:* "public buildings intended for utilitarian purposes." Here *opportunitate* must be interpreted by Vitruvius's own definition of the word in 15, 9 ff: *publicorum autem distributiones sunt tres, e quibus est una defensionis, altera religionis, tertia opportunitatis. . . . Opportunitatis communium locorum ad usum publicum dispositio, uti portus fora porticus balineae theatra inambulationes ceteraque quae isdem rationibus in publicis locis designantur,* that is: "there are three classes of public buildings,

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Woch., **27,** 1372 (1907). Sontheimer (see above, note 1) revives it in a somewhat different form, holding that the work was ready in 32 B. C., but that publication was delayed until some time between August of the year 29 and January of the year 27, when it was published with the addition of the prefaces to the various books, but without any other additions.

[19] Rev. Arch., **41,** 46 (1902).

the first for defensive, the second for religious, and the third for utilitarian purposes. . . . Under utility, the provision of meeting places for public use, such as harbors, markets, colonnades, baths, theatres, promenades, and all other similar arrangements in public places." With this compare the use of the same word in 128, 22, and 134, 9.

12. *ut civitas . . . auctoritates:* "so that not only should the State have been enriched with provinces by your means, but that the greatness of its power might likewise be attended with distinguished authority in its public buildings." Here *civitas*, the main subject, is thrust forward, and *maiestas imperii*, "the greatness of its power," refers to it. This phrase does not mean "the majestic empire," nor does it necessarily convey any other idea inconsistent with republican times, for it is found in Cicero, R. A. 131, *Sullam, cum solus republicam gubernaret imperique maiestatem quam armis receperat, iam legibus confirmaret.* For another example of *maiestas* referring literally to size, cf. Vitr. 52, 18, *in ea autem maiestate urbis et civium infinita frequentia.*

provinciis esset aucta: If strictly interpreted, the completed tense *esset aucta* seems to show that the provinces had already been added, while the following *haberet* may indicate that the buildings were not yet finished. Egypt became a province in 30 B. C., and Cyprus in 27 B. C. while Moesia was at least an administrative district as early as 29 B. C.[20]

14. *auctoritates:* Here Mortet [21] has this note: "Vitruve revient à plusieurs reprises, à propos d'édifices, sur ce qu'il appelle des modèles d'architecture, *auctoritas, auctoritates aedificii,* c'est-à-dire conformes aux règles de l'art et aux meilleures traditions architectoniques (Voy. l'Index de Nohl, v° *auctoritas*)." That is to say, he would render *publicorum aedificiorum egregias auctoritates* by some such phrase as "unsurpassed models of public buildings." [22] But I have carefully examined all the occurrences cited in Nohl's Index, and do not find one in which the word means "a model" or "models." It occurs twenty times besides here. In nine, it is applied to scholars or architects or to their writings, and it signifies their "influence" or "authority" (2, 26; 3, 3; 11, 9; 62, 25; 63, 8; 103, 4 and 5; 173, 19; 218, 12). In one, it refers to the severe dignity of a certain kind

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[20] On all these, see Marquardt, Röm. Staatsverw.,^{2} I, pp. 439, 391, 302. The existence of Galatia and Pamphylia as provinces cannot be certified before 25 B. C. (Marquardt, ib., 358, 375).

[21] Rev. Arch., **41**, 58, n. 1 (1902).

[22] Marini in his note to the passage had already rendered the word by *exempla*, without citing any parallels.

of music (111, 18). In the other ten passages it refers to buildings, and denotes their dignity or imposing effect (e. g., 72, 22, *conservavit auctoritatem totius operis,* and cf. 12, 25; 72, 1; 73, 1; 81, 11; 107, 26; 154, 17; 161, 15; 162, 4; 175, 5). So Turnebus, Advers. 1195, 45, explains our passage by "*dignitates et pulchritudines.*"

non putavi: On this phrase I have already written elsewhere.[23] Schmalz in a private letter to me compares the Ciceronian use of *nego, nolo, veto* (Acad. 2, 121; Mur. 59; Off. 1, 30), where the negative idea does not really belong to the main verb.

15. *de his rebus ea:* "my writings on this theme." Here *ea* refers to *scripta et explicata* in line 7, though the identity should not be too closely pressed; nor should *his rebus* be thought of as referring only to *publicorum aedificiorum,* since it includes also the ideas expressed in *opportunitate* and *egregias auctoritates.* Hence it must be rendered generally, as I have suggested in the phrase "this theme."

ideo quod: For this phrase used at the beginning of a sentence like a particle of inference, cf. Vitr. 88, 21. I do not know any other exact parallel.

16. *parenti tuo:* i. e. Julius Caesar, here and two lines below, called the *parens* of the person to whom Vitruvius writes, while in 203, 13, the word *pater*[24] is used of him. But nothing is to be argued seriously from the different words,[25] since fortunately Augustus himself in the *Monumentum Ancyranum* calls his adopted father both *parens* (1, 10) and *pater* (2, 24; 3, 7; 4, 14). It may be convenient to assemble here the other passages in which Vitruvius refers to Julius Caesar. There are two of them. In one he calls him *divus Caesar* (59, 18); four lines further *imperator* (59, 22), and a little below simply *Caesar* (60, 4). In that passage he is relating an anecdote about a campaign in the Alps. In the other passage, where he is giving examples of pycnostyle temples, we find the clause *quemadmodum est divi Iulii et in Caesaris foro Veneris* (70, 18). Both these passages, therefore, like the words which follow in the preface which we are studying, show that Vitruvius

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[23] Language of Vitruvius, p. 487.

[24] Retaining, as I think we must, the reading *patre Caesare* (so Mortet, Rev. Arch., **41**, 69 (1902); Degering, Berl. Phil. Woch., **27**, 1468 (1907)), instead of Rose's emendation *patre Caesari.* The word *patre* is inserted here by Vitruvius for fear that readers should think he meant the living Caesar (Augustus); so Cicero, Phil., 5, 49, *utinam C. Caesari, patri dico, contigisset,* etc.; ib. 39, *Pompeio enim patre.*

[25] Though Degering (l. c.), arguing against Mortet's hypothesis, suggests that *parens* is a more appropriate term for the adoptive father and uncle of Augustus than for the actual father of Titus.

wrote after the deification of Julius, which took place by decree not long after his death (Plut. Caes. 67; cf. CIL. 1, 626; 9, 2628).

de eo: The singular *eo* is used rather loosely here after *ea* and *his rebus*, but "that thing" can mean nothing except architecture, so that there is no danger of confusion here any more than in Cic. Att. 9, 10, 10, *perlegi omnes tuas* (*litteras*) *et in eo acquievi.* As for the use of causal *de*, I have defended it against Ussing's strictures in another place.[26]

fueram notus: On this use of *fueram* with the pf. partc., see Landgraf, Hist. Gramm., Heft 1, 220 ff., who says that it is found ten times in Vitruvius against seven occurrences of the regular formation with *eram.*

eius virtutis studiosus: This awkwardness of the dependence of one genetive (*eius*) upon another (*virtutis*) is found elsewhere in Vitruvius: cf. *a leone transiens in virginem progrediensque ad sinum vestis eius* (227, 9); *timore eorum fortitudinis effectus*, "for fear of the effect of their courage" (three genitives! 5, 7). The expression "devoted to his *virtus*," though logically correct in Latin, means in idiomatic English, "devoted to him on account of his *virtus*," and in this way I have rendered it. In cod. S, cod. Estensis,[27] and in eight codd. of Marini, as well as in the Venetian edition of 1497, the word *erat* stands between *virtutis* and *studiosus.* If this meant anything, it would mean that Julius Caesar, "was interested in the excellence of architecture" (*eius* referring to *eo*, and cf. 64, 15, *nostrae scientiae virtutem*). But *studiosus* is resumed just below (2, 2) by *idem studium meum*, so that the reading *erat* hardly deserves further attention. The word *virtutis* in this clause is not to be confined to military valor (as in 1, 2), nor to moral worth, but is used in a much more general sense; hence I have rendered it by "great qualities."

17. *concilium caelestium:* cf. Cic. Off. 3, 25, *Herculem quem hominum fama in concilio caelestium collocavit.* But as Schneider notes: "satis dextre adulatur Octaviano Vitruvius, dum patrem non a Romanis inter deorum numerum relatum, sed ab ipso deorum concilio allectum et dedicatum fuisse ait." Vitruvius uses *caelestes* as a substantive again in 102, 22; cf. Cic. Phil. 4, 10.

Page 2, 1. *imperium parentis in tuam potestatem transtulisset:* "transferred your father's power to your hands." Here Mortet[28] has this observation: "La manière dont Vitruve parle de la translation de la dignité impériale appelle aussi une remarque qui n'est pas sans intérêt. Ce n'est pas à Auguste, pensons-nous avec W. Newton, que Vitruve aurait parlé d'une translation régulière de l'empire. Le langage de l'auteur de la Préface s'applique à une époque ou l'on était déjà habitué à des changements réguliers dans la première fonction de l'État: Auguste ne l'aurait point toléré pour des raisons politiques qu'il est facile de comprendre." But it is a pure assumption that Vitruvius is speaking of "a regular transmission of the empire," and the very use of the word "empire" in this connection is a part of the difficulty created, as I have suggested above, by modern commentators and not really existing in the Latin of Vitruvius. I have already pointed out (in my note on 1, 2) the republican meaning of *imperium*. Julius Caesar had *imperium*, and we know that Octavian received it in 43 or 42 B. C. (see on 1, 1). The language of our preface is therefore no more "imperial" than is the language of the unknown republican orator in Ad Herennium, 4, 13: *imperium orbis terrae . . . ad se transferre;* cf. Caes. B. G. 7, 63, 5, *ut ipsis summa imperi transdatur.* The verb *transfero* was the regular one to use of transfers of power; cf. Cic. L. A. 2, 54, *earum rerum omnium potestatem ad decemviros esse translatam;* Mur. 2, *cum omnis deorum immortalium potestas aut translata sit ad vos;* and Mon. Ancyr. 6, 15, *rempublicam ex mea potestate in senatus populique Romani arbitrium transtuli.* When we get down to Tacitus we do indeed find: *suscepere duo manipulares imperium populi Romani transferendum, et transtulerunt* (H. 1, 25). But there was nothing "regular" in this transfer!

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[26] Language of Vitruvius, p. 485.
[27] See Sola, Riv. d. Biblioteche, **11**, 35 ff. (1900).
[28] Rev. Arch., **41**, 47 (1902).

2. *idem studium meum in eius memoria permanens:* These words should not be separated with Mortet,[29] who punctuates thus: *idem studium meum, in eius memoria, permanens in te, contulit favorem,* and translates, "Le même zèle que j'avais de sons temps, subsistant envers vous, m'a apporté votre faveur." He compares 63, 12, *aeterna memoria ad posteritatem sunt permanentes.* But I believe that the idea which Vitruvius was struggling to express was this: "While Caesar was among us, I was devoted to his person; now that he is gone, my devotion continuing unchanged as I remembered him," etc. He expresses it obscurely, but for a somewhat similar use of *in memoria,* cf. Cic. Att. 9, 11 A, 3, *pius . . . in maximi beneficii memoria,* "loyal as I remember my extreme obligation"; and for the mere syntax of *permanens* with *in* and the ablative, cf. for instance Cic. Fam. 5, 2, 10, *ut in mea erga te voluntate permanerem,* and Quint. 3, 4, 4, *mihi in illa vetere persuasione permanenti.* Ussing[30] renders the phrase thus:

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[29] Rev. Arch., **41,** 49 (1902).

[30] Observations, p. 9.

"this ardor of mine in clinging to his memory"; but even if *in memoria* is really Latin in this sense (which may be doubted), it is surely not in accordance with the usage of Vitruvius. He has the word *memoria* sixteen times besides here. In six passages it denotes literally the faculty of memory (3, 18; 7, 23; 10, 10; 103, 22; 104, 11; 157, 12). In five, it refers to the future, — to the record which one is to leave for posterity, as in the phrase *posteris memoriae tradi* (cf. 2, 12; 4, 22; 63, 12; 155, 11 and 19). Once it means "fame" (63, 18); twice we have the common *nostra memoria*, "in our time" (162, 7; 251, 3), and once *post nostram memoriam* (218, 4).[31] Finally there is a peculiar usage of the plural, probably in the sense of "history" (217, 20). It is obvious that the idea of "remembering" and of "memory" in the literal sense is the prevalent meaning in Vitruvius, and so I have taken it in our passage.

3. *in te contulit favorem:* Schneider has this note: "Displicet in sermone Vitruvii *favor*, quem is transtulit ad filium, cum potius ex nostrorum hominum sensu petere ab Octaviano deberet, ut is in memoria patris permanens ad Vitruvium favorem transferret." And Ussing[32] translates: "This ardor of mine has transferred its favor to thee," and then he remarks upon the idea as "coarse and out of taste." These criticisms seem based upon a mistaken notion of the meaning of the Latin word *favor*. It is not at all a common word, particularly in republican Latin. It is not found in Ennius, Plautus, Terence, Caesar, or Nepos. Cooper[33] speaks of it as one of the seven substantives in *-or* that are found in Cicero and not in earlier writers. In its meaning it is very restricted; indeed, it is almost technical until well on in the imperial period, and the English word "favor" is consequently an exceedingly unfortunate one to employ in the translation of it. In republican and early imperial times it appears to be confined to the theatrical and political spheres, in which it denotes the "applause" or "support" which is given to an actor or to a politician by his well wishers. Cicero uses it only four times. In Rosc. Com. 29, speaking of the actor Panurgus, he says: *quam enim spem et expectationem, quod studium et quem favorem secum in scaenam attulit Panurgus, quod Rosci fuit discipulus. Qui diligebant hunc, illi favebant.* And in Sest. 115, in a passage where he is speaking of expressions of popular opinion at theatrical or other shows, we find: *qui rumore et, ut ipsi loquuntur, favore populi tenetur*

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[31] These last three occurrences really afford no support to Mortet's strange interpretation of *in eius memoria*.

[32] Observations, 9 f.

[33] Word Formation in the Sermo Plebeius, 25.

et ducitur. Here the use of the technical term *favore* is excused by *ut ipsi loquuntur.* And similarly in the very significant quotation by Quintilian (8, 3, 34) from a lost letter of Cicero's we have "*favorem*" *et* "*urbanum*" *Cicero nova credit. Nam et in epistula ad Brutum eum, inquit, amorem et eum, ut hoc verbo utar, favorem in consilium advocabo.* Obviously Cicero is here transferring the theatrical usage of the word to the political sphere.[34] And the same is true of the fourth passage in which he employs it, Legg. 2, 11, *quae* (*leges*) *sunt varie et ad tempus discriptae populis, favore magis quam re legum nomen tenent.* This same idea is found in the author who is the next to employ the word, Sallust: cf. J. 13, 7, *in gratiam et favorem nobilitatis;* J. 73, 4, *generis humilitas favorem addiderat* (said of Marius). So in Livy, who perhaps has the word only once, we find *regimen totius magistratus penes Appium erat favore plebis* (3, 33, 7). And finally I may cite Vell. Pat. 2, 54, 2, *ingens partium eius* (*Pompei*) *favor bellum excitaverat Africanum;* cf. also 2, 43, 3; 89, 1; 92, 4. In none of these authors is there anything like the condescending tone which is often implied by the English word "favor" or the German "Gunst," and which is what gives offence to Ussing and Schneider. But we may go further and observe that the same restricted interpretation will usually hold good in republican Latin for the related words *fautor* and *faveo.* The theatrical sense of *fautor* (in the form *favitor*) comes out very clearly three times in the prologue to the Amphitruo of Plautus (67; 78; 79).[35] It denotes a political supporter in Cic. Fam. 1, 9, 11, *cuius* (*Pompei*) *dignitatis ego ab adulescentia fautor;* cf. 10, 12, 5; Att. 1, 16, 11. In the orations of Cicero it occurs nine times in this sense: e. g., *nobilitatis fautor* (R. A. 16); *fautores Antoni* (Phil. 12, 2). So Sallust, H. 3, 88 (M.), *Pompeius . . . sermone fautorum similem fore se credens Alexandro;* cf. J. 15, 2, *fautores legatorum.* And Livy uses it in the sense of "partisans" in 1, 48, 2, *clamor ab utrisque fautoribus oritur.* The verb *faveo* occurs earlier than either *favor* or *fautor.* It is found in Naevius (ap. Non. 205, 27), but here we have not context enough to help us to its meaning. In another fragment (ap. Front. Ep. II, 10, p. 33 Nab.), which begins *regum filiis linguis faveant,* the verb seems already to convey the idea of "support." This comes out clearly in Ennius, Ann. 291 (Vahlen) *Romanis Iuno*

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[34] See Holden in his edition of Pro Sestio, 115, where he gives a note by Reid. And for further illustration cf. Hor. Ep. 2, 1, 9; C. 4, 8, 26; Verg. A. 5, 343.

[35] In two fragments of Lucilius we have not enough of the context to assure us of the exact meaning of the word. But see Marx on frag. 269 f., and cf. 902.

coepit placata favere; and the theatrical usage seems to me to appear in Ann. 419, *matronae moeros complent spectare faventes.* In Terence, Eun. 916, *illi faveo virgini* is said by a "supporter" (though not political) of the maiden in question, and in Andr. Prol. 24, *favete, adeste aequo animo,* we have again the theatrical meaning of "applaud." But when we reach the classical period, the political meaning is very prominent. Caesar uses the verb five times, and always in this sense: e. g., B. C. 2, 18, 6, *provinciam omnem Caesaris rebus favere cognoverat* (cf. 1, 7, 1; 1, 28, 1; B. G. 6, 7, 7; 1, 18, 8). See also Cicero, Fam. 12, 7, 1, *favebam et rei publicae, cui semper favi, et dignitati tuae* (cf. 10, 1, 3, and 3, 2; Att. 12, 49, 1). And in his orations, Cicero employs the verb some twenty-five times in this sense: [36] e. g., Sest. 21, *omnes boni semper nobilitati favemus;* cf. Planc. 18. Sallust uses *faveo* in the political sense in Cat. 17, 6, *iuventus pleraque Catilinae inceptis favebant;* cf. 48, 1; J. 85, 5. Finally I may cite Vell. Pat. 2, 26, 2, *faventis* (acc. pl.) *Sullae partibus.* In view of all this, I think that it should be granted that when Vitruvius uses the word in our passage,[37] he is thinking of this technical political sense. He had served under Julius Caesar and was devoted (*studiosus*) to him. When Caesar was gone, "my devotion, continuing unchanged as I remembered him (*idem studium meum in eius memoria permanens*), bestowed its support upon you (*in te contulit favorem*)." This is a literal translation of the passage. Vitruvius may take a clumsy way of saying "inclined me to support you," but certainly no statesman to-day or in antiquity would see anything coarse or out of taste in an author's recalling the fact that, at a critical period, he had lent that statesman his support. And this interpretation of the passage involves no distortion of the plain intent of the Latin; for the construction and meaning of *in te contulit favorem* is illustrated by Cic. Fam. 13, 50, 2, *in me officia et studia Brundisi contulisti;* cf. Att. 1, 1, 4; Fam. 10, 1, 3; 15, 2, 8.[38] The usage of Vitruvius himself offers us no exact parallel,[39] but many examples similar to those which I have cited are given in the new Thesaurus s. v. *confero* (184, 30–72) under the lemma "beneficia sim. in aliquem conferre." [40] There is, however, an entirely different interpretation of *in te contulit favorem* which should be mentioned here, although I consider it erroneous. It has the support of Newton, Gwilt, Reber, and Mortet. Newton translates: "procured me thy favor"; Gwilt: "has been the cause of your goodwill towards me"; Reber: "mir auch Deine Gunst erworben hat"; Mortet: "m'apporté votre faveur." It will be observed that these versions, all practically the same, are probably due in the first instance to that misconception of the meaning of the word *favorem* to which I have already referred. But even taking *favorem* in its correct sense and extending it a little so as to apply to Augustus's "support" of Vitruvius, I do not see how *in te contulit favorem* can mean "acquired" or "procured me thy support." There are some examples of the use of *confero* gathered in the Thesaurus (175, 16 ff.) under the lemma "iungendo efficere aliquid, componere, acquirere," but, after a careful examination of them, I do not find one which confirms that meaning here, and to adopt it would oblige us to take *te* as ablative, not accusative, which in this context seems impossible. Marini evidently felt this strongly, for he emended *in te* to *in me*. At first thought, the following *itaque* might seem logically to call for this interpretation. Perhaps it would, if *itaque fui praesto* must be rendered "hence I have been appointed" (Gwilt, cf. Terquem, p. 76); but there is nothing of this sort necessarily implied in *praesto*. Vitruvius merely says: "I became one of your supporters, and hence I was ready," etc.

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[36] In the theatrical sense he employs it (as well as the substantive *favor*) in R. C., 29, which I have already quoted (p. 162).

[37] He has it nowhere else, nor *faveo,* nor *fautor.*

[38] Mortet, Rev. Arch., **41,** 50 (1902), has this note: "La vraie forme classique serait ici *conciliavit* et l'on attendrait même plutôt à *attulit* qu'à *contulit.*" But the difference between *contulit* and *attulit* is excellently shown by Cic. Fam., 10, 5, 1, *itaque commemoratio tua paternae necessitudinis benevolentiaeque eius quam erga me a pueritia contulisses, ceterarumque rerum . . . incredibilem mihi laetitiam attulerunt.* However, Mortet is supporting a different translation for our passage, of which I shall speak later (p. 165).

[39] The nearest is 159, 12, *quibus felicitas maximum summumque contulit munus,* where we have the dative instead of *in* and the accusative. Else-

Aurelio . . . Minidio . . . Cornelio: These men cannot be identified with any persons otherwise known to us. The *nomina* Aurelius and Cornelius were of course common under the republic, but the *gens Minidia* is elsewhere known, so far as I am aware, only from a tombstone found at Ostia (CIL. 14, 1356), and presumably of the imperial period. There is no MS. evidence for the reading *Numisio* substituted in our passage by Schneider, Stratico, and some earlier editors in order to identify the colleague of Vitruvius with the architect of the theatre of Herculaneum (CIL. 10, 1446).

4. *ad apparationem . . . fui praesto:* For the meaning and the syntax

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where Vitruvius has the verb five times in the literal sense of "bring together" (33, 5; 43, 10; 158, 12; 168, 14; 280, 11); once meaning "compare" (157, 13); and once each in the common phrases *se conferre* (105, 26) and *sermonen conferre* (218, 7).

[40] Our passage is not included here, but is wrongly, as I believe, placed under the lemma "potestatem, honores, sim. deferre" (182, 30).

of *praesto* with *ad* and accusative, cf. Cic. Fam. 4, 8, 1, *ad omnia quae tui velint ita sim praesto;* Deiot. 24, *non solum ad hospitium sed ad periculum etiam atque ad aciem praesto fuit;* and for *ad* with the gerundive, Cic. Caec. 29. While Vitruvius does not distinctly say that he was appointed to any particular post in the army of Octavian, it is natural to think that he and the other three men whom he mentions were *praefecti fabrum.* The office of *praefectus fabrum* later became a very high one (something like that of engineer in chief to a great modern army), and among its duties was the supervision of those *qui arma, vehicula, ceteraque genera tormentorum vel nova facerent vel quassata repararent* (Veget. 2, 11), a passage the latter part of which recalls Vitruvius's description of the functions which he was ready to perform. But that such a functionary accompanied the smaller detached armies of the republic is clear from Cic. Fam. 3, 7, 4, *Q. Leptam, praefectum fabrum meum.* Sometimes there were more than one; cf. Caesar ap. Cic. Att. 9, 7, C, 2, *duo praefecti fabrum Pompei in meam potestatem venerunt.* Further information about such officers is given by Marquardt (Röm. Staatsv. 2, 516), and by Mommsen (Röm. Staatsrecht, 1, 120; 2, 98).

5. *refectionem:* Syntactically this word seems to belong only with *scorpionum reliquorumque tormentorum,* and therefore Vitruvius, strictly taken, does not say that he was ready to repair *ballistae,* or to supply *scorpiones* and other *tormenta.* But I can hardly believe that he was really such a specialist, and I fancy that in his eagerness to produce the fine example of chiastic order displayed in *apparationem . . . refectionem,* he overlooked the exact sense. Hence I have taken a liberty in my translation. Still it may be observed that in the tenth book (269, 10, *ipse faciundo*) Vitruvius speaks of his practical experience in constructing *ballistae* and that he does not say anywhere that he ever made other kinds of artillery. For *refectio* in the literal sense of "repair," cf. 140, 21, and Columella, 12, 3, 9; also in inscriptions, cf. Olcott, Studies in Word Formation, 28. For *apparatio,* cf. 54, 5; 124, 21; Cic. Off. 2, 56.

6. *commoda accepi:* To discover the meaning of the word *commoda* here is important, because upon it and the next two sentences is based the commonly accepted view that Vitruvius, when he wrote this preface, was in retirement, and some have gone so far as to translate *commoda* by "pension." I am not aware that its meaning has ever been thoroughly studied, and I do not find the word treated in the books on military antiquities. Let us therefore examine the different ways in which it is employed. Three may be distinguished. In the first place, *commoda* is used of the emoluments, allowances, or advantages which

civil or military officers, or certain public slaves, received while still in service or working. It is thus applied to a quaestor by Cicero, Red. in Sen. 35, *Plancius qui omnibus provincialibus ornamentis commodisque depositis totam suam quaesturam in me sustentando et conservando collocavit.* And again of a military tribune, Fam. 7, 8, 1, *sum admiratus cur tribunatus commoda, dempto praesertim labore militiae, contempseris* (in this case Caesar had apparently offered Trebatius a military tribuneship, with exemption from duties). Frontinus in his work on the Roman aqueducts describes (116 ff.) the two gangs of public slaves employed upon them; one was the *familia publica,* the other the *familia Caesaris.* Then he goes on (119): *commoda publicae familiae ex aerario dantur . . . Caesaris familia ex fisco accipit commoda.* Here the word *commoda* is not equivalent to our "wages," which are paid at regular short intervals, but it seems to denote an annual lump sum given to these public slaves every year.[41] And in the case of the quaestor and the tribune mentioned by Cicero, the word does not mean "pay," for we know that officials and officers of these and the higher ranks were not, in republican times, paid what we understand by salaries. Instead, they got free quarters and transport, rations, their outfit or a lump sum covering it (*vasarium*), certain rights of requisitioning for necessaries when in the provinces, and officers on the staff or in the employ of higher magistrates expected to receive from them, or from the treasury, good service rewards in the way of "gratifications" or free gifts (*congiaria, beneficia*) which also seem to have been paid annually in a lump sum.[42] It was "*chommoda*" of this or any other sort [43] for which Arrius was looking when he went out on the staff of Crassus to Syria (Catullus 84). In the second place, *commoda* is used in the sense of some form of gratuity presented to soldiers on their retirement from service. So in the letter of Brutus and Cassius to Antony (Cic. Fam. 11, 2, 3): *ea re denuntiatum esse veteranis quod de commodis eorum mense Iunio laturus esses;* and probably the word has this meaning in Cicero himself, L. A. 2, 54, *putant si quam spem in Cn. Pompeio exercitus habeat aut agrorum aut aliorum commodorum.* Suetonius certainly thus employs it several times: cf. Aug. 49, *quidquid autem ubique militum esset ad certam stipendiorum praemiorumque formulam adstrinxit, definitis pro gradu cuiusque et temporibus militiae et commodis*

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[41] Mommsen, Staatsrecht,^{3} 1, 323; cf. 299, n. 2.

[42] On all this see Mommsen, ib., 294–300, and on *commoda tribunatus,* 300, n. 4.

[43] No doubt it covered a good deal of what we now call "graft."

missionum; Cal. 44, *commoda emeritae militiae;* Nero 32, *commoda veteranorum;* Vit. 15, *veteranorum iustaeque militiae commoda.* See also an African inscription (CIL. 8, 792): *P. Ennius T. F. Epilli N. Quir. Paccianus commodis acceptis ex leg. II Aug. ab imp. Domitiano Caesare Aug. Ger. cos. VIII.* These gratuities, though not mentioned in the books on Roman military antiquities under the name *commoda,* do appear in such books under the name *praemia,* and this indeed is the term employed by Augustus in the Monumentum Ancyranum 3, 31 ff.: *militibus quos emeriteis stipendis in sua municipia remisi praemia numerato persolvi* (cf. also 3, 37). And Suetonius combines the two words in Aug. 24, *alias (legiones) immodeste missionem postulantes citra commoda emeritorum praemiorum exauctoravit* (cf. also Aug. 49, cited just above). There is no evidence that these *commoda* or *praemia* ever took the form of a stipend paid annually or at more frequent intervals like our military pensions. A lump sum paid at the time of discharge is what is meant by them,[44] and we know that Augustus gave 5000 denarii to praetorians and 3000 denarii to legionaries (Dio C. 55, 23; cf. Suet. Aug. 49, *certam praemiorum formulam,* more fully cited above). It is also well known that Augustus (at least in his earlier period) had distributed lands to retiring soldiers; cf. Mon. Anc. 1, 19, *iis omnibus agros aut pecuniam pro praediis dedi;* and Dio C. 54, 25, *διέταξε τά τε ἔτη ὅσα οἱ πολῖται στρατεύσοιντο, καὶ τὰ χρήματα ὅσα παυσάμενοι πῆς στρατείας, ἀντὶ τῆς χώρας ἣν ἀεί ποτε ᾔτουν, λήψοιντο.* This statement by Dio is made of the year 741 (13 B. C.), after which time Mommsen[45] thought that Augustus determined to recompense his discharged soldiers in money. Finally there is no evidence that *commoda* in this sense were given to retired officers of higher grades, though we may readily imagine that centurions and lower officers received them. We come now to the third usage of the word *commoda,* still somewhat technical, but approaching more closely to the very common general meaning of "advantages" than does either of the other two. In this usage it denotes special "privileges," and perhaps it does not occur in republican Latin. But it comes out in Suetonius, Aug. 31, *sacerdotum et numerum et dignitatem sed et commoda auxit, praecipue Vestalium virginum.* Such privileges might include public land or money.[46] In another place Suetonius himself makes clear what privileges he means; cf. Cl. 18 f., *naves mercaturae causa fabricantibus magna commoda constituit pro condicione cuiusque: civi vacationem legis Papiae Poppaeae, Latino ius Quiritium, feminis ius IIII liberorum.* Ovid seems to be aware of this sense of *commoda* when in his account of the rape of the Sabine women (A. A. 1, 131) he jestingly exclaims: *Romule, militibus scisti dare commoda solus! Haec mihi si dederis commoda, miles ero.* And Juvenal in his sixteenth satire speaks of the privileges of a military career (the civilian won't venture to strike the soldier whom *esprit de corps* protects; the soldier is not subject to the delays of law courts; he can make a will while his father is alive), and he calls these privileges once *commoda* (7) and twice *praemia* (1 and 35). In another satire (9, 89) Juvenal uses *commoda* of the privileges of the *ius trium liberorum*. Now out of these three distinct usages of *commoda*, which does Vitruvius employ in our preface? What he received was something substantial, for in the next sentence but one he says that it relieved him from the fear of poverty for the rest of his life. We have no evidence that *commoda* in the third sense of "privileges" would apply to his case; but in its first and second senses it might apply. For while he was in active service he may have received *commoda* of the first kind which I have mentioned, that is emoluments or allowances, and perhaps also good service rewards; cf. Cic. Fam. 5, 20, 7, *quod scribis de beneficiis,*[47] *scito a me et tribunos militaris et praefectos et contubernalis dumtaxat meos delatos esse.* We do not know at all how much money or land was given as a good service reward to any officer, but it seems improbable that a functionary so humble as Vitruvius would have received much. And so perhaps, when the general peace was made, Octavian bestowed upon him *commoda* of the second kind, a good service reward in the form of a retiring gratuity (although, as I have said, we have no evidence that such was given to any except common soldiers), or he may have continued him in office without any actual duties, just as Julius Caesar offered a sinecure tribuneship to Trebatius. And the word *primo* in the next sentence in Vitruvius shows that he had received *commoda* more than once. But obviously all this is pure speculation. The word *commoda* in itself does not tell us whether Vitruvius had retired or not; therefore it cannot be rendered by "pay" or "emoluments"; or by "pension," for this implies the modern practice of paying a stipend at regular intervals. The translator must select a word or phrase which will cover all the contingencies, and hence I have selected "rewards for good service."

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[44] Mommsen, Res. Gestae Aug., 9 and 67; Marquardt, Röm. Staatsv.,^{3} 1, 122; 2, 564.

[45] Res. Gestae Aug., 9 and 65.

[46] Marquardt, Staatsv.,^{2} 2, 80 f.; 3, 223 ff. For *commoda* in this usage in inscriptions, cf. CIL., 6, 971 (a *collegium victimariorum* in the time of Hadrian), and CIL., 6, 955.

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[47] It is perhaps a mere coincidence that Vitruvius uses this same word just below: *eo beneficio obligatus* (2, 8). On *beneficia*, see Mommsen, Staatsr.,^{3} 2, 1126, n. 1.

primo: "for the first time," "originally." So in 209, 25, *cum primo aqua a capite inmittitur;* 36, 2, *cum ergo haec ita fuerint primo constituta.*

7. *cum tribuisti . . . servasti:* these two verbs do not denote coincidence of action, but here, as well as in three other passages in Vitruvius (50, 12; 59, 26; 157, 2), we have the perfect indicative in both parts of a sentence, the protasis of which is a survival of the old indicative narrative *cum*-clause. On such sentence, see Hale, The *cum*-construction, 204 ff., where he cites the same combination occurring, for instance, in Caes. B. C. 3, 87, 7; Bell. Hisp. 18, 2; Galba ap. Cic. Fam. 10, 30, 4.

recognitionem: This is a rare word, and it occurs first in Vitruvius. Paucker (Meletemata Altera, 48) cites only Livy for it, and Cooper in his Sermo Plebeius (4 ff.) does not include it in the list of the ninety-four abstracts in *-tio* which Vitruvius added to the Latin language. It is not found in Cicero [48] (though he added hundreds of such abstracts) nor in Caesar. Our study of its meaning must begin with the remark that it seems never to signify a "recognition" in the modern sense of an acknowledgment of a person's services, standing, or the like. Neither does it mean "favor" ("Gewogenheit," Reber). In the other sense in which we use "recognition," that is, to denote a "knowing again" of somebody whom we have known before, it is found twice in Latin, — both times in that form of the well-known story of Androcles and the lion as it is related by Gellius; cf. Index Capit. 5, 14, *recognitionem inter se mutuam ex vetere notitia hominis et leonis;* and 5, 14, 14, *tum quasi mutua recognitione facta.* This meaning of the substantive is found also in the verb *recognosco;* cf. Cic. Fam. 12, 12, 1, and T. D. 1, 57; and particularly Livy 5, 16, 7, *receptis agrorum suorum spoliis Romam revertuntur. Biduum ad recognoscendas res datum dominis; tertio incognita sub hasta veniere.* But it is at once clear that this meaning of *recognitio* will not suit the passage in Vitruvius, where there is no question of the renewal of an acquaintance between him and Augustus. We must therefore seek another meaning, and we find at once that, except in Gellius, it conveys but one idea, — that of an investigation, inspection, or review. Thus Livy has it in 42, 19, 1, *per recognitionem Postumi consulis magna pars agri Campani recuperata in publicum erat* (cf. 42, 1, 6, *senatui placuit L. Postumium consulem ad agrum publicum a privato*

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[48] Unless the reading of inferior codd. be accepted in Verr., 4, 110.

terminandum in Campaniam ire). Similarly of an inspection of clothing and tools in Col. 11, 1, 21, and of the *equites* in Suet. Claud. 16. Seneca has it of self-examination (*recognitionem sui*, Ira 3, 36, 2). The elder Pliny, in his celebrated account of the habits of ants (N. H. 11, 109), says that they have regular times on which they meet and inspect together the stock which they have gathered: *et quoniam ex diverso convehunt altera alterius ignara, certi dies ad recognitionem mutuam nundinis dantur*. Here the context shows that *recognitionem* does not mean a recognition of the ants by each other, and as ants live a community life it does not signify the identification or "knowing again" of individual property, as in the Livian passage (5, 16, 7) already quoted. This same idea of an investigation or inquiry survived in low Latin; cf. Du Cange (ed. Favre) s. v., where we find that the word was used in charters to denote inquiries into cases of disputed lands (cf. Livy 42, 19, 1, quoted above). These are the only meanings of *recognitio* which I have found in ancient Latin. Although Vitruvius does not use the word elsewhere, yet he has the participle *recognoscentes* once (213, 11), where, after speaking of the useful discoveries made by great men, he adds: *quae recognoscentes necessario his tribui honores oportere homines confitebuntur*, "on reviewing these discoveries, people will admit that honors ought to be bestowed upon them." In this sense, *recognosco*, though a less technical word, is often a synonym of *recenseo*, as a glance at any good lexicon will show. This is well illustrated by Columella, 11, 1, 20, *tum etiam per ferias instrumentum rusticum* (*vilicus*) *recognoscat et saepius inspiciat ferramenta* as compared with 11, 1, 21, *tam vestem servitiorum quam, ut dixi, ferramenta bis debebit singulis mensibus recensere. Nam frequens recognitio nec impunitatis spem nec peccandi locum praebet.* Now in the passage in our preface, to what does *recognitio* refer? Obviously to *commoda*, for Vitruvius says: "after originally bestowing these upon me, you continued (*servasti*, see below) your *recognitio*" — which can only mean "your *recognitio* of these *commoda*." It is natural to suppose that the Roman ruler reviewed or revised at intervals the list of persons who were receiving *commoda*, and that at such times suggestions for additions to the list might be made. Persons whose names were in the list might well be described as *recogniti*, just as *recensi* was used of persons in the list of those who received corn at the public cost; cf. Suet. Caes. 41, *in demortuorum locum ex iis qui recensi non essent*. And the act of setting a name in the list would thus, by a slight extension of meaning, be expressed by the word *recognitio*. But as Vitruvius had at some earlier time (*primo*) received *commoda*, the act in his case was a renewal, and this to his mind may have been further indicated by the prefix *re-* in *recognitio*, especially as contrasted with *primo*. And we may perhaps also compare the common phrase found in the diplomata of discharged soldiers: *descriptum et recognitum ex tabula aenea*, etc. (Dessau, Inscr. Lat. 1, 1986 ff). Our whole sentence, then, may best be rendered: "After your first bestowal of these upon me, you continued to renew them on the recommendation of your sister."

commendationem: cf. Cic. Cat. 1, 28, *hominem per te cognitum, nulla commendatione maiorum*. The word is used elsewhere three times by Vitruvius: 31, 9; 32, 26; 63, 11.

sororis: Octavia, the sister of Augustus, died in 11 B. C. (Liv. Per. 140; Dio C. 54, 35). We know that she had influence with her brother; cf. her successful appeal for the proscribed husband of Tanusia (Dio C. 47, 7). A book was dedicated to her by Athenodorus, son of Sandon (cf. Plut. Popl. 17, Ἀθηνόδωρος ὁ Σάνδωνος ἐν τῷ πρὸς Ὀκταουίαν τὴν Καίσαρος ἀδελφήν. See also Gardthausen, Aug. u. seine Zeit, 1, 217. In regard to the theory that Vitruvius wrote under Titus, it may be remarked that he also had a sister, Domitilla, but that she died before Vespasian came to the throne (Suet. Vesp. 3), and consequently before Titus attained to much power.

servasti: "you continued." For this meaning cf. Caes. B. C. 3, 89, 1, *superius institutum servans* (so also 3, 84, 3, and 75, 2); Cic. Clu. 89, *ut consuetudinem servem*. Similarly in Vitruvius 240, 21, *servat administrationem;* "keeps the works going," etc. This use of *servo* is not found elsewhere in Vitruvius, who happens to employ it, except in these two passages, only in connection with concrete things (*poma*, 16, 20; *fructus*, 145, 20; *frumenta*, 147, 23; *structuras*, 53, 11; *crassitudo*, 75, 19; *cavo*, 47, 11).

8. *beneficio:* It is true that this word may possibly convey here the technical sense of Cic. Fam. 5, 20, 7 (see above, p. 169 and note 47); but as Vitruvius elsewhere employs it only generally (85, 11; 133, 15; 151, 11), I render it by "favor," which fits both usages.

9. *haec tibi scribere coepi:* "I began to write this work for you." Here *haec* refers to the *De Architectura* as now fully completed, not to Vitruvius's preliminary collections (see above on *scripta et explicata*, 1, 7). For this preface was written,[49] or at least professes to have been written, after the whole treatise was finished. The dative *tibi* is supported by Cic. Top. 4, *cum tu mihi meisque multo saepe scripsisses*, although *ad* and the accusative seems to be commoner in dedica-

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[49] Mommsen's expression, to the contrary (Res. Gestae Augusti, 81), seems to me very strange. If Sontheimer's theory (see above, note 18) be adopted, perhaps we should translate: "I set about dedicating this work to you."

tions; cf. Cic. Att. 14, 20, 3, *cum scripsissem ad eum de optimo genere dicendi;* so Lael. 4 (*scriptus ad te*); Off. 1, 4. The work was intended, Vitruvius says here, for the personal use of his patron, to assist him in the ways indicated by lines 10–16. But another reason is given in 160, 6 ff., namely the lack of writings on architecture in the Latin language.

10. *te aedificavisse et nunc aedificare:* among the important early buildings of Octavian which Vitruvius may have in mind are the *aedes divi Iuli* (cf. 70, 18), begun in 42 B. C. and finished at least as early as the year 37, when it appears on coins;[50] and the *curia Iulia,* projected by Julius Caesar and dedicated by Octavian in 29 (Dio C. 51, 22). Other buildings of course had been planned, and some of them may have been finished before Vitruvius published his work.[51]

animadverti . . . te . . . curam habiturum: Schneider found fault with the use of the fut. inf. with the verb *animadverto* and thought that some such word as *spero* or *confido* had dropped out in the latter part of this long sentence. But Vitruvius has the future also in 32, 7, *animadverto fore ut,* etc.; and cf. Cic. Div. 1, 112, *animadverterat olearum ubertatem fore.*

12. *tradantur:* the emendation of Schneider; *traderentur,* codd. The error, as Rose suggests in his second edition, may be due to the preceding *gestarum.*

13. *conscripsi:* "I have composed," "drawn up"; cf. the Thesaurus, s. v., 375, 36, under the lemma "scribendo componere, litteris mandare." It seems unlikely that this word ever means "compile" in Vitruvius. It might possibly have this meaning in 218, 14, *his auctoribus fretus sensibus eorum adhibitis et consiliis ea volumina conscripsi;* but this is improbable in view of all the other passages in which it appears (5, 28; 134, 7; 142, 7; 151, 20; 159, 21), and of the use of *conscriptio,* "treatise," three times (103, 14; 104, 4; 155, 10). Cf. also Cic. Top. 5, *itaque haec, cum mecum libros non haberem, memoria repetita in ipsa navigatione conscripsi tibique ex itinere misi;* Verr. 2, 122, *leges conscribere;* Brut. 46, *praecepta conscribere* (and so Vitr. 5, 28; 159, 21).

praescriptiones terminatas: "definite rules"; cf. "bestimmte Vorschriften" (Reber). Vitruvius always uses *praescriptio* in this sense: cf. 62, 8; 121, 23; 204, 13; 280, 10. In all these passages he promises success to those who follow the "rules." See also his use of the verb *praescribo* in 5, 19, and 83, 17; also Cic. Acad. 2, 140, *praescriptionem*

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[50] Mommsen, ibid., 80.

[51] See Mommsen, ibid., 79–82, and Sontheimer, 120.

naturae; T. D. 4, 22, *praescriptione rationis.* The verb *termino* appears in only one other place in Vitruvius, 64, 20, *terminavi finitionibus,* "I have defined the limits"; but cf. Cic. Fin. 1, 46, *ipsa natura divitias . . . et parabiles et terminatas.* Further light on the meaning of the verb may be got from the use of the substantive *terminatio,* which occurs thirteen times in Vitruvius. In five of these it means "limits" (36, 24, *finire terminationibus,* cf. 64, 20, *terminavi finitionibus* just quoted above; 28, 8; 67, 20; 112, 6; 113, 21); "end" in 103, 13; "terminating point," 135, 21; "boundary," 203, 5; 232, 2; "departments," 12, 8; "extremities," 111, 2; "rules" or "laws," 155, 16; "scope," 32, 28.

16. *disciplinae:* "art," used of architecture in 133, 26; 160, 9; of other arts in 6, 20; 10, 11, and 14; 36, 6; 224, 23.

TRANSLATION.

While your divine intelligence and will, Imperator Caesar, were engaged in acquiring the right to command the world, and while your fellow citizens, when all their enemies had been laid low by your invincible valor, were glorying in your triumph and victory, — while all foreign nations were in subjection awaiting your beck and call, and the Roman people and senate, released from their alarm, were beginning to be guided by your most noble conceptions and policies, I hardly dared, in view of your serious employments, to publish my writings and long considered ideas on architecture, for fear of subjecting myself to your displeasure by an unseasonable interruption. But when I saw that you were giving your attention not only to the welfare of society in general and to the establishment of public order, but also to the providing of public buildings intended for utilitarian purposes, so that not only should the State have been enriched with provinces by your means, but that the greatness of its power might likewise be attended with distinguished authority in its public buildings, I thought that I ought to take the first opportunity to lay before you my writings on this theme. For in the first place it was this subject which made me known to your father, to whom I was devoted on account of his great qualities. After the council of heaven gave him a place in the dwellings of immortal life and transferred your father's power to your hands, my devotion continuing unchanged as I remembered him inclined me to support you. And so with Marcus Aurelius, Publius Minidius, and Gnaeus Cornelius, I was ready to supply and repair ballistae, scorpiones, and other artillery, and I have received rewards for good service with them. After your first bestowal of these upon me, you continued to renew them on the recommendation of your sister.

Owing to this favor I need have no fear of want to the end of my life, and being thus laid under obligation I began to write this work for you, because I saw that you have built and are now building extensively, and that in future also you will take care that our public and private buildings shall be worthy to go down to posterity by the side of your other splendid achievements. I have drawn up definite rules to enable you, by observing them, to have personal knowledge of the quality both of existing buildings and of those which are yet to be constructed. For in the following books I have disclosed all the principles of the art.

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OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF ARSENIC.

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Below is a summary of the previous work upon the atomic weight of arsenic,[1] the results obtained by the several investigators having been recalculated with the use of the following atomic weights:[2] O = 16.000; Ag = 107.880; Cl = 35.457; Br = 79.916; S = 32.07 K = 39.096; Na = 22.977; Cr = 52.01; Pb = 207.09.

1816	Thomson,	Schweigger Jour., **17**, 421,	$2As : As_2O_5$	76.35
1818	Berzelius,	Pogg. Ann., **8**, 1,	$2As_2O_3 : 3SO_2$	75.03
1845	Pelouze,	Compt. Rend., **20**, 1047,	$AsCl_3 : 3Ag$	74.93
1855	Kessler,	Pogg. Ann., **95**, 204,	$3As_2O_3 : 2K_2Cr_2O_7$	74.95
			$3As_2O_3 : 2KClO_3$	75.23
1859	Dumas,	Ann. Chim. Phys., (3), **55**, 174,	$AsCl_3 : 3Ag$	74.87
1859	Wallace,	Phil. Mag., (4), **18**, 279,	$AsBr_3 : 3Ag$	74.20
1861	Kessler,	Pogg. Ann., **113**, 140,	$3As_2O_3 : 2K_2Cr_2O_7$	75.01
1896	Hibbs,	Doctoral Thesis, Univ. of Penn.,	$Na_4As_2O_7 : 4NaCl$	74.88
1902	Ebaugh,	Jour. Amer. Chem. Soc., **24**, 489,	$Ag_3AsO_4 : 3AgCl$	75.02
			$Ag_3AsO_4 : 3Ag$	74.92
			$Pb_3(AsO_4)_2 : 3PbCl_2$	75.06
			$Pb_3(AsO_4)_2 : 3PbBr_2$	74.88

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[1] Clarke, A Recalculation of the Atomic Weights, Smith. Misc. Coll., Constants of Nature, Part V, p. 213 (1897). For an excellent critical discussion of previous work, by Brauner, see Abegg's Handbuch der anorganischen Chemie, **3**, (2), 491 (1907).

[2] Richards, Jour. Chim. Phys., **6**, 130 (1908).

A glance at this rather discordant series of results shows the necessity for a redetermination of the atomic weight of arsenic. Even in the more recent investigations of Hibbs and Ebaugh there exists an extreme variation of nearly two tenths of a unit in the averages of the five series.

In this research silver arsenate was chosen for analysis, first, because the compound is unchanged by moderate heating, and hence may be dried at a temperature high enough to expel all but a very small amount of moisture. In the second place, silver compounds may be analyzed with great ease as well as accuracy by precipitation of the silver as silver halogen compounds. Furthermore, preliminary experiments confirmed the statement by Ebaugh that it is possible completely to convert the arsenate into chloride by heating in a current of hydrochloric acid gas. Such a process has the advantage that no transfer of material is involved.

PURIFICATION OF MATERIALS.

Silver arsenate. — All the samples of silver arsenate were prepared by adding to a fifteenth normal solution of silver nitrate a solution of similar concentration of an equivalent amount of an arsenate of sodium or ammonium, the differences between the different samples consisting chiefly in the nature of the soluble arsenate employed. Precipitation was carried out in a room lighted only with ruby light. After the silver arsenate had been washed by decantation many times with pure water, it was dried in a preliminary way by centrifugal settling in platinum crucibles, and then by being heated in an electric oven at a temperature of about 130° C. The salt was powdered in an agate mortar before the final heating in a quartz tube or platinum boat, as explained later. It was shown by tests with diphenylamine that the arsenate could be washed free from nitrates.

Although one of the hydrogens of arsenic acid resembles the hydrogen of strong acids in its dissociating tendency, the other two hydrogens are those of weak acids.[3] Hence perceptible hydrolysis takes place in solutions of salts of this acid, even when the base is strong, that of the tertiary salts being of course greatest in extent. It is not an easy matter to predict the effect of this hydrolysis upon the composition of a precipitate of silver arsenate; for while the Phase Rule allows the existence of only one solid in equilibrium with the arsenate solution except at certain fixed concentrations, the possibility of the occlusion of either basic or acid arsenates by the silver arsenate still exists. Experiments only are able to throw light on this point. Accordingly arsenate solutions of different conditions of acidity and alkalinity were used in the precipitations, and the compositions of the different precipitates were compared.

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[3] Washburn calculates from Walden's conductivity measurements the constant for the first hydrogen of arsenic acid to be $4.8 \cdot 10^{-3}$. Jour. Amer. Chem. Soc., **30**, 35 (1908). The constants for the second and third hydrogens are probably lower than those of phosphoric acid, $2.1 \cdot 10^{-7}$ and $5.6 \cdot 10^{-13}$. Ibid., 38.

Sample A. Commercial C. P. disodium arsenate was recrystallized four times, all but the first crystallization being conducted in platinum vessels. The mother liquor from the fourth crystallization, after the removal of the arsenic by hydrogen sulphide, gave no test for phosphate. The calculated amount of redistilled ammonia to make disodium ammonium arsenate was added to a solution of the purified salt before the precipitation of the silver arsenate.

Sample B. This sample was made from disodium arsenate which had been recrystallized five times in platinum vessels. Silver arsenate was precipitated with a solution of this salt without the addition of ammonia.

Sample C. Commercial C. P. arsenic trioxide was recrystallized three times from dilute hydrochloric acid solution, and, after being rinsed with water and centrifugally drained, it was converted into arsenic acid by means of nitric and hydrochloric acids in a porcelain dish. The hydrochloric and nitric acids were expelled by evaporation nearly to dryness, and the residue was twice evaporated to dryness with nitric acid in a platinum dish. After the residue had been dissolved in water, the solution was allowed to stand for some time in order to allow pyro and meta arsenic acids to be converted as completely as possible into ortho arsenic acid. Then sodium carbonate which had been twice crystallized in platinum was added to the solution in amount sufficient to form disodium arsenate, and the product was crystallized four times in platinum vessels.

Sample D. A portion of the arsenic acid made for the preparation of Sample C was converted into ammonium dihydrogen arsenate by adding the calculated amount of redistilled ammonia, and the salt was recrystallized five times in platinum. A sufficient quantity of ammonia to form triammonium arsenate was added to a solution of this salt before the precipitation of the silver arsenate. One specimen of silver arsenate made in this way was discarded, since its composition was very irregular.

Sample E. To a portion of the arsenic acid used for Sample C recrystallized sodium carbonate was added in amount sufficient to form disodium arsenate. After the solution had been evaporated to dryness, the salt was recrystallized four times in platinum. Enough ammonia to form disodium ammonium arsenate was added to a solution of this salt before the precipitation of the silver arsenate.

Sample F. A portion of the disodium arsenate prepared for Sample B was converted into trisodium arsenate by means of recrystallized sodium carbonate, and the trisodium arsenate was recrystallized six times in platinum vessels.

Sample G. Arsenic trioxide was twice resublimed in a current of pure dry air and then once crystallized from dilute hydrochloric acid solution. Next the arsenious acid was oxidized to arsenic acid exactly as described under Sample C. Finally the arsenic acid was converted into trisodium arsenate by means of pure sodium carbonate, and the salt was crystallized four times in platinum.

In all the foregoing crystallizations the crystals were thoroughly drained in a centrifugal machine employing large platinum Gooch crucibles as baskets,[4] and each crop of crystals was once rinsed with a small quantity of pure water and subsequently drained in the centrifugal machine.

Silver nitrate. — The silver nitrate used in the preparation of the different samples of silver arsenate was recrystallized several times in platinum vessels, with centrifugal drainage, until the mother liquor gave no opalescence upon dilution when tested in the nephelometer.

Hydrobromic acid. — One quarter pound of commercial bromine was converted into potassium bromide by addition to recrystallized potassium oxalate. In the concentrated solution of this bromide, in a distilling flask cooled with ice, three pounds of bromine were dissolved, in several separate portions, each portion being distilled from the solution into a flask cooled with ice before the addition of the next succeeding portion. A portion of the purified bromine was then converted into potassium bromide with pure potassium oxalate as before, and the remainder of the bromine was distilled in small portions from solution in this pure potassium bromide. The product obtained was thus twice distilled from a bromide, the bromide in the second distillation being essentially free from chlorine. This treatment has already been proved sufficient to free bromine from chlorine.[5]

Hydrobromic acid was synthesized from the pure bromine by bubbling hydrogen gas (made by the action of water on "hydrone") through the bromine warmed to 40°–44° and passing the mixed gases over hot platinized asbestos in a glass tube. The apparatus was constructed wholly of glass. The hydrogen was cleansed by being passed through two wash bottles containing dilute sulphuric acid, and through a tower filled with beads also moistened with dilute sulphuric acid. The hydrobromic acid gas was absorbed in pure water contained in a cooled flask. In order to remove iodine the solution of hydrobromic acid was diluted with water and twice boiled with a small quantity of free bromine. Then a small quantity of recrystallized potassium permanganate was added to the hybrobromic acid solution, and the bromine set free was expelled by boiling. Finally the acid was distilled with the use of a quartz condenser, the first third being rejected. It was preserved in a bottle of Nonsol glass provided with a ground-glass stopper.

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[4] Baxter, Jour. Amer. Chem. Soc., **30**, 286 (1908).

[5] Baxter, These Proceedings, **42**, 201 (1906).

The purification of the hydrobromic acid was carried on in conjunction with Dr. Grinnell Jones, who was engaged in a parallel research upon the atomic weight of phosphorus. Using this acid, he found that 10.48627 grams of silver bromide were obtained from 6.02386 grams of the purest silver. This ratio of silver bromide to silver of 100.0000 to 57.4452 is in close agreement with the most probable value, 100.0000 to 57.4453.[6]

Hydrochloric acid. — A solution of this acid was purified by distillation after dilution.

Hydrochloric acid gas was generated by dropping C. P. concentrated sulphuric acid into C. P. concentrated hydrochloric acid. The acids were shown to be essentially free from arsenic.

Water. — All the water used in the research was purified by distilling the ordinary distilled water of the laboratory, once with alkaline permanganate and then once alone, in both cases with the use of block tin condensers which required no cork or rubber connections to the distilling flasks.

Utensils. — Quartz or platinum vessels were always employed in place of glass, whenever glass was unsuitable.

METHODS OF ANALYSIS.

The first method of analysis employed was that of converting the silver arsenate into silver chloride by heating in a current of hydrochloric acid gas. Since this process does not involve transfer of material it should be capable of giving results of great accuracy. Glass and porcelain are unsuitable for containing the arsenate during this process on account of the certainty of their being attacked. The first attempts at using quartz for the purpose resulted in slight etching of the surface of the tube where it came in contact with the salt. Experience showed, however, that with careful management the attacking of the quartz could be wholly prevented. The vessel used to contain the arsenate was a quartz tube nearly two centimeters in diameter but joined to small tubes at each end. These tubes were exactly like those employed by Richards and Jones in the conversion of silver sulphate into silver chloride.[7] After the tube had been weighed by substitution for a counterpoise similar in shape and size, a suitable quantity of silver arsenate was introduced, and the tube and contents were heated in a current of pure dry air for between seven and eight hours at 250° C. Although this treatment is not sufficient to expel last traces of moisture, it was hoped that by uniform treatment of the arsenate in all the analyses the proportion of water retained by the salt could be reduced to a constant percentage, which could be determined in separate experiments. The complete drying of the salt by fusion was not permissible because of decomposition of the arsenate at temperatures in the neighborhood of its fusing point. During the drying of the arsenate the quartz tube was surrounded with a cylinder of thin platinum foil and was contained in a hard glass tube connected with an apparatus for furnishing a current of pure dry air. The hard glass tube was heated by means of two aluminum blocks 15 centimeters by 13 centimeters by 5 centimeters, one placed above the other, the upper surface of the lower block and the lower surface of the upper being suitably grooved to contain the tube. The blocks were bored to contain a thermometer the bulb of which was located near the middle of the tube. This oven could be readily maintained at constant temperature within a very few degrees by means of a small Bunsen flame. We are indebted to Dr. Arthur Stähler of the University of Berlin for the suggestion of this method of heating. In order to purify and dry the air it was passed through a tower filled with beads moistened with dilute silver nitrate solution, through a tower filled with small lumps of solid potassium hydroxide, then through three towers filled with beads moistened with concentrated sulphuric acid, and finally through a tube filled with resublimed phosphorus pentoxide. The apparatus was constructed wholly of glass, with ground joints.

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[6] Baxter, These Proceedings, **42**, 201 (1906).

After being heated, the quartz tube was transferred to a desiccator and allowed to come to the temperature of the balance case before being weighed. The quartz tube was then placed upon hard glass supports, in a horizontal position, one end being slipped into a larger tube through which could be passed a current of either dry hydrochloric acid gas or dry air. The other end of the quartz tube slipped into one of the arms of a large U-tube filled with glass pearls, which served to condense any silver chloride vapor which might escape from the quartz tube. The other arm of the U-tube was connected with the flue of a hood, the suction thus caused being sufficient to prevent the escape of gaseous arsenic compounds from the apparatus. The quartz tube was protected from dust by a covering of sheet mica.

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[7] Pub. Carnegie Institution, No. 69, 69 (1907).

The usual method of procedure was as follows: The quartz tube containing the silver arsenate being in place, a current of hydrochloric acid gas was passed through the tube, and the tube was slowly revolved with pincers tipped with platinum wire in order that the salt might be thoroughly exposed to the action of the acid. Neglect to do this at the commencement of the reaction always resulted in the caking of the salt in the tube, thereby rendering the action of the acid less rapid. The hydrochloric acid was dried by passing through three towers containing beads moistened with concentrated sulphuric acid. The apparatus for generating and purifying the acid was constructed wholly of glass.

In the earlier experiments the salt was gently heated from the commencement of the reaction. To all outward appearance it was entirely converted into silver chloride in a few hours. Upon fusion, however, it presented a very cloudy appearance, owing to the presence of considerable arsenic, which could not be completely removed even by keeping the silver chloride fused in the current of hydrochloric acid for as long as eight hours. This is the cause of the larger quantities of arsenic found in the chloride obtained in the earlier analyses. Furthermore, the longer period of heating at a temperature above the fusing point of silver chloride accounts for the larger amounts of volatilized silver chloride found in these experiments.

As experience was gained it was found best to expose the salt first in the cold for about eight hours to the action of the hydrochloric acid gas, next to heat the salt gently below its fusing point for from ten to fifteen hours, and finally to keep it barely fused for from five to ten hours longer. When the reaction was apparently at an end, the current of hydrochloric acid gas was stopped, and dry air was passed through the tube for about fifteen minutes in order to eliminate hydrochloric acid. The silver chloride was then allowed to solidify in a uniform thin layer around the inside of the quartz tube by slowly revolving the tube during solidification. The platinum wire used in weighing the tube was slipped on, the tube was transferred to its desiccator, and after standing several hours beside the balance it was weighed.

In order to make sure that the reaction was complete the silver chloride was again fused, and exposed to the action of hydrochloric acid for several hours longer. As a rule, no change in weight was observed. In all cases constant weight was obtained upon heating in the same way for a third time.

After making certain that only a small quantity of arsenic, if any, remained in the silver chloride, the contents of the quartz tube were dissolved in ammonia, and the silver chloride was reprecipitated by boiling the solution to expel the ammonia and adding a small quantity of sulphuric acid. The solution, after evaporation, was added to a Berzelius-Marsh apparatus containing arsenic-free zinc and sulphuric acid, and a mirror of arsenic was deposited in a hard glass capillary tube in the usual way. The hydrogen was dried by calcium chloride before passing into the hard glass tube, and the generating flask was cooled with water to prevent the evolution of hydrogen sulphide.

The arsenic mirror formed was compared with a photograph of standard arsenic mirrors,[8] the original mirrors showing that comparison with the photograph was equally satisfactory. The correction was applied by assuming that the arsenic was present in the silver chloride as arsenic trichloride, although if present as silver arsenate the correction would be much smaller. In any case the correction for residual arsenic is so small as to be almost without effect upon the final result.

Ebaugh used essentially the same method of heating the arsenate in hydrochloric acid, although the periods were shorter, so that it is probable that the small quantities of arsenate used (scarcely over one gram in any analysis) did not retain weighable amounts of arsenic.

The U-tube beyond the quartz tube was washed out thoroughly with dilute ammonia, and the solution was made up to definite volume after nearly all the ammonia had been expelled by boiling. The silver content of the solution was then compared in the nephelometer with that of standard solutions of silver, care being taken that the tubes were treated in as nearly as possible the same way.

The second method of analysis consisted in heating the silver arsenate in a platinum boat and, after weighing, dissolving the arsenate in nitric acid and precipitating the silver as chloride or bromide. The platinum boat was heated in a hard glass tube forming part of a bottling apparatus,[9] by means of which the boat could be transferred to the weighing bottle in which it was always weighed without exposure to moist air. The boat and bottle were weighed by substitution by comparison with a counterpoise similar both in shape and volume.

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[8] Sanger, These Proceedings, **26**, 24 (1891).

[9] Richards and Parker, These Proceedings, **32**, 59 (1896).

After the silver arsenate had been weighed, the boat with its contents was transferred to a flask, and the salt was dissolved in warm nitric acid of density about 1.15. The weighing bottle was rinsed with acid, and the rinsings were added to the main solution; then the solution was carefully transferred to a large glass-stoppered precipitating flask and diluted to a volume of about one litre.

From the weight of silver arsenate the amount of either hydrochloric or hydrobromic acid necessary to precipitate the silver was calculated. A slight excess of one acid or the other was then diluted to about six hundred cubic centimeters, and the solution was slowly poured into the solution of silver arsenate in the precipitating flask. After a few moments' shaking the precipitate was allowed to stand for several days, with occasional agitation.

The precipitated silver chloride or silver bromide was next collected upon a weighed Gooch crucible, after it had been washed by decantation about ten times with dilute hydrochloric acid in the case of silver chloride, with water in the case of silver bromide. After several hours' heating in an electric air bath at 150° C., and about two hours' heating at 200° C., the precipitate was cooled in a desiccator and weighed.

In order to determine the moisture retained by the precipitate it was transferred as completely as possible to a small porcelain crucible and weighed. Then the salt was fused by heating the small covered crucible contained in a large crucible and was again weighed.

The asbestos mechanically detached from the Gooch crucible, together with a small quantity of silver chloride or silver bromide which escaped the crucible, was collected upon a small filter through which the filtrate and wash waters were passed, and the filter paper was ignited in a small weighed porcelain crucible. Before being weighed the ash was treated with a drop of nitric and a drop of either hydrochloric or hydrobromic acid and again heated.

The filtrate and wash waters were evaporated to small bulk. The precipitating flask was rinsed with ammonia, and the rinsing was added to the evaporated filtrate and wash water. Then the solution was diluted to definite volume, and the silver content was determined by comparison with standard silver solutions in the nephelometer.

The operations of precipitating and collecting the silver halides were all carried out in a room lighted only with ruby light.

Insoluble Residue.

All the specimens of silver arsenate, after being heated at 250° C., when dissolved in dilute nitric acid, were found to contain a small amount of insoluble residue, which would dissolve only in rather concentrated nitric acid. Although the proportion of this residue was apparently increased by exposure to light, specimens of the arsenate which had been prepared wholly in the dark room were not free from it. No process of purification to which the soluble arsenate used in the preparation of the silver arsenate was subjected seemed to have the slightest effect upon the proportion of insoluble matter. A similar phenomenon was met by Dr. Grinnell Jones in the preparation of silver phosphate.

Although the amount of this residue in one gram of silver arsenate which had been treated as in the analyses for silver was not over 0.00005 gram, it was important to determine its silver content. This was done in three cases in which the proportion of residue had been purposely increased as much as possible by exposure to light. The arsenate was dissolved in dilute nitric acid, and the residue was collected upon a weighed platinum Gooch crucible, the detached asbestos shreds being carefully determined by filtration upon a filter paper. The weight of residue was found by reweighing the crucible. After the residue had been dissolved in concentrated nitric acid and the solution had been diluted to definite volume, the silver content of the solution was ascertained by comparison with standard silver solutions in a nephelometer.

The first of the above determinations was made with a sample of silver arsenate which had been exposed to bright light inside a desiccator for a month. During this time the quartz tube containing the

Weight of Silver Arsenate.	Weight of Insoluble Residue.	Weight of Silver.	Per cent of Silver.
grams.	gram.	gram.	
4.26	0.00198	0.00143	72.3
10.00	0.00228	0.00162	71.1
9.28	0.00657	0.00500	76.1

Average 73.2
Theoretical per cent of silver in silver arsenate . 70.0

salt showed no perceptible change in weight. The third determination also was made with a sample of salt which had been exposed to bright light for three weeks in a dry state. In the second determination the salt was exposed to light under water for one week.

Two facts show that the presence of the small proportion of the residue in the arsenate could have had no important effect upon the results. In the first place, the formation of the insoluble matter under the influence of light is not attended by change in weight. In the second place, the silver content of the residue is very near that of silver arsenate. Nevertheless care was taken to protect the arsenate as far as possible from exposure to light.

The Determination of Moisture in Silver Arsenate.

T. W. Richards [10] and others have already drawn attention to the fact that it is not possible, without fusion, to dry completely a substance formed in aqueous solution, owing to the mechanical retention of liquid in pockets within the solid. In the case of silver arsenate, although it is possible to fuse the salt, the temperature necessary is so high that decomposition of the salt takes place to some extent. Hence the loss in weight on fusion cannot be used as a true measure of the water content of the salt. Since decomposition of the salt could produce only easily condensible substances and oxygen, the difficulty was overcome in the present instance by fusing weighed quantities of the salt in a current of pure dry air and collecting the water vapor in a weighed phosphorus pentoxide tube. Of course great pains were taken to treat the salt used in the water determinations in exactly the same way as that used in the analyses for silver.

The procedure was as follows: A sample of salt very nearly as pure as that used in the silver analyses was weighed out in a copper boat which had been previously cleaned and ignited in the blast lamp to remove organic matter. The boat was placed in a hard glass tube and was heated for between seven and eight hours at 250° C. in a current of dry air. In these experiments, before passing through the drying towers the air had first been passed over hot copper oxide in order to oxidize any organic matter it might contain. Furthermore, the concentrated sulphuric acid in the drying towers had been heated with a small quantity of potassium dichromate. One end of the hard glass tube was connected to the apparatus for supplying pure air, by means of a well-fitting ground joint upon which no lubricant was used. The other end was sealed to a small hard glass tube which was surrounded with a damp cloth during the fusion of the salt in order to facilitate condensation of any silver or arsenic compounds vaporized from the salt. As a matter of fact, very little sublimation actually took place.

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[10] Zeit. physik. Chem., **46**, 194 (1903).

In order to fuse silver arsenate within the hard glass tube it was necessary to use the hottest flame of the blast lamp, the tube being covered with a semi-cylinder of sheet iron. Furthermore, since at this temperature even the hard glass became very soft, it was found necessary to wrap the tube with asbestos and closely wound iron wire for several inches at the point where fusion took place. This also served to distribute the heat more evenly and to prevent the tube from cracking during the experiment.

Just before the salt was fused a carefully weighed U-tube containing resublimed phosphorus pentoxide was attached to the end of the tube, and beyond this was joined another similar tube which served as a protection against any moisture which might diffuse back into the weighed tube from the outside air. These phosphorus pentoxide tubes were provided with one way ground glass stopcocks lubricated with Ramsay desiccator grease.

The salt was heated for twenty-five minutes with the hottest flame of the blast lamp, being then completely fused, and was further heated for thirty-five minutes at a considerably lower temperature in order to make certain that all moisture was carried into the absorption tube by the current of air. Finally the phosphorus pentoxide tube was reweighed.

The pentoxide tube was weighed by substitution with the use of a counterpoise of the same size and weight filled with glass beads. Before being weighed both tubes were carefully wiped with a damp cloth and were allowed to stand near the balance case for one hour. One stopcock in each tube was opened immediately before the tube was hung upon the balance, in order to insure equilibrium between the internal and external air pressure. The stopcock of the counterpoise was left open during the weighing. Owing to the considerable length of time required for the tubes to come to equilibrium on the balance, it was considered unsafe to leave the stopcock of the pentoxide tube open during the weighing. As a check on the first weight of the pentoxide tube one stopcock was opened and closed and its weight determined a second time. Ordinarily no change in weight was observed.

Since it seemed possible that the hard glass tube itself, when heated nearly to fusion, might give off traces of water vapor, two blank determinations were made by heating the empty hard glass tube in exactly the same way as in the water determinations. These determinations showed a gain in weight of the pentoxide tube of 0.00022 and 0.00037 gram respectively, the average being 0.00030 gram. This correction was confirmed in another experiment in which the hard glass tube was kept at the highest temperature obtainable with the blast lamp for one hour. The observed gain in weight of the absorption tube was 0.00048 gram. A negative correction of 0.00030 gram was applied in each water determination.

Weight of Silver Arsenate.	Weight of Water.	Weight of Water per Gram of Salt.
grams. 11.09	gram. 0.00083	0.000075
13.59	0.00073	0.000054
17.23	0.00085	0.000049
12.57	0.00057	0.000045
Average		0.000056

In order to allow for moisture the weight of the arsenate was therefore always corrected by subtracting 0.000056 gram per gram of salt. Ebaugh took no notice of the water contained in silver arsenate which had been dried at only 170°.

THE SPECIFIC GRAVITY OF SILVER ARSENATE.

In order that the apparent weight of the silver arsenate might be corrected to a vacuum standard, the specific gravity of the arsenate was found by determining the weight of toluol displaced by a known quantity of salt. The toluol was first dried by means of stick soda and was then distilled, with rejection of the first portion of the distillate. Its

Weight of Silver Arsenate in Vacuum.	Weight of Displaced Toluol in Vacuum.	Specific Gravity of Silver Arsenate 25°/4°.
grams. 5.1690	gram. 0.6688	6.662
5.6729	0.7350	6.652
Average		6.657

specific gravity at 25° referred to water at 4° was found to be 0.8620. Pains was taken to remove air from the arsenate when covered with toluol by placing the pycnometer in an exhausted desiccator.

The following vacuum corrections were applied:

	Specific Gravity.	Vacuum Correction.
Weights	8.3	
Toluol	0.862	+0.00126
Silver arsenate	6.657	+0.000036
Silver chloride	5.56	+0.000071
Silver bromide	6.473	+0.000041

Balance and Weights.

All weighings were made upon a nearly new short-armed Troemner balance, easily sensitive to one fiftieth of a milligram with a load of fifty grams.

The gold-plated Sartorius weights were several times carefully standardized to hundredths of a milligram by the method described by Richards,[11] and were used for no other work.

SERIES I.

3 AgCl : Ag_3AsO_4.

No. of Analysis.	Sample of Ag_3AsO_4.	Corrected Weight of Ag_3AsO_4 in Vacuum.	Weight of AgCl in Vacuum.	Residual $AsCl_3$.	Volatilized AgCl.	Corrected Weight of AgCl in Vacuum.	Ratio 3AgCl : Ag_3AsO_4.
		grams.	grams.	gram.	gram.	grams.	
1	A	3.17276	2.94912	0.00004	0.00014	2.94922	0.929544
2	A	2.65042	2.46364	0.00004	0.00007	2.46367	0.929539
3	A	3.51128	3.26395	0.00001	0.00002	3.26396	0.929564
4	B	5.83614	5.42499	0.00001	0.00005	5.42503	0.929558
5	C	5.72252	5.31947	0.00001	0.00001	5.31947	0.929568
Average							0.929555

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[11] Jour. Amer. Chem. Soc., **22**, 144 (1900)

SERIES II.

$3\ AgCl : Ag_3AsO_4$.

No. of Analysis.	Sample of Ag_3AsO_4	Corrected W'ght of Ag_3AsO_4 in Vacuum.	Weight of AgCl in Vacuum.	Weight of Asbestos.	Dissolved AgCl from Filtrate.	Loss on Fusion.	Corrected Wt. of AgCl in Vacuum.	Ratio $3AgCl : Ag_3AsO_4$.
		grams.	grams.	gram.	gram.	gram.	grams.	
6	C	4.59149	4.26815	0.00008	0.00012	0.00039	4.26796	0.929537
7	D	3.38270	3.14401	0.00037	0.00013	0.00015	3.14436	0.929542

Average 0.929540

Average of all analyses in Series I and II 0.929550

Per cent of Ag in Ag_3AsO_4 . . . . 69.9609[12]

SERIES III.

$3\ AgBr : Ag_3AsO_4$.

No. of Analysis.	Sample of Ag_3AsO_4.	Corrected W'ght of Ag_3AsO_4 in Vacuum.	Weight of AgBr in Vacuum.	Weight of Asbestos.	Dissolved AgBr from Filtrate.	Loss on Fusion.	Corrected Weight of AgBr in Vacuum.	Ratio $3AgBr : Ag_3AsO_4$
		grams.	grams.	gram.	gram.	gram.	grams.	
8	C	8.75751	10.66581	0.00008	0.00004	0.00040	10.66553	1.21787
9	D	6.76988	8.24529	0.00024	0.00007	0.00015	8.24545	1.21796
10	D	5.19424	6.32569	0.00017	0.00009	0.00005	6.32590	1.21787
11	D	5.33914	6.50251	0.00009	0.00006	0.00008	6.50258	1.21791
12	E	8.24054	10.03497	0.00053	0.00014	0.00012	10.03552	1.21782
13	E	7.57962	9.23134	0.00021	0.00005	0.00013	9.23147	1.21793
14	E	6.05230	7.37066	0.00038	0.00005	0.00003	7.37106	1.21789

Average 1.21789

Per cent of Ag in Ag_3AsO_4 . . . . 69.9622[13]

Average per cent of Ag in Ag_3AsO_4 . . . . 69.9616

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[12] Ag : AgCl = 0.752632 : 1.000000. Richards and Wells, Pub. Car. Inst., No. 28 (1905).

[13] Ag : AgBr = 0.574453 : 1.000000. Baxter, These Proceedings, **42**, 201 (1906).

SERIES IV.

3 AgCl : Ag_3AsO_4.

No. of Analysis.	Sample of Ag_3AsO_4.	Corrected W'ght of Ag_3AsO_4 in Vacuum.	Weight of AgCl in Vacuum.	Residual $AsCl_3$.	Volatilized AgCl.	Corrected W'ght of AgCl in Vacuum.	Ratio 3AgCl : Ag_3AsO_4.
		grams.	grams.	gram.	gram.	grams.	
15	F	4.67268	4.34393	0.00006	0.00002	4.34389	0.929636
16	F	7.71882	7.17602	0.00007	0.00002	7.17597	0.929672
17	G	5.28049	4.90908	0.00001	0.00001	4.90908	0.929664
18	G	4.25346	3.95422	0.00000	0.00002	3.95424	0.929652
19	G	3.47340	3.22892	0.00000	0.00001	3.22893	0.929616
20	G	5.17269	4.80877	0.00000	0.00002	4.80879	0.929650
21	G	4.10766	3.81856	0.00000	0.00002	3.81858	0.929624
			Average				0.929645

SERIES V.

3 AgCl : Ag_3AsO_4

No. of Analysis.	Sample of Ag_3AsO_4.	Corrected Wt. of Ag_3AsO_4 in Vacuum.	Weight of AgCl in Vacuum.	Weight of Asbestos.	Dissolved AgCl from Filtrate.	Loss on Fusion.	Corrected Wt. of AgCl in Vacuum.	Ratio 3AgCl : Ag_3AsO_4.
		grams.	grams.	gram.	gram.	gram.	grams.	
22	G	5.47133	5.08686	0.00009	0.00014	0.00066	5.08643	0.929652
		Average of all analyses in Series V and VI						0.929646
		Per cent of Ag in Ag_3AsO_4						69.9681

SERIES VI.

3 AgBr : Ag_3AsO_4

No. of Analysis.	Sample of Ag_3AsO_4.	Corrected Wt. of Ag_3AsO_4 in Vacuum.	Weight of AgBr in Vacuum.	Weight of Asbestos.	Dissolved AgBr from Filtrate.	Loss on Fusion.	Corrected Wt. of AgBr in Vacuum.	Ratio 3AgBr : Ag_3AsO_4.
		grams.	grams.	gram.	gram.	gram.	grams.	
23	G	4.96261	6.04438	0.00004	0.00010	0.00012	6.04440	1.217988
24	G	5.31743	6.47645	0.00015	0.00009	0.00011	6.47658	1.217991
25	G	4.46882	5.44273	0.00026	0.00011	0.00010	5.44300	1.217995
26	G	4.16702	5.07533	0.00010	0.00004	0.00008	5.07539	1.217990

Average 1.217991
Per cent of Ag in Ag_3AsO_4 . . . . 69.9678
Average per cent of Ag in Ag_3AsO_4 . . . . 69.9680

Discussion of Results.

The first point to be noted in the foregoing tables is that the results can be divided into two distinct groups according to the samples of arsenate employed, Series I, II, and III, with Samples A to E, giving values for the per cent of silver in the arsenate lower than Series IV, V, and VI, with Samples F and G.

In the second place, both methods for determining the ratio of the arsenate to the chloride give essentially identical results. This is shown by the agreement of Series I and II, and that of Series IV and V.

Finally, the per cent of silver in silver arsenate found in Series I and II agrees within less than two thousandths of a per cent with that found in Series III. This agreement, together with that of the individual analyses of each series, indicates both uniformity in the material employed and purity of the hydrochloric and hydrobromic acids, as well as accuracy in the analytical work. The agreement of Series IV and V with Series VI is closer still.

In the following table are given the sources of the various samples of silver arsenate:

Sample A	$Na_2NH_4AsO_4$	Sample E	$Na_2NH_4AsO_4$
Sample B	Na_2HAsO_4	Sample F	Na_3AsO_4
Sample C	Na_2HAsO_4	Sample G	Na_3AsO_4
Sample D	$(NH_4)_3AsO_4$		

It is to be expected that the basicity due to hydrolysis would be most marked with Samples F and G, less in the case of Samples A and E, still less with Sample D, and least in the case of Samples B and C. In the case of Samples B and C, acid accumulates in the solution during the precipitation of the silver arsenate. In comparing the results from the different samples of silver arsenate it must be noted that occluded basic salt would increase the apparent percentage of silver in the arsenate. In the case of Samples F and G the conditions are most favorable for the occlusion of basic salts, and *these two samples actually yield a higher percentage of silver than the other samples.* On the other hand accumulation of acid in the solution in which the precipitation of the silver arsenate is taking place is found to have no tendency to promote occlusion of acid salts, since Samples B and C give results agreeing closely with those of Samples A, D, and E. These two facts lead to the conclusion that Samples A to E represent normal trisilver arsenate, and that Samples F and G contain basic impurities.

In order to calculate the atomic weight of arsenic from the per cent of silver in silver arsenate a knowledge of the ratio of the atomic weights of silver and oxygen is necessary. Some uncertainty exists as to this ratio, hence calculations have been made upon the basis of several possible values for silver, oxygen being assumed to have the value 16.000. This has been done only with the results of Series I, II, and III, since, as has been pointed out, they are probably nearer the truth than those of Series IV, V, and VI. The difference between the two sets of results amounts only to six one hundredths of a per cent in the atomic weight of arsenic.

	Series I. and II.	Series III.
If Ag = 107.93, As =	75.026	75.017
If Ag = 107.88, As =	74.961	74.953
If Ag = 107.85, As =	74.923	74.914

When the results of Series I and II are averaged with those of Series III, it is found that

If Ag = 107.930 As = 75.021
If Ag = 107.880 As = 74.957
If Ag = 107.850 As = 74.918

The atomic weight of arsenic will be further investigated in this laboratory.

The most important results of this research may be briefly summed up as follows:

1. Methods for the preparation of normal trisilver arsenate were devised.
2. It is shown that trisilver arsenate precipitated by means of trisodium arsenate probably contains occluded basic impurity.
3. It is shown that silver arsenate cannot be completely dried without fusion.
4. The specific gravity of unfused trisilver arsenate is found to be 6.66 at 25° C., referred to water at 4° C.
5. The per cent of silver in silver arsenate is found to be 69.9616 by three closely agreeing methods.
6. With several assumed values for the atomic weight of silver referred to oxygen 16.000, the atomic weight of arsenic is found to have the following values:

If Ag = 107.93 As = 75.02
If Ag = 107.88 As = 74.96
If Ag = 107.85 As = 74.92

A grant from the Carnegie Institution of Washington has been of great assistance in the pursuit of this investigation. We are also indebted to the Cyrus M. Warren Fund for Research in Harvard University for much indispensable platinum apparatus.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL
LABORATORY, HARVARD UNIVERSITY.

THE MEASUREMENT OF HIGH HYDROSTATIC PRESSURE.

I.—*A SIMPLE PRIMARY GAUGE.*

BY P. W. BRIDGMAN.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE MEASUREMENT OF HIGH HYDROSTATIC PRESSURE.

I. A SIMPLE PRIMARY GAUGE.

By P. W. Bridgman.

Presented by W. C. Sabine, December 9, 1908. Received December 16, 1908.

Introduction.

The classical work of Amagat on various physical effects of high hydrostatic pressure is practically the only work we have in which the pressure has been accurately measured with a direct reading gauge over any considerable pressure range. Amagat's pressure measurements were made with his *manometre à pistons libres,* which is too well known to need description here. The gauge gives consistent results, and throughout the pressure range the indications are proportional to the pressure. In fact, the accuracy of the pressure measurements is limited only by the accuracy with which the dimensions of the pistons can be measured. With this primary gauge Amagat measured a number of secondary pressure effects, principally the compressibilities of various liquids and gases over a pressure range of about 3000 kgm. per sq. cm. The value of the compressibility found in this way has in turn been used by other experimenters as a means of calibrating whatever secondary gauge they found it convenient to use. It is thus possible to avoid the direct measurement of pressure with a *manometre à pistons libres,* which is in most cases inconvenient, because of the unavoidable leak and the time necessary to make the readings. The care with which the ground surfaces of piston and cylinder must be kept free from grit, and the expense of the instrument, are other objections to its common use.

With increasing experience in methods of reaching high pressures, and increasing excellence of commercial steels, it has been found possible, however, to exceed the pressure limit set by Amagat. Thus

Tammann[1] on one occasion reached 5000 kgm. per sq. cm., and Carnazzi,[2] working with Lussana's[3] apparatus, has also attained 5000 kgm. Both of these observers measured the pressure with a secondary gauge involving directly the compressibility of water as found by Amagat. But because Amagat's data run to only 3000 kgm., the pressure measurements of both Tammann and Carnazzi must be uncertain at these higher pressures. Tammann had to content himself with an extrapolation beyond 3000, and Carnazzi does not give any results beyond 3000.

The purpose of this paper is to provide data which shall enable others to work, if they desire, beyond Amagat's pressure range with a reasonable degree of confidence in the accuracy of the pressure measurements. It seems that the most feasible way of doing this is to determine, under conditions that may be reproduced with accuracy, the variation with pressure of some easily measurable physical property. Compressibility does not seem to be the best secondary property for this purpose, for it cannot be measured with much accuracy conveniently. In this paper, advantage has been taken of a suggestion of de Forest Palmer's,[4] and the variation of the electrical resistance of pure mercury under pressure has been determined. The secondary gauge, involving the variation of the resistance of mercury, has proved itself trustworthy and accurate.

This matter of a secondary standard is discussed in the second part of this paper. The first part is occupied with a discussion of the slightly novel form of gauge with which the fundamental direct measurements of pressure were made. Amagat's *manometre à pistons libres* is not well adapted for high pressures. Amagat himself was accustomed to use it to only 3000 kgm. per sq. cm., and others following him have not been so high; thus Barus found that above 2000 kgm. the leak became troublesome. In this paper a gauge is described with which, by modifying the design and decreasing the dimensions, it has been found possible to reach higher pressures than Amagat, frequently without perceptible leak. In fact the primary gauge proved itself so manageable, and is so simple to construct, that if it were not for the greater convenience of the secondary gauge, the primary gauge could be used directly in any high pressure investigation. This paper gives results that have been obtained with this gauge up to 6800 kgm. per sq. cm. The first part is occupied with a description of the gauge, calculation of the corrections to be applied, and a comparison of two gauges to determine the accuracy and sensitiveness.

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[1] Tammann, Kristallisieren und Schmelzen, p. 201 (Leipzig, Barth, 1903).

[2] Carnazzi, Nuov. Cim., (5), **5**, 180–189 (1903).

[3] Lussana, Nuov. Cim., (5), **4**, 371–389 (1902).

[4] de Forest Palmer, Amer. Jour. Sci., **6**, 451–454 (1898).

Description of the Gauge.

Besides Amagat's [5] manometer, other forms of direct pressure gauge have been used, examples of which are the pressure balance at Stuckrath,[6] and the differential manometer at the National Physical Laboratory at London.[7] Lisell,[8] in his measurement of the pressure coefficient of resistance of wires, used a gauge much like that at Stuckrath. These gauges differ in the manner in which the pressure exerted on the piston is measured. Amagat measures it by measuring with a mercury column the hydrostatic pressure acting on a larger piston which balances the total thrust exerted by the high unknown pressure on a much smaller piston. The thrust is measured at Stuckrath or by Lisell by hanging weights on the piston either directly or with the aid of a lever. At London the action of weights is used to equilibrate the differential effect of the pressure on two pistons of nearly the size. A common feature of all these gauges is the piston fitting accurately in the cylinder, which is subjected to pressure on the inside. The distortion produced by the pressure is, therefore, a compression of the piston, accompanied by a stretching of the cylinder, the resultant effect being to increase the breadth of the crack between piston and cylinder. The leak, therefore, at higher pressures increases because of the increased pressure expelling the liquid and the increased breadth of the crack.

This effect is avoided in the gauge used in this work by subjecting the cylinder in which the piston plays to pressure on the outside as well as on the inside. It is well known that a cylinder subjected to the same pressure externally and internally shrinks to the same extent as a solid cylinder subjected to the same external pressure. By properly decreasing the external pressure on the hollow cylinder, the shrinkage at the inner surface may be made as small as we please, or may be made an expansion. Practically the same result may be obtained by subjecting only a portion of the external surface of the cylinder to pressure. By suitably changing the area subjected to pressure, the shrinkage of the interior may be controlled. This is the method adopted with the present gauge.

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[5] Amagat, Ann. de Chim. et Phys., (6), **29**, 544 (1893).

[6] Zeit. f. Instrk., **14**, 307 (1894). Manometer für hohe Druck.

[7] Engineering, **75**, 31 (1903). The Estimation of High Pressures.

[8] Lisell. Om Tryckets Imflytande på det Elektriska Ledningmotståndet hos Metaller, samt en ny Metod att Mäta Höga Tryck. Upsala, 1903 (C. J. Lundström).

The leak may be further decreased by decreasing as far as possible the dimensions of the piston and cylinder, thus decreasing the circumference of the crack through which leak occurs. Decreasing the size has the additional advantage of making the whole gauge more compact and manageable. In particular, the total thrust becomes small enough to be balanced directly by hanging weights on the free end of the piston. Where the magnitude of the weights is not so great as to make this infeasible, the direct application of weights seems preferable to the usual indirect methods of measuring the thrust. In the gauge adopted in this work, the piston is only $\frac{1}{16}$ in. (0.159 cm.) in diameter, requiring at the maximum pressure of 6800 kgm. an equilibrating weight of about 130 kgm.

The cylinder and piston are shown in Figure 1. In Figure 2 they are shown in place in a large steel block which serves as a reservoir between the gauge and the pressure pump. The dimensions of the important parts are indicated in Figure 3. The thrust on the piston P (Figure 1) is taken by the large cylindrical rod A joined to the piston by a forced fit. A terminates in a hardened point B, on which the weights are hung by a stirrup supporting the scale pan underneath the large steel block. The upper end of the cylinder acts as a guide for the rod A, as does also the attachment screwed onto the top of the cylinder shown in Figure 2. It is essential that fitting here should be accurate, so that the small piston may move freely in a vertical line without danger of any bending of the top end when projecting some distance from the cylinder.

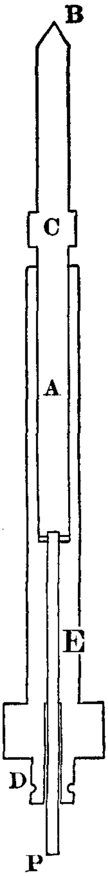

FIGURE 1. The direct reading gauge. P, piston; E, cylinder; A, larger steel rod through which the pressure of the equilibrating weights is transmitted to the piston; B, hardened steel point on which the stirrup carrying the weight pan is hung; C, stop (see Figure 2); D, groove by which the rubber tube containing the viscous mixture of molasses and glycerine is attached.

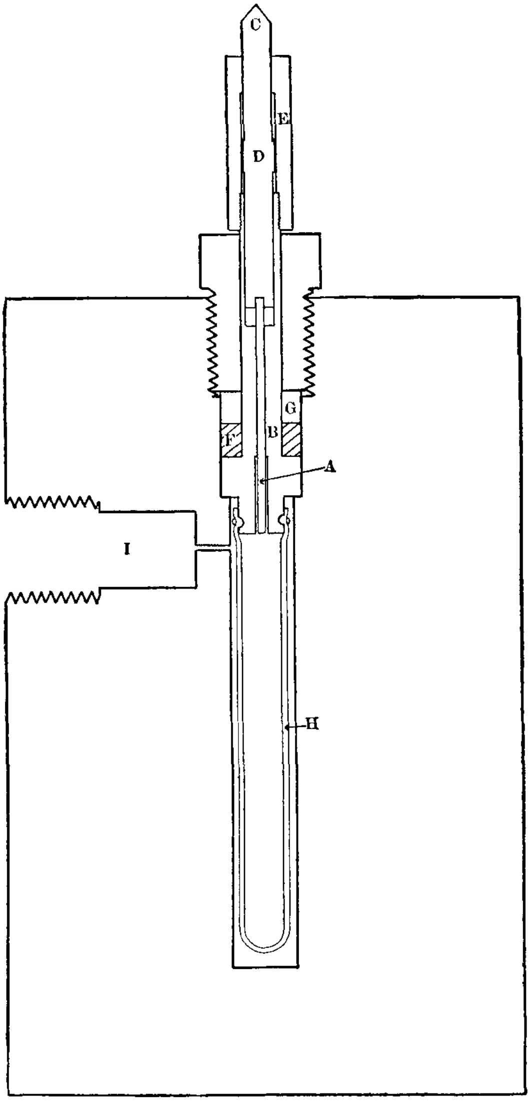

FIGURE 2. A, piston; B, cylinder; C, hardened steel point, on which the equilibrating weights are hung; D, stop, preventing too long a stroke of the piston either up or down. In this stop is placed the rod by which the rotary motion is imparted to the piston to increase sensitiveness. E, guide to insure the upper part of the piston moving rigidly in a straight line. F, rubber packing. G, steel washer, retaining the rubber packing. H, easily collapsible rubber tube, containing the viscous mixture of molasses and glycerine. I, connection to the high pressure pump. The thin mixture of water and glycerine transmitting the pressure is injected through this hole, acts on the outside of the rubber tube H, and so transmits the pressure to the piston A.

The enlargement C, on the rod A, serves as a stop at either end of the stroke, which in this case was ½ in. (1.3 cm.). The piston was made at least ½ in. (1.3 cm.) longer than the hole in the cylinder in which it fits, so that at no part of the stroke is any part of the hole empty. This insures the constancy of the crack through which leak occurs, and ought to increase the accuracy of the results. To diminish friction between piston and cylinder the piston was kept in slow rotary motion through 30° by a rod inserted in a hole in the enlargement C. The rod was driven by a small motor.

The purpose of the shoulder at the bottom of the cylinder will be plain on an inspection of Figure 2. The disposition of packing, shown by the shading, is one that has proved itself serviceable in other high pressure work. It is obvious from the figure that the pressure on the outside of the cylinder mentioned above as preventing the enlargement of the crack between cylinder and piston is the pressure exerted by this packing. The portion of the cylinder over which it acts may obviously be varied by varying the quantity of packing. With dimensions of cylinder, etc., shown above, ¼ in. (0.64 cm.) thickness of packing proved satisfactory.

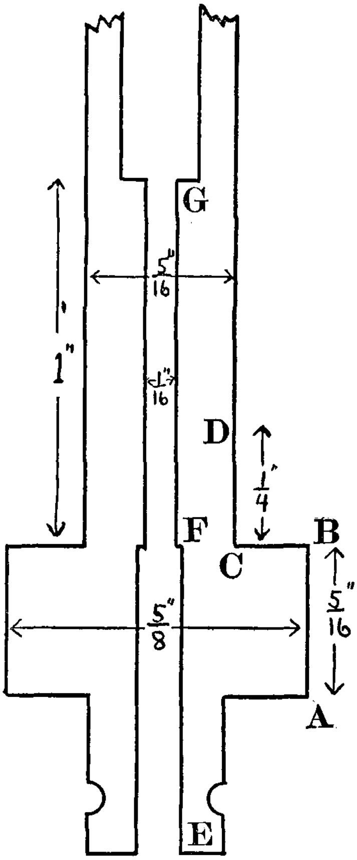

FIGURE 3. Detail, giving the dimensions of the cylinder.

To go into this question of packing at any length would be beyond the scope of this paper. Neither can any description be given here of the apparatus with which the pressure was produced. Briefly, pressure was produced by a small piston pushed by hydrostatic pressure on a larger piston. Pressure was transmitted to various parts of the apparatus by heavy steel tubing. It is hoped that methods of producing high pressures may be made the subject of another paper.

The cylinder (E, Figure 1) was turned in a lathe from a rod of about 1.25 per cent carbon tool steel. The drilling of the hole in which the piston moves demanded care. This was drilled first with a drill about 0.002 in. (0.05 mm.) under $\frac{1}{16}$ in. (1.59 mm.), and then enlarged to full size with a two-lipped $\frac{1}{16}$ in. twist drill. The hole made in this manner proved round, uniform, and satisfactory in every particular. It is a matter of common experience that a two-lipped twist drill hugs the hole very tightly when used as a following drill. For this reason, care is necessary not to push the drill too hard, as otherwise the sharp cutting corners are quickly blunted. After turning and drilling, the cylinder was hardened in water, and the temper drawn below a blue. Drawing the temper is a necessary precaution in the interests of safety, as glass hard steel proved itself very treacherous. The cylinder is so small that with the exercise of a little care in heating and quenching it is not distorted appreciably by the hardening.

The piston was a piece of $\frac{1}{16}$ in. (1.59 mm.) "Crescent" drill rod, hardened in oil, the temper not being drawn further. This drill rod was found to be remarkably round and uniform in diameter, variations of so much as 0.0001 in. (0.0025 mm.) being rare from end to end of the same length. Different pieces, however, of nominally the same size rod may differ by 0.0005 in. (0.0125 mm.) in diameter. It was merely necessary, then, to select from several lengths of drill rod a piece fitting the hole in the cylinder. No grinding whatever was necessary, either on the cylinder or the piston, except rubbing with the finest emery paper to remove the film of oxide after hardening. In fact, it is the salvation of this device that no grinding is necessary, accurate grinding of a piston so small as $\frac{1}{16}$ in. being out of the question, to say nothing of the $\frac{1}{16}$ in. hole in the cylinder. Because of its slenderness, considerable care is necessary in hardening the piston without warping. Several attempts were usually necessary before a perfectly straight piston was obtained. This, however, is a matter of no consequence, because a piston can be made in a few minutes. The writer has himself made two cylinders and pistons complete in one day.

Leak was reduced to a very low value by using a liquid of great viscosity to transmit pressure to the piston. A mixture of molasses and glycerine proved suitable. The viscosity can be given any desired value by boiling away enough water from the molasses before adding the glycerine. Besides increasing the viscosity, the glycerine serves the useful purpose of preventing the molasses from drying where it leaks out between piston and cylinder. The liquid used to transmit pressure from the high pressure pump to the gauge was a mixture of two parts glycerine to one part water. This was prevented from coming into contact with the molasses and glycerine by enclosing the latter in an easily collapsible rubber tube, closed at the lower end, and at the upper end tied over the mouth of the cylinder, as shown in Figure 2.

Molasses was the liquid used by Amagat in his manometer. A heavy mineral oil, such as Barus used in a gauge of Amagat's type, was found to be unsuitable for high pressure work, because it freezes at room temperature under pressure. One grade of heavy oil tried in this experiment froze at 20° under a pressure of 4500 kgm. Presumably vaseline and such soft solids become unsuitable for the same reason, although this point was not tested. For the same reason the glycerine transmitting pressure from the pump had to be diluted with water. The ease with which glycerine subcools, and the difficulty of getting it pure, made any exact determinations impossible; but it was found that commercially pure glycerine was very apt to solidify at 6000 kgm. and 20°.

CORRECTIONS TO BE APPLIED TO THE ABSOLUTE GAUGE

In spite of the simplicity of this gauge, and the directness with which it carries the measurement of pressure back to the fundamental definition, there are two corrections which must be applied in practical use. These corrections are both so small, however, that neither need be determined with much accuracy.

The first correction is introduced by the slow leak, and is in amount equal to the frictional force of the escaping liquid on the piston. The equilibrating force must balance both the hydrostatic pressure on the end of the piston and this frictional force. The effect of the correction, therefore, is to increase slightly the effective area of the piston. If we assume that both cylinder and piston are perfectly cylindrical, and that the crack between them is so narrow that the friction exerted by the escaping liquid is equally divided between cylinder and piston, then we easily see by writing down the equations of steady motion of the escaping liquid that the friction increases the effective diameter of the piston to the mean of the diameter of the piston and cylinder. It appears from the equations that this correction is independent of both the rapidity of leak and pressure. This is usually determined by measuring the diameter of the piston directly, and the diameter of the hole in the cylinder by some such indirect method as weighing the quantity of mercury required to fill it. The dimensions of the gauge used here were so small, however, that direct measurement of even the piston could not be made with the desired percentage accuracy, and accordingly the effective diameter was determined in another way, to be described later.

The second correction is a correction for the distortion of the gauge under pressure, and increases in percentage value directly with the pressure. This correction, of course, varies with the type of gauge, but in the types of gauge described above, and the pressure gauge employed, the correction is practically negligible. A rough calculation showed that at 3000 kgm. the correction in Amagat's manometer is about $\frac{1}{10}$ per cent. Since, however, it was desired in this work to reach an accuracy of $\frac{1}{10}$ per cent, and since the pressure range is 6800 kgm., some approximate evaluation seemed desirable.

No easy experimental method of determining this correction suggested itself, so recourse was had to a calculation, using the theory of elasticity. This was done only as a last resort, because of the doubtful accuracy of the mathematical theory at these pressures, and of the fact that the solution obtained is only an approximation, instead of a rigorous mathematical solution. In fact, the general problem involved has not been solved mathematically, and even if it could be, its application here would be doubtful, because slight irregularities in either cylinder or piston would destroy the ideal boundary conditions of the mathematical problem. In spite of all these objections, however, the magnitude of the approximate correction turned out to be so slight, $\frac{1}{10}$ per cent, that the calculated value can probably be applied with a fair degree of confidence.

The facts used in the following calculation are taken from the most elementary parts of the theory of elasticity, and may be found stated in any book under the calculation of the strains produced in a cylinder by external or internal hydrostatic pressure. It will be noticed that the correction for distortion found below includes the effect of the friction of the escaping liquid.

The strain in the piston can be broken up into two components. The first is that due to the longitudinal compression of the piston by the hydrostatic pressure at one end and the equilibrating weights at the other, and is uniform throughout the piston. The radius increases from this effect by the amount

$$\Delta r = \frac{3\kappa - 2\mu}{18\mu\kappa} \times r \times P,$$

where P is pressure in kgm. per sq. cm., κ the compressibility modulus, and μ the shear modulus. These elastic constants vary only slightly in different grades of steel. If we assume as average values that we find that

$$\mu = 7.8 \times 10^5 \text{ kgm./cm.}^2,$$
$$\kappa = 15 \times 10^5 \text{ kgm./cm.}^2,$$
$$\Delta r = 1.4 \times 10^{-7} \times r \times P.$$

The second component part of the strain is that due to the pressure of the escaping liquid over the curved surface of the piston. Here an approximation must be introduced, for the determination of the strain in a cylinder under a given system of normal stresses on the curved surface seems to be a mathematical problem not yet solved, while in this case the problem is additionally complicated because the stress system is not given but depends in turn on the strain. The approximation made is the assumption that the radial displacement at any point is proportional to the normal pressure at that point, and is the same as that in an infinite cylinder subjected to the same pressure over its entire length. This assumption is probably fairly close to the truth where the extent of the cylinder exposed to the pressure is long compared with the radius, and the pressure varies gradually from point to point, as is the case here.

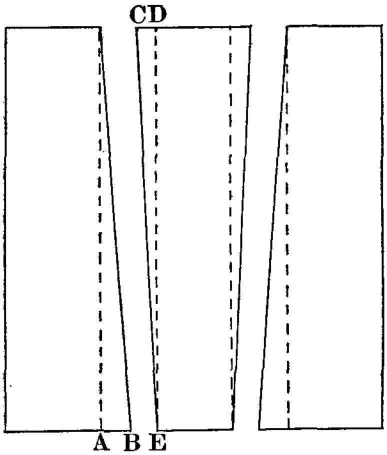

Figure 4. Exaggerated effect of the pressure in distorting the cylinder and piston.

The piston then assumes under the external pressure the form of a frustum of cone, as is shown in Figure 4. It will appear in the following that it is absolutely immaterial whether the generating lines of the frustrum of the cone into which the piston has been deformed are straight, as drawn in Figure 4, or not. The displacement at any point due to the external pressure is, on the above assumptions,

$$\Delta r = -\frac{4\mu + 3\kappa}{18\mu\kappa} \times r \times p$$
$$= -3.5 \times 10^{-7} \times r \times p.$$

p increases along the piston from its full value, P, at the inner end, E, to zero at the outer end, C. Now by adding these two components of strain, we find that the total radial displacement of the piston consists of a shrinkage of $2.1 \times 10^{-7} \times r \times P$ at the inner end, and a swelling of $1.4 \times 10^{-7} \times r \times P$ at the outer end.

The strain in the cylinder is more difficult to compute because of the uncertainty in the external boundary conditions introduced by the packing. Upon the portion of surface DCB (Figure 3) there is a normal pressure exerted by the packing, equal to 1.32 of the internal hydrostatic pressure. On BAEF there is the normal hydrostatic pressure, and from F to G the same distribution of pressure as on the piston, decreasing from the full value at F to zero at G. The maximum radial displacement due to external pressure may be taken as somewhat less than that from a pressure equal to 1.32 P over the entire external surface, because of the supporting action of the part AB, which is subjected to P only, and of the part beyond D, on which there is no pressure. An upper limit to the distortion is probably set by the distortion of an infinite cylinder subjected to 1.32 P on the outside, and P on the inside. This gives

$$\Delta r = \left(- \frac{0.32\, a^2}{2\,\mu\,(a^2 - b^2)} + \frac{4\,\mu + 3\,\kappa}{18\,\mu\kappa} \cdot \frac{b^2 - 1.32\, a^2}{a^2 - b^2} \right) \times b \times P$$
$$= -\,6.9 \times 10^{-7} \times b \times P$$

where a is the external radius, $\frac{5}{16}$ in. (0.79 cm.), and b the internal radius, $\frac{1}{16}$ in. (0.16 cm.). A value probably nearer the truth is found by assuming for the effective external pressure 1.16 P, i. e., a mean between the maximum and the pressure on AB. This gives

$$\Delta r = \left(- \frac{0.16\, a^2}{2\,\mu(a^2 - b^2)} + \frac{4\,\mu + 3\,\kappa}{18\,\mu\kappa} \cdot \frac{b^2 - 1.16\, a^2}{a^2 - b^2} \right) b \times P$$
$$= -\,5.3 \times 10^{-7} \times b \times P$$

and this value will be used in this computation. This represents the maximum radial displacement of the cylinder, which occurs at the inner end; at the outer end there is no pressure either external or internal, and the displacement will be assumed to vanish. Throughout the length of the cylinder the displacement at the inner surface will be assumed proportional to the internal pressure at that point, although the approximation is not so good here as for the piston.

From these displacements of piston and cylinder it is now required to correct for the change in the effective area of the piston. We do this by considering the equilibrium of the escaping liquid. The piston and cylinder each exert on the liquid approximately the same frictional force (F). Furthermore, the cylinder exerts on the escaping liquid a pressure P_1, which is the negative of the component in the direction of the axis of the pressure of the liquid in the crack on the cylinder. P_1 corresponds, therefore, to the axial component of pressure on a ring of breadth AB (Figure 4). Similarly the piston exerts a pressure P_2 equivalent to that on a ring CD. The free liquid at the inner end exerts P_3 on the ring BE. Since the liquid escapes steadily without acceleration, we have

$$2F + P_2 = P_1 + P_3.$$

The effective force on the piston is $F + P_2$

$$F + P_2 = \frac{P_1 + P_3 + P_2}{2}.$$

We now can calculate P_1 and P_2 without any assumption as to the distribution of pressure in the crack if we assume only that at every point the radial displacement is proportional to the pressure at that point. This gives

$$P_1 = 2\pi R \int_{r_1}^{r_2} pdr,$$

where r_1 is the value of r at the end ABE of the cylinder, and r_2 at the end CD. R is the average of r_1 and r_2. But

$$\begin{aligned} r_2 - r &= Cp, \\ dr &= -Cdp, \end{aligned}$$

$$\begin{aligned} P_1 &= -2\pi CR \int_{P_A}^{P_C} pdp \\ &= 2\pi C \frac{RP^2}{2} = 2\pi \frac{P(r_2 - r_1)}{2} R. \end{aligned}$$

That is, P_1 is equal to the pressure exerted by the total internal pressure P on a ring of half the breadth of AB. Similarly, P_2 is the pressure on a ring of half the breadth of CD. If now we put R equal original radius of piston, and $R + \Delta R$ equal original radius of cylinder,

$$\begin{aligned} AB &= 5.3 \times 10^{-7} \times (R + \Delta R) \times P, \\ CD &= 3.5 \times 10^{-7} \times R \times P, \\ BE &= \Delta R + (2.1 \times 10^{-7} - 5.3 \times 10^{-7}) \times R \times P, \\ &= \Delta R - 3.2 \times 10^{-7} \times R \times P. \end{aligned}$$

Hence, $$F + P_2 = 2\pi R \frac{(2.6 + 1.8 - 3.2)\, 10^{-7} \times R + \Delta R}{2} \times P$$

$$= 2\pi R \left(\frac{\Delta R}{2} + 1.2 \times 10^{-7} \times R\right) \times P.$$

This force, $F + P_2$, acts in addition to the hydrostatic pressure on the inner end of the piston, which is now decreased in radius by $2.1 \times 10^{-7} \times R \times P$. The new effective radius is therefore

$$R + \frac{\Delta R}{2} - (2.1 - 1.2) \times 10^{-7} \times R \times P,$$

as compared with the original effective radius $R + \Delta R/2$. The correction on the area is therefore $2 \times (2.1 - 1.2) \times 10^{-7} \times P$, or 0.018 per cent per 1000 kgm. The correction turns out, as was to be expected, independent of the size of the crack.

If the maximum value given above for the distortion of the cylinder is used, the effective radius will be found to be

$$R + \frac{\Delta R}{2} - 1.7 \times 10^{-7} \times R \times P,$$

which gives a maximum correction of 0.034 per cent per 1000 kgm. per sq. cm. Experimental reasons will be given later for preferring the lower value for the correction. This value, 0.018 per cent per 1000 kgm., was therefore the correction applied in all the subsequent work.

The Gauge in Practical Use.

The first essential in making an actual measurement with this gauge is a knowledge of the effective area of the piston. As has been intimated above, this could not be determined directly because of the smallness of the parts, and an indirect method was therefore adopted. Briefly, this consisted in subjecting simultaneously to the same hydrostatic pressure the small piston and another piston large enough to be measured accurately, and finding the equilibrating weights required on the two pistons. The effective areas are then in the ratio of the equilibrating weights.

The larger piston was ¼ in. (0.635 cm.) in diameter, 2 in. (5.18 cm.) long, ground to fit a reamed ¼ in. hole in a large cylinder of Bessemer steel. As this larger gauge was intended for use only to 1000 kgm., the increased breadth of crack produced by exerting the pressure on the interior only of the cylinder was not great enough to give troublesome leak. Also the correction to the effective cross section due to distortion is small enough to be entirely neglected at 1000 kgm. The diameter of the ¼ in. piston could be measured certainly to one part in 2500 with a Brown and Sharpe micrometer. The hole in the cylinder was not measured by filling with mercury and weighing, or by any such frequently employed device. It was instead carefully tested against the piston while the latter was in process of being ground to size. The piston was too large to enter the hole except by forcing, when 0.0001 in (0.00025 cm.) larger than the final size. This allowance is probably too much, but still probably not so high as to make the error introduced here in the effective area as much as $\frac{1}{10}$ per cent. This method of measuring the diameter of a hole by testing against plugs of known size is the method used by Brown and Sharpe themselves, and is probably the most accurate that we have, when it is possible to obtain the comparison plugs. The comparison of piston and cylinder was easy in this case because all the work was done in the machine shop of this laboratory.

As preliminary work with this larger gauge, a Bourdon gauge by the Société Genevoise was calibrated to 1000 kgm., and showed a maximum error of 5 kgm. per sq. cm. Various liquids were used to transmit pressure to the $\frac{1}{4}$ in. piston, from vaseline which gave a barely perceptible leak, to a thin mixture of water and glycerine, with which the leak was so rapid that pressure could be maintained only with difficulty. The indications of the gauge, as compared with the Bourdon gauge, proved independent of the rapidity of leak, as they should. In the use of the gauge, sensitiveness was secured as usual, by keeping the piston in continual rotation. Made sensitive in this way, the gauge was very much more sensitive than the Bourdon gauge, responding to about one part in 20,000 at 1000 kgm.

Two high pressure gauges of the type described above were compared with this $\frac{1}{4}$ in. gauge at 1000 kgm. Pressure was kept constant during the comparison by the rise or fall of the $\frac{1}{4}$ in. piston, which had a long enough stroke to accomplish this. As was to be expected, the larger piston proved more sensitive than the smaller ones. The certainty of rise or fall of the small pistons was made greater by observing them with the telescope of a cathetometer. The method of proceeding was to apply a constant weight to the small piston, and then find the two weights on the large piston for which the small piston just began to rise or fall. To accomplish this, the weight on the large piston had to be changed by 0.4 kgm. with a total load of 300 kgm. The mean of these two extreme values gives, therefore, the true equilibrating weight to certainly $\frac{1}{10}$ per cent, and probably much better than this.

From the effective area of either piston found in this way, and the measured diameter, the size of crack between piston and cylinder can be computed. It turned out to be 0.0001 in. (0.00025 cm.) for one gauge, and 0.0003 in. (0.00075 cm.) for the other. This was roughly verified by the more rapid leak shown at higher pressures by the latter gauge. With the former gauge the leak was almost imperceptible after pressure had been kept at 7000 kgm. for an hour. It is a curious fact that the leak around the more loosely fitting piston was distinctly most rapid at 2000 kgm. The decreased leak at higher pressures may probably be taken as proof of the efficiency of the application of pressure to the outside of the cylinder in decreasing the size of the crack, although there is a slight possibility that the effect is due to increased viscosity of the molasses under pressure.

With this calibration, the critical examination of the behavior of the gauges might have been terminated, because the simplicity of the construction is such as to make improbable any error in their use. As a matter of fact, the indications of the various types of gauge described above have usually been accepted at their face value, without comparing with any other absolute gauge. There were means at hand in the present case, however, of so easily comparing the one gauge with the other that it seemed worth while doing. The method adopted was an indirect one, depending on the secondary mercury gauge described in the second part of this paper. It had been found from a great many preliminary comparisons of different mercury gauges that the indications of the mercury gauge were constant, giving a trustworthy measurement of pressure, if once the calibration with a primary gauge could be effected. More detailed proof of this statement will be found in the second part. The two absolute gauges described above were, therefore, compared at different times against the same mercury gauge, and the two sets of readings compared.

The results of the comparisons are shown in Table I. Gauge I was compared twice with the mercury resistance, and Gauge II once. Each number entered in the table is the mean of two or four readings made at increasing or decreasing pressures. The agreement of the two readings under increasing or decreasing pressure, as also of the readings of Guage I on two separate occasions, was as close as it was possible to make the measurements of change of resistance, and, therefore, only averages have been tabulated. The change of resistance could be read to one part in 3000, at the maximum pressure. The average divergence of the readings of either gauge from the mean is well under $\frac{1}{10}$ per cent. The readings of Gauge II are consistently higher than those of Gauge I, a discrepancy which would point to a slight error in determining the effective area of the pistons. The discrepancies also show a tendency to become larger at the higher pressures. This is probably no fault of the gauges themselves, but may be due to the increased difficulty of making fine adjustments of pressure at the higher values. The method of procedure was to apply a known weight to the piston, and then vary the pressure until equilibrium was produced. Setting on this equilibrium pressure was made more difficult by the fact that pressure always showed a tendency to fall after an increase, and to rise after a decrease, a fact that may be explained by thermal effects of compression, but is more probably due to elastic after effects in the containing steel vessels. It may be concluded, therefore, from the agreement of these comparisons, that even if all the error is in the absolute gauge and none in the mercury resistance, that this type of gauge is good to about $\frac{1}{10}$ per cent.

TABLE I.

Comparison of Two Absolute Gauges against the Same Mercury Gauge.

Gauge I.		Gauge II.		$\frac{\Delta R}{R_0}$ from Gauge I at Gauge II Pressures.	Percentage Divergence from Mean.
Pressure kgm./cm.^{2}	$\frac{\Delta R}{R_0}$.	Pressure kgm./cm.^{2}	$\frac{\Delta R}{R_0}$.		
917	0.002862	929	0.002898	0.002897	−0.015
1501	0.004555	1519	0.004605	0.004604	−0.012
2018	0.005960	2043	0.006032	0.006025	−0.05
2602	0.007491	2634	0.007577	0.007572	−0.03
3196	0.008989	3235	0.009095	0.009083	−0.05
3779	0.010390	3825	0.010530	0.010500	−0.10
4233	0.011420	4285	0.011560	0.011530	−0.15
4816	0.012740	4864	0.012840	0.012840	−0.00
5348	0.013860	5414	0.014020	0.013990	−0.10
5932	0.015030	6005	0.015220	0.015180	−0.13
6452	0.016070	6531	0.016290	0.016230	−0.20
6841	0.016820	....	....	....	....

The absolute gauges were not corrected for distortion, as this is not necessary for the comparison.

The comparison with mercury gauges also furnished an estimate of the sensitiveness of the gauge. It was found that throughout the entire pressure range the pistons would respond to differences of pressure that could not be detected by the change of electrical resistance. At 7000 kgm., therefore, the gauges remain sensitive to at least 2 kgm. per sq. cm. The continued sensitiveness of the piston with the crack only 0.0001 in. furnishes an argument against the maximum value set, in the discussion above, on the distortion of the cylinder. For, if we accept the above maximum, we shall find that at 7000 the crack must decrease 0.00018 in., or in this case completely close up. There cannot well be an error of this magnitude in the micrometer measurement of the diameter, and the probable correctness of the average value of the distortion used above is thus increased.

Conclusion.

In this first part of the present paper a description has been given of an absolute gauge, so designed that leak does not become troublesome, at least to 6800 kgm. per sq. cm. The various corrections to be applied have been discussed, and the method by which the dimensions were determined has been described. From a comparison of two gauges of this type with one of another type, the probable accuracy of the gauge is estimated to be at least $\frac{1}{10}$ per cent, and the sensitiveness, 2 kgm. per sq. cm., at 7000 kgm. per sq. cm.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE MEASUREMENT OF HIGH HYDROSTATIC PRESSURE.

II. — *A SECONDARY MERCURY RESISTANCE GAUGE.*

By P. W. Bridgman.

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THE MEASUREMENT OF HIGH HYDROSTATIC PRESSURE.

II. A SECONDARY MERCURY RESISTANCE GAUGE.

By P. W. Bridgman.

Presented by W. C. Sabine, December 9, 1908. Received December 16, 1908.

In the introduction to the first part of this paper it was stated that the end sought in designing the primary gauge was the calibration by means of it of some secondary gauge which should be easily reproducible. The secondary gauge that it was proposed to adopt is one involving the variation of mercury resistance with pressure. This is of an entirely different character from the type of secondary gauge in common use, which is usually some form of metallic deformation gauge like that of Bourdon. Undoubtedly the Bourdon is one of the most convenient forms of secondary gauge that it would be possible to devise, being almost immediate in its action, and capable of standing considerable rough handling. If it were applicable over the wide pressure range contemplated for the mercury gauge, its greater convenience would certainly overbalance the fact that every such Bourdon gauge must be initially calibrated against some direct standard.

It seems to be a fact, however, that any elastic deformation gauge becomes unsuitable at high pressures, even when once calibrated, because of the entrance of hysteresis effects. It is true that the existence of elastic hysteresis effects has frequently been doubted, and it has even been stated that proof of their existence would give us knowledge of a new elastic property. It nevertheless seems to be a fact that hysteresis may be inappreciable at low values of the stress, but become increasingly important at higher pressures. This is not the place, however, to enter into a discussion of this point, which will afford the subject for another paper. But in this paper there will be given a somewhat detailed examination of the behavior under pressure of one Bourdon gauge, which will at least show that this type of gauge is irregular at high pressures, whatever the true explanation of this irregularity may be. This paper will be chiefly concerned with a careful examination of the suitability of the proposed mercury standard, and a determination of the constants necessary to its use up to 6800 kgm. per sq. cm. At the end will be found a calculation from the constants of the mercury gauge of the variation of the specific resistance of mercury under pressure. This calculation involves a knowledge of several compressibilities, which had to be independently determined. In order, however, not to group together in one paper unrelated matter, the determination of compressibilities under high pressures is made the subject of another paper, and only the numerical results there found are used here.

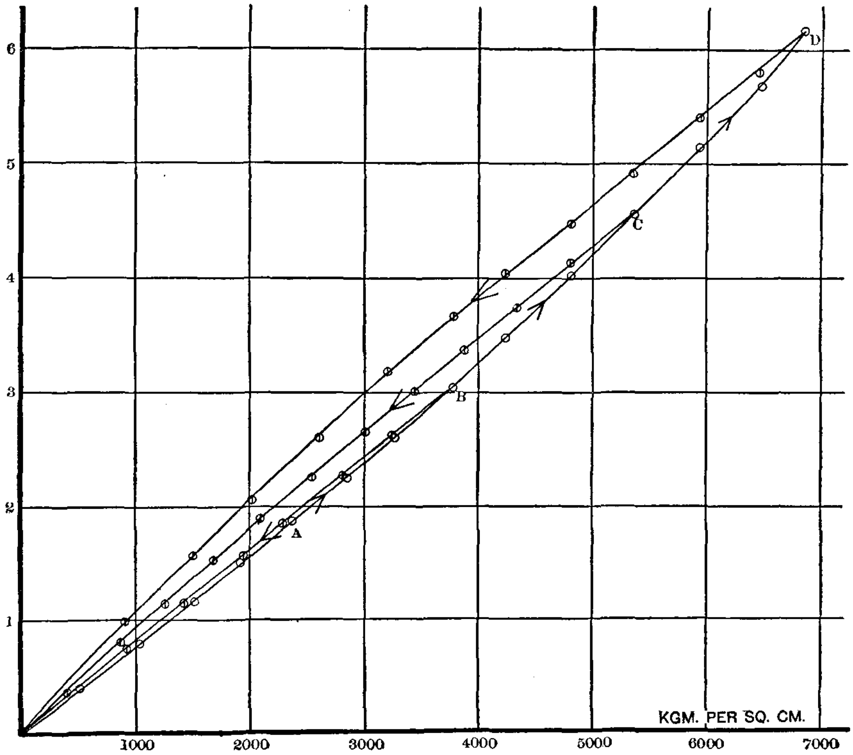

FIGURE 1. Deflection of free end of Bourdon gauge plotted against pressure. Four complete cycles are represented, the points A, B, C, and D being the successive turning points. The figure shows the increasing importance of hysteresis at higher pressures.

The Bourdon gauge used consisted of hard drawn Shelby steel tubing $\frac{5}{16}$ in. (0.79 cm.) outside diameter and $\frac{1}{16}$ in. (0.159 cm.) inside diameter, wound into a helix of five turns of 5 in. (12.7 cm.) diameter. The tube was not flattened into an elliptic cross section, as in the ordinary Bourdon gauge, since to do this would have too greatly decreased the strength. Even when the cross section is left round, however, the tube unwinds upon the application of pressure, like the ordinary Bourdon. The amount of unwinding was read directly by observing the position of the free end with a microscope, a method of reading which proved more satisfactory than any multiplying mechanism. Thus gauge had been in use for upward of six months before the readings shown in Figure 1 were made. The gauge had been so thoroughly seasoned by the many applications of pressure in this interval that the deflections on many subsequent occasions were found to agree within the errors of reading. Initially, the gauge showed some slight set under the maximum pressure, but after the first few applications of pressure no further set appeared. Elastic after effects, which might be expected to be troublesome over this wide pressure range, could be noticed at every stage of the pressure variations, but were too small to appear on the diagram.

In Figure 1 the deflection of the free end (mm.) is plotted against pressure in kgm., which was measured with a mercury resistance that had been calibrated against an absolute standard, as will be described later. The figure shows the effect of applying four cycles of pressure, from zero by steps to the maximum and by steps back to zero, each subsequent maximum being higher than the preceding. Pressure was first applied in steps from zero to A, and then reduced to zero. The return path coincides so closely with the initial path that the difference cannot be shown on the diagram. Pressure was now increased from zero to B and decreased to zero. The first part of the path zero–B coincides exactly with the path from zero to A. The return path B–zero is sensibly linear, but does not coincide with the path zero–B. We have here, then, the beginning of departure from linearity, and also the beginning of hysteresis. Two more loops, zero–C–zero, and zero–D–zero, reaching to higher pressures, were now described. The essential characteristics are the same, but departure from linearity and hysteresis both increase rapidly with the rise of the range. The return paths for these longer loops do not continue linear, as for zero–B–zero, but they both start as straight lines and run for about the same distance before beginning to curve down to meet the origin. The increasing importance of hysteresis is shown by the fact that the greatest error introduced by hysteresis in the loop zero–B is

4 per cent, while in the loop zero–D it is 40 per cent, an increase of tenfold for a doubling of the pressure range. The return path D–zero was not described at the same time as the part zero–D, because an explosion occurred when the maximum D was reached. It is, however, the return path described on another occasion when the initial path zero–D was identical with the above.

Other types of gauge have shown the same characteristics at high pressures. Whatever the true explanation may be, it has been found in every case that an elastic deformation gauge does show behavior like the above. This type of gauge appears, then, to be unsuitable for the accurate measurement of high pressures, and must be replaced by some form not showing hysteresis; for even if this gauge were readily reproducible, the fact that it shows hysteresis would make its indications such a complicated function of pressure, both present and past, that the meaning of the indications could not be conveniently deciphered.

Any scalar physical property when changed by a strain the same in every direction, such as is produced by hydrostatic pressure in a perfectly homogeneous solid, or a liquid, may be expected to show no hysteresis relative to the stress. Such a property, which has the advantage of being easily measured, is electrical resistance. This has been proposed at least twice as a pressure indicator.

Lisell[1] measured the resistance of a number of metals, drawn out into wires, when subjected to hydrostatic pressures up to 3000 kgm. Pressure was measured on an absolute gauge in which the pressure on the freely moving piston was balanced by weights on a lever. Lisell found no evidence of hysteresis, and proposed the measurement of electrical resistance as a satisfactory means of measuring pressure. The variation of resistance of metallic wires, however, was found by Lisell to have the fatal disadvantage for the present purpose of being so greatly influenced by slight impurities in the metal that specimens of the same metal from different sources gave very different results. This gauge, then, would not be reproducible, but each new specimen of wire would have to be calibrated individually against some absolute standard. In addition, the pressure coefficient is inconveniently small, so that great care must be taken to avoid other effects in measuring the slight change of resistance brought about by pressure. Lisell claims as an advantage of this method that the heat of compression of the metallic wires is smaller than for most substances.

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[1] Lisell, Om Tryckets Inflytande på det Elektriska Ledningsmotståndet hos Metaller, samt en ny Metod att Mäta Höga Tryck. Upsala, 1903. (C. J. Lundström.)

De Forest Palmer,[2] working with the high pressure apparatus of Barus, made measurements of the electrical resistance of mercury up to 2000 kgm., and suggested it as a suitable secondary standard. He gives data from which the pressure can be calculated if the change of resistance is known. It appears from his work that the pressure coefficient is large enough to make accurate measurements of the change of resistance easy. The additional advantage of presumable reproducibility made it seem worth while to examine with some care its suitability as a secondary standard. The conclusion reached is that with ordinary care the mercury resistance gauge is good to about $\frac{1}{10}$ per cent.

In order to attain this probable degree of accuracy, however, it was necessary to examine several minor points with somewhat greater detail than de Forest Palmer found necessary for the purpose of his work. The probable error in de Forest Palmer's work was $\frac{1}{10}$ per cent on the total resistance, which means an error of 1.5 per cent on the pressure at 2000 kgm. The percentage error at lower pressures is of course proportionally greater. Within these limits of error he found the pressure coefficient to be constant. Furthermore, the mercury was placed in a capillary of some glass not specified, so that the data given will not apply to other mercury gauges with a greater degree of accuracy than the possible error introduced by variations in the compressibility of the glass. It is known that different grades of glass may differ in compressibility by as much as 100 per cent.

In fact, this matter of the glass containing vessel proved to be the chief source of possible error. Pure mercury may with confidence be assumed to be perfectly reproducible, and since internal strains cannot be set up in it, to be also perfectly free from hysteresis. The glass, however, is a solid in which it is particularly difficult to get rid of internal strains. It cannot be assumed, therefore, that a pure hydrostatic pressure will not produce hysteresis, or even set analogous to the volume set shown in thermometers after exposure to changes of temperature. It is an advantage, however, that the total effect of the glass envelope is unusually small, both because of the comparative largeness of the pressure effect on the resistance of the mercury, and because the correction factor is only $\frac{1}{3}$ instead of the whole of the compressibility. This latter fact is due to the simultaneous shortening of the capillary which contains the mercury, and the decrease of the bore, the one resulting in an increase of resistance and the other in a decrease. The total correction on the observed change of resistance introduced by the glass envelope is only 2.5 per cent as against 60 per cent in determinations of the compressibility of mercury. Hysteresis and other irregular action will appear, therefore, simply as perturbations of this 2.5 per cent correction. There are a number of smaller sources of error, which, even though very obvious, will be mentioned as occasion presents, because in the justification of a new standard it seems well to record all the sources of error that were considered or guarded against.

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[2] de Forest Palmer, Amer. Jour. Sci., **4,** 1–9 (1897), and **6,** 451 (1898).

The electrical measurements were carried out on a bridge of the Carey Foster type provided with an eight point mercury switch. The variable mercury resistance took the place of one extension coil, and the other was a manganin coil of approximately ten ohms. Measurements were made by setting the slider for no deflection, this being preferable to measuring the current by ballistic or steady throw of the galvanometer. A D'Arsonval galvanometer of low resistance was used, of sensitiveness great enough to indicate changes in the position of the slider of less than $\frac{1}{10}$ millimeter. Extension coils and balancing coils were of seasoned manganin, all approximately ten ohms. In comparing together two mercury resistances the same balancing and extension coils were used, the bridge being provided through leads of $\frac{1}{4}$ in. copper wire of negligible resistance with two slide wires one meter long. The slide wires were interchanged by mercury switches frequently cleaned. The resistance of the extension and balancing coils, as well of the bridge wire, was measured against standard manganin coils known to be correct to 0.01 per cent, which were kindly loaned for the purpose by Professor B. O. Peirce. The bridge wire was calibrated for uniformity by stepping off on it a resistance equivalent to approximately 10 cm. at 3 cm. intervals. The maximum correction of one wire was 0.4 mm., of the other 0.7 mm. The average arithmetic correction of the first was 0.17 mm., of the latter 0.4 mm. Approximately 33 cm. of either wire has a resistance of one ohm. All the connections in the circuit were either soldered without acid for a flux or were through mercury cups, except two connections at the insulating plug leading to the mercury resistance, which were made with nuts. As it was found that induction effects were unnoticeable, the bridge was operated with the galvanometer circuit permanently closed, thus eliminating the principal sources of thermal currents. Two readings of every resistance with the extension coils interchanged were really unnecessary, therefore; but they were always made so as to secure the increased accuracy of two independent settings. Current was supplied by a single Samson cell of about one volt, and was decreased by inserting 100 ohms in the battery circuit. The current through the mercury resistance was therefore about $\frac{1}{220}$ ampere. It was necessary that the current be about as small as this to avoid heating effects in the very fine mercury thread. With this low current, however, the key might be closed indefinitely, with no apparent change in the resistance of the mercury.

In carrying out the measurements, the first and most considerable difficulty that presented itself was the designing of a suitable insulating plug for leading the electrical connections into the pressure chamber. Amagat, and most investigators following him, have used as insulating plug a cone of steel (B, Figure 2) separated from the surrounding walls of the pressure chamber by a thin layer (A) of hard rubber or ivory. Any such arrangement as this proved unsuitable for the pressures dealt with here, the hard rubber flowing completely out of the conical crevice, and exuding in the form of a more or less continuous cylindrical tube. Various modifications of this, using the tougher red fibre instead of hard rubber, were tried with little success. Silk also was used as an insulating material and with better success. The silk was cut out in the form of a number of discs and placed around the shank of the cone, which was then forced into its seat. It was found advisable to make the cone and its shank from one piece of steel, otherwise they were pulled apart by the friction of the silk. This form of plug has a high enough insulating resistance and is tight, but has the disadvantage of not being permanent. After ten or twenty applications of pressure the silk loses all semblance of structure, and leaks more and more rapidly with every successive application of pressure.

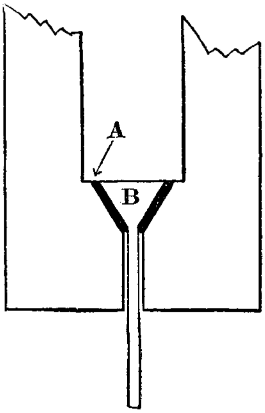

FIGURE 2. Amagat's insulating plug. A, insulating shell of hard rubber or ivory; B, cone of steel. At high pressures the insulating material, A, flows out of the crack.

The cone was now given up and mica tried for insulation, tightness being secured by a layer of marine glue (G, Figure 3). The mica showed no tendency to flow or crumble at the unsupported edge at A. This device was much better than the silk, but it too was not permanent, the marine glue being eventually forced past the mica washers which were a drive fit in the hole. In the form finally adopted (Figure 4) the mica insulation is kept, but tightness was secured by a layer of soft rubber, R, between the mica washers, M. The small steel washer S was necessary to prevent the rubber forcing its way past the mica next the stem, where it is unsupported by the steel at the rear surface. G is an insulating tube of glass. It is well to secure the steel piece B against working loose by the nut and hard rubber washer at A. This plug is the most permanent so far found; one has been subjected to 6500 kgm. upward of seventy times with no sign of leak. The insulation resistance of these plugs is high enough for the work in hand. Initially it is over 10 meg-ohms. With successive applications of pressure the resistance drops considerably, finally

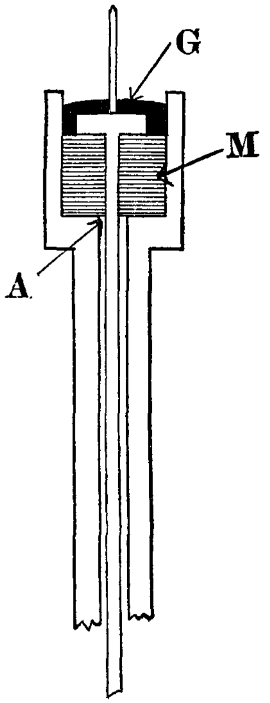

Figure 3. Preliminary form of insulating plug for higher pressures. M, mica washers; G, marine glue to prevent leak. Eventually the glue is forced by the pressure past the mica washers.

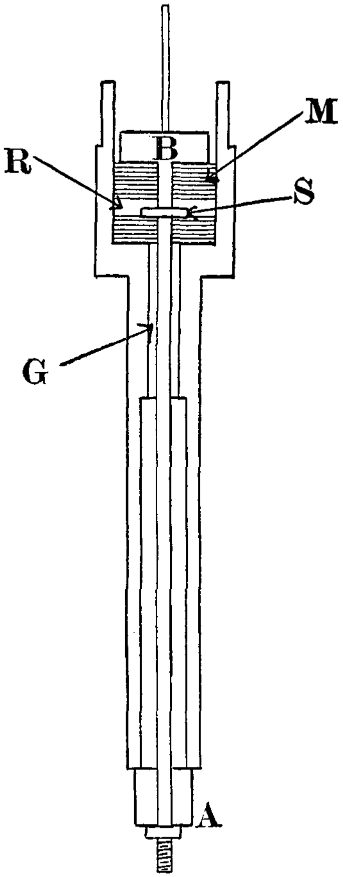

Figure 4. Final form of insulating plug. M, mica washers; R, soft rubber to prevent leak; S, steel washer to prevent leak of the rubber past the mica; G, insulating tube of glass; A, nut to keep the steel stem and the enlargement B from working loose.

reaching a steady value which is of the order of 100,000 ohms. The lowest resistance found in any of these plugs was 30,000 ohms. The resistance of these plugs was measured under pressure, all the conditions of the actual experiment as to position of the electrodes, etc., being reproduced, except for a dummy glass capillary to hold the mercury. When in use, the insulation resistance sometimes increased under pressure, the increase being sometimes as much as 100 per cent. This is still outside the limits of error, the error introduced in the above most unfavorable case being only one part in 6000 on the apparent resistance of the mercury. The performance was usually much better than this. Thus the insulation resistance of one plug which seemed to settle down after several applications of pressure at 150,000 ohms was found to be 220,000 after seven more applications of 7000 kgm.

In devising a form of vessel for holding the mercury, endeavor was made to keep the mercury as much as possible from contact with all sources of contamination by the use of platinum electrodes and a containing vessel entirely of glass. Other experimenters have allowed the mercury to come in contact with the steel of the containing vessel, using the vessel as one electrode, but this seems undesirable in view of the somewhat large effect of minute quantities of impurity. Many forms of glass containing vessel which readily suggest themselves are impractical because of the impossibility of using platinum electrodes sealed into the glass, the difference of compressibility between platinum and glass being sufficiently great to crack the glass around the electrodes. Two forms were finally adopted and used. The form first used was a U capillary (Figure 5), the electrodes dipping into the two cups at the upper end. In the form originally used this was made of thermometer tube of about 6 mm. outside diameter and 0.1 mm. bore. Several times, however, even when carefully annealed, the glass cracked at the bend, apparently because of the unequal strains set up by the hydrostatic pressure within

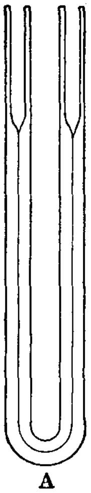

FIGURE 5. Original and final form of the receptacle for holding the mercury whose resistance is to be measured. If the glass is too thick, it invariably breaks under pressure at the bend A.

the glass, which must have been initially strained. This led to the adoption of a form in which there were no bends in the glass (Figure 6). The glass capillary (A) with the cup on the upper end for an electrode dips into the thin walled tube B containing mercury into which the other electrode dips. This form worked perfectly well, but was somewhat less convenient to handle than the U form. It was finally found that by making the stem of the U capillary very slender, about 1.5 mm., there was no tendency to crack at the bend, and this was the form with which the final determinations were made.

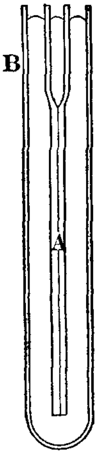

FIGURE 6. Alternative form of containing vessel for the mercury resistance. The resistance of the thin thread of mercury in the capillary A is measured. The containing vessel B must be of thin glass to insure freedom from breakage.

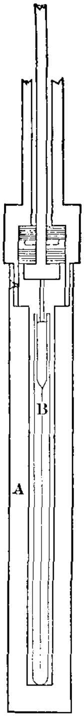

FIGURE 7. Manner of mounting the mercury resistance. The steel envelope A speedily conducts away the heat of compression.

The U capillary (B, Figure 7) is mounted in a split cylindrical piece of steel (A, Figure 7), which is attached to the lower end of the insulating plug. The capillary and plug may thus be connected together and inserted as one piece into the pressure chamber with the certainty that none of the connections will be disarranged in assembling. By making the split steel cylinder containing the U a snug fit, the glass is closely surrounded by metal on all sides, and the quantity of liquid transmitting the pressure is greatly diminished. This has the double advantage of decreasing the total change of volume of liquid necessary to reach a given pressure, and of decreasing the total heat of compression. The heat of compression generated in the small volume of liquid is so speedily conducted away by the metal that one has to work with inconvenient rapidity after increasing the pressure to find any trace of this effect. This seems to dispose of the only real advantage claimed by Lisell for the solid metallic resistance over the mercury gauge.

The electrodes are of platinum, one soldered to the outside shell of the plug, and the other to the inner stem, which is insulated from contact with the liquid by a layer of marine glue. The electrode leading from this stem is insulated with a soft fine rubber tube, except where it enters the cup of the capillary, where it is covered with a piece of glass tubing, joined continuously to the rubber above it with gutta percha. The electrode from the outer shell of the plug is also protected with glass where it enters the other glass cup. This precaution showed itself necessary, for otherwise if the platinum is not kept from contact with the walls of the cup the liquid above shows an appreciable tendency, with the successive lowerings and raisings of the surface by each application of pressure, to creep down the glass past the mercury.

There are several sources of error here that must be guarded against. Possible short-circuiting from one electrode to the other through the liquid has already been excluded by the measurements of the insulation resistance of the plug with a dummy capillary. In addition, the resistance of the electrodes between the mercury and the plug may change because of (1) lengthening of the free part of the electrode by depression of the mercury surface under pressure or distortions in the containing vessel, (2) pressure effects on the specific resistance of the platinum, (3) and change in resistance at the soldered connection between the electrodes and the plug. The first two sources of error may evidently be eliminated by using heavy enough electrodes. In this work electrodes 0.8 mm. in diameter were large enough. The third effect was found to be troublesome by Lisell, who avoided it by using long metal wires of resistance high in comparison with the resistance of the joint. No trace of this effect could be found, however, in this investigation. The absence of all three effects was tested by measuring the resistance when the terminals were short-circuited by dipping into a large tube of mercury, the resistance of the mercury now being negligible. In this case, the depression of the mercury due to compression is much greater than in the U capillary actually used. In the form tried, this depression may amount to 0.2 mm. Measurements were made up to 7000 kgm., and no change in resistance of the platinum terminal occurred of so much as $\frac{1}{1500}$ ohm, the smallest quantity that could be detected on the bridge. The possible error here, therefore, when the resistance to be measured is 10 ohms, is less than one part in 15000.

During the course of the experiments the steel cylinders containing the mercury resistance were placed in thermostats by which the temperature was usually kept constant within 0.01° during a day's work. Such constancy of temperature as this was not necessary, differences of temperature in the mercury of 0.06° being just perceptible on the bridge wire. Most of this work was carried out at temperatures of about 25°, which was high enough above room temperatures to insure the satisfactory performance of the thermostat. The temperature of the bath was read by a small Goetze thermometer graduated to tenths of a degree and calibrated at the temperature of the bath against a standard Tonnelot thermometer.

Before making the final calibration against the absolute gauge, many preliminary experiments carried out with varying success showed the necessity of observing rather carefully certain apparently insignificant matters of detail.

These preliminary tests were made by comparing together a number of pairs of mercury resistances, there being for this purpose two steel pressure cylinders to contain the resistances, two thermostats, and, as has already been mentioned, two bridge wires, either of which could be connected to the extension and balancing coils. The procedure in comparing two mercury resistances was: read resistance No. 1 on slide wire No. 1; throw in slide wire No. 2 and measure resistance No. 2; interchange the extension coils with the eight point switch and measure resistance No. 2 again; and finally throw in slide wire No. 1 and measure resistance No. 1 again. If these readings were made at equal intervals of time, as they usually were, the average of the two determinations of each resistance gives the value at the same instance of time. In this way the effects of slight changes of pressure due to dissipation of heat of compression and elastic after effects are eliminated. There was no leak. The pressure was roughly measured with the Bourdon gauge described above. These preliminary tests are competent to decide the question of the reproducibility of the mercury resistance gauge. The question of entire freedom from hysteresis, however, cannot be settled merely by a comparison of two gauges, for complete agreement would indicate only that hysteresis in the glass envelope was the same in either gauge. Entire freedom from hysteresis, within the limits of error, can be shown only by a comparison with the absolute gauge.

The results first obtained in the comparison of the two gauges were irregular beyond possibility of experimental error, discrepancies of 1 per cent being not uncommon. This was found to be due principally to three causes: minute impurities in the mercury, the effect of which will be discussed more in detail later; corrections due to air occluded in the mercury; and variations of elastic behavior of the glass envelope under pressure.

With the first few applications of pressure to the glass capillary directly after drawing, the zero value of the mercury resistance undergoes a permanent change, the magnitude of the change decreasing with successive applications of pressure until finally after four or five applications no further change is perceptible. This set is almost certainly due to a change of form of the glass vessel. This initial change has been observed as large as 3 mm. of bridge wire, that is, $\frac{1}{500}$ of the total resistance, and is always in the direction of decreased resistance, that is, toward an increase of cross section of the glass, contrary to what one might expect. If, however, this change of zero is caused by a relieving of the internal strains in the glass, it is in the direction one might expect, because the strains set up by drawing the capillary down from a larger size might decrease upon increasing the size toward its initial value. Not only is there zero change on the first application of pressure, but the elastic behavior over the entire pressure range, as shown by comparison with a well seasoned gauge, is irregular. This irregularity of behavior is shown independently of the resistance measurements by measurements of the compressibility of the glass, which will be given in another paper. The remedy for this defect is to season the glass by gradually applying and relieving the pressure several times. Sudden changes in pressure, such as have sometimes occurred when parts of the apparatus have exploded, are accompanied by large changes in the glass. If the glass has been subjected to considerable temperature changes after being seasoned in this way, it must be seasoned again before its indications are trustworthy.

Occlusion of air in the mercury is likely to cause considerable trouble if present in much quantity. Occluded air, as de Forest Palmer remarks, was doubtless responsible for the surprisingly large pressure coefficient of mercury resistance found by Lenz,[3] 0.0002. The complete removal of the air is difficult and was accomplished only once or twice. Boiling the mercury into the capillary several times is a fairly efficient method, but is open to the objection, as suggested above, that the glass must be seasoned again after each filling. Finally, after several attempts, the following somewhat extravagant method of procedure was found to work satisfactorily: One of the cups of the U capillary was nearly closed by a glass stopper, and the whole U tube was then placed in one of the two compartments of a glass vessel which was connected to a mercury pump and exhausted. Heat was then applied to the other compartment of the vessel which was full of mercury, and the mercury slowly distilled over until it covered the capillary, as high a vacuum as possible being maintained all the while by constant operation of the pump. This distillation acts as an additional purification of the mercury, that coming over being dry and presumably free from air. When the capillary was covered with mercury, air was admitted though the pump and mercury forced into the capillary through the open cup, any small possible bubble of air rising to the top of the other cup. In this way nearly all the air can be removed, the slight quantity remaining having probably clung to the inner walls of the capillary throughout the exhaustion. The quantity of air left was usually large enough to introduce an appreciable correction. This correction was determined by measuring the resistance of the mercury at low pressures compared with a calibrated Bourdon gauge of the Société Genevoise, and extrapolating back for the zero from 50 kgm. The tube must be refilled if the correction is large, because it will not remain constant, as obviously the effect of the occluded air on the resistance depends on its position as well as on its quantity. If the correction is small, however, it remains constant apparently indefinitely. In the tubes used, the correction ranged from 0.6 mm. to 0.1 mm. of the bridge wire, that is, a mean correction of about $\frac{1}{5000}$ of the total resistance. That the permanent change of the zero mentioned above was due to set in the glass and not to a curious behavior of the contained air, is proved by the fact that no set was found after filling in the manner above a tube once seasoned, but the correction for air assumed at once its final value.

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[3] Lenz, Wied. Beibl., **6**, 802 (1882).

In addition to showing the necessity of seasoning the glass and removing all air from the mercury, the preliminary comparisons showed that the mercury must be purified with some care. Later, a quantitive determination of the effect of two common impurities will be given. It was found that the mercury could be got sufficiently pure for present purposes by distilling commercial mercury, cleaning with acid, washing and drying, and finally distilling into the U capillary as described above.

When all these precautions are taken, the mercury gauge seems to be reproducible at pleasure. The results of a comparison of two such gauges is shown in Table I. The two mercury resistances compared were each contained in capillaries of the same kind of glass, Jena No. 3880 a. One capillary (R 9), however, was twice the linear dimensions of the other (R 10) because it seemed desirable to eliminate any possible effect of the size of the capillary on its elastic behavior. The smaller capillary, of course, was drawn down farther from the original piece, and so it is conceivable that the internal strains might be enough greater to result in different elastic behavior. In Table I the displacements of the slider of the bridge wire corresponding to the changes of R 9 and R 10, together with the ratio of the displacements, are tabulated against the approximate pressure, which was calculated from the comparison of R 10 later against an absolute gauge. The ratio is constant at 1.007, excepting two values, either of which could be brought to 1.007 by an error of only 0.1 mm. in the slider settings. The ratio of the resistance R 10 to R 9 multiplied into a constant expressing the different linear resistances of the bridge wires is also 1.007. Within the limits of error of the electrical measurements, therefore, or within $\frac{1}{10}$ per cent, the mercury resistance gauge may be assumed to be reproducible.

TABLE I.

Comparison of Two Mercury Gauges to show Reproducibility.

Approximate Pressure kgm./cm.^{2}	Displacement of Slider in Cm.		
	R 10.	R 9.	Ratio.
1040	5.53	5.51	1.003
1930	9.81	9.74	1.007
2870	14.08	13.98	1.007
3750	17.47	17.35	1.007
4390	20.73	20.60	1.007
5650	24.61	24.45	1.007
6600	27.95	27.72	1.008
4250	19.59	19.45	1.007
1990	10.07	10.00	1.007

There is now left only one point in regard to the suitability of the mercury resistance as a secondary standard to be cleared up by the comparison of the mercury with an absolute gauge, namely complete freedom from hysteresis. The absolute gauge is that described in the first part of this paper. The steel parts of this gauge may of course show hysteresis, but if we assume that the liquid transmitting the pressure shows no hysteresis, which is almost certainly true, it is evident that any hysteresis effects in the steel parts will merely affect the correction for distortion of the gauge. The largest value of this is about $\frac{1}{10}$ per cent. Within the limits of error, therefore, the absolute gauge shows no hysteresis. Freedom of the mercury from hysteresis will be shown by agreement of the resistance measurements under increasing and decreasing pressure.

TABLE II.

Comparison of Mercury Gauge against Absolute Gauge at Increasing and Decreasing Pressures, to show Freedom from Hysteresis.

Pressure kgm. /cm.^{2}	Slider Displacement Cm.	
	Increasing Pressure.	Decreasing Pressure.
917	4.89	4.89
1501	7.77	7.79
2018	10.17	10.19
2602	12.80	12.79
3196	15.35	15.37
3779	17.73	17.74
4233	19.49	19.51
4816	21.75	21.75
5348	23.70	23.65
5932	25.65	25.67
6452	27.45	27.43
6841	28.71	. . .

Comparisons of mercury resistance and absolute gauge were carried out with one mercury resistance (R 9) of soft Jena glass tubing No. 3880 a, and two absolute gauges, as has already been mentioned in the first part of this paper. The results of one of the comparisons under increasing and decreasing pressure, to determine freedom from hysteresis, are given in Table II. Here the displacements of the slider in cm. are tabulated against pressure, calculated from the corrected dimensions of the absolute gauge as described in the first part. The displacements under increasing or decreasing pressure agree within the limits of error of reading the position of the slider. Another comparison of R 9 against the same absolute gauge, as also a comparison against another absolute gauge, led to the same result. These comparisons were taken to afford sufficient proof of freedom from hysteresis of the mercury resistance in the soft Jena glass capillary within errors of $\frac{1}{10}$ per cent.

Having established the reproducibility and freedom from hysteresis of the mercury, we pass to the more important results to be obtained from the comparison with the absolute gauge, namely the final translation of the indications of the mercury gauge into kgm. per cm. The data used for this were those obtained from the two comparisons of R 9 against absolute gauge No. 1, and the one comparison against gauge No. 2. The results of these comparisons have already been given in Part I of this paper, where it appears that the two absolute gauges do not differ on the average so much as $\frac{1}{10}$ per cent from the mean. The average of these two comparisons is taken as the true value and is used in the following computations.

If the change of resistance is to be used as a practical standard of pressure, some empirical formula is desirable connecting the change of proportional resistance with the pressure. In the following, two formulas will be given, the first expressing the change of resistance in terms of the pressure, and the second, which will be more useful in practice, expressing pressure in terms of observed change of resistance. $\frac{\Delta R}{R_0}$ will be abbreviated by ρ, where ΔR is the observed change of the resistance in the soft envelope of Jena glass No. 3880 a, and R_0 the initial resistance measured in this envelope. Then ρ is some function of the pressure, approximately linear. A number of forms of this function were tried, it being desirable for convenience in computation to choose such a form that the number of empirically determined constants is small. It was at once obvious that the ordinary power series representation of the relationship was totally inadequate, at least five and probably more arbitrary constants being necessary to obtain $\frac{1}{10}$ per cent agreement over the entire range. Several other forms of power series tried, with fractional instead of integral exponents, were better, but not sufficiently approximate. Several exponential forms of the type $\rho = ap\,10^{P}$, where P is a power series in p, gave still better results. The form finally adopted was, $\rho = ap\,10^{bp^c}$, where

$$\log a = 5.5242 - 10,$$
$$\log (-b) = 6.2486 - 10,$$
$$c = 0.75.$$

This form does not lend itself to computation by least squares, and the best values for a, b, and c were found by trial. Table III shows the

TABLE III.

COMPARISON OF OBSERVED AND CALCULATED CHANGE OF RESISTANCE WITH PRESSURE.

Pressure kgm./cm.^{2}	$\frac{\Delta R}{R_0 p}$.		
	Calculated.	Observed.	Difference.
923	0.00003123	0.00003120	+3
1510	0.00003029	0.00003032	−3
2031	0.00002955	0.00002952	+3
2619	0.00002879	0.00002876	+3
3217	0.00002808	0.00002811	−3
3804	0.00002745	0.00002747	−2
4262	0.00002696	0.00002697	−1
4843	0.00002639	0.00002643	−4
5385	0.00002587	0.00002588	−1
5974	0.00002534	0.00002531	+3
6495	0.00002489	0.00002491	−2
6848	0.00002460	0.00002454	+6

$$\frac{\Delta R}{R_0} = -ap\,10^{bp^{\frac{3}{4}}}$$
$$\log a = 5.5242 - 10$$
$$\log (-b) = 6.2486 - 10$$

observed values and the values calculated by the above formula, together with the discrepancies. The divergence is rarely more than $\frac{1}{10}$ per cent and seems irregular in sign. The fairly high discrepancy at 6800 is doubtless because this pressure was reached with only one of the absolute gauges, while all the other values are means of two

TABLE IV.

COMPARISON OF OBSERVED PRESSURE WITH THAT CALCULATED FROM THE CHANGE OF RESISTANCE.

$\frac{\Delta R}{R_0}$.	Pressure kgm./cm.^{2}		Difference.	
ρ	Calc.	Obs.	Actual.	Nearest tenth %.
0.02880	925	923	+ 2	+2
0.04578	1512	1510	+ 2	+2
0.05995	2028	2031	− 3	−2
0.07532	2614	2619	− 5	−2
0.09044	3221	3217	+ 4	+1
0.10450	3810	3804	+ 6	+1
0.11490	4266	4264	+ 4	+1
9.12800	4856	4843	+13	+3
0.13940	5393	5385	+ 8	+2
0.15120	5969	5974	− 5	−1
0.16185	6507	6497	+10	+2
0.16810	6831	6848	−17	−3

$p = \rho\alpha 10^{\beta\rho^{1.03}}$ $\alpha = \log 4.4871$ $\beta = \log 9.8836 - 10$

determinations. The probable error of the formula itself, calculated by the formula for the error of the mean, is $\frac{1}{30}$ per cent.

The above formula gives the measured change in the resistance of mercury in a specified glass envelope at 25° in terms of the pressure. In practice, it will be necessary to compute the pressure, given the measured change of resistance. The above formula cannot be easily solved for p, and another was set up giving p in terms of ρ. The form of this is exactly the same as for ρ in terms of p, and the procedure in determining the coefficients was the same. It was not found possible to get quite so good an approximation, however, partly because of the shape of the curve itself, which was such that a given percentage error in p produces less percentage error in ρ than the same percentage error in ρ produces in p. In practice, it will be found most convenient to find p graphically from a curve representing the relation between pressure and resistance. The form adopted was

$$p = \alpha \rho \, 10^{\beta \rho^{1.03}},$$

$$\alpha = \log^{-1} 4.4871,$$

$$\beta = \log^{-1} 9.8836 - 10.$$

Table IV shows the observed and computed values for p with the discrepancies. The probable error of a single reading is 0.12 per cent; that of the formula itself much less. This formula holds for mercury in soft Jena glass No. 3880 a at 25°.

At first sight it seems that the two empirical formulas may be combined by eliminating $\frac{p}{\rho}$ so as to give a single purely exponential relation between p and ρ which may be readily solved for either. This is not practical, however, because the exponential parts of the above expressions are only slightly affected as to percentage accuracy by relatively large percentage errors in the arguments, and therefore, inversely, small errors in the exponential part may produce large errors in the unknown (p or ρ) calculated from it. Errors of as much as 20 per cent were found to be introduced by the suggested elimination.

The above formulas are only empirical representations of the facts throughout a given pressure range, and their use by extrapolation over any considerably greater range is doubtful. No theoretical value is claimed for them, and it is evident that they cannot represent the actual form of the unknown function. Thus the formula for resistance in terms of pressure predicts a negative minimum of resistance of about -6 at 48,000 kgm. per sq. cm. Neither can extrapolation be carried entirely to the origin of pressure, for the formula demands that $\frac{d}{dp}\left(\frac{\rho}{p}\right)$ be infinite when $p = 0$, which is almost certainly not the case. The error here is slight, however, and confined to the immediate neighborhood of $p = 0$. $\frac{\rho}{p}$ at the origin remains finite, with nearly the same values as may be deduced from the formula for p in terms of ρ.

The above formula holds only when the mercury resistance is enclosed in a glass capillary of Jena glass No. 3880 a. If a different glass is used, it will be possible to use the formula by introducing a correction factor. This factor for one other glass, hard Jena combustion tubing No. 3883, was determined by comparing two mercury resistances. The comparison was made not so much with the idea that this hard glass would prove more convenient for practical use, but rather in the hope that these two different kinds of glass, one very infusible and the other very fusible, would show a comparatively large difference of compressibility. Table V shows the ratio of the observed changes of resistance in the hard and soft envelopes at different pressure. The mean of the ratios, weighted according to the magnitude of the effect measured, is 1.0266, while the ratio of the initial resistances is 1.0253. The difference between these two numbers is presumably due to the difference of compressibility of the envelopes, which turns out not to be as large as was expected from the character of the glass. The fact that the ratio of the change of resistances is greater than the ratio of the total resistances shows that

TABLE V.

EFFECT OF DIFFERENT GLASS ENVELOPES.

Pressure kgm./cm.^{2}	Ratio $\frac{\Delta R7}{\Delta R9}$.	Pressure kgm./cm.^{2}	Ratio $\frac{\Delta R7}{\Delta R9}$.
1170	1.025	5800	1.027
1950	1.025	6520	1.026
2960	1.027	4370	1.028
3830	1.028	2100	1.028
4700	1.025	. . .	. . .

Mean of ratios of change of resistance weighted according to pressure is 1.0266.
Ratio of initial resistances is 1.0253.
$R7$ is enclosed in hard Jena glass 3883.
$R9$ is enclosed in soft Jena glass 3880 a.

the hard glass is more compressible than the soft. That the difference is actually due to the difference of compressibility of the glass and is not an experimental error will receive experimental confirmation later by actual measurement of the compressibility of the glass. Resistances in hard as well as in soft glass envelopes may be used as standards, therefore, multiplying, however, the proportional changes of resistance in hard glass by 1.0013 to reduce to soft glass. But it will be noticed from Table V that the ratio of the changes of resistance in the hard and soft glass capillaries varies much more irregularly than the ratio for two capillaries of soft glass (Table I). That this is actually due to irregularities in the deformation of the hard glass will receive confirmation in the paper on compressibility. The hard glass is not so suitable, then, for the capillary as the soft Jena glass.

In practical applications of this gauge it will doubtless be inconvenient to work at the temperature above, 25°, and accordingly the temperature coefficient was determined over a range from 0° to 50°. The determination was made by comparing R 7, which was kept at the standard temperature 25°, with R 9, which was maintained during one set of readings at the given temperature over the entire pressure range. Comparisons were made at six different temperatures, 50.35°, 43.75°, 36.95°, 30.32°, 15.00°, 0.00°. At each temperature seven readings were made with increasing pressure and two with decreasing pressure to avoid all possibility of hysteresis, no evidence of which was found. In making this comparison it appeared necessary after each change of temperature to season the glass by preliminary subjection to the entire pressure range, the irregularities thus eliminated being greater the greater the temperature range. It was found that pressure may be calculated from temperature and the observed proportional change of resistance by the formula:

$$p = \alpha\rho\ 10^{\beta\rho^{1.03}}\,[1 - a_1\ (t - 25°) - b_1\ (t - 25°)^2],$$

where α and β have the values previously given, and

$$a_1 = \log^{-1} 7.1253 - 10,$$

$$b_1 = \log^{-1} 4.4487 - 10.$$

a_1 and b_1 were computed by least squares. It was evident on plotting the various points, that a_1 and b_1 are variable with the pressure, becoming less with increasing pressure, but the effect is very slight, and no systematic variation over the entire temperature range could be found. Attempts to introduce such a variation into the general formula

$$\frac{\Delta V}{V_0} = bp + cp^2,$$

$$b = \log^{-1} 4.5681 - 10,$$

$$-c = \log^{-1} 9.2977 - 20.$$

Now to find the changed specific volume resistance of mercury we have

$$\frac{\Delta R_s}{R_0} = \rho + a\, p,$$

where ΔR_s is the observed decrease of resistance corrected for changed shape of glass, R_0 is the initial resistance measured in the same glass, a is the linear compressibility of the glass, and ρ has the meaning already given, namely the observed proportional decrease of resistance in the given capillary. But ρ has already been found in terms of p, and a has just been given, so that we have the empirical formula

$$\frac{1}{R_0}\,\frac{\Delta R_s}{p} = a\,[0.02168 + 10^{bp^{\frac{3}{4}}}],$$

where a and b have the values already given, namely,

$$a = \log 5.5242 - 10$$

$$b = -\log 6.2486 - 10$$

The slope of the curve, i. e., the instantaneous pressure coefficient at any point, is:

$$\frac{1}{R_0}\,\frac{dR_s}{dp} = -a\,[0.02168 + 10^{bp^{\frac{3}{4}}}\,\{1 + \tfrac{3}{4}\,bp^{\frac{3}{4}} \log_e 10\}],$$

where R_s is the variable resistance corrected for the glass. The instantaneous coefficient per unit resistance is at any point:

$$\frac{1}{R_s}\,\frac{dR_s}{dp} = -\frac{a\,[0.02168 + 10^{bp^{\frac{3}{4}}}\,\{1 + \frac{3}{4}\,bp^{\frac{3}{4}} \log_e 10\}]}{1 - ap\,[0.02168 + 10^{bp^{\frac{3}{4}}}]}.$$

These three quantities were computed by the above formula and are given in Table VII. They are also shown graphically in Figure 8, which indicates the general behavior without, of course, the accuracy of the formula. The general character of all these curves is the same, showing a continually decreasing effect of pressure on change of resistance as the pressure increases, this decrease itself also decreasing.

TABLE VII.

SPECIFIC VOLUME RESISTANCE OF MERCURY.

Pressure kgm./cm.^{2}	$\frac{1}{R_0} \frac{\Delta R_s}{p}$.	$\frac{1}{R_0} \frac{dR_s}{dp}$.	$\frac{1}{R_s} \frac{dR_s}{dp}$.
...	0.00003344	0.00003344	0.00003344
500	0.00003276	0.00003171	0.00003223
1000	0.00003182	0.00003011	0.00003111
1500	0.00003102	0.00002878	0.00003018
2000	0.00003031	0.00002760	0.00002938
2500	0.00002966	0.00002653	0.00002865
3000	0.00002906	0.00002552	0.00002796
3500	0.00002849	0.00002461	0.00002735
4000	0.00002795	0.00002374	0.00002674
4500	0.00002744	0.00002293	0.00002616
5000	0.00002696	0.00002216	0.00002561
5500	0.00002655	0.00002148	0 00002515
6000	0.00002603	0.00002073	0.00002457
6500	0.00002562	0.00002006	0.00002407

The curves do not run to high enough pressures to justify any speculation as to their ultimate behavior.

De Forest Palmer's are the only results with which these can be compared. He found $\frac{1}{R_0} \frac{\Delta R_s}{p}$ to have the constant value 3.224×10^{-5} between 0 and 2000 kgm.[4] There is, however, as already stated, a probable error of 1.5 per cent at 2000 kgm., and proportionally more

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[4] de Forest Palmer, Amer. Jour. Sci., **4**, 8 (1897).

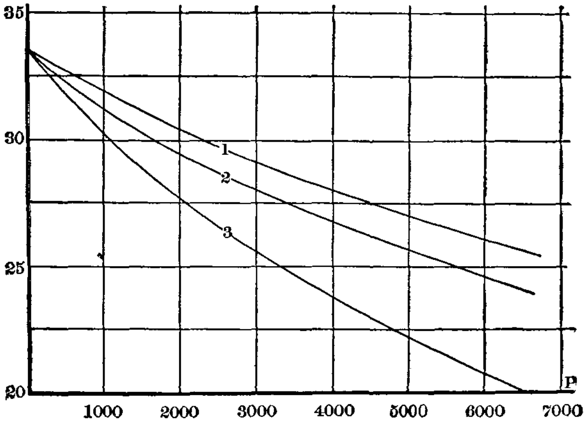

FIGURE 8. Various functions of the specific resistance of mercury plotted against pressure. 1 shows $\frac{1}{R_0}\frac{\Delta R_s}{p}$, 2, $\frac{1}{R_s}\frac{dR_s}{dp}$, and 3, $\frac{dR_s}{dp}$, where R_0 is the initial resistance and R_s is the variable resistance under pressure, corrected for the distortion of the glass containing vessel.

at lower pressures. According to the results above, $\frac{1}{R_0}\frac{\Delta R_s}{p}$ varies from 3.344 to 3.031 × 10^{-5} between 0 and 2000 kgm., giving a mean value of 3.187 × 10^{-5}, which agrees within 1.1 per cent with de Forest

TABLE VIII.

SPECIFIC VOLUME RESISTANCE AND SPECIFIC MASS RESISTANCE OF MERCURY UNDER PRESSURE.

Pressure kgm./cm.^{2}	R_s.	$R_s \times D$.	Pressure kgm./cm.^{2}	R_s.	$R_s \times D$.
0	1.0000	1.0000	3500	0.9003	0.9114
500	0.9836	0.9854	4000	0.8882	0.9010
1000	0.9682	0.9716	4500	0.8765	0.8904
1500	0.9535	0.9588	5000	0.8652	0.8806
2000	0.9394	0.9462	5500	0.8540	0.8708
2500	0.9258	0.9342	6000	0.8438	0.8616
3000	0.9128	0.9228	6500	0.8335	0.8527

Palmer's value. In view of the magnitude of the variation found in the coefficient over the pressure range, the uncertain correction for the glass introduced by de Forest Palmer, and the magnitude of his probable error, this agreement is better than could be expected.

By combining the two empirical formulas for change of specific volume resistance and change of volume of the mercury, the value of resistance times density (*R·D*), i. e., the specific mass resistance, may be found. The departure of this from constancy may be described as the pure pressure effect on mercury resistance. In Table VIII the specific volume resistance and the specific volume resistance multiplied by the density are given for various pressures. They are also shown graphically in Figure 9. The curves are similar in all respects and show no indications of any remarkable behavior at higher pressures. The comparatively small part played by the change of density in the total change of resistance under pressure is of interest.

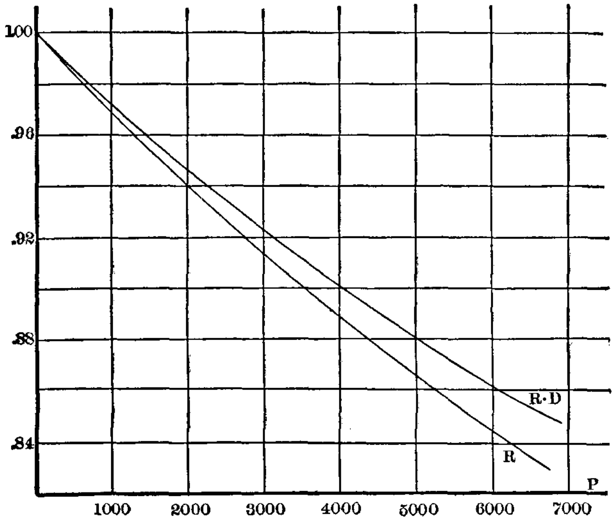

FIGURE 9. The changed resistance of mercury under pressure in terms of the resistance under zero pressure. The curve shows the measured resistance corrected for the distortion of the glass containing vessel. The curve R·D shows the former curve corrected for the changed density of mercury. It shows the pure pressure effect on resistance, that is, the resistance corrected for the increased conductivity due the increased concentration of the molecules. The smallness of the change of resistance due to this concentrating of the molecules is to be noticed.

Finally, the variation of specific resistance with temperature may be calculated from the formula given for the variation with temperature of p as determined by the measurement of ρ. Retaining only the term of the first degree in p, we have to the degree of experimental accuracy reached in these results:

$$\frac{\Delta R_s\,(p, t)}{R_s\,(0, t)} = \frac{\Delta R_s\,(p, t_0)}{R\,(0, t_0)} + a^2\, a_1\, \alpha\, p\, 10^{2bp^{\frac{3}{4}}}\,(t - t_0),$$

where a, a_1, α, and b have the values already assigned, and t_0 equals 25°. In the deduction of this formula the variation of the compressibility of the glass with the temperature was neglected. This variation is beyond the limits of error if the glass used has a temperature coefficient of the same order as that found by Amagat,[5] who found a change of 10 per cent for 100°. From this formula $R(p, t)$ was calculated for a number of pressures and for the temperatures 125°, 25°, and — 75°, assuming R (0, 25°) equal to unity, and taking for the temperature coefficient of specific conductivity the value 0.000888. These results are given in Table IX and plotted in Figure 10. This large temperature range was taken merely for convenience in showing

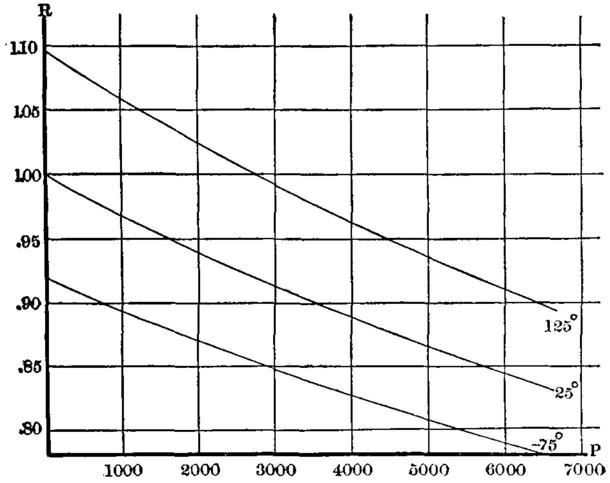

FIGURE 10. The resistance of mercury at various temperatures and pressures in terms of the resistance at zero pressure and 25°.

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[5] Amagat, C. R., **110,** 1248 (1890).

diagrammatically the general tendency of the results. The formula actually does not give results better than $\frac{1}{10}$ per cent beyond the range 0° to 50°. The temperature coefficient found above is nearly ten times de Forest Palmer's value, who, however, worked only at the extremes of a wider temperature range than that used here, namely, 9° to 100°.

TABLE IX.

VARIATION OF MERCURY RESISTANCE WITH PRESSURE AND TEMPERATURE.

Pressure kgm./cm.^{2}	R (p, −75°).	R (p, 25°).	R (p, 125°).
. . .	0.9186	1.0000	1.0970
500	0.9055	0.9831	1.0770
1000	0.8930	0.9682	1.0580
1500	0.8818	0.9535	1.0400
2000	0.8714	0.9394	1.0230
2500	0.8582	0.9258	1.0070
3000	0.8478	0.9128	0.9908
3500	0.8369	0.9003	0.9759
4000	0.8268	0.8882	0.9614
4500	0.8174	0.8765	0.9475
5000	0.8076	0.8652	0.9342
5500	0.7982	0.8540	0.9208
6000	0.7896	0.8438	0.9086
6500	0.7807	0.8335	0.8966

No theoretical discussion of the way in which these curves might be expected to behave has been attempted. Only a few points require remark. For instance, it is obvious from the table that temperature has a greater effect on the pressure coefficient of resistance than it does on the resistance itself. The temperature coefficient of the former is 0.00137, and of the latter 0.000888. In other respects the curves behave as one would expect, i. e., at higher pressures the proportionate effect of temperature is reduced. This is shown by the temperature effect both on resistance and on pressures coefficient of resistance. Thus the temperature coefficient of the pressure coefficient has become reduced at 6500 kgm. to 0.7 of its initial value, while the temperature coefficient of resistance is reduced from 0.0009 to 0.0007. This latter effect shows itself in a tendency of the curves for different temperatures to draw together with increasing pressure toward some value of resistance greater than zero. That is, for a large enough value of pressure, the resistance acts as if it might have a definite value independent of temperature.

CONCLUSION.

In this paper it has been found that the mercury resistance gauge is a reliable secondary standard of pressure if proper precautions are used. The mercury must be pure and free from air. The irregular behavior under pressure of the containing glass capillary is the principal source of error. An easily fusible glass in which the strains left after drawing are presumably small, is better than an infusible glass. The glass must be seasoned by several applications of pressure over the entire range before it becomes regular in behavior. If after this it is exposed to considerable changes of temperature or to sudden changes of pressure, it must be reseasoned. The maximum error that can be introduced by irregularities in the glass is about 2.5 per cent. The dependence of pressure on the measured proportional change of resistance (p) and temperature is given by the equation

$$p = \alpha \rho \, 10^{\beta \rho^{1,03}} \left[1 - a_1 (t - 25^\circ) - b_1 (t - 25^\circ)^2\right],$$

where

$$\alpha = \log^{-1} 4.4871;$$

$$\beta = \log^{-1} 9.8836 - 10;$$

$$a_1 = \log^{-1} 7.1253 - 10;$$

$$b_1 = \log^{-1} 4.4487 - 10.$$

This formula, which applies to mercury in a capillary of Jena glass No. 3880 a, gives the pressure correctly to $\frac{1}{10}$ per cent between 500 and 6800 kgm. and 0° and 50° C.

Empirical expressions have also been deduced connecting the specific volume resistance and the specific mass resistance of mercury with the pressure.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

AN EXPERIMENTAL DETERMINATION OF CERTAIN COMPRESSIBILITIES.

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By P. W. Bridgman.

Presented by W. C. Sabine, December 9, 1908. Received December 16, 1908.

In a preceding paper the change of resistance produced by hydrostatic pressure on a fine thread of mercury in a capillary of a specified glass was measured. This change of resistance is the sum of two effects: the change of resistance produced by the changed dimensions of the glass capillary, and the change of resistance due to the changed electrical properties of the mercury under pressure. The change of resistance produced by the distortion of the glass is simultaneously an increase of resistance because of the decreased bore of the capillary, and a decrease because of the decreased length. The total fractional change of resistance is easily seen to be the linear compressibility of the glass. The change of resistance due to the changed electrical properties of the mercury may be further divided into two effects: that due to the change in the conducting power of the separate molecules, and that due to the change in the number of molecules occupying a given space. This latter effect is determined directly by the compressibility of the mercury.

A complete description of the phenomena involved in the measured change of resistance of the mercury involves, therefore, a knowledge of the compressibility of both the glass and the mercury. This paper is occupied with a description of the methods by which these were determined. As the pressure range employed here (6500 kgm.) is somewhat higher than that usually used, modifications of the methods in common use were necessary. It seemed undesirable, however, to bury a description of these methods in a paper on the unrelated subject of the electrical resistance of mercury, and the matter has therefore been made the subject of a separate paper, although the method has been applied to only a few substances, and all the data have been collected solely with a view to the above discussion of the effect of pressure on the resistance of mercury. However, the paper contains an investigation of several minor points that came up in the course of the work, that may be of interest on their own account. Among these is an experimental determination of the difference of linear compressibility of a piece of commercial rolled steel along and perpendicular to the direction of rolling, and some account of the seasoning effect of successive applications of pressure on the elastical behavior of glass. In detail, the paper contains a determination by one method of the compressibility of two kinds of Jena glass, of a piece of commercial aluminum rod, and of several grades of steel; and by another method, the compressibility of mercury, all up to about 6500 kgm. per sq. cm.

In determining the compressibility of a solid the method adopted was to measure the change of length of a rod of the substance produced by hydrostatic pressure applied all over the external surface. This method applies, therefore, only to those solids that can be obtained in the form of a cylindrical rod or tube. The cubic compressibility is found by multiplying the linear compressibility by three. It is a fundamental assumption throughout all the following determinations of the compressibility of solids, therefore, that the substance is so homogeneous and isotropic that the compression under hydrostatic pressure is sensibly the same in all directions. Some experimental proof of the justifiability of this assumption has been attempted in the case of a piece of rolled steel boiler plate.

It is a feature common to all the compressibility methods used in this paper, that the distortion produced by pressure is measured by the displacement of a ring sliding with slight friction on some movable part of the apparatus. The method is not continuous reading, therefore, but the apparatus has to be taken apart and readings made after each application of pressure, the reading obtained corresponding to the maximum pressure. A method of this kind is doubtless inconvenient, but it has the advantage of simplicity and directness over any continuous reading method that would be practical over so wide a pressure range.

In the determination of the compressibility of solids two slightly different methods may be used, according as the solid is of relatively low or high compressibility. The first method, not so accurate as the second, applies to iron and metals of the same order of compressibility. The second applies to substances of higher compressibility, and involves directly the compressibility of iron as determined by the first method.

The first method measures the relative change of length of a rod of the substance and a heavy cylinder of steel. The rod is enclosed in the cylinder, throughout the interior of which hydrostatic pressure would be beyond the accuracy of this work. Table VI shows the value of p computed by the formula for the two extremes of the temperature

TABLE VI.

TEMPERATURE CORRECTION FOR PRESSURE IN TERMS OF RESISTANCE.

51.35°.			0°.		
Pressure kgm./cm.^{2}			Pressure kgm./cm.^{2}		
Obs.	Calc.	Diff.	Obs.	Calc.	Diff.
1074	1080	+6	1042	1037	− 5
1869	1864	−5	1879	1881	+ 2
2824	2825	+1	2845	2840	− 5
3641	3641	0	3637	3644	+ 7
4478	4478	0	4522	4524	+ 2
5470	5479	+9	5518	5522	+ 4
6527	6528	+1	6560	6573	+13
4249	4243	−6	4262	4256	− 6
1976	1969	−7	0015	2010	− 5

range. The observed pressures tabulated are the pressures computed from the change of R 7 after correction is applied reducing to soft glass. The difference column really contains, therefore, two sources of error. The differences are fairly small and irregular in sign. The irregularity is doubtless due to the incomplete seasoning of the glass by the previous single excursion through the pressure range, and the less regular behavior of the comparison resistance in the hard glass capillary.

During the preliminary comparisons of different mercury resistances, the effect of a known slight quantity of impurity in the mercury was determined. The numerical values thus obtained are given here, as they may be of interest as showing the degree of purity which it is necessary to attain. It was found that metallic impurities have the greatest effect. Impurities that may be absorbed from the glycerine and water unavoidably in contact with the mercury appear to have no effect, as is shown by the constancy of behavior of the gauge over long intervals of time. To test the effect of small metallic impurities, two experiments were made on pure mercury contaminated with known quantities of foreign metal, in the one case 0.1 per cent of zinc, and in the other 0.1 per cent of lead. This is a very large quantity of impurity, much larger than can possibly occur in practice. On standing a short while in the air, the surface of the mercury becomes positively filthy with oxides. The effect of 0.1 per cent zinc is to decrease the resistance by about 1.4 per cent, but the pressure coefficient of resistance by about 5 per cent. Furthermore, the departure from the linear relation between total change of resistance and pressure is less than for pure mercury, being 3 per cent less at 6500 kgm. The results with the lead were not so satisfactory as those with the zinc. It was pretty certain, however, that the effect of the lead is less on the total resistance and greater on the pressure coefficient.

The formulas given above connect the change of resistance of mercury in a capillary of specified glass with the pressure, and are all that is required for use with a secondary standard of pressure. The observed change of resistance, however, is due to a combination of two unrelated effects; the change of dimensions of the glass, and the changed specific resistance of mercury. The results given above will not possess theoretical value, therefore, until the two effects are separated. In the following an experimental determination of these two effects is given.

We may distinguish two specific resistances of mercury, both of which are altered by pressure. The first may be called the specific volume resistance, and is the resistance of a body of mercury of invariable form, but of mass variable with the pressure. The second may be called the specific mass resistance of mercury, and is the specific volume resistance multiplied by the ratio of the masses within the given surface at the variable and standard pressure, i. e., the density. The specific mass resistance seeks to correct for the increased conductivity to be expected at any pressure because of the increased number of conducting particles in a given volume. In order to determine the specific volume resistance, the above results have to be corrected for the compressibility of the glass envelope; to determine the mass resistance, an additional correction must be applied for the compressibility of the mercury. These compressibilities are determined in another paper, to which reference must be made for the methods used. Only the results there found will be used here. It was found that for Jena glass No. 3880 a, $\kappa = 2.17 \times 10^{-6}$, and that the change of volume of mercury is connected with pressure by the relation is applied. The rod shortens, therefore, under the uniform external pressure, while the cylinder lengthens under the interior pressure. The lengthening of the cylinder is very much less than the shortening of the rod. In the present experiment it was only 5 per cent. The strain in the cylinder is complicated, consisting of a radial displacement away from the centre, and of a longitudinal extension which may produce warping of the originally plain sections. This warping is greatest at the ends, and must vanish at the mid section if the cylinder is symmetrical at the two ends. The warping cannot be easily calculated, and was neglected in the present work. It can in any event constitute only a correction for the above 5 per cent correction term. The method consists, therefore, in subtracting from the relative change of length of the rod and the cylinder the increase of length of the cylinder as obtained from the measurement of external change of length under pressure. The result is the linear compressibility of the rod, from which the cubic compressibility is calculated.

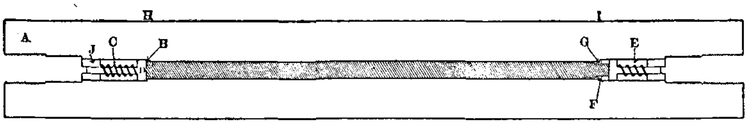

FIGURE 1. Apparatus for measuring the linear compressibility of rods. The rod to be measured is indicated by the shading. The stop D is held permanently against the shoulder B by the spring C, which is kept compressed by the pump connections, not shown. The brass ring F is kept in contact with the shoulder G during increase of pressure by the spring E, which pushes the shortening rod through the ring F, so as always to be in contact with the stop D. When pressure is released the ring comes back with the rod and the displacement is measured. The rod is removed through the end E to make these measurements; the connections at A to the pressure pump are not disturbed during the measurements. The elongation of the cylinder is measured externally at the scratches H and I.

The cylinder used is shown in Figure 1. It is made of annealed tool steel, 18 in. (45.7 cm.) long, and 2 in. (5.1 cm.) in diameter. It is pierced through the entire length by a reamed ⅜ in. (0.95 cm.) hole, in which the rod to be tested is placed. At either end the ⅜ in. hole is enlarged in several steps in the manner indicated, in order to afford room for the various connections. The enlargements of the holes are precisely alike at the two ends, so as to insure symmetrical warping of the cylinder. The rod to be tested is indicated by the shading. It is carefully turned so as to slide without lateral play into the reamed hole. Three shallow grooves, milled the entire length of this rod, allow the compressing fluid to flow freely from one end of the cylinder to the other. The change of length of the rod is obtained by keeping one end of the rod always fixed opposite the same part of the cylinder, and measuring the relative displacement of the other end, which is free. The fixed end of the rod is kept so by the action of the spring at E, which keeps the rod pressed against the stop at D. This stop D is kept immovable by the spring at C, which keeps D pressed against the shoulder B. This spring C is very much stiffer than the spring E, and is kept permanently compressed by the pump connections (not shown) which are screwed into the end A, and keep the ring J fast in the position shown. This method of securing the invariable position of the stop seemed preferable to any plug arrangement screwed fast into the cylinder, for the latter might shift slightly, owing to the change produced by the pressure in the dimensions of the thread.

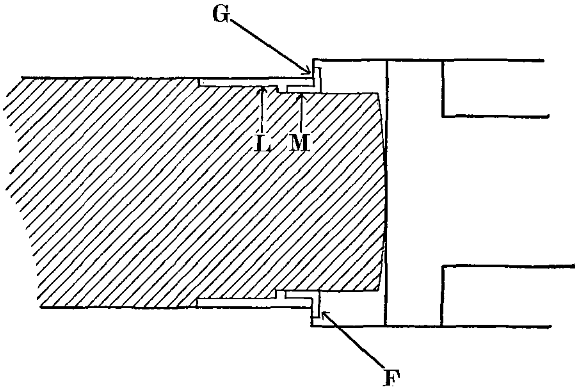

Figure 2. Enlarged view of the brass ring, etc., of Figure 1. The displacement of the ring is measured by measuring the distance between the scratches at L and M on the rod and the ring respectively.

The shift of the free end of the rod relatively to the cylinder was obtained by measuring the displacement on the brass ring F, which is pushed back by the shoulder G. An enlarged view of the ring is shown in Figure 2. The brass ring F is split so as to slide without too great friction on the end of the rod, which is turned down to about $\frac{5}{16}$ in. (8 mm.). There is a fine scratch on the ring at M, and also a scratch on the corresponding ledge L of the rod. The ring and rod are turned in the lathe so that these two scratches are at the same radial distance from the axis of the rod, thus enabling both scratches to be in focus simultaneously under a high power microscope. The effect of an application of pressure is to shorten the rod, pushing up the ring, which stays in its extreme position. The rod is then taken out by unscrewing the plug at the end I and the distance between the scratches L and M measured. The increase of distance over the zero position gives the relative change of length of rod and cylinder. There is here a small source of error in finding the effective length of the rod, which terminates at some unknown place within the brass ring. The effective length used was the length from the fixed end to the middle of the ring when in the zero position. As the breadth of the bearing surface of the ring was only about 2 mm., and the length of the rod was 30 cm., the maximum error here is only 1/300.

It is at once obvious that any slight error in replacing the rod after each measurement in exactly its former position will produce considerable error in the result, since the change of length produced by pressure is small. In the form used, in which the rod is 30 cm. long, the change of length for 1000 kgm. is only 0.05 mm. Slight particles of grit are likely, therefore, to produce considerable irregularities. By working with some care it was found possible, however, to secure fairly uniform results. Particular attention must be given to washing out the cylinder after each application of pressure. The effect of pressure is, of course, to flood the interior of the cylinder with the pump liquid, in this case glycerine and water, which may carry considerable grit in suspension. After each measurement the cylinder was thoroughly washed several times by a jet of water violently expelled from a glass tube reaching into the cylinder as far as the stop D. No cloth or other substance must be used for wiping out the hole. The rod to be tested was also carefully washed under the tap after each measurement, again taking care not to wipe with a cloth or to bring into contact with any possible source of grit. It was found that by decreasing the diameter of the rod for a short distance at the end B, there is less tendency for grit to collect between the end of the rod and the stop D when the rod is replaced in the hole after each measurement.

The change of length of the steel cylinder was not measured at the same time as the relative change of length of rod and cylinder, but was, instead, determined independently as a function of the pressure. Three determinations of this extension were made, one preliminary to, one in the course of, and one after the series of compressibility measurements. The last two agreed within the limits of error; the first was slightly different, as has always been found to be the case when the deformation of a metal is measured on the first application of pressure. In making these measurements, the cylinder was clamped to a heavy comparator bed, which carried two microscopes. The cylinder was clamped at only one point, the middle, so as to avoid any possible distortion of the comparator by the lengthening of the cylinder under pressure. The close contact of cylinder and comparator insured the practical equality of temperature of the two, and the coefficients of expansion of the two pieces proved so close that the few tenths of a degree variation which occurred in the temperature of the room introduced no appreciable error. The microscopes were focussed on fine fortuitous scratches on the cylinder at the points H and I (Figure 1). Change of length was measured by a micrometer eyepiece in either microscope, which had been previously calibrated. Settings on the fine scratches could be made with a maximum error of 0.0003 mm., thus introducing a possible error of reading of the change of length of 0.0006 mm. The total change of length was found to be 0.02 mm. at 6000 kgm. The maximum error here possible on the extension coefficient of the cylinder is, therefore, 6 parts in 200. The mean of several readings, of course, has a much less probable error.

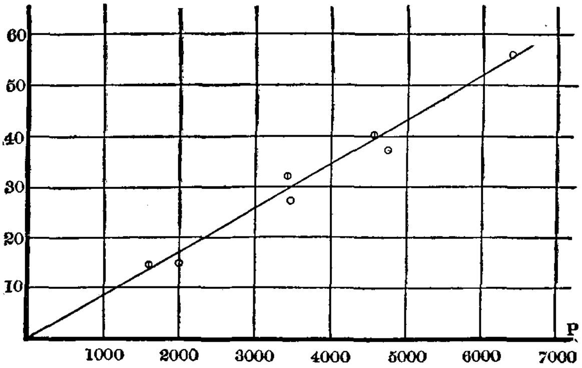

FIGURE 3. The elongation of the cylinder of figure 1, as a function of the pressure. ○, observations at increasing pressures; ⦶, at decreasing pressures. The ordinates give the proportional elongation multiplied by 10^6. That is at a pressure of 6400 kgm. per sq. cm. the elongation of the cylinder is 0.000056 per unit length.

The results obtained are shown in Figure 3, in which extension of the cylinder is plotted against pressure. The pressure was measured here, as in all subsequent work in this paper, by a secondary gauge depending on the variation of the resistance of mercury under pressure. The justification and calibration of this gauge has been made the subject of another paper. The figure shows distinct evidence of hysteresis, the extension under decreasing pressure being greater than the corresponding extension under increasing pressure. This is the more surprising as the total extension of the cylinder is only $\frac{1}{30}$ of the value of the extension at the elastic limit under pure tension. The departure of the points from a straight line representing the mean is comparatively slight, however, and in applying the corrections determined in this way the relation between extension and pressure was assumed to be linear.

With this apparatus the linear compressibility of a piece of commercial aluminum rod and several specimens of iron and steel were made. In Figure 4 is shown the fractional change of length of the aluminum rod corrected for the extension of the steel cylinder, plotted against pressure. This figure does not include the first observation which was made with a pressure slightly higher than any subsequently reached. The rod took a distinct set on this first application, being permanently shortened by one part in 30,000. No evidence of further set was found on subsequent applications of pressure. This is the first occasion on which a set in any dimension by the application of hydrostatic pressure has been directly observed. No attempt was made to find whether this linear set is accompanied by volume set. The displacement was measured from the mean of several determinations of the position of the ring at zero pressure. But this determination is obviously affected by the same errors as displacement measurements at higher pressures. It is evident from the figure that within the limits of error the points lie on a straight line. This was assumed to be of the form $a + bp$, and a and b determined by least squares, discarding the most discordant results. a is the true zero position and b the pressure coefficient of contraction. In this way every measurement at any pressure contributes to the more accurate determining of

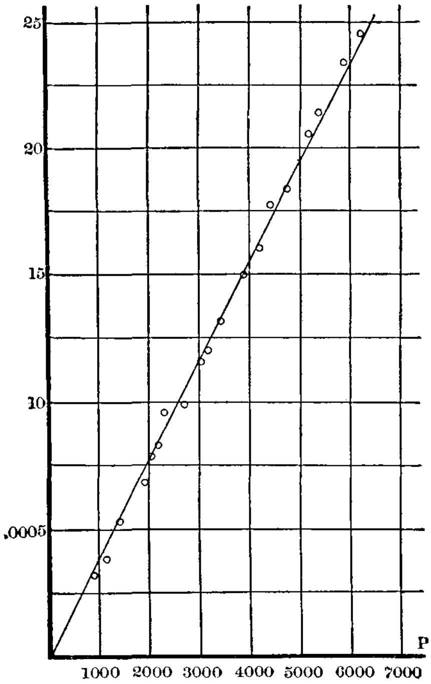

FIGURE 4. The observed proportional change of length of an aluminum rod plotted against pressure.

TABLE I.

Compressibility of Aluminum Rod.

Order of Observation.	Pressure kgm./cm.^{2}	$\frac{\Delta l}{l_0}$.		
		Observed.	Calculated.	Difference.
18	900	0.000320	0.000346	+26
6	1154	0.000386*	0.000445	+59
12	1436	0.000545	0.000555	+10
5	1910	0.000684*	0.000741	+57
19	2050	0.000790	0.000796	+ 6
11	2180	0.000867	0.000847	−20
17	2396	0.000960	0.000910	−50
4	2694	0.000990*	0.001048	+58
13	3030	0.001163	0.001179	+16
7	3180	0.001202	0.001238	+36
8	3416	0.001318	0.001330	+12
1	3890	0.001509	0.001515	+ 6
10	4230	0.001605	0.001648	+43
14	4418	0.001775*	0.001722	−53
2	4760	0.001838	0.001855	+17
9	5200	0.002054	0.002027	−27
16	5384	0.002140	0.002099	−41
3	5892	0.002339	0.002297	−42
15	6240	0.002450	0.002434	−16

$\frac{\Delta l}{l_0} = a + bp.$ $a = -0.0000056.$ $b = 0.0000003910.$

* Discarded in the calculation.

the zero position, the necessity of a large number of determinations of which are therefore avoided. It was found that

$$\frac{\Delta l}{l_0} = -0.0000056 + 0.0000003910\, p.$$

The cubic compressibility is, therefore, 0.000001173 kgm. per sq. cm. In Table I are shown the observed and calculated results. The probable error of a single observation is less than one per cent at the higher pressures. The probable error of *b*, the compressibility, is about $\frac{1}{3}$ per cent. The value found by Richards[1] for the compressibility of aluminum is 1.28×10^{-6}. He does not state the chemical purity of the aluminum. The specimen used above was commercial aluminum rod, which is usually very pure. No chemical analysis was made, however, and the discrepancy may be due to impurities.

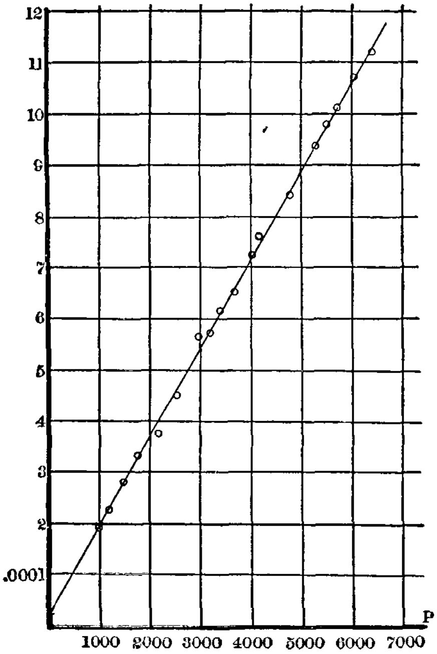

FIGURE 5. The observed proportional change of length of a rod of Bessemer steel plotted against pressure. The zero is here arbitrary.

In an exactly similar manner the compressibilities of several samples of iron or steel were determined. The first piece was from a piece of $\frac{1}{2}$ in. (1.27 cm.) Bessemer rod annealed by heating to redness and cooling slowly, and then turned down to $\frac{3}{8}$ in. (0.95 cm.). It was from the same piece of rod as a piezometer for determining the compressibility of mercury, as will be described later. The results obtained for this steel corrected for the extension of the cylinder are plotted in Figure 5, the zero being arbitrary as formerly. The results are better proportionately than for

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[1] Compressibilities of the Elements and their Periodic Relations. Richards, Carnegie Inst., Washington, p. 61 (1907).

TABLE II.

COMPRESSIBILITY OF BESSEMER ROD. SAME MATERIAL AS MERCURY PIEZOMETER.

Pressure kgm./cm.	$\frac{\Delta l}{l_0}$.		
	Observed.	Calculated.	Difference.
994	0.000195	0.000196	+ 1
1190	0.000228	0.000230	+ 2
1488	0.000281	0.000281	. .
1770	0.000332	0.000330	− 2
2174	0.000374*	0.000399	+25
2540	0.000452*	0.000462	+10
2980	0.000565*	0.000538	−27
3176	0.000570	0.000572	+ 2
3400	0.000615	0.000611	− 4
3670	0.000652	0.000657	+ 5
4040	0.000724	0.000721	− 3
4176	0.000769*	0.000744	−25
4760	0.000839	0.000845	+ 6
5294	0.000938	0.000937	− 1
5506	0.000977	0.000973	− 4
5730	0.001013	0.001012	− 1
6060	0.001072	0.001068	− 4
6430	0.001127	0.001133	+ 6

$\frac{\Delta l}{l_0} = a + bp.$ $a = 0.0000249.$ $b = 0.0000001722.$

* Discarded in the calculation.

the aluminum, although, because of the smaller size of the total effect, one would expect greater percentage variation from the particles of grit. Probably the improvement is due to the increased familiarity with the method, which seems capable of giving accurate results. A straight line through the observations, discarding the four worst, was computed by least squares, giving as the linear compressibility 1.722×10^{-7}, and the cubic compressibility 5.166×10^{-7} kgm. per sq. cm. Table II shows the differences between the observed and the computed values. The four starred points are the ones discarded in the computation. The probable error of a single observation, excepting the four discarded ones, is 2.3, less than $\frac{1}{4}$ per cent at the higher pressures. The probable error of the compressibility is $\frac{1}{10}$ per cent, which therefore does not vary more than this from constancy throughout the pressure range. No set was observed in this piece of steel on the first application of pressure, which is perhaps evidence of the freedom from internal strain, and to a less degree evidence for equal compressibility in all directions.

An attempt was made to get some light on the possible magnitude of differences of compressibility in different directions by the following method: Two strips were cut from a very homogeneous piece of $\frac{5}{8}$ in. (1.59 cm.) Bessemer boiler plate, respectively along and perpendicular to the direction of rolling; these were turned down to $\frac{3}{8}$ in. (0.95 cm.) like the other test pieces of steel or aluminum, and the compressibility of each determined. The results are given in Tables III and IV. The compressibility of each was calculated by least squares, discarding only one observation from each set. The probable error of a single observation is approximately the same in either set, $\frac{7}{10}$ per cent at the higher pressures. The probable error of the compressibility in either case is about $\frac{2}{10}$ per cent. The compressibility of the lengthwise piece was 5.298×10^{-7}, and of the transverse 5.303×10^{-7}, agreeing within the limits of error. No claim is made that this settles the question of the equal compressibility of metals in all directions. Doubtless with metals of different character there are internal strains left from working that would produce such a difference.

There are only a few other direct determinations of the compressibility of steel. Amagat[2] measured the change of length by an electric contact device, but does not publish his data. He states that the results agree with a determination by an indirect method involving the theory of elasticity and gives 6.8×10^{-7} as the best value. Richards,[3]

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[2] Amagat, C. R., **108**, 1199 (1888).

[3] Richards, loc. cit., p. 50.

COMPRESSIBILITY OF BESSEMER BOILER PLATE.

TABLE III. LONGITUDINAL.

Pressure $\frac{\text{kgm.}}{\text{cm.}^2}$	$\frac{\Delta l}{l_0}$. Obs.	Calc.	Diff.
794	0.000120	0.000120	0
984	0.000156	0.000154	− 2
1150	0.000176	0.000186	+10
1396	0.000233	0.000227	− 6
1660	0.000271	0.000273	+ 2
2016	0.000314	0.000336	+22
2228	0.000362	0.000373	+11
2480	0.000431	0.000418	−13
2834	0.000481	0.000480	− 1
3040	0.000500	0.000517	+17
3272	0.000591	0.000558	−33
3540	[0.000663]	0.000595	−68
3646	0.000625	0.000624	− 1
3920	0.000672	0.000672	0
4398	0.000748	0.000757	+49
4400	0.000761	0.000757	− 4
4740	0.000835	0.000817	−18
4920	0.000847	0.000849	+ 2
5340	0.000954	0.000923	−31
5440	0.000938	0.000941	+ 3
5690	0.000988	0.000985	− 3
6164	0.001061	0.001069	+ 8
6430	0.001099	0.001116	+17

$$\frac{\Delta l}{l_0} = a + bp.$$

$a = -0.000020.$

$b = \log^{-1} 3.2470 - 10.$

Cubic compressibility $= 0.0_6 5298$.

TABLE IV. TRANSVERSE.

Pressure $\frac{\text{kgm.}}{\text{cm.}^2}$	$\frac{\Delta l}{l_0}$. Obs.	Calc.	Diff.
1000	0.000174	0.000173	− 1
1190	0.000197	0.000206	+ 9
1222	0.000215	0.000212	− 3
1446	[0.000192]	0.000252	+60
1680	0.000280	0.000293	+13
2014	0.000345	0.000353	+ 8
2180	0.000384	0.000381	− 3
2526	0.000439	0.000442	+ 3
2816	0.000518	0.000494	−24
3060	0.000537	0.000537	0
3346	0.000593	0.000586	− 7
3660	0.000635	0.000643	+ 8
3980	0.000703	0.000699	− 4
4186	0.000729	0.000736	+ 7
4472	0.000789	0.000786	− 3
4988	0.000906	0.000877	−29
5294	0.000929	0.000932	+ 3
5456	0.000966	0.000960	− 6
5668	0.000994	0.000998	+ 4
6034	0.001044	0.001063	+19
6210	0.001099	0.001093	− 6
6400	0.001099	0.001128	+29
. . .	. . . .	. . . .	. .

$$\frac{\Delta l}{l_0} = a + bp.$$

$a = -0.000004.$

$b = \log^{-1} 3.2474 - 10.$

Cubic compressibility $= 0.0_6 5303$.

also observing the change of length by an electrical contact device, finds 3.9×10^{-7}. The iron used by Richards was commercial wrought iron, chemical analysis of which is not given. The mild Bessemer steel used in this investigation is usually as free from carbon as wrought iron, and is very much more likely to be homogeneous. The absence of set is evidence of the closeness of texture, while Richards states that the wrought iron used by him was porous and had to be hammered to give satisfactory results. This possibly may account for some of the difference in the results.

To get some idea of the effect of varying percentage of carbon, the compressibility of a piece of high carbon (1.25 per cent) annealed tool steel was determined with the same probable error as in the other determinations, and was found to be 0.000000525. The discrepancies between Richards' values and the values found in this paper can hardly be explained by impurities of this nature.

It is to be noted that neither the steel nor the aluminum shows any tendency to become decreasingly compressible at higher pressures, in analogy with the behavior of more compressible substances, particularly liquids. In fact, as will be seen from an inspection of either the curves or the table, the aluminum shows a distinct though slight tendency to become more compressible at higher pressures. However, it did not seem that this single example would justify the conclusion that this paradoxical behavior was due to anything except errors of observation, and accordingly the coefficient was calculated by least squares on the assumption that it was constant.

The second modification of the above method for measuring linear compressibility consists in comparing the change of length of a tube of the substance in question with the simultaneous change of length of a piece of steel, both the substance and the steel suffering uniform contraction by the hydrostatic pressure over the whole exterior surface. From the relative change of length the absolute linear compressibility may be found if the linear compressibility of the comparison piece of steel is known. This latter may be found by the first method given above.

The apparatus with which the relative change of length of the tube (in this case of glass) and the steel were determined is shown in Figure 6. The glass tube C was kept pressed against the bottom B of the cylindrical hole in the steel cylinder A, by the spring at G, through the medium of the tie rod H and the nuts E and F. A split brass ring D slides on the glass tube easily, but tightly enough to remain securely in position under moderate jarring. Fine scratches were made on the steel at I and the flange of the brass ring. The whole combination was placed in the pressure chamber, and subjected to hydrostatic pressure all over. Both glass and steel shrink, the glass shrinking the more, and hence the ring D is pushed up on the tube. When pressure is released, D comes back with the tube, and the increased distance between the scratches, measured with two microscopes, gives the relative change of length for the highest pressure reached. The glass tube was taken out of the steel jacket and everything washed carefully after each application of pressure, in order to insure freedom from small particles of grit. It is an advantage of this method over the first, that because of the greater accessibility of the parts, complete freedom from grit is secured by washing after each application of pressure. Repeated measurements of the zero position of the ring gave results agreeing within 0.001 mm., which in this case was about the magnitude of possible errors of reading. The total displacement at 6500 kgm. was about 0.35 mm. in the form above.

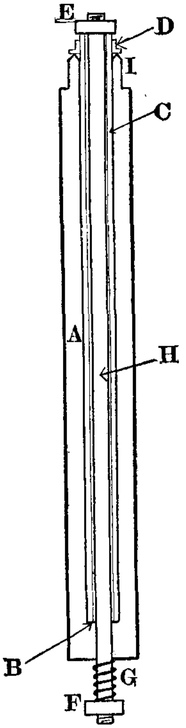

FIGURE 6. Apparatus for comparing the linear compressibility of glass and steel. The glass tube C is compared with the enveloping steel tube A. The relative change of length is measured by measuring the displacement of the ring at D, sliding on the glass tube. The glass tube is kept in contact with the shoulder B by the spring G, acting on the nut F through the tie rod H, which in turn presses on the glass tube by the nut E.

Among possible sources of error we have here again a maximum uncertainty in the effective length of the glass tube of $\frac{1}{2}$ the width of the ring D. In the form used the total length was about 8 cm., and the width of the ring 2 mm. The results may, therefore, be in error by $\frac{1}{80}$, but probably by less than this. This source of error may obviously be decreased at pleasure by increasing the length of the tube. Another possible source of error is temperature change. Error from heat of compression was avoided by operating slowly, applying pressure nearly to the maximum, waiting for the equalization of the temperature, and then applying the last few hundred kilograms. Differences of temperature at different times of measuring the displacement did not prove great enough to introduce perceptible error, since the difference of dilation between the glass and the steel is small. To secure good results, it was found necessary that the glass tube fit closely in the steel cylinder without play sidewise. As it was found difficult to draw a tube accurately enough, this desired freedom from play was secured by wrapping tin foil at either end.

Measurements were made in this way of the change of length of two kinds of Jena glass: a hard combustion tubing No. 3883, and a very fusible glass No. 3880 a. The results at first were discouragingly irregular. After repeated trials, however, they settled down into a fairly regular final form. It became evident on trial with different pieces of glass that there is here, directly observed, the same seasoning effect of successive applications of pressure that was noticed in measurements of electrical resistance. The final behavior never became entirely regular, however. The general effect of frequent applications was to slightly increase, in a totally irregular fashion, however, the observed change in length. In Figure 7 the observed changes of length are plotted against pressures. The irregularity of the results is noticeable, particularly for the hard glass; it approaches, or may sometimes exceed, 5 per cent of the total effect to be expected. The results with the soft Jena glass were only one third as irregular. The explanation suggests itself that the less regularity of the results with the hard infusible glass is because of the greater internal strains set up in this by the long temperature range through which it cools after passing plasticity.

In order to find whether there is any appreciable change in the linear compressibility of glass when it is drawn down from larger sizes, the above form of apparatus was modified by placing the comparison piece of steel inside, instead of outside, the glass tube. The tubes tested were 1 cm. in diameter, which is the original size from which the test pieces mentioned above were drawn down to 0.5 cm. Within the limits of error, no variation of compressibility with absolute size could be detected.

The linear compressibility of the steel against which the glass was compared was determined indirectly by finding the relative change of length in the same manner as for the glass, of the steel and a piece of aluminum cut from the rod whose absolute compressibility was determined above. These readings of the relative change of steel and aluminum are shown in the lower line in Figure 7. The points, except one, lie on a straight line within errors of reading. The one discordant point represents a discrepancy of only 0.0003 mm., and no importance is attached to it. The regularity of these measurements of the aluminum, made with the same apparatus as the measurements of the glass, furnishes additional presumptive evidence, therefore, that the irregularity of the latter is not due to errors of measurement, but is an actual property of the glass. The lower line in Figure 7 was computed by least squares, giving the relative compressibility of the aluminum and the steel. From this and the known absolute compressibility of the aluminum, the cubic compressibility of the steel was

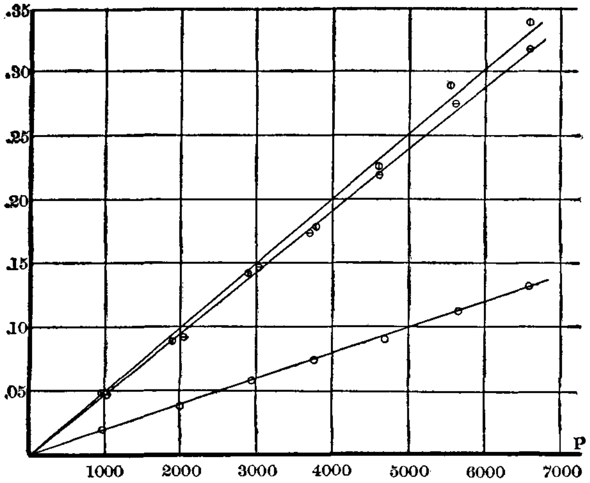

Figure 7. Observed relative change of length of steel, and glass or aluminum. The ordinates give the change of length in millimeters, the total length being about 8 cm. ⦶ shows hard Jena glass; ⊖ shows soft Jena glass, and ○ the aluminum.

found to be 4.74 × 10^{-7}, a value somewhat lower than the values found directly for the other specimens of steel. Similarly, the other lines of Figure 7, connecting relative change of length of glass and steel with the pressure, were computed by least squares. The irregularity of the results is too great to warrant the assumption of any other than a linear relation, although the hard Jena glass in particular shows a tendency toward the paradoxical behavior of higher compressibility at higher pressures already remarked in the aluminum. From these constants calculated by least squares, and the compressibility of the comparison piece of steel determined as above, the compressibility of the glass was found to be:

for Jena glass No. 3880 a	2.17×10^{-6} kgm. per sq. cm.
for Jena glass No. 3883	2.23×10^{-6} kgm. per sq. cm.

The hard glass, contrary to what one might expect, is therefore the more compressible, a result that has already received confirmation by measurements of electrical resistance.

Beside these determinations of the compressibility of glass, it was also necessary to find the compressibility of mercury, in order to find the pressure coefficient of the molecular conductivity of mercury. None of the data at hand reach over a sufficient pressure range for the purpose of this paper, and the data had, therefore, to be extended up to 6800 kgm. The correction introduced by the compressibility of mercury is only 10 per cent of the total change of resistance, so that a highly accurate value of the compressibility was not necessary. The interest of this determination lay rather in finding whether there is any marked decrease of compressibility over the pressure range used. To make this determination, a method was adopted which gives promise of being a considerably better means of determining compressibility even at comparatively low pressures than those methods at present in common use.

The compressibility of mercury at low pressures has been the subject of a number of investigations, and the results which have been obtained recently have been fairly concordant. It is a common feature of all earlier determinations that the mercury has been enclosed in a glass piezometer, the correction for the compressibility of which is 60 per cent of the total effect. The correction for the glass is unusually large in this case because of the comparatively small compressibility of the mercury. For many liquids, the correction for the piezometer is considerably less (6 per cent for water, for example), and the objections urged in the following have proportionally less weight. This correction may be determined in various ways, depending in general on the theory of elasticity, which makes, among others, the assumption of the uniform compressibility of the glass in all directions. Too often, however, the compressibility of the glass has been merely assumed from the work of other investigators on a glass presumably of the same general character as the glass used in the experiments. The correction for compressibility determined by elastic experiments on the same or other pieces of glass seems doubtful in view of the large correction involved. Thus if the behavior of the glass were as irregular as that observed in the case of the hard Jena glass, discrepancies in the compressibility determined with the same piezometer of as much as 3 per cent are to be expected, at least over any considerable pressure range. Doubtless this uncertain correction for the envelope is the cause of the discordant results previously obtained.

The work of Amagat and de Metz along this line seems the most credible. Each gives the mean of the results with several piezometers, where others have used only one. The results of de Metz [4] with four piezometers vary as much as 5 per cent, while those of Amagat [5] with seven piezometers vary 2 per cent. The value of Amagat at 20° is 0.00000380 kgm./cm.^{2} while that of de Metz is 0.00000379 kgm./cm^2. Lately Richards [6] obtained the value 0.00000371, working with a glass piezometer by an electric contact device, but in such a fashion as to eliminate the necessity for calculating the compressibility of the glass, this step being replaced by a calculation from the observed linear compressibility of steel, in which large percentage errors are of much less importance. The values above are for a small pressure range: de Metz and Amagat 50 kgm., and Richards 500 kgm. The results all agree within a unit in the last place, when correction is made for the difference in pressure range.

The only work over a wider range seems to have been done by Carnazzi,[7] who worked between 0 and 200° and went up to 3000 atmos. He used a glass piezometer, assuming Amagat's mean value for the compressibility of the glass, and a manometer depending in a way not entirely free from objection on the compressibility of water as determined by Amagat. Only two significant figures are given in the results, compressibility at 23° being 0.0000038 from 0 to 500 atmos., and 0.0000034 from 2500 to 3000 atmos. These results must be decreased about 3 per cent, becoming 0.0000037 and 0.0000033 respectively, to reduce from atmospheres to kilograms.

In the present determination, a steel instead of a glass envelope was used. The advantages of a steel over a glass piezometer are manifold. The correction for the compressibility of the steel is only 15 per cent of the total effect against 60 per cent when glass is used. Again, the steel is very much more regular in its elastic behavior than the glass; this is obvious at once from an inspection of the curves showing the compressibility of the glass and of the steel. It has been already stated that the irregular behavior of the glass might introduce discrepancies of 3 per cent. Finally, the correction for the glass must be determined from the theory of elasticity, assuming uniform compressibility in all directions. The difficulty of obtaining glass free from internal strain makes the validity of this assumption at least doubtful. Many anomalous results may be explained by this effect. Thus in one case [8] an actual increase of the internal volume of the piezometer under hydrostatic pressure has been recorded. On the other hand, the great homogeneity of steel makes its uniform compression apriori more probable, and here the probability has been greatly increased by an experimental proof of the uniformity of strain in a piece of rolled steel plate, of the same grade of steel as that used in the mercury piezometer.

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[4] de Metz, Wied. Ann., **47**, 706 (1892).

[5] Amagat, C. R., **108**, 228 (1888).

[6] Richards, loc. cit., p. 51.

[7] Carnazzi, Nouv. Cim., **5**, 180 (1903).

The method is essentially a revival of one used by Perkins [9] in 1825. Possibly the bad results obtained by Perkins, which were 250 per cent too large, accounts for the subsequent neglect of the method. Several slight modifications were suggested by Professor Sabine, however, so that it has been possible to obtain very satisfactory results. The method consists essentially in observing the extent to which a freely moving piston is pushed into a cylinder containing the liquid to be examined, by the application of hydrostatic pressure all over the exterior of the piston and cylinder. The arrangement used is shown in Figure 8. The containing cylinder C is of

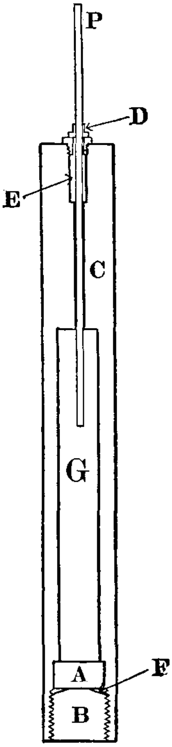

FIGURE 8. Piezometer for determining the compressibility of mercury. C, containing cylinder of steel; G, mercury; P, easily moving piston; D, movable brass ring by which the displacement of the piston is measured. The packing of molasses and glycerine is placed at E. The piezometer is closed at the lower end by the steel plug A, held in place by the screw B. The crack at F is filled with solder.

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[8] M. Schumann, Wied. Ann., **31**, 22 (1887).

[9] Perkins, Phil. Trans. Royal Society, London, p. 324 (1819–1820).

Bessemer steel $\frac{1}{2}$ in. (1.27 cm.) diameter and $3\frac{1}{2}$ in. (8.89 cm.) long. The piston P is $\frac{1}{16}$ in. (0.16 cm.) in diameter, made in exactly the same way as the piston of the absolute gauge described in a previous paper. The piston accurately fits the hole within 0.0002 or 0.0003 in. (0.00051–0.00076 cm.). The cavity G, which is filled with mercury, is $\frac{1}{4}$ in. (0.635 cm.) in diameter and 2 in. (5.08 cm.) long. The lower end is closed with a plug of steel driven into place and soldered on the outside at F and held additionally by the screw B. The piston P is a slightly looser fit than that used in the absolute gauges, a few ounces without rotation sufficing to displace it. The displacement produced by pressure is indicated by the use of a sliding brass ring at D, exactly as in measuring the change of length of rods. The piezometer was filled by pouring recently distilled mercury through the small hole at the top by a fine glass capillary. The inside of the piezometer was first wet with a few drops of water to insure filling of all the crevices. After filling in this way it was placed under an air pump as an additional precaution against the inclusion of air. The whole was now heated until the mercury rose from the top of the piston hole. The piston, smeared to insure tightness with the same mixture of molasses and glycerine used in the absolute gauge, was inserted and follows the mercury down as it cools. The inside of the enlargement at E was now smeared with molasses, and mercury was poured over the whole to prevent contact of the molasses and the mixture of glycerine and water transmitting the pressure. This packing of viscous molasses very much improved the behavior of the piezometer, reducing the leak past the piston to a minimum. If, however, this packing is used, its protection by the mercury is absolutely necessary, for otherwise the glycerine diffuses through the molasses on each application of pressure, rapidly changing the amount of liquid inside the cylinder.

The method of making the readings was to place the cylinder in the pressure chamber and subject it to hydrostatic pressure all over. By means of the freely moving piston this pressure is transmitted immediately to the interior of the cylinder, the amount of motion of the piston, and so the apparent loss of volume, being indicated by the displacement of the ring D, which is measured after pressure is released and the cylinder removed again from the pressure chamber. This displacement, together with the cross section of the piston and volume of the mercury, gives, therefore, the difference of compressibility between the mercury and the steel of the envelope. The volume of the mercury was obtained by weighing, and the diameter of the piston was measured with a Brown and Sharpe micrometer, the error here not being more than 0.00005 in. on a total of 0.062 in., introducing a possible error of $\frac{1}{600}$ in the area. The determination of the compressibility of the steel, which must be made independently, takes the place of the determination of the compressibility of the glass in previous work.

A variation of temperature of one degree is equivalent in displacement of the piston to about 50 kgm. The pressure chamber in which the cylinder was placed was inserted in a water bath as nearly as possible at room temperature, and the small variations of this temperature were read to 0.01° after every determination. The temperature at the time of measuring the displacement, which was done with a reading microscope, was also recorded and corrections applied for variations. The observations were carried out at temperatures varying only slightly from 20°, and the final results are for this temperature. The error from temperature variations, which were hardly as much as 0.1°, becomes entirely negligible at the higher pressures, in which the principal interest of this work lay. For accurate work at lower pressures it would, of course, be necessary to take more elaborate temperature precautions.

Another correction necessary to apply is a correction on the measured diameter of the piston, because the piston in advancing into the inner cavity draws with it some of the molasses in the crack between piston and cylinder. The effect of this is to increase the effective diameter of the piston. The question has already been discussed in connection with the absolute gauge and a method given for determining the correction, which, however, is not applicable here. In this case the correction was determined by first smearing the hole in the cylinder with a heavy oil, inserting the piston, and then withdrawing it again. A film of oil adheres to the piston equal approximately to one half of the volume of the oil originally in the crack between piston and cylinder. The quantity of oil thus clinging to the piston was determined by weighing, and the crack in this manner found to be 0.0003 in. (0.00076 cm.) wide. The method of course is very inaccurate, but seemed the only practical way of getting any idea of this small quantity. The total correction thus introduced is only 1 per cent, so that fairly large errors in the correction are unimportant.

It seemed necessary to investigate one other source of possible error before confidence could be placed in the results. There has been expressed a feeling that metals might be porous under high pressures, the experience of Amagat in forcing mercury through 8 cm. of cast steel being adduced as evidence on this point. To test this, a piece of steel from the same piece as the piezometer was weighed before and after subjection to pressure, in an endeavor to detect possible increase of weight from the absorbed liquid. No change of weight of more than one part in 400,000 could be detected. On a previous occasion a piece of drawn copper had been found to suffer no increase of weight of one part in 1,800,000. It may be confidently expected, therefore, that ordinary commercial bar metal shows no considerable porosity. Amagat's result was probably due to flaws in the casting.

In Figure 9 the observed proportional changes of volume of mercury measured from an arbitrary zero, as in the case of the determination of the compressibility of rods, are plotted against pressure, measured in the usual way with a mercury resistance. The maximum ordinate corresponds to a displacement of the piston of 1.5 cm. Results obtained with a preliminary piezometer, not so well made as the final one, agree with the curve given within the somewhat larger limits of experimental error. The principal source of error seems to be the inclusion of minute air bubbles. Measurement from an arbitrary zero, determined by backward extrapolation as above, removes this as a consistent source of error, but the measurement of the actual displacement becomes irregular from the lack of certainty with which the piston is returned after release of pressure to the initial position by the comparatively feeble expansive action of the bubble of air. All the precautions described above to remove this bubble appear necessary.

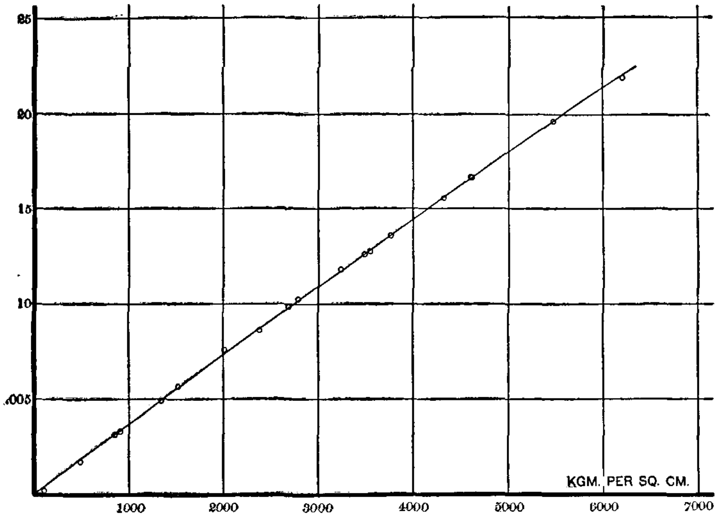

Figure 9. The proportional change of volume of mercury, as determined with the piezometer of Figure 8, plotted against pressure.

The compressibility of the steel envelope has already been determined, and hence the proportional change of volume of the mercury can be corrected and the true compressibility found. It was assumed that

$$\frac{\Delta V}{V_0} = a + bp + cp^2,$$

and the constants were calculated by least squares. The results are shown in Table V. The constant a has the same significance as in the case of the steel and aluminum rods, the constants b and c alone having significance for the mercury itself. The values found were

$$a = 0.001252,$$

$$b = 3.699 \times 10^{-6},$$

$$c = -1.985 \times 10^{-11}.$$

The compressibility at low pressures is b, 3.70×10^{-6} compared with 3.80×10^{-6}, found by Amagat, de Metz, and Richards, and 3.7×10^{-6} found by Carnazzi. It is to be remarked, however, that the purpose of this investigation was not to find the compressibility at low pressures, only two observations being made at less than 800 kgm. Both the dimensions of the piezometer and the temperature changes make the low pressure values of this determination doubtful. There is, moreover, obvious on inspection of the table a tendency for the low pressure values to lie below the values given by the formula. This would increase the initial compressibility. The experimental error is sufficient, however, to make illusory a more accurate determination of the initial b by passing a curve of the above type through the lower values only. The probable error of a single observation, discarding the first, is $\frac{1}{3}$ per cent at the highest pressure. The probable percentage error of values determined by the formula is 0.25 per cent, discarding the lowest value, or 0.18 per cent, discarding the lowest two.

The departure of the compressibility from constancy is shown by the constant c, which is very small, in fact much smaller than has been found by either Carnazzi or Richards. It may be found from the above formula that the instantaneous compressibility at 2700 kgm. has decreased to 3.58×10^{-6} from its initial value of 3.70×10^{-6}. Carnazzi finds the average compressibility between 2500 and 3000 to be 3.3×10^{-6} against 3.7×10^{-6} between 0 and 500. Richards finds a decrease of compressibility from 3.80×10^{-6} to 3.64×10^{-6} over a pressure range of 500 kgm. However, Richards himself recognized the possibility that his pressure unit might be in error at the higher

TABLE V.

Compressibility of Mercury.

Pressure kgm./cm.^{2}	$\frac{\Delta V}{V_0}$.		
	Observed.	Calculated.	Difference.
116	0.000140	0.000168	+28
496	0.000297	0.000308	+11
850	0.000440	0.000439	− 1
916	0.000458	0.000462	+ 4
1346	0.000619	0.000619	0
1536	0.000691	0.000689	− 2
2050	0.000892	0.000875	−17
2380	0.000990	0.000994	+ 4
2690	0.001117	0.001106	−11
2792	0.001157	0.001142	−15
3224	0.001314	0.001297	−17
3492	0.001393	0.001393	0
3550	0.001408	0.001413	+ 5
3760	0.001497	0.001487	−10
4320	0.001679	0.001486	+ 7
4600	0.001796	0.001784	−12
4610	0.001788	0.001787	− 1
5490	0.002097	0.002096	− 1
6216	0.002329	0.002347	+18

$\frac{\Delta V}{V_0} = a + bp + cp^2.$ $b = \log^{-1} 4.5681 - 10.$

$a = 0.0012523.$ $-c = \log^{-1} 9.2977 - 20.$

pressures. He finds, e. g., for the compressibility of water at 200 and 400 kgm. 42.5 and 39.6 respectively, against 42.4 and 40.6 as found by Amagat. This points, therefore, to an error in Richards' standard in the right direction, and of approximately the right magnitude to bring his result into agreement with the above. It may also be remarked in this connection that the quantity c is essentially a difference of the second order, and that consequently any increase of the pressure range will give a more than proportionate increase in the probable accuracy of *c*, other things being equal.

The form of steel piezometer described above may be applied with a few obvious modifications to the determination of the compressibility of other liquids than mercury, and even of liquids that attack the steel. In fact, it seems probable that some such form will prove most useful for high pressure work in general, because the forms of glass piezometer in common use become impracticable at high pressures by the cracking of the glass around any pieces of sealed-in platinum, or even by the cracking of the glass alone, when blown into at all complicated shapes.

CONCLUSION.

In this paper there have been presented methods applicable over a wide pressure range for finding the compressibility of solids in the form of rods or tubes, and also of liquids. These methods have been applied to the determination of a few compressibilities which were needed for another purpose. The pressure range employed was 6500 kgm. The compressibilities found were as follows: two pieces of Jena glass

No. 3880 a, 2.17×10^{-6} kgm. per sq. cm.

No. 3883, 2.23×10^{-6} kgm. per sq. cm.

Four pieces of steel: two of Bessemer boiler plate, one of Bessemer rod, and one of tool steel, respectively,

$$5.298 \times 10^{-7},\ 5.303 \times 10^{-7},\ 5.16 \times 10^{-7},\ \text{and}\ 5.25 \times 10^{-7}.$$

Another piece of Bessemer by an indirect method, not so accurate, gave 4.7×10^{-7}. Compressibility of commercial aluminum rod, 11.7×10^{-7}. The change of volume of mercury is connected with pressure by the relation

$$\frac{\Delta V}{V_0} = bp + cp^2$$

$$b = \log^{-1}(4.5681 - 10);$$

$$-c = \log^{-1}(9.2977 - 20).$$

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL
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THE THEORY OF BALLISTIC GALVANOMETERS OF LONG PERIOD.

By B. Osgood Peirce.

With a Plate.

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THE THEORY OF BALLISTIC GALVANOMETERS OF LONG PERIOD.

By B. Osgood Peirce.

Presented November 11, 1908. Received December 22, 1908.

If a ballistic galvanometer is to be used to measure the whole quantity of electricity which flows impulsively in a circuit when a condenser is discharged through it, or when the flux of magnetic induction through the circuit is suddenly changed, it can generally be assumed that the time during which the current lasts is so short that the flow practically ceases before the suspended system of the instrument has moved sensibly from its position of rest. That is, that the whole time of flow is not greater than, say, one fiftieth part of the time required for the needle or suspended coil to reach the end of its throw.

It is often desirable, however, to determine with accuracy the change of magnetic flux in a massive closed iron frame caused by a given change of excitation, and in such a case it usually happens that eddy currents in the metal or the inductance of the exciting coil so retard the change that the process lasts for a number of seconds at least. Under these circumstances a ballistic galvanometer of any ordinary form is practically useless. Indeed, according to the experiences[1] of Du Bois with such galvanometers as are to be found in most laboratories, the ballistic method fails when the time required for the change exceeds about one second.

Slow flux changes can be measured, nevertheless, with the aid of photographic records from a suitable oscillograph[2] either in the main circuit of the magnet or in the circuit of a testing coil wound about the iron. My experience with hundreds of such records seems to show, however, that the thickness of the photographed line obscures somewhat the slow changes when the exciting current has nearly reached its new value, and in the very sensitive instruments sometimes required for use in a secondary circuit there is a small but occasionally troublesome lag just at the beginning of the motion. For all ordinary purposes this method is wholly satisfactory if not always easy or convenient to carry out.

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[1] Du Bois, The Magnetic Circuit in Theory and Practice, Atkinson's translation, § 216, London, 1896.

[2] T. Gray, Phil. Trans., **184** (1893); Thornton, Electrical Engineer, **29** (1902); Phil. Mag., **8** (1904); Electrician, 1903; Peirce, These Proceedings, **41** (1906); **43** (1907).

Such fluxmeters as I have been able to procure, though admirable in many ways, have not been so free from crawling, due apparently to the paramagnetic properties of their copper coils, that their indications can be trusted for very slow magnetic changes. If the fluxmeter coil[3] is not wound on a metal frame, the mutual damping caused by the action of currents in the coil, and the core which it surrounds, are not always effective unless the resistance of the exterior circuit is small, and this frequently makes an instrument which works very well for one piece of work, nearly useless for another.

When the excitation of the core of a large electromagnet initially in a given magnetic condition and under a given excitation is changed by a predetermined amount, it sometimes happens — as is well known — that the resulting change in the magnetic flux through the iron depends somewhat upon the manner in which the exciting current is changed; that is, the flux change is different when the current in the magnet coil is changed very gradually or in short steps from what it is when the change is made very suddenly. This difference is generally small, and seems to depend upon a variety of circumstances[4] in a way not yet very well understood, but it must be determined for every large magnet if the behavior of the core under given conditions is to be predicted with any great accuracy.

I have recently had occasion to inquire how the changes of magnetic flux in each of a number of large cores, of which two are represented by Figures A and B, corresponding to given changes in the current in the exciting coil, depend upon the manner of growth of that current, and since such oscillograph records as I was able to make were not wholly satisfactory for the purpose, I found it desirable to attempt to procure a ballistic galvanometer (preferably of the d'Arsonval type, to avoid disturbances due to changes in outside magnetic fields) of period so long that the throw of the coil due to a change of flux of the usual sort, lasting for say thirty seconds, should not be sensibly different from the throw due to the same amount of electricity sent impulsively through the coil when at rest in its position of equilibrium.

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[3] Beattie, Electrician, Dec., 1902; Peirce, These Proceedings, **43** (1907).

[4] G. Wiedemann, Galvanismus, **3**, 738; Gumlich und Schmidt, Electrotechnische Zeitschrift, **21** (1900); Ruecker, Inaugural Dissertation, Halle, 1905; A. Hoyt Taylor, Phys. Rev., **23** (1906).

The galvanometer I sought did not need to be very sensitive, but it must have one property which, according to my experience, is rare in suspended coil instruments; that is, there must not be the slightest sensible shift of the zero point due to thermal currents or to chemical action at the junctions when the galvanometer circuit should be closed on itself. This condition forbade the leading of the current into the galvanometer coil through the phosphor bronze or steel gimp by which the coil was suspended, and required that the whole galvanometer circuit, even to the binding posts and connectors, should be of one metal, copper.

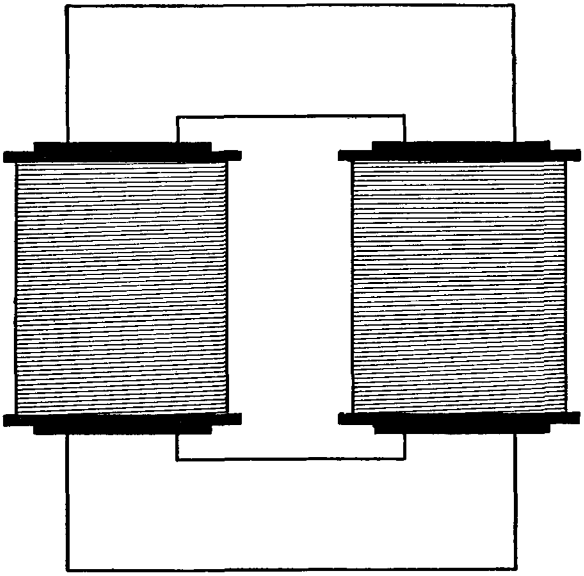

FIGURE A.

This electromagnet has a laminated core made of sheet iron one third of a millimeter thick and weighs about 300 kilograms.

It is of course not desirable to make the period of a ballistic galvanometer long by making the righting moment due to the suspending fibre small, for a weak fibre makes the zero point uncertain, and a large throw on one side usually shifts the zero point slightly in that direction unless the gimp is even stouter than that commonly used in sensitive instruments. It seemed necessary, therefore, to increase the moment of the suspended system so much that in spite of a stiff suspending gimp the period should be long.

In the case of a galvanometer coil with a period several minutes long, it is difficult to tell by mere inspection for a few seconds whether the coil is really at rest at its zero or whether it has a very slight velocity which in the course of its slow swing will lead to an addition of two or three millimeters to the amplitude of the throw. For this reason it was desirable that the coil should be subjected to some slight electromagnetic damping, though, as will appear later on, it was not possible to damp the coil critically.

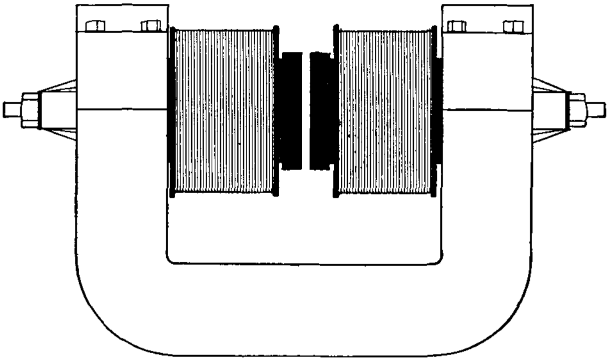

Figure B.

This magnet has a solid core which weighs about 1500 kilograms.

The requirements enumerated above are so simple that it seemed at first that there would be no difficulty in meeting them all, and this would have been the case if it were not for the fact that the best copper and silver wire, and the best copper, silver, and aluminium sheet to be had in the market are usually so highly paramagnetic that in an intense magnetic field the galvanometer coil and the metal frame on which it is wound, if a frame be used, often acquire a large magnetic moment, and this increases in an irregular way the righting moment of the suspended system — perhaps to many times the value due to the gimp alone. The difficulty is an old one; many persons have struggled with it, and some have succeeded in overcoming it more or less completely, by great care in the preparation of special wire for the purpose. The difficulties are, however, very much increased when it is necessary to provide a sufficient electromagnetic damping (air damping is sometimes objectionable) for a suspended system which in order to have the requisite moment of inertia must weigh perhaps 300 grams. Silk insulating material is generally magnetic, and so is most paraffine wax. A certain closed frame made by Mr. G. W. Thompson, the mechanician of the Jefferson Physical Laboratory, of the best obtainable sheet copper, to hold the coil of a d'Arsonval galvanometer of the common cored type, had a period of oscillation of about 2 minutes when suspended by a certain piece of gimp in free space, but a period of only 9 seconds when put in place in the instrument. In this case the righting moment due to the fibre was clearly wholly overshadowed by that due to the magnetism of the copper. When copper was wound on this frame, the magnetic moment of the whole, if placed between the poles of the permanent magnet, became so large that the whole suspended system could be deflected at will, when the circuit was open, by a bar magnet held in the hand outside the frame of the instrument.

It is easy to make the period of an ordinary d'Arsonval galvanometer of the Ayrton and Mather form as long as, say, 120 seconds, by attaching two small brass masses symmetrically to the ends of a horizontal aluminium wire centred on the axis of suspension of the coil (Figure C), though it is not always easy to balance the coils and its weights so exactly that the throws shall be symmetrical on both sides of the zero point and that the instrument shall not be unpleasantly affected by changes of level. Galvanometers of this kind are often useful: several (one with a period of 158 seconds) have been used for years in the Jefferson Laboratory, and Professor A. Zeleny has lately employed a loaded coil galvanometer in his investigations of the properties of condensers. When the case of a d'Arsonval galvanometer is large enough, it is obviously better to load the coil with a disk centred on the axis of suspension than by several small masses, and in the instruments to be described in this paper thin disks with strongly reinforced rims were employed.

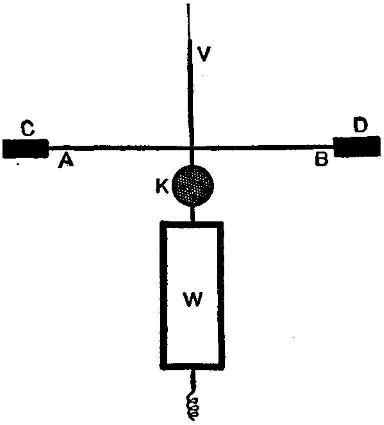

FIGURE C.

The horizontal rod AB is threaded, and the brass masses C, D can be screwed on the rod as far as is necessary. The system must be accurately balanced.

Two loaded coil d'Arsonval galvanometers have been constructed for me by Mr. Thompson. The first (V), shown in Figure 1, Plate 1, is about 76 centimeters high over all, and the gimp by which the coil is hung is 32 centimeters long. The brass disk, which is 11.4 centimeters in diameter, is rigidly attached to the rectangular frame (3 centimeters × 7 centimeters) upon which the copper wire coil is wound, and is accurately perpendicular to the axis of suspension.

After the copper frame constructed for this instrument had proved unsatisfactory, a cast type-metal frame was made to take its place. Of course this frame is not nearly so effective in damping the swings of the coil as a copper frame would be, but, on the other hand, its magnetic moment when it lies between the poles of the magnet of the galvanometer is not large. The insulated copper wire on the frame, however, gives a comparatively high moment to the whole suspended system, and the period of the galvanometer is much shorter — only about 140 seconds — than we supposed it would be with so large and heavy a disk. The binding posts and all the other connections are of copper, and the current is led into and out of the coil by two copper spirals under the disk, so fine that they do not exert any appreciable righting moment when the coil is deflected. The gimp is of steel, just stout enough to hold up securely the loaded coil.

The second galvanometer (W), represented in Figure 2, Plate 1, is about 111 centimeters high over all and 30 centimeters in diameter; the suspension gimp is about 80 centimeters long. It seemed nearly hopeless to attempt to get a sufficiently small righting moment with a hollow coil made of such wire as was to be obtained in the open market, so a coil of the Ayrton and Mather form was made for this instrument. The disk is accurately mounted on a metallic rod upon which the coil is fastened. The disk is built up of a thin sheet of flat aluminium with a brass rim about 24 centimeters in outside diameter and 15 millimeters in width. The current enters and leaves the coil through very fine copper spirals, one above and one below. If No. 44 or No. 46 copper wire be rolled out flat between jewellers' rolls or other similar device the resulting gimp serves to make a spiral which has extremely little torsional rigidity. It is possible to increase the number of field magnets in this instrument at pleasure. The logarithmic decrement of the galvanometer is small, but it has proved to be possible to bring the coil to rest at its zero point without much difficulty. The complete time of swing of the coil is about ten minutes, and the throws due to successive impulses of the same intensity agree with each other very closely indeed. I am much indebted to Mr. Thompson for the great skill and patience he has used in making these instruments. The apparatus was mounted for use by Mr. John Coulson, who has helped me in all the work.

When the coil of a d'Arsonval galvanometer is disturbed from its position of equilibrium and is then allowed to swing under the action of a righting moment proportional to the angular deviation from its original place, the damping effects of the resistance of the air and of the induced currents in the frame and the coil, as they move between the poles of the permanent magnet of the instrument, may usually be accounted for, with an accuracy sufficient for most practical purposes, on the assumption that the motion of the suspended system is hindered at every instant by a force-couple of moment proportional to the angular velocity. Gauss and Weber showed that this hypothesis served to explain very well the motion of the magnets which they used in their measurements at Göttingen, and they put the mathematical theory of motion resisted in this way into the form in which it appears in most treatises on Physics[5] at the present day. When, however, a system swings under strong air damping, the motion sometimes[6] departs pretty widely from the Gaussian law at the beginning, at least, and it is not always safe to apply Gauss's equations to a ballistic galvanometer which has air damping as well as electromagnetic damping until one has found out whether the ratio of successive amplitudes is fairly constant during the whole motion, as Gauss's hypothesis demands. Even in the case of one of Gauss's own magnets, the logarithmic decrement of the amplitudes increased on a certain occasion from 1168 × 10^{-6} to

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[5] Gauss, Resultate des Magnetischen Vereins, 1837; W. Weber, Resultate des Magnetischen Vereins, 1836, 1838; Maassbestimmungen, **2**; Math-phys. Abhandlungen der K. Sächs. Gesellschaft, 1852; Du Bois-Reymond, Monatsberichte der Berl. Acad., 1869, 1870; Chwolson, Bulletin de St. Petersbourg, 1881; Dorn, Ann. der Physik, **17** (1882); **35** (1888); Maxwell, Treatise on Electricity and Magnetism, **2**; G. Wiedemann, Lehre von der Elektricität, **3**; Deprez et d'Arsonval, Comptes Rendus, **94** (1882); Riecke, Abhandlungen der K. Gesellschaft der Wissenschaften zu Göttingen, **30**; Rachniewsky, Lumière Élect., **17** (1885); see also Lumière Élect., **29** (1888); **33** (1889); **45** (1892); Ledeboer, Comptes Rendus **102** (1886); Ayrton, Mather and Sumpner, Philosophical Magazine, **30** (1890); **42** (1896); **46** (1898); Classen, Electrotechnische Zeitschrift, **16** (1895); Sack, Electrotechnische Zeitschrift, **17** (1896); Des Coudres, Zeitschrift für Electrochemie, **3** (1897); Barus, Phys. Rev., **7** (1898); Salomon, Philosophical Magazine, **49** (1900); Robertson, Electrician, **46**, 901–904; **47**, 17–20 (1901); G. Kummell, Zeitschrift für Electrochemie, **7** (1901); Diesselhorst, Ann. der Physik, **9** (1902); Jaeger, Instrumentenkunde, 1903; Stewart, Phys. Rev., **16** (1903); White, Phys. Rev., **19** (1904); **22** (1906); **23** (1906); Shedd, Phys. Rev., **19** (1904); Smith, Phys. Rev., **22** (1906); A. Zeleny, Phys. Rev., **23** (1906); Wenner, Phys. Rev., **22** (1906); **25** (1907).

[6] Peirce, These Proceedings, **44** (1908).

1301×10^{-6} in 422 oscillations. It will appear in the sequel that the two long period galvanometers described in this paper follow the Gaussian law, if not exactly, still quite nearly enough to make it worth while to study their characteristics in the light of the usual theory.

The behavior of a damped ballistic galvanometer through which impulsive currents flow when the suspended system is away from its position of equilibrium and is in motion was first treated thoroughly by Dorn in a paper[7] written before d'Arsonval galvanometers were much used. In this paper Dorn studies the error introduced into observations made by Weber's methods of multiplication and of recoil, when the impulses are not properly timed. He also considers the case where the galvanometer is subjected to the action, not of a series of impulses, but of a continuous current, which lasts with given varying strength for a considerable time, and some of his equations have lately been put into other convenient forms by Diesselhorst. We shall find it desirable to derive from the beginning the special equations which we need in this paper.

The equation of motion of the coil of a d'Arsonval galvanometer, when the resisting moment is proportional to the angular velocity, is of the form

$$K \cdot \frac{d^2\theta}{dt^2} + 2\,a \cdot \frac{d\theta}{dt} + b^2\theta = 0, \tag{1}$$

where K is the moment of inertia of the suspended system about the axis of suspension. If this equation be written in the form

$$\frac{d^2\theta}{dt^2} + 2\,\alpha \cdot \frac{d\theta}{dt} + \beta^2\theta = 0, \tag{2}$$

α may be called the "damping coefficient," and β^2 the "restoring coefficient." It will be convenient to represent $d\theta/dt$ by ω, $(\beta^2 - \alpha^2)$ by ρ^2, and the complete time of swing of the coil by T.

If when $t = 0$, θ and ω have the given values θ' and ω', the general solution of (2) takes the form

$$\theta = e^{-\alpha t}\left[\theta' \cdot \cos \rho t + \frac{\omega' + \alpha\theta'}{\rho} \cdot \sin \rho t\right], \tag{3}$$

whence

$$\omega = e^{-\alpha t}\left[\omega' \cdot \cos \rho t - \frac{\alpha\omega' + \beta^2\theta'}{\rho} \cdot \sin \rho t\right]. \tag{4}$$

If, when the system is at rest in its position of equilibrium, an impulsive angular velocity ω_0 be given to it, and if after t_1 seconds have elapsed and the angular velocity has become ω_1, this velocity be impulsively increased by the amount ω_2, θ and ω are given during the first stage of the motion by the equations

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[7] Dorn, Ann. der Physik, **17** (1882); Diesselhorst, Ann. der Physik, **9** (1902).

$$\theta = \frac{\omega_0}{\rho} \cdot e^{-at} \cdot \sin \rho t, \tag{5}$$

$$\omega = e^{-at} \left[\omega_0 \cdot \cos \rho t - \frac{\alpha\omega_0}{\rho} \cdot \sin \rho t\right], \tag{6}$$

and

$$\theta_1 = \frac{\omega_0}{\rho} \cdot e^{-at_1} \cdot \sin \rho t_1, \tag{7}$$

$$\omega_1 = e^{-at_1} \left[\omega_0 \cdot \cos \rho t_1 - \frac{\alpha\omega_0}{\rho} \cdot \sin \rho t_1\right], \tag{8}$$

$$\rho = 2\,\pi/T,\ \alpha = 2\,\lambda/T,\ \alpha/\rho = \lambda/\pi,\ \beta^2 = \rho^2 + \alpha^2.$$

If, then, for θ' and ω' in (3) and (4) we substitute θ_1 and ω_1 as given by (7) and (8), and for t in (3) and (4) put $(t - t_1)$, in order that the origin of time shall be that of (5) and (6), we shall get

$$\theta = \frac{\omega_0}{\rho}\, e^{-at} \cdot \sin \rho t + \frac{\omega_2}{\rho}\, e^{-a(t-t_1)} \sin \rho\, (t - t_1), \tag{9}$$

$$\omega = \omega_0\, e^{-at} \left[\cos \rho t - \frac{a}{\rho} \cdot \sin \rho t\right]$$

$$+ \omega_2\, e^{-a(t-t_1)} \left[\cos \rho\, (t - t_1) - \frac{a}{\rho} \cdot \sin \rho\, (t - t_1)\right]. \tag{10}$$

Dorn points out that after the second impulse at $t = t_1$, the motion is the same as it would have been if there had been no such impulse, but if when $t = 0$, the values of θ and ω had been

$$-\frac{\omega_2}{\rho} \cdot e^{at_1} \cdot \sin \rho t_1, \tag{11}$$

and

$$\omega_0 + \omega_2 \cdot e^{at_1} \left[\cos \rho t_1 + \frac{a}{\rho} \cdot \sin \rho t_1\right], \tag{12}$$

and shows that the formulas can easily be generalized to fit the case in which there are a number of belated impulsive changes in the angular velocity, instead of one.

In the motion represented by (3) and (4), the angular velocity vanishes at the time t' defined by the equation

$$\tan \rho t' = \frac{\rho \omega'}{\alpha \omega' + \beta^2 \theta'}, \tag{13}$$

and if the first root be used, the amplitude at the first elongation is

$$e^{-\alpha t'} \left[\theta' \cdot \cos \rho t' + \frac{\omega' + \alpha \theta'}{\rho} \cdot \sin \rho t'\right]. \tag{14}$$

For the motion defined by (5), (6), (9), and (10), therefore, the first amplitude can be found by substituting for θ' and ω' in (13) and (14) the values given by (11) and (12). The computation is, however, not very simple, and we shall do well to treat the matter graphically, using equation (9) as the basis of our work.

If we define the function $F(t)$ by the equation

$$F(t) = e^{-\alpha t} \sin \rho t \tag{15}$$

and denote the constants $\frac{\omega_0}{\rho}$, $\frac{\omega_2}{\rho}$ by p and q, (9) may be written in the form

$$\theta = p \cdot F(t) + q \cdot F(t - t_1). \tag{16}$$

For any given galvanometer with a given resistance of the coil circuit α and ρ are definite, easily determined constants, and $F(t)$ is therefore determined. For the galvanometer represented by Figure 1, Plate 1, for instance, ρ is twice α for a coil circuit resistance of about 150 ohms. If we represent ρt by x, ρt_1 by x_1, and the ratio of α to ρ by μ, then

$$\theta = p \cdot e^{-\mu x} \sin x + q \cdot e^{-\mu(x - x_1)} \sin (x - x_1) = p \cdot f(x) + q \cdot f(x - x_1). \tag{17}$$

If then we draw the curves $y = p \cdot f(x)$, $y = q \cdot f(x)$, the ordinates of which are in the constant ratio p/q, and displace the second curve bodily to the right through the distance x_1, the sum of the ordinates of the first curve and the displaced curve will represent θ. For most purposes only the ratio (r) of q to p is important, and in plotting the curves we may make $p = 1$ and q, r.

To illustrate the process just described, let us suppose that when the galvanometer coil is at rest in its position of equilibrium, an impulsive current is sent through it, and after the coil, in response to this impulse, has had about half time enough to reach its elongation, a second impulse is given it half as strong as the first. The general form of the diagram will be much the same whether the damping be very slight or so strong that the motion is just aperiodic, but in Figure D the lines are drawn to scale for the case $a/\rho = 1/2$.

OEUD is the curve $y = e^{-x/2} \cdot \sin x$, which reaches its maximum at M. OPFC is the curve $y = \frac{1}{2} \cdot e^{-x/2} \cdot \sin x$, and AFB is the last curve moved to the right through the distance $x = \rho t_1$. The angular deviation of the coil is given as a function of ρt by the broken curve OEGH, the ordinates of which are the sums of the corresponding ordinates of OEMD and AFB. The maximum of this curve belongs to a point slightly to the left of G and measures the throw of the coil under the circumstances. If both impulses had been given to the coil when it was at rest, the deviation would have been given by the curve OKQGL. The actual throw is about 96 per cent of the throw which would be obtained if both impulses came together at the beginning. The actual values of a and ρ are not needed, and one does not need to know the period of the coil, the actual intensities of the impulses, or anything else, besides λ and r. In this case it is easy to find out by trial in two or three minutes how great the lag OA may be if the difference of the throws is not to be greater than one half per cent, for instance.

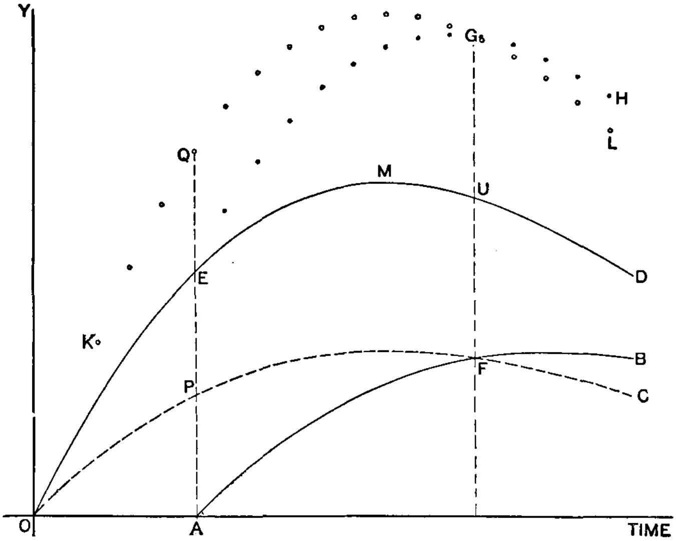

FIGURE D.

If the secondary of an induction coil which has no iron core be connected with the coil of the galvanometer represented by Figure 1, Plate 1, and if when the current I is running steadily through the primary of the induction apparatus the primary circuit be first broken and then, after the coil has had just one quarter enough time to reach its elongation, closed in reverse direction, the angular deviation of the coil will be given as a function of ρt by the curve OBMVJC, Figure E. The ordinates of this curve are the sums of the corresponding ordinates of OBDL and ADK. If the current in the primary circuit of the induction apparatus were suddenly reversed while the galvanometer coil was at rest in its position of equilibrium, the deviation would be given by the curve OWFPH, the ordinates of which are double those of the curve OBDL. The throw with the lag OA is nearly 99 per cent of that when the current is suddenly reversed.

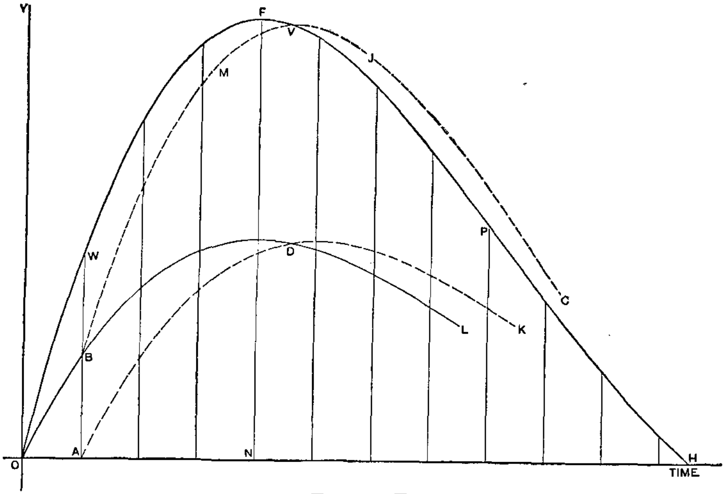

FIGURE E.

This graphical process is especially convenient when the allowable decrease of throw is given and one wishes to find the maximum lag which will not make the throw difference too great. If the lag is given and the throw difference is wanted, this may be found by computation, though the graphical treatment has solid advantages. It is evident that the curve $y = e^{-\lambda x} \cdot \sin x$ serves for a given galvanometer with a given coil circuit for throws of all magnitudes.

It often happens that one has to work with a galvanometer the period of which is rather too short for the purpose in hand, but it is usually possible to determine, in the manner pointed out above, a correction factor to be applied to all throws, which will make the instrument trustworthy.

When a galvanometer is critically damped $\beta^2 = \alpha^2$, $\rho = 0$, and the equation of motion is

$$\frac{d^2\theta}{dt^2} + 2\,\alpha\,\frac{d\theta}{dt} + \alpha^2\theta = 0, \tag{18}$$

and the general solution of this is

$$\theta = (A + Bt)\,e^{-\alpha t}. \tag{19}$$

If when $t = 0$, $\theta = \theta'$, and $\omega = \omega'$;

$$\theta = [\theta' + (\omega' + \alpha\theta')\,t]\,e^{-\alpha t}. \tag{20}$$

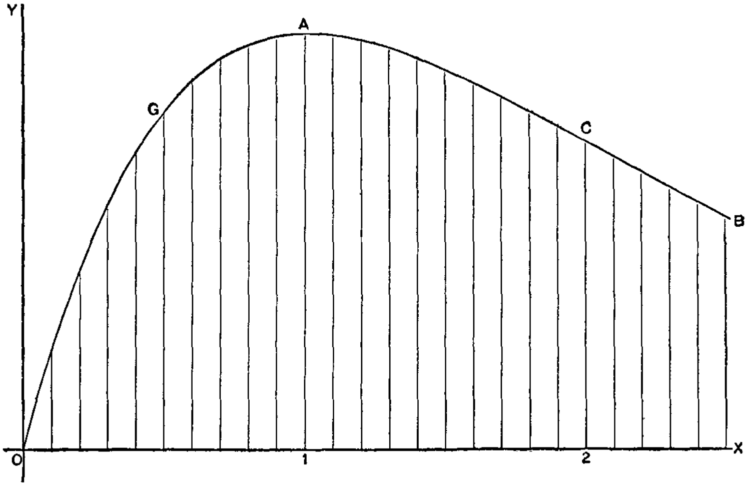

Figure F.

If when the coil is at rest in its position of equilibrium, an impulsive current sent through the instrument gives the coil an initial angular velocity ω_0,

$$\theta = \omega_0 \cdot t \cdot e^{-\alpha t}, \quad \omega = \omega_0 \cdot e^{-\alpha t}\,(1 - \alpha t), \tag{21}$$

and if after this motion has gone on until the time t_1 a second impulse increases the angular velocity by the amount ω_2, then after the second impulse

$$\theta = \omega_0 t\,e^{-\alpha t} + \omega_2\,(t - t_1)\,e^{-\alpha(t-t_1)}. \tag{22}$$

It is possible to give to this equation also a graphical treatment similar to that which we have discussed above for the case where α is less than β. If $\phi(t)$ is defined by the equation

$$\phi(t) = \alpha t\, e^{-\alpha t},$$

$$\theta = \frac{\omega_0}{\alpha} \cdot \phi(t) + \frac{\omega_2}{\alpha} \cdot \phi\,(t - t_1). \tag{23}$$

Figure F shows the form of the curve $y = xe^{-x}$.

In considering the magnitude of the throw of a damped ballistic galvanometer due to a given continuously varying current which flows through the coil for a finite time interval, we shall do well to use Dorn's results in nearly the forms into which they have been put by Diesselhorst in his important paper on the subject.

When the suspended system is at rest in its position of equilibrium, a short-lived current shall flow through the coil and shall have the intensity, I, which is a given function of the time. From the epoch $t = \tau$, I shall have the value zero. The product of the strength of the magnetic field between the poles of the permanent magnet, at the place where the coil is, and the effective area of the turns of the coil shall be denoted by q, so that while the current is flowing, the equation of motion of the coil, for such small angles as are used in mirror instruments, has the form

$$K \cdot \frac{d^2\theta}{dt^2} + 2\,a \cdot \frac{d\theta}{dt} + b^2 \cdot \theta = qI \tag{24}$$

or

$$\frac{d^2\theta}{dt^2} + 2\,\alpha \cdot \frac{d\theta}{dt} + \beta^2 \cdot \theta = \frac{q}{K} \cdot I = \mu \cdot I. \tag{25}$$

If, as before, m and n are the roots of the equation $x^2 + 2\,\alpha x + \beta^2 = 0$, if Q_t represents the whole flux of electricity through the coil from $t = 0$ to $t = t$, and if M_t, N_t represent the ratios to Q_t of the integrals

$$\int_0^t I \cdot e^{-mt} \cdot dt, \qquad \int_0^t I \cdot e^{-nt} \cdot dt, \tag{26}$$

respectively, then the solution of (25) is

$$\theta = \frac{\mu}{m - n}\,[e^{mt} \cdot \int_0^t I \cdot e^{-mt} \cdot dt - e^{nt} \int_0^t I \cdot e^{-nt} \cdot dt] \tag{27}$$

$$= \frac{\mu Q_t}{m - n}\,[M_t \cdot e^{mt} - N_t \cdot e^{nt}]. \tag{28}$$

After the time $t = \tau$, M_t, N_t have the constant values M, N, and Q_t becomes Q, the total amount of electricity carried by the current from $t = 0$ until it ceases to flow at $t = \tau$, so that

$$\theta = \frac{\mu Q}{m - n} \left[M \cdot e^{mt} - N \cdot e^{nt} \right]. \tag{29}$$

If, as is usually true in practice, β is greater than α, ρ is positive, $m = -\alpha + \rho i$, $n = -\alpha - \rho i$, $m/(m-n) = \frac{1}{2} + \alpha i / 2\rho$, $n/(n-m) = \frac{1}{2} - \alpha i / 2\rho$, but the results are, of course, real.

If we determine $d\theta/dt$ from (29) and equate it to zero, we learn that at a time of elongation

$$t = \frac{1}{m-n} \cdot \log \left(\frac{nN}{mM} \right), \tag{30}$$

and this value of t substituted in (29) gives the amplitude at elongation in the form

$$A = \frac{\mu Q}{m-n} \left[\left(\frac{n}{m} \right)^{\frac{m}{m-n}} - \left(\frac{n}{m} \right)^{\frac{n}{m-n}} \right] M^{\frac{n}{n-m}} \cdot N^{\frac{m}{m-n}} \tag{31}$$

$$= C \cdot M^{\frac{n}{n-m}} \cdot N^{\frac{m}{m-n}} \tag{32}$$

where C is a function of the constants of the galvanometer and is independent of the manner in which the whole flux Q of electricity is sent through the circuit. If A_0 denote the amplitude at the first elongation when Q is sent impulsively through the coil at the origin of time,

$$\frac{A}{A_0} = M^{\frac{n}{n-m}} \cdot N^{\frac{m}{m-n}}. \tag{33}$$

If I happens to be given in analytic form as a function of t, it is possible, as Diesselhorst shows in a general case, to obtain a convergent series for A/A_0. For the purposes of this paper, however, where the form of I is shown merely by an oscillograph record, we shall find it desirable if m and n are real, to plot the curves $y = Ie^{-mt}$, $y = Ie^{-nt}$ directly from this record and then to find the values of M and N by mechanical integration.

If β is greater than α, (27) may be written

$$\theta = \frac{\mu}{\rho} e^{-\alpha t} \left[\sin \rho t \cdot \int_0^t I \cdot e^{\alpha t} \cdot \cos \rho t \cdot dt - \cos \rho t \cdot \int_0^t I \cdot e^{\alpha t} \cdot \sin \rho t \cdot dt \right]. \tag{34}$$

$$\text{If } R \cdot Q = \int_0^\tau I \cdot e^{at} \cdot \cos \rho t \cdot dt \text{ and } S \cdot Q = \int_0^\tau I \cdot e^{at} \cdot \sin \rho t \cdot dt, \tag{35}$$

the value of θ after the current has ceased is

$$\theta = \frac{\mu Q e^{-at}}{\rho} [R \cdot \sin \rho t - S \cos \rho t] \tag{36}$$

where Q, R, and S are constants.

At the first elongation,

$$\tan \rho t = \frac{\rho R + a S}{a R - \rho S} \tag{37}$$

or

$$\cos \rho t = \frac{a R - \rho S}{\beta \sqrt{R^2 + S^2}} \qquad \sin \rho t = \frac{\rho R + a S}{\beta \sqrt{R^2 + S^2}}, \tag{38}$$

and if the first root of these equations be substituted for t in (36), it appears that the first elongation is given by the expression

$$A = \frac{\mu Q \sqrt{R^2 + S^2}}{\beta} \cdot e^{-u} \tag{39}$$

where

$$u = \frac{a}{\rho} \cdot \tan^{-1} \frac{\rho R + a S}{a R - \rho S}. \tag{40}$$

If the quantity Q of electricity had been sent impulsively through the galvanometer when the coil was at rest in the position of equilibrium, the throw would have been as (5) shows

$$A_0 = \frac{\mu Q}{\beta} \cdot e^{-v} \tag{41}$$

where $v = \frac{a}{\rho} \cdot \tan^{-1} \frac{\rho}{a}$.

Hence

$$\frac{A}{A_0} = \sqrt{R^2 + S^2} \cdot e^{-(u-v)} = \sqrt{R^2 + S^2} \cdot e^{-w}, \tag{42}$$

where $w = \frac{a}{\rho} \cdot \tan^{-1} \frac{S}{R}$.

If $\frac{1}{2}$ Q were sent impulsively through the circuit at $t = 0$, and $\frac{1}{2}$ Q at $t = \tau$, the values of R and S to be used in (42) would be

$$R = \tfrac{1}{2} (1 + e^{a\tau} \cdot \cos \rho\tau), \quad S = \tfrac{1}{2} e^{a\tau} \cdot \sin \rho\tau. \tag{43}$$

With some of the forms of short period, critically damped d'Arsonval galvanometers commonly used in American laboratories, it is difficult to reverse the current in the primary of an induction apparatus with air core by a large double throw switch so quickly as to avoid a decrease in the throw of the galvanometer coil owing to the lag in the second impulse.

If a current of constant intensity (Q/τ) flowing for the time interval τ conveys a quantity, Q, of electricity through the circuit, the values of R and S are

$$R = \frac{1}{\beta^2\tau}\left[e^{\alpha\tau}(\rho\cdot\sin\rho\tau + \alpha\cdot\cos\rho\tau) - \alpha\right] \tag{44}$$

$$S = \frac{1}{\beta^2\tau}\left[e^{\alpha\tau}(\alpha\sin\rho\tau - \rho\cdot\cos\rho\tau) + \rho\right] \tag{45}$$

$$\sqrt{R^2 + S^2} = \frac{1}{\beta\tau}\sqrt{e^{2\alpha\tau} - 2\,e^{\alpha\tau}\cos\rho\tau + 1}. \tag{46}$$

In the case of a critically damped instrument

$$\theta = \mu e^{-}\;\left[t\int_0^t I\cdot e^{\alpha t}\cdot\,dt - \int_0^t I\cdot t\cdot\;e^{\alpha t}\cdot dt\right].$$

If there were no damping, α would be zero, e^{-w} would be equal to unity, and R and S would satisfy the equations

$$RQ = \int_0^\tau I\cdot\cos\rho t\cdot dt, \qquad SQ = \int_0^\tau I\cdot\sin\rho t\cdot dt.$$

The foregoing theory rests, of course, upon the assumption that the swinging system of a galvanometer meets with a resistance to its motion which may be attributed to a force couple of moment equal at any instant to the product of a fixed constant and the angular velocity which the system then has. It is evident, however, that this condition cannot be exactly fulfilled during the whole motion of the needle or coil of any instrument in which the damping soon brings the swinging system absolutely to rest. In the case of a horizontal bar magnet swinging without sensible friction about a vertical axis through its centre, the ratio of successive half amplitudes usually remains nearly constant for a large portion of the motion, though the actual value of the ratio often depends upon the atmospheric conditions, as Gauss showed. The logarithmic decrement of the oscillations of a magnetic needle swinging in a strong field under the damping action of a mica vane of the usual kind usually diminishes as the amplitudes grow smaller. The same tendency often shows itself in the case of a d'Arsonval galvanometer when the damping, either electromagnetic or atmospheric, is fairly large.

In a galvanometer of any of the common forms in which the restoring moment is due, not to the mutual action of a magnet and the external field, but to torsional forces in a spring or suspending fibre, even though the system comes to rest sensibly at its old position of equilibrium, the swings are often one-sided in a fashion best described, perhaps, with the help of an example or two.

A certain d'Arsonval galvanometer (Y) of the Ayrton and Mather type was connected in series with a rheostat of resistance R and the coil of a small magneto-inductor. The period of the galvanometer coil was dependent of course upon the value of R: when the circuit was broken, its value was about 16.5 seconds. The same flux change in the coil of the inductor might be made over and over again at pleasure by slipping the coil in one direction or the other between two fixed stops. The resistance of the galvanometer and the inductor coil together was about 96.6 ohms. When the galvanometer coil was at rest in its position of equilibrium (scale reading 711), and the value of R was 600 ohms, the inductor coil was moved quickly from one stop to the other and a short series of turning points, 329, 886, 623, 750, 689, were observed. When the inductor coil was slipped back to its original place, the readings were 1095, 534, 799, 672, 733. Using the first set of turning points and the zero 711, the successive half amplitudes were 382, 175, 88, 39, 22, and the ratios of the successive pairs were 2.18, 1.99, 2.26, 1.77. The other set of turning points give the half amplitudes 384, 177, 88, 39, 22, and the ratios, 217, 2.01, 2.26, 177. The half sums of corresponding numbers in the two observed sets are 712, 710, 711, 711, 711, and there is no obvious bias in favor of deflections on one side of the zero point. There was no sensible "set" when the system came to rest, but during the swings there seemed to be a very slight movement of the zero point towards the side of the first excursion, at the end of which the whole angle of twist in the long gimp was only about 1°. When R was made 400 ohms, the time of swing fell from 8.6 seconds to 8.2 seconds, the throw due to the same movement of the inductor coil rose to 483, and the ratios of successive pairs of half amplitudes became 3.16, 2.68, 3.17. When the twist in the gimp per centimeter of its length is made as large as in many of the instruments in common use, the tendency here noted becomes very troublesome, and it is difficult to determine from a short set of throws corresponding to a fairly strong damping what the value of the logarithmic decrement should be.

A certain d'Arsonval galvanometer (X), of the type represented in Figure C, which was formerly in use in the Jefferson Laboratory, had a period of 149 seconds. When the coil was given a deflection corresponding to a scale reading of 14.15 cms., and was then allowed to swing, the ratios of the successive half amplitudes were 1.066, 1.061, 1.067, 1.061, 1.066, 1.060, etc.

TABLE I.

R.	T.	ρ.	λ.	α.	β.
3000	7.00	1.030	1.207	0.396	1.104
4000	5.95	1.056	0.699	0.234	1.082
10000	5.78	1.086	0.398	0.137	1.096
20000	5.74	1.094	0.224	0.128	1.097
Infinity	5.73	1.097	0.032	0.011	1.097

The galvanometers (X, Y) just mentioned, unlike most of those which are usually available in a laboratory, were almost exactly symmetrical in their throws on opposite sides of the zero. In most large instruments in which the coils are wound on open metal frames, there is a slight bias, so that a given flow of electricity sent impulsively through the circuit causes a little larger throw on one side than on the other. Sometimes the bias, when the always small throw is increased by increasing the discharge, changes sign; sometimes levelling the instrument will help a trifle, but usually the lack of symmetry seems to be connected with the magnetism induced in the frame or the coil by the field of the magnet.

Mr. John Coulson, who has studied in the Jefferson Laboratory the characteristics of an excellent short period d'Arsonval galvanometer of the very best make, has found a bias of about 2 per cent in favor of the throws on one side of the zero point. In this instrument there is also the same irregularity in the ratios of successive amplitudes which has been already noticed. For a given impulse, which caused a throw on one side, after which the coil oscillated with decreasing amplitude, the ratios were 2.16, 2.03, 2.15, 2.08, while the same impulse reversed in direction gave the ratios 2.09, 2.12, 2.09, 1.97. These values were persistent and could be obtained over and over, and their differences were quite large enough to disturb a person who is attempting to get an accurate value of the so-called damping coefficient for use in the differential equation.

Some of the constants of this galvanometer as determined by Mr. Coulson are given in Table I.

Such slight departures from symmetry as these seem, however, not to affect in the least the usefulness of a good d'Arsonval galvanometer in measuring quantities of electricity sent through its coil; the mean of throws on opposite sides of the zero point due to a given impulsive discharge remains practically constant, and a good calibration might often be made to serve for a long time, though the instrument should be tested, of course, every time it is used.

In view of the fact that the motion of the coil of a d'Arsonval galvanometer usually deviates somewhat, as we have seen, from the course laid down by the Gaussian theory, we may inquire whether such equations as (14), (33), (42), based on that theory, agree with the results of observations on ordinary instruments. It may be well to say at the outset that, according to my experience, the agreement is wonderfully close.

To support this assertion I may adduce first a simple test made a long time ago upon the galvanometer X mentioned above. If we assume for λ the value 0.0611, the natural logarithm of 1.063, and for T the value 149, it appears that $\alpha = 0.00082$ and $\rho = 0.0422$. The time required for the swing out from the zero to the turning point is then $\frac{1}{\rho}\tan^{-1}\left(\frac{\rho}{\alpha}\right)$ or 36.4 seconds: the return to the zero requires 38.1 seconds. If under these circumstances a given impulse be sent through the coil, and after an interval $\tau = 10$ seconds, another equal impulse, the resulting throw should bear to that which would be caused if both impulses came together at the beginning, the ratio given by (42) when $\alpha\tau = 0.082$, and $\rho\tau = 0.422$, which corresponds to 24.18°. In this case $R = 0.9597$, $S = 0.2064$, $\sqrt{R^2 + S^2} = 0.982$, $\log e^{-w} = 9.9980$, and A/A_0 is about $0.977 +$. Now when a single impulse from an induction apparatus without iron was sent through the coil, and after a delay of ten seconds another equal to the first, the throw as given by a number of readings was 1144, but the reading when both came together was 1170. The ratio of these numbers is 0.978. It is easy to show by a little computation that if the delay were 5 seconds, the ratio of A to A_0 would be 0.994; but if it were 30 seconds, the ratio would be about 0.806.

TABLE II.

R.	∞	500.	300.	200.	100.	50.	30.	0.
T	132.0	132.1	132.4	132.8	133.7	136.0	137.8	143.0
r	1.23	1.34	1.40	1.46	1.68	1.94	2.19	3.12
λ	0.207	0.293	0.336	0.378	0.519	0.663	0.784	1.138
$\alpha = 2\lambda/T$	0.00314	0.00443	0.00508	0.00570	0.00776	0.00975	0.01138	0.01592
$\rho = 2\pi/T$	0.0476	0.0476	0.0474	0.0473	0.0470	0.0462	0.0456	0.0439
α/ρ	0.066	0.093	0.107	0.120	0.165	0.211	0.250	0.362
β	0.0477	0.0477	0.0477	0.0476	0.0476	0.0472	0.0470	0.0468

Table II gives some of the results of several days' study of the characteristics of the galvanometer V. The periodic time, which was determined with the help of a chronograph, is given in round numbers, because slight differences of dampness in the air or of barometric pressure seemed to affect the period somewhat. With small values of R, the ratio (r) of successive half amplitudes was usually somewhat variable in the manner described above, though the values were persistent. Under these circumstances the average value is given. If the instrument followed the Gaussian law exactly, the value of β should be the same throughout.

As this galvanometer was to be used in an important series of magnetic measurements during which it was necessary to determine with accuracy the change of flux in the solid core of a fairly large electromagnet when the exciting current should be reversed in direction, it was desirable to study with some care the effect upon the throw due to the duration of the induced currents. If under all ordinary cases the area beneath the curve in the record of an oscillograph in series with the galvanometer is proportional to the corresponding throw of the galvanometer, one may assume that the performance of the galvanometer will continue to be satisfactory; but this test is not easy to make. It is comparatively easy, however, to give to the galvanometer coil, by aid of a large induction apparatus with air core, such a series of given impulses at given time intervals as shall give all necessary information. In fact the simple device of determining the throw due to two equal impulses separated by the interval τ for a number of different values of τ will usually serve to decide sharply whether or not the galvanometer coil follows the Gaussian law closely enough to make it possible to predict its behavior under ordinary circumstances from the equations proved above. This kind of experiment was made with Galvanometer V: an adjustable commutator, driven through a train of wheels by a motor running very steadily at just under 30 revolutions per second, served to give the impulses at the right time interval apart. A series of

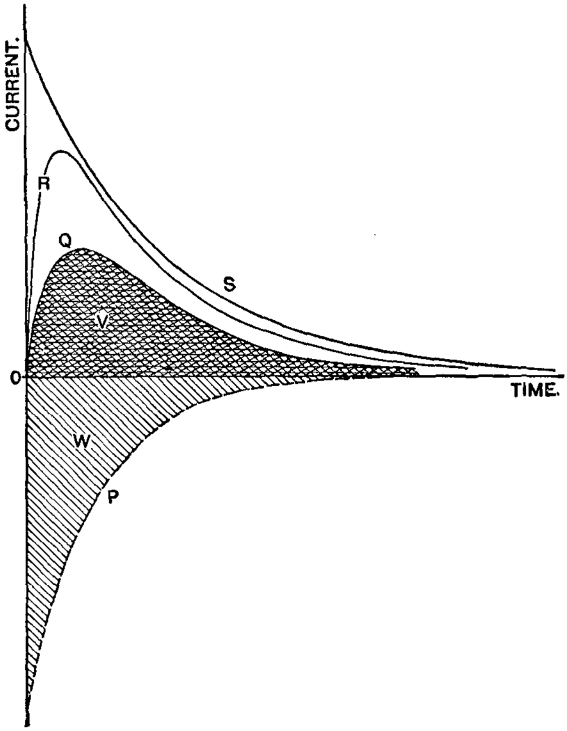

Figure G.

The curves Q, R, S represent for different relative values of the mutual inductance the current induced in the secondary circuit of a certain induction coil without iron, when the primary circuit is suddenly closed.

careful observations showed that the throw was 1471, 1470, 1468, 1464, 1458, 1452, 1444, according as the interval between the impulses was 0, 1, 2, 4, 6, 7, or 8 seconds. At this circuit resistance, $T = 139$, $\rho = 0.0450$, $\alpha = 0.0125$, and if we assume the interval to be 8 seconds, $\alpha\tau = 0.1$, and $\rho\tau = 0.360$, which corresponds to 20.63°. According to (43) under these conditions, $R = 1.017$, $S = 0.195$, $\sqrt{R^2 + S^2} = 1.035$, $\tan^{-1}(S/R) = 0.1891$, and $A/A_0 = 0.982$. That is, the throw when the second impulse follows the first at the interval of eight seconds should theoretically be only 982 thousandths of the throw due to the two impulses coming together. The results of experiment give 1444/1417 or 0.982. This exact coincidence is, of course, a matter of chance.

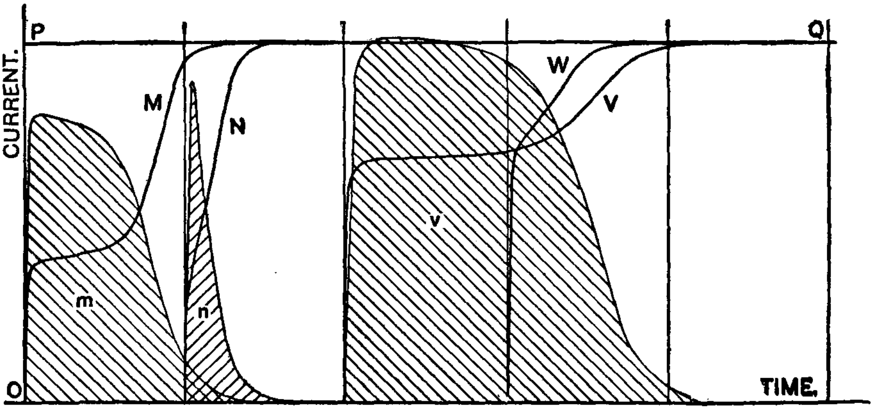

FIGURE H.

When the interval is 4 seconds, $\alpha\tau = 0.05$, $\rho\tau = 0.180$, and $A/A = 0.995$; here again the agreement with observation is exact for $1464/1471 = 0.995$. For an interval of 6 seconds, theory gives for A/A_0 the value 0.992+ and experiment, 0.992—, so that the experimental results, which were obtained long before any computations were made, point to a complete agreement, within the limits of observation, with theory.

With this damping, corresponding to a value for R of about 25 ohms, the time required for the coil to reach its elongation from the zero point is about 28.9 seconds; the return takes 40.6 seconds. When R is 500, the time from the zero is 32.9 seconds, and the time back is 33.1 seconds.

When the circuit of the exciting coil of a large electromagnet is suddenly broken, the induced current in a test coil wound around the core rises very quickly to a maximum value and then falls away gradually: indeed the form of the current is usually much like that in the secondary circuit of an induction coil with air core when the primary current is suddenly interrupted. Such a current is shown by curve P of Figure G, which is drawn for the case $M = L/2$ when the self-inductances of the two circuits are equal. If, after the current in the exciting coil of an electromagnet has been running steadily, its circuit be broken and after a short interval closed again, the induced current in the test coil will be very different according to the direction of the current in the main circuit. If the new direction is the same as that of the current before the break, the new current is called "direct," but if the new direction is opposed to the old, the new current is said to be "reversed." The curves M, N in Figure H, which are reproduced to scale from the records of an oscillograph, show the manners of growth of reversed and direct currents, respectively, in the exciting circuit of a certain electromagnet; and the boundaries of the shaded portions of the diagram show the forms of the induced currents. The shaded areas give the

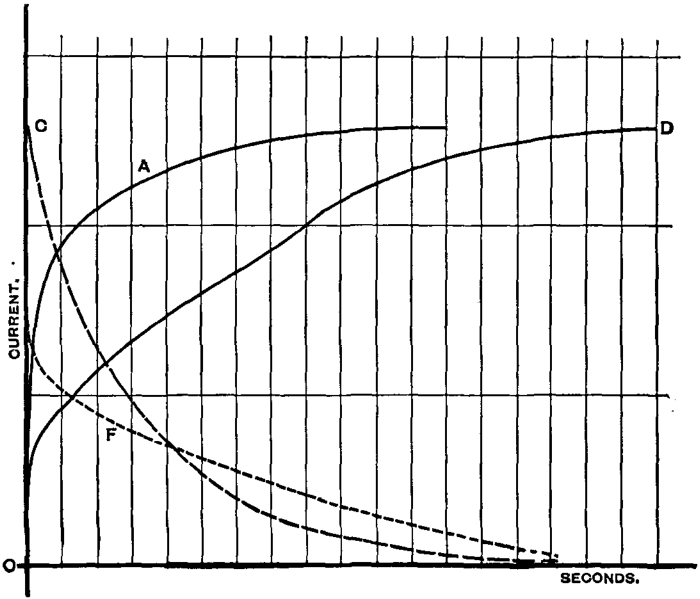

FIGURE I.

whole transfer of electricity in the induced currents in the two cases. Besides the exciting coil, this magnet had another similar coil wound about the core. Curves V and W show the growth of reversed and direct currents in the exciting circuit when the last named coil was closed on itself, and the currents induced in it hindered the establishment of the

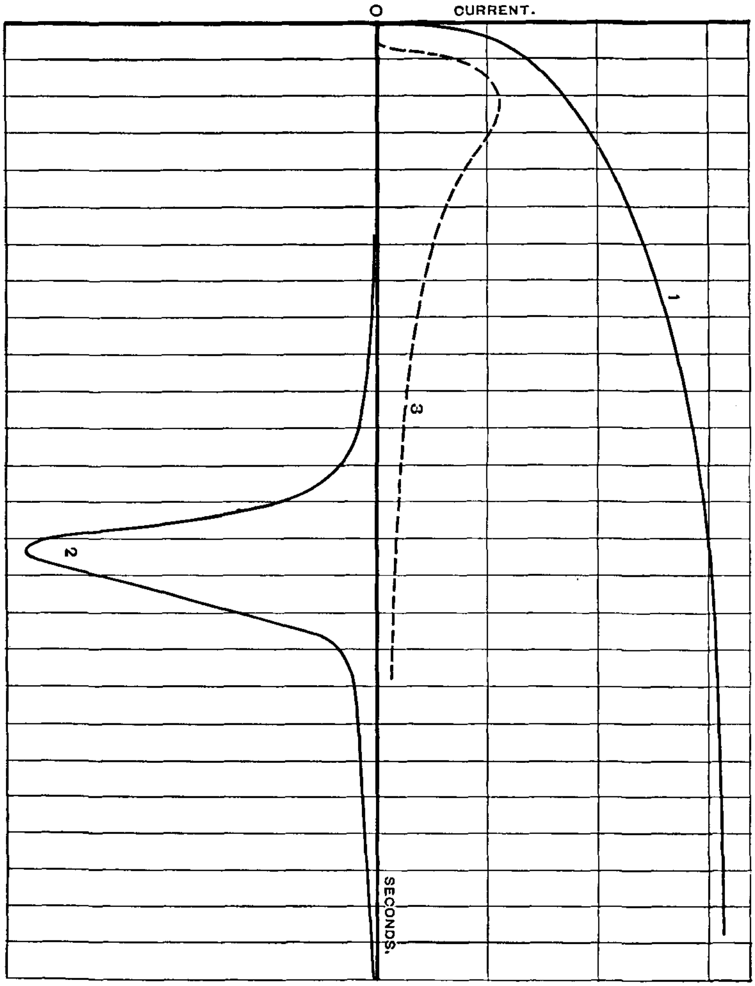

FIGURE J.

main current. The scale of the oscillograph in the secondary circuit was different from that used before, but the general shape of the induced current is shown by the boundary of the shaded area v. Curves C and F of Figure I show the forms of induced currents in the testing coil in the case of a very large magnet the cross section of the solid core of which had an area of about 500 square centimeters. A and D show the corresponding currents in the main circuit: in the first case the generator was a battery of 40 storage cells, and a considerable amount of extra resistance was used in the circuit; in the second case the same final current was caused by a battery of 10 cells, and very little extra resistance was needed. This particular engraving, which was made by the "Wax Process," does not reproduce the original exactly, for the upper portions of A and D are here too nearly horizontal.

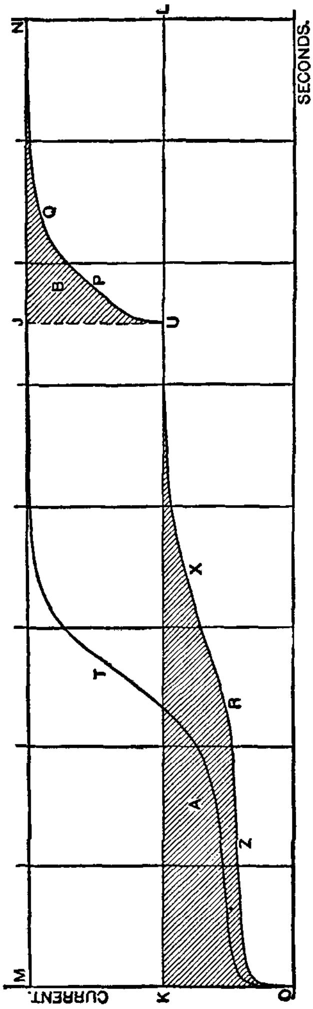

FIGURE K.

A very uncommon form of secondary current is shown in Figure J. Curve 1 represents the form of the main current of a very large electromagnet with massive core. At the axis of a portion of the core was a longitudinal hole about an inch in diameter, and in this hole was inserted an iron rod around which a layer of insulated wire was wound to serve as a test coil. Curve 2 shows the form of the induced current in this coil when the main circuit was closed; the dotted curve gives the form of the induced current when the main circuit was suddenly broken. The crest of the curve 2 does not come until fourteen seconds after the main current starts.

Figure K shows the manner of growth of a current of final intensity 2.3 amperes, under a voltage of perhaps 60, in a coil of 1388 turns about the core of the magnet depicted in Figure A. The curve OTJN is a copy of the record of an oscillograph in the circuit when the electromotive force was suddenly applied at $t = 0$. The area between this curve and its asymptote up to any value of the time represents the whole change of the flux of magnetic induction through the coil, and the difference between the ordinate of the asymptote and that of the curve is proportional to the instantaneous rate of change of this flux, and, therefore, to the induced electromotive force in a test loop passed around the core. The general form of the induced current in such a secondary circuit might be seen by looking at the curve just mentioned upside down and through the paper. In this case the induced current would practically come to an end in about five and one half seconds. The line OZRXUPQN shows the growth of the main current when there was an extra non-inductively wound resistance in the circuit which was suddenly shunted out after about five and one half seconds. Here, again, the general shape of the induced current in the secondary circuit might be seen by looking at this line upside down, from behind. The intensity of the induced current was inappreciable after about eight seconds.

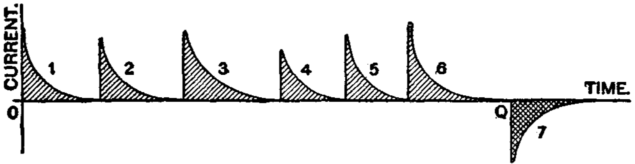

FIGURE L.

A portion of the record of an oscillograph in the circuit of a secondary coil wound on the core of an electromagnet when the current in the exciting coil is made to change by sudden steps in the determination of a hysteresis cycle.

Figure L shows the general shape of the induced currents in the circuit of a test coil of a few turns wound on the core of an electromagnet when the current in the exciting circuit is made to grow by shunting out a part of the resistance of this circuit by steps. If the currents, up to the time OQ were sent through the coil of a long period ballistic galvanometer, the resulting throw would not fall so much below the throw due to the whole quantity of electricity carried by the currents, sent instantaneously through the galvanometer at the origin of time, as would the throw due to a steady current lasting for the time OQ and carrying the same total amount.

The examples already given will serve well enough to show what is required of a galvanometer which shall measure accurately the whole quantity of electricity which flows in the test coil. Of course, the induced current may last with an extremely feeble intensity for a long time, but in any practical case it is easy to set a limit of time after which no sensible flow will occur.

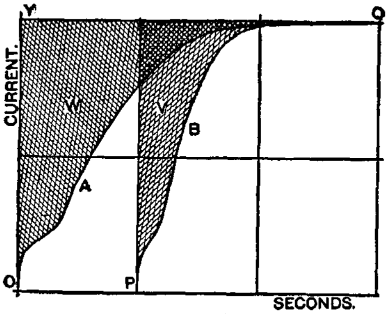

Figure M.

Figure M shows two reverse current curves for a toroidal magnet. The final strength of the current was the same in both cases, but the applied electromotive force was twice as great in the case of the curve B as in the case of the curve A.

If A_0 is the throw which would be caused by an instantaneous discharge of Q units of electricity through a galvanometer at the beginning of motion, A' the throw caused by an instantaneous discharge of $\frac{1}{2}$ Q units at the beginning and another discharge of $\frac{1}{2}$ Q units seconds later, and A'' the throw due to a steady current of Q/τ units intensity lasting from $t = 0$ to $t = \tau$, then A' is less than A'', and this in turn is less than A. Occasionally one encounters an induction current which has a form much like that indicated in Figure N by the curve KLG, and we shall find it interesting to determine the ratio A''/A for one or two practical cases. It is well to notice that the *second member of* (42) *depends only upon the ratios* $\lambda = \alpha/\rho$ and

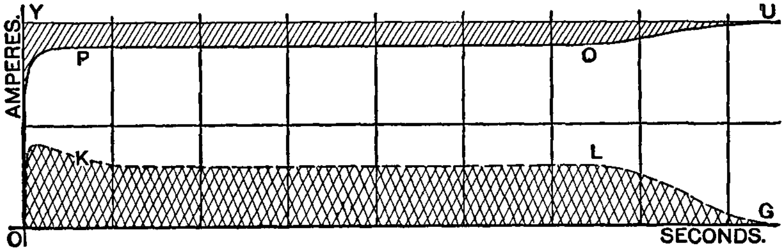

Figure N.

$\delta = \tau/T$, *and not at all upon the other constants of the instrument;* for if we write $z = \rho t$ and $I = f(t) = \phi(z)$, we shall find that

$$R = \frac{\int_0^{2\pi\delta} \phi(z) \cdot e^{\lambda z} \cdot \cos z \cdot dz}{\int_0^{2\pi\delta} \phi(z) \cdot dz}, \quad S = \frac{\int_0^{2\pi\delta} \phi(z) \cdot e_{\lambda z} \cdot \sin z \cdot dz}{\int_0^{2\pi\delta} \phi(z) \cdot dz}, \tag{47}$$

and these expressions involve λ and δ but are independent of the sensitiveness of the galvanometer and of its time of swing.

It is possible to show from equations (44) and (45), after some computation, that for the case of the galvanometer V, for which we may take $\alpha = 0.0125$, $\rho = 0.0450$; $A''/A_0 = 0.994$, or 0.998, according as τ is 8 seconds or 4 seconds. It is well to recall the fact mentioned above, that $A'/A = 0.982$ or 0.995, according as $\tau = 8$ seconds or 4 seconds.

Perhaps most of the induction currents which one meets in making magnetic measurements have forms similar to those of the curves S or P in Figure G, and it is worth while to compute the value of the ratio A/A_0 on the supposition that the current flows from $t = 0$ to $t = \tau$ with the intensity $I = k\,(\tau - t)$ where it is clear that $k = 2Q/\tau^2$.

Since

$$\int x \cdot e^{\lambda x} \cdot \sin x \cdot dx = \frac{e^{\lambda x}}{(1+\lambda^2)^2}\left[(\lambda \cdot \sin x - \cos x)(\lambda^2 x + x - \lambda) + (\sin x + \lambda \cdot \cos x)\right], \quad (48)$$

and

$$\int x \cdot e^{\lambda x} \cdot \cos x \cdot dx = \frac{e^{\lambda x}}{(1+\lambda^2)^2}\left[(\sin x + \lambda \cdot \cos x)(\lambda^2 x + x - \lambda) - (\lambda \cdot \sin x - \cos x)\right], \quad (49)$$

it is not difficult to prove that when $I = k\,(\tau - t)$,

$$R = \frac{2}{\beta^4 \cdot \tau^2}\left[\alpha \cdot e^{\alpha\tau}(\rho \cdot \sin \rho\tau + \alpha \cdot \cos \rho\tau) + \rho \cdot e^{\alpha\tau}(\alpha \cdot \sin \rho\tau - \rho \cdot \cos \rho\tau) + \rho^2 - \alpha^2 - \alpha\beta^2\tau\right], \quad (50)$$

$$S = \frac{2}{\beta^4 \cdot \tau^2}\left[\alpha \cdot e^{\alpha\tau}(\alpha \cdot \sin \rho\tau - \rho \cdot \cos \rho\tau) - \rho \cdot e^{\alpha\tau}(\rho \cdot \sin \rho\tau + \alpha \cdot \cos \rho\tau) + \beta^2\rho\tau + 2\,\alpha\rho\right]. \quad (51)$$

These formulas are not very well adapted for easy computation, and in many practical cases in which the quantities in the brackets are very small and the coefficient $2/\beta^4\tau^2$ very large it is desirable to use five or six place logarithms in the work. As an illustration of the use of these equations we may consider the instance of the galvanometer V through which a current of the form $I = k\,(\tau - t)$ shall flow for 8 seconds. Here $\alpha = 0.0125$, $\rho = 0.0450$, $\beta^2 = 0.0021812$, $2/\beta^4\tau^2 = 6568.39$, $R = 1.04723$, $S = 0.12545$, and $A/A_0 = 0.9974$. The throw due to this current is the same within about one quarter of one per cent as if the whole amount of electricity conveyed by the current had been sent instantaneously through the coil at the time $t = 0$. For a galvanometer of the same period with practically no damping the value of A/A_0 under the circumstance just mentioned would be about 0.9964. A current of the form $I = k\,(\tau - t)$ and lasting for 34 seconds would, in the case of the galvanometer W, give a throw within about one third of one per cent the same as an impulsive discharge of the same total amount would cause if sent through the coil at the origin of the motion.

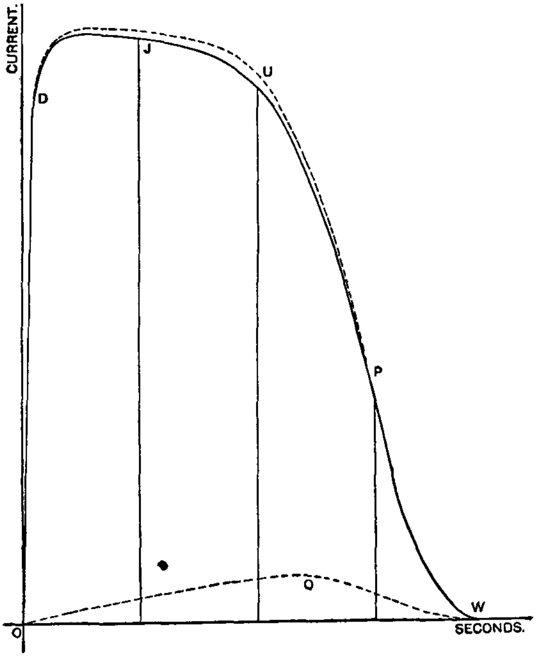

FIGURE O.

For a current of the general shape of S (Figure G) regarded as stopping at the time $t = \tau$, the ratio of A/A_0 would be much more nearly unity than for a current of the form $I = k\,(\tau - t)$.

If as in the case of an induction coil without iron, when the primary circuit is suddenly broken, I is of the form $I_0 \cdot e^{-kt}$, and if we write $g = a - k$,

$$RQ = \frac{I_0}{g^2 + \rho^2}\left[e^{g\tau}\left(\rho \cdot \sin \rho\tau + g \cdot \cos \rho\tau\right) - g\right], \qquad (52)$$

$$SQ = \frac{I_0}{g^2 + \rho^2}[e^{g\tau}(g \cdot \sin \rho\tau - \rho \cdot \cos \rho\tau) - \rho], \tag{53}$$

$$Q = \frac{I_0}{k}(1 - e^{-k\tau}). \tag{54}$$

If $g = -\frac{1}{2}$, $\alpha = 0.0125$, and $\rho = 0.0450$; the value of A/A_0 will be 0.989, if the current flows until the needle reaches its elongation, say for 29 seconds.

When the shape of an induced current which is to pass through a ballistic galvanometer of long period is not analytically simple, it is always possible to determine by mechanical integration, with sufficient accuracy, the ratio of the throw caused by the current to the throw which the same total quantity of electricity sent instantaneously through the instrument would give. As an example, we may consider the form of current represented by the curve ODJPW of Figure O, which is a fairly close copy of an oscillogram. If we assume that the duration of the current is to be 4 seconds and that galvanometer V is to be used, so damped that

$$\alpha = 0.0125, \qquad \rho = 0.0450,$$

it is easy to measure a number of ordinates of the current curve, multiply each by the corresponding values of $e^{at} \cdot \cos pt$, $e^{at} \cdot \sin pt$, and thus compute the ordinates of the curves OUPW and OQW. The areas under these curves obtained by a good planimeter represent RQ and SQ of (35) and (42), and the area under the current curve gives Q on the same scale. An actual trial would show that A falls below A_0 by about one seventh of one per cent. If the galvanometer W were used, it would be quite impossible to detect the difference between A_0 and A, even if the duration of the current, of the form shown, were as much as 16 seconds.

The galvanometers V and W are to be used in making determinations by the "Isthmus Method" of the ultimate values of the intensity of magnetization in a large number of specimens of magnetic metals, in cases where it is necessary to reverse the direction of the exciting currents. When a rather small yoke which weighs about 300 kilograms is used under a fairly high voltage, V works very well: the whole duration of the induced current is practically less than 5 seconds, and the intensity falls off rapidly after the first, so that the difference between A and A_0 is wholly inappreciable. For very high values of the induction a solid yoke of the form shown in Figure B is to be employed. In this case the smallest cross section of the core has an area of 450 square centimeters, and it is not possible sensibly to reverse an excitation of say one hundred and fifty thousand ampere turns about this core in less than about 30 seconds under any practicable voltage. Of course the process is not completed even in this time, but the amount of electricity carried by the induced current after 30 seconds can be made relatively very small. Indeed for the shape of current practically encountered with this apparatus, the duration of the flow might be 60 seconds without causing a decrease of more than a fraction of one per cent in the throw of the galvanometer W.

I wish to express my obligation to the Trustees of the Bache Fund of the National Academy of Sciences for the loan of apparatus used in studying for this paper some of the induction current diagrams.

The Jefferson Laboratory,
Cambridge, Mass.

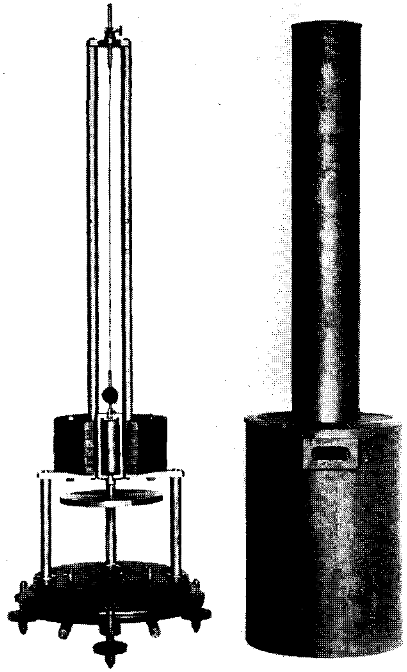

FIGURE 1.

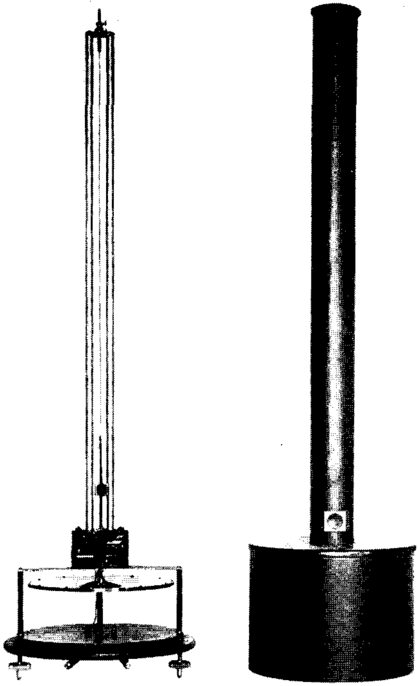

FIGURE 2.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

CRYSTAL RECTIFIERS FOR ELECTRIC CURRENTS AND ELECTRIC OSCILLATIONS.

II. — *CARBORUNDUM, MOLYBDENITE, ANATASE, BROOKITE.*

BY GEORGE W. PIERCE.

WITH A PLATE.

INVESTIGATIONS ON LIGHT AND HEAT MADE AND PUBLISHED, WHOLLY OR IN PART, WITH APPROPRIATION FROM THE RUMFORD FUND.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

CRYSTAL RECTIFIERS FOR ELECTRIC CURRENTS AND ELECTRIC OSCILLATIONS.

II. CARBORUNDUM, MOLYBDENITE, ANATASE, BROOKITE.

By George W. Pierce.

Presented December 9, 1908. Received December 22, 1908.

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Introduction.

Concerning Part I. — Carborundum had been found by General Dunwoody[1] to be capable of acting as a receiver for the electric waves of wireless telegraphy. Having learned of this property of carborundum, it occurred to the writer that a further study of the electrical behavior of this substance would be interesting. In the course of this study, an account of which has been published in the Physical Review[2] for July, 1907, it was discovered that when a piece of carborundum is placed in a clamp between contact electrodes, the heterogeneous conductor consisting of the carborundum and the electrodes permits the passage of a greater current in one direction than in the reverse direction under the same applied voltage. The device can be used as a rectifier for small alternating currents and oscillations. The phenomenon is very striking. For example, with one specimen under an electromotive force of 30 volts the current in one direction is 4000 times as great as the current in the opposite direction under the same external voltage.

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[1] Dunwoody: U. S. Patent, No. 837,616, issued Dec. 4, 1906.

Although the rectified current is not large (in the case just cited, 3 milliamperes in one direction and .00075 milliamperes in the opposite direction) such a rectifier, being constructed entirely of solid parts, possesses sufficient permanence and constancy to permit of many useful applications, where the detection and measurement of small alternating currents is required. As an example of such applications details are given in Part I of the employment of the rectifier in the construction of an alternating current voltmeter operable with an extremely small consumption of energy.

Questions arising in Connection with the Phenomenon. — Many questions of theoretical interest arise in connection with the phenomenon. Is the action localized at the surface of contact between the crystal and the metallic electrode? Is the action due to electrolytic polarization? Is the action thermoelectric, conditioned on unequal heating of the two electrode contacts? If the phenomenon is novel, how is it related to the hitherto studied properties of conductors?

In the experiments on carborundum performed by the writer the investigation of these questions met with limitations on account of the form of occurrence of the carborundum in discrete masses to which electrodes could not be rigidly attached; so that the conditions at the electrodes could not be widely varied. However, by increasing the pressure of the electrodes against the carborundum beyond a certain limit, and by cathodically platinizing the surfaces of the carborundum at both the contact areas, the rectification, though not entirely eliminated, was rendered very imperfect; that is to say, the ratio of the strength of the current in one direction to that in the reverse direction approached unity. On the other hand, platinizing one only of the surfaces of contact, while the other surface was left unplatinized, generally rendered the rectification more nearly perfect. This fact indicated that the seat of the action was the area of contact with the electrodes, and that the action at the two contacts were in opposition to each other, so that when the action at one of the contacts was reduced by platinizing, the rectification at the other contact appeared more pronounced.

---

[2] Pierce: Physical Review, **25**, 31-60 (1907).

These characteristics of the phenomenon are consistent with the view that *the rectification is conditioned on the localization of the energy of the circuit at the high resistance boundary between the two different classes of conductors*, the crystal and the metallic electrode.

Now such a localization of energy at the boundary of the two conductors is favorable to the production of electrolytic polarization, if we may have electrolytic polarization in solids, and is also favorable to the production of a thermoelectromotive force, either of which might result in rectification.

Nevertheless, in Part I, a number of experiments are described which were taken to indicate that neither electrolysis nor thermoelectricity plays an important part in the phenomenon.

On the question of electrolysis, the following experiment, performed since the publication of Part I, has a bearing.

Experiment showing Permanence of the Carborundum Rectifier. — In confirmation of the absence of electrolytic polarization, a durability test of the rectifier has later been made as follows: A crystal of carborundum enclosed in a glass tube with a few drops of oil[3] and held between brass electrodes, one of which was under tension of a spiral spring, was kept under almost daily observation[4] from October 23, 1907, until March 18, 1908. During this time more than 1200 measurements were made of the direct current obtained through the crystal under different direct and alternating voltages. The rectifier was kept in a thermostat and subjected to various long periods of heating and cooling ranging from 0° to 80° C. Notwithstanding the long continued exposure of the crystal to large changes of temperature, and notwithstanding the frequent loading and occasional overloading of the rectifier with current, it was found at the end of the series that the values of the direct current obtained from the crystal under a given applied alternating voltage over a range of current from 4 to 400 microamperes (direct) and a range of voltage between 1.5 and 6 volts (alternating) did not differ from the corresponding values at the beginning of the series by an amount exceeding the limit of accuracy of the experiment, which was about $\frac{1}{3}$ of 1 per cent.

---

[3] The oil served to prevent accumulation of moisture.

[4] This series of measurements was carried out by Mr. K. S. Johnson, to whom the writer wishes to express his sincere thanks. The experiment was finally discontinued on account of the accidental melting of the cement holding in the ends of the tube.

This experiment shows that if there is any kind of electrolytic action, it must be of such a character as not to change the nature of the electrodes or of the crystal.

On the Question of a Possible Thermoelectric Origin of the Phenomenon. — It is apparent that the disposition of the carborundum for the best rectification is exactly the most favorable disposition for the development of a thermoelectric voltage at the high resistance contact. This voltage, being always in one direction, by superposition on an alternating current through the crystal, might give rise to a unilateral cycle through the crystal. In Part I, several experiments are described which present evidence adverse to this explanation, and the opinion is expressed that "heat is practically a negligible factor in the process."

However, since it is very important to exclude the possibility of bringing the experiments into consistent relation with thermoelectricity before admitting that we are dealing with a new phenomenon, the question of the applicability of the thermoelectric explanation is taken up anew in the present account.

Extension of the Experiments to Other Crystals. — Prior to the publication of Part I, the writer had found a number of other crystals showing the rectifying property similar to carborundum. These have now been under investigation for a period of more than a year, and though the work is by no means completed, it is thought that an account of the experiments as far as they have gone may be of interest. The present account deals with the rectifying action of Anatase, Brookite, and Molybdenite in contact with a metallic electrode.

ANATASE AND BROOKITE.

Anatase. — Anatase, an octahedral crystal of oxide of titanium with the chemical formula TiO_2, was found to rectify quite markedly when placed in a clamp, under a contact pressure of 1 to 3 kilograms. Current-voltage curves[5] of anatase, with a diagram of the disposition of the crystal in the experiment, are given in Figure 1. The upper curve was obtained when the current was through the crystal in one direction, the lower curve was with the current in the opposite direction, as indicated by the arrows. The contact pressure in this experiment was 2 kilograms. These curves have the same general form as those obtained in the experiments on carborundum. By a comparison with Part I, it is seen, however, that the anatase gives much larger currents with a small applied voltage than does the carborundum. This characterizes the anatase as a much more sensitive rectifier for small alternating voltages and as a much more sensitive detector for electric waves than is the carborundum.

---

[5] The current-voltage curves were drawn in Part I with positive co-ordinates when the current was in one direction and negative co-ordinates when the current was in the opposite direction. In order to economize space in the present account, both the positive and negative currents are drawn in the same co-ordinate quadrant. This has the advantage of permitting an easier comparison.

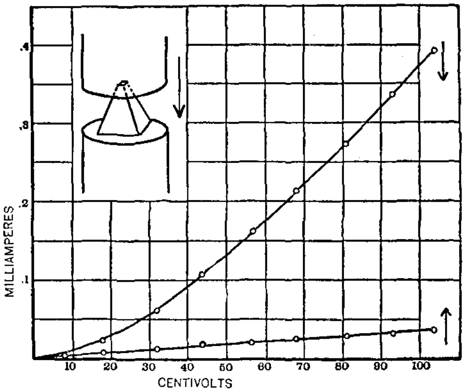

Figure 1. Current-voltage curves for anatase, with direct current.

Brookite. — This is another crystal form of TiO_2, which was found to serve as a rectifier of small alternating currents with about the same sensitiveness as anatase. Although a considerable amount of time was spent in experimenting with anatase and brookite, these substances, occurring like carborundum in discrete pieces to which terminals could not be attached, did not serve to throw much light on the phenomenon. Numerical data in regard to them are, therefore, omitted.

Molybdenite.

One of the most sensitive of the rectifiers thus far investigated makes use of molybdenite as a member.[6] Molybdenite, with the chemical formula MoS_2, is a mineral occurring in nature in the form of tabular hexagonal prisms with eminent cleavage parallel to the base of the prism. The cleavage of the crystal resembles that of mica, and thin sheets of the mineral several square centimeters in area may be scaled off from a large crystal of molybdenite. These sheets have a metallic lustre and look not unlike sheets of lead foil. They can be readily electroplated with copper, so that connecting wires may be soldered to them. This property, together with the thinness of the sheets and the ease with which the thermoelectric property of the substance may be studied, admirably adapts it to the present experiments.

---

[6] See also G. W. Pierce: A simple Method of Measuring the Intensity of Sound, These proceedings, **43,** 377 (Feb., 1908), in which the Molybdenite Rectifier was employed.

The Molybdenite Rectifier. — The rectifying action of the molybdenite was first obtained with a thin, flat specimen of the mineral held between flat contact electrodes in a clamp of which the two jaws were insulated from one another. With this form of mounting the molybdenite also acts as a receiver for electric waves with or without a battery in the local circuit.

It was soon found, however, that the apparatus was more sensitive as a receiver for electric waves and as a rectifier, when one of the contacts between the molybdenite and the electrode had a high resistance. A form of mounting in which this is attained is shown in section in Figure 2. T is a threaded brass post on the top of which is placed a disc of mica, N. On top of the mica is a thin circular disc of the molybdenite M, with an area of about 1 square centimeter, leaving a projection of the mica beyond the periphery of the molybdenite. A hollow cap, D, threaded inside and having a conical hole at the top, is screwed down on the post T so as to clamp the molybdenite between the mica disc[7] and the annular shoulder of the cap, with the upper surface of the molybdenite exposed above. At the free surface of the molybdenite contact is made with the metallic rod P.[8]

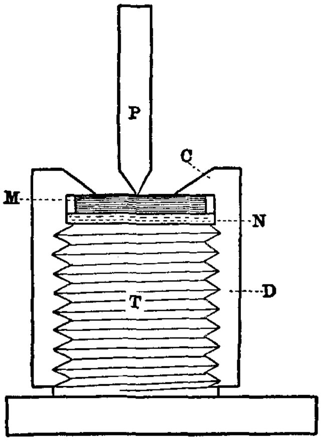

FIGURE 2. Holder for molybdenite.

The rod P was either supported unadjustably, as in the author's experiments on sound,[9] or it was mounted in a manner to permit of ready adjustment, as is shown in Figure 3. The clamp K containing the molybdenite is metallically connected with the binding post H (Figure 3). Another binding post is attached to the metallic block A, on top of which is supported a stout spring B. Through a hole in B provided with a set screw, the rod P is allowed to drop down into contact with K. The set screw is then tightened against P, and the final adjustment is made by the slow motion screw S. The apparatus is connected in circuit by means of the binding posts, so that the current of the circuit is made to enter the molybdenite through the contact area between P and the molybdenite and leave by way of the contact between the molybdenite and the cap C, or the reverse. It is found that a larger current flows in one direction than in the reverse direction for a given applied electromotive force.

---

[7] The purpose of the mica disc under the molybdenite is to confine the current as much as possible to the upper layer of the molybdenite. This was done so as not to complicate the phenomenon by conduction across the laminae of the substance, and also so that when the detector is immersed in oil in some of the later experiments, the oil shall have free play over the conducting surface and over the contacts, and serve the better to avoid possible changes of temperature of the essential parts of the apparatus.

[8] In the diagrams of Figure 2 and Figure 3 the lower end of the rod P is shown pointed. It is found, however, that the end of the rod P may be blunt or even flat with an area as great as 4 sq. mm. without much loss of sensitiveness of the instrument as a receiver for electric waves or as a rectifier.

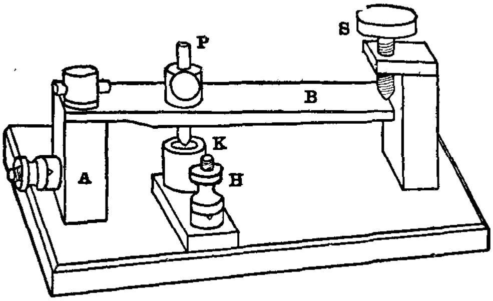

FIGURE 3. Mounting of molybdenite.

Current-Voltage Characteristic of the Molybdenite Rectifier. — A large number of current-voltage curves of the molybdenite rectifier with the form of mounting shown in Figure 3 have been taken both with direct and alternating applied voltages. Two sets of these curves, with the corresponding tables, are here given. In taking the observations of Figure 4, Table I, the rectifier was submerged in a constant temperature oil bath. The oil was rapidly stirred and had free access to the surface of the molybdenite and to the point contact between the molybdenite and the copper rod. A steady voltage was applied to the terminals of the rectifier, and the current through the crystal was measured. The voltage was then reversed and the current again measured. The process was repeated with various values of the voltage. These values thus obtained in the oil bath were found to be the same as the corresponding values when the rectifier was in air at the same temperature. That is, the presence of the oil about the rectifying contact did not materially affect the process.

The values of Table I are plotted in the curves A and B of Figure 4. A is the curve obtained when the current was sent from the copper to the molybdenite, B the corresponding curve when the current was sent from the molybdenite to the copper. These curves resemble those obtained in Part I with carborundum. The molybdenite rectifier is, however, seen to operate with a much smaller resistance than the carborundum rectifier. This makes the molybdenite rectifier applicable to use with smaller voltages than the carborundum, consequently the molybdenite rectifier is a more sensitive detector for electric waves or for small alternating voltages than the carborundum rectifier. In fact, the molybdenite rectifier, as a detector for electric waves, is, so far as

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[9] Loc. cit.

TABLE I.

Current-Voltage Values for the Molybdenite Rectifier.

Current from Copper to Molybdenite.		Current from Molybdenite to Copper.	
Volts.	Microamperes.	Volts.	Microamperes.
.0407	0.012	0.082	0.020
.0815	0.025	0.203	0.038
.122	0.043	0.363	0.058
.163	0.068	0.651	0.090
.203	0.102	0.815	0.114
.244	0.147	1.140	0.185
.285	0.202	1.300	0.261
.326	0.262	1.465	0.375
.363	0.337	1.630	0.534
.407	0.415	1.790	0.732
.447	0.504	1.96	0.947
.488	0.600	2.03	1.056
.529	0.700	2.12	1.180
.570	0.812	2.18	1.306
.651	1.062		
.710	1.306		

the writer can judge, equal in sensitiveness with the most sensitive detectors heretofore employed in wireless telegraphy. Also the molybdenite rectifier, giving comparatively large values of direct current for small values of applied alternating voltage, affords a sensitive method of measuring the small alternating voltages arising in telephony and in experiments on sound. Application of the rectifier to the measurement of sound has been made in a paper entitled "A Simple Method of Measuring the Intensity of Sound." [10]

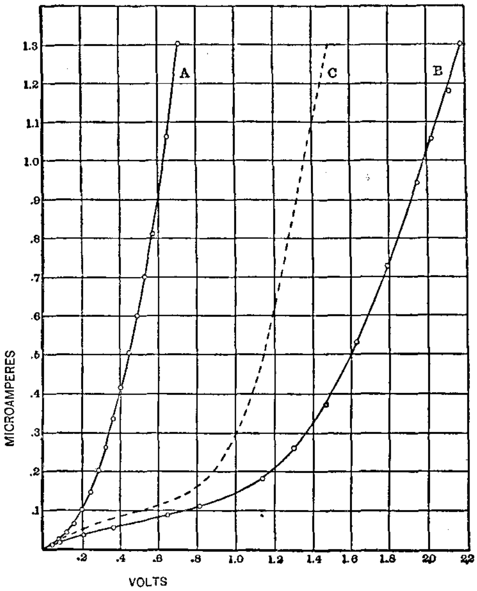

FIGURE 4. Current-voltage curves of the molybdenite rectifier. A, current from copper to molybdenite; B, current from molybdenite to copper; C, excess voltage.

Referring again to Figure 4, attention is called to the dotted curve C. This curve is calculated from the curves A and B by subtraction of corresponding abscissas. The curve C, therefore, represents the excess of voltage required to force the current from the molybdenite to the copper above that required to send an equal current in the opposite direction. The numerical values for curve C are given in Table II.

The current-voltage values for the molybdenite rectifier differ for different specimens and for different adjustments of the same specimen. The results of another set of experiments, in which larger values of the current and voltage are employed, are given in Table III. These values were obtained with a specimen mounted somewhat differently from the mounting of Figure 3, in that, in order to eliminate any possible uncertainty from the use of the clamp holder K (Figure 3), the tight contact terminal was soldered to a copper-plated area on the molybdenite, and the sheet of molybdenite with its soldered terminal was held down upon a block of wood by means of a mica covering screwed to the block. A hole through the mica covering admitted the contact rod P.

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[10] G. W. Pierce: These proceedings, **43**, 337 (Feb., 1908).

The values recorded in Table III are plotted in Figure 5. By a reference to the curves or to the table it is seen that the rectification at 10 milliamperes is practically perfect, since the current from the molybdenite at 2.2 volts is 10 milliamperes, while the current in the opposite direction at the same voltage is about .02 milliamperes. This is a larger value of the rectified current, at practically perfect rectification, than I was able to obtain with the carborundum rectifier. It was, therefore, decided to recur to the attempt to obtain an oscillographic record of the phenomenon with the aid of Braun's tube, as had been attempted with only partial success in the study of carborundum. The result in the present experiment is highly satisfactory.

TABLE II.

EXCESS OF VOLTAGE TO SEND CURRENT FROM MoS_2 TO Cu ABOVE THAT TO SEND CURRENT FROM Cu TO MoS_2.

Microamperes.	Excess Volts.	Microamperes.	Excess Volts.
0.05	0.18	0.70	1.24
0.10	0.515	0.80	1.27
0.20	0.89	0.90	1.32
0.30	1.01	1.00	1.36
0.40	1.09	1.10	1.40
0.50	1.15	1.20	1.45
0.60	1.19	1.30	1.48

OSCILLOGRAPHIC RECORDS OF RECTIFIED CYCLE.

Method of obtaining the Oscillograms. — The Braun's tube oscillograph was employed. A sketch of the oscillographic apparatus is given in Figure 6. The Braun's tube was filled with hydrogen and was pumped to the vacuum at which it has its highest sensitiveness.[11] The high-potential current through the tube was supplied by Professor Trowbridge's 40,000 volt storage battery, which he kindly placed at my disposal. Usually only 20,000 volts of the battery were employed, and this was controlled by means of a running-water rheostat in series with the battery and the tube.

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[11] My thanks are due to Mr. E. L. Chaffee for very carefully pumping out the tube for me, and for other valuable assistance with the oscillographs.

TABLE III.

Current-Voltage Values for the Molybdenite Rectifier.
Larger Currents.

Current from Copper to Molybdenite.		Current from Molybdenite to Copper.	
Volts.	Milliamperes.	Volts.	Milliamperes.
0.5	0.20	2.0	0.02
0.6	0.50	4.5	0.10
0.77	1.00	5.27	0.25
0.84	1.50	7.1	0.55
0.92	2.00	8.6	1.15
1.07	2.50	10.1	2.20
1.15	3.00		
1.32	4.00		
1.52	5.00		
1.70	6.00		
1.88	7.00		
2.00	8.00		
2.15	9.00		
2.22	10.00		

The cathode beam in the tube produced a luminescent spot on the fluorescent screen at O. The electromagnets, through which the current to be oscillographed was sent, were placed above and below the Braun's tube at MM. Therefore the deflection of the spot was in a horizontal line perpendicular to the plane of the Figure. The photograph of the moving luminescent spot was taken on a sheet of bromide paper carried by a rotating drum F, which made 20 revolutions per second about a horizontal axis. This drum was enclosed in a light-tight box at the back of an improvised camera. A horizontal slit S, immediately in front of the rotating drum, shut off all luminescence in the tube except that in the line of motion of the spot.

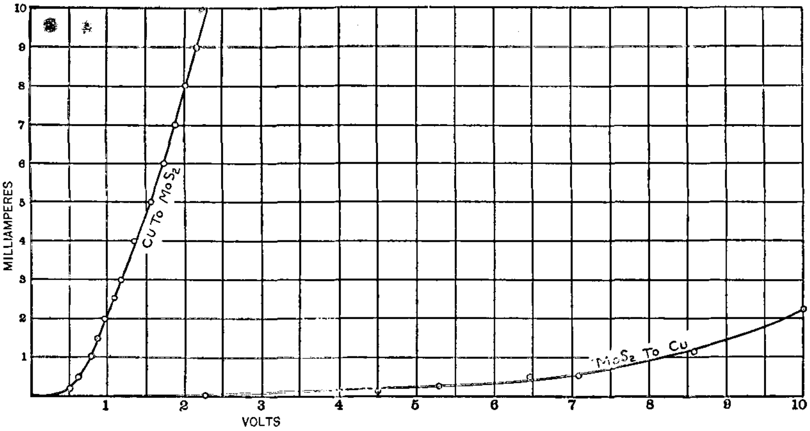

Figure 5. Current-voltage curves of molybdenite rectifier, with large current.

The rotating drum was driven by a synchronous motor operating on the 60-cycle alternating current mains of the laboratory. The alternating current sent through the rectifier and the deflecting magnets was taken from the same supply. The synchronism of the drum with the deflections of the luminescent spot was so perfect that exposures of four minutes could be made, during which time the image of the spot

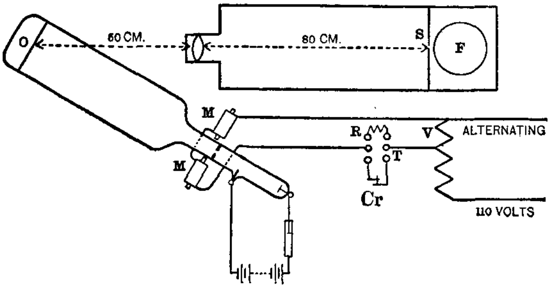

Figure 6. Oscillographic apparatus.

moved over the sensitive paper 4800 times, without any failure of perfect superposition, and without any appreciable fogging of the paper.

The deflecting electromagnets MM had a combined resistance of 436 ohms, and were provided with soft iron cores about 6 millimeters in diameter. With these deflecting coils a direct current of 1.5 milliamperes gave a deflection of 1 cm. on a ground glass put in the place of the sensitive paper at the back of the camera. A calibration for different values of direct current through the coils showed the deflections of the light spot to be proportional to the current, for the small values of the current employed, and showed no evidence of hysteresis in the iron.

The Oscillographic Records. — Reproductions (reduced to $\frac{1}{3}$) of a characteristic set of the oscillographic records obtained are given in the Plate. Oscillograph No. 1 was taken with the molybdenite rectifier adjusted to give practically perfect rectification. No. 2 is with the same rectifier slightly out of adjustment (overloaded), so that the rectification is less perfect. No. 3 is with the same rectifier further out of adjustment. No. 4 is an oscillographic record with the carborundum rectifier. No. 5 is with the rectifier of brookite. In taking No. 2 the rectifier was submerged in oil, to test the effect of cooling.

In making these records the following steps were taken : The drum carrying the film was set rotating. The high-potential current was started in the tube. The potential V (Figure 6) and the contact of the rectifier were adjusted so that the deflection of the luminescent spot on the fluorescent screen was wholly or chiefly to one side of the zero position. Exposure of about 2 minutes was then made. This exposure gave the heavy line of the oscillograms. The switch at T was then thrown open, so that the luminescent spot came to its zero position. The exposure in this position was made for a shorter time of about 40 seconds. This traced the light straight line along the centre of the picture, and gave the axis of zero current. The switch at T was then thrown to the position to put the resistance R in the circuit in place of the crystal. The resistance R had been previously adjusted so that the amplitude of the deflection with R in the circuit should coincide with the maximum amplitude with the crystal in the circuit. With the resistance R in circuit an exposure of about 1 minute was made, giving the light sinusoidal curve of the picture.

On each picture the three exposures give, therefore, (1) the form of the rectified cycle as a heavy line, (2) the position of the axis of zero current, as a straight line through the figure, and (3) the form and position of the alternating current cycle when an equivalent resistance R is substituted for the rectifier. The last-named cycle appears in the pictures as a thin-lined sine curve. This curve is in phase with the impressed voltage immediately about the crystal, and is referred to below as the " voltage-phase curve."

In tracing all the curves, the motion of the light spot over the paper is from left to right; the time co-ordinate is, therefore, the abscissa of the curves and is drawn as usual from left to right.

The scale drawn in ink at the left-hand margin of each picture gives the value of the current, one division being one milliampere.

A tabular description of the conditions under which each of the records was taken is contained in Table IV.

TABLE IV.

Tabular Description of the Oscillographic Records of the Plate.

No.	Material of Rectifier.	Condition.	Maximum Rectified Current in Milliamperes.	R. M. S. Alternating Volts.	Equivalent Resistance in Ohms.
1	Molybdenite	Good adjustment	4.9	3.54	400
2	"	Out of best adjustment. Submerged in oil and overloaded	4.9	3.54	400
3	"	Out of best adjustment	4.5	. . .	. . .
4	Carborundum platinized on one side	Overloaded	5.4	22.0	6000
5	Brookite	"	3.0	2.22	992

A discussion of the records follows.

Oscillogram Nos. 1, 2, *and* 3 — *Molybdenite.* — The pressure of the copper rod[12] against the molybdenite for good rectification is slight, and is somewhat difficult to attain. Some points of the crystal are more sensitive than others, and the crystal has to be moved around under the copper contact and tried at several different points before the best adjustment can be found. Oscillogram No. 1 was taken with a molybdenite rectifier in good adjustment. The rectification in this case is seen to be practically perfect; the cycle through the specimen consists of a nearly sinusoidal curve for one half period and a practically straight line for the other half period. The large current flows from the copper to the molybdenite, and the zero current from the molybdenite to the copper.

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[12] The end of the copper rod in contact with the molybdenite had an area of 4 sq. mm.

When the pressure on the contact was increased until a small negative current was permitted to pass, oscillogram No. 2 was obtained. Increasing the pressure still more so as to get a larger negative current gave oscillogram No. 3.

One object in taking these oscillograms, together with the voltage-phase cycle, was to see if there is any evidence of lag of the rectified cycle with respect to the voltage-phase cycle. *No such lag* appears. On the other hand, the rectified cycles *lead* their respective voltage-phase cycles at three positions :

The first of these positions of lead is at the part of the cycle in which the rectified current approaches the zero axis after having traversed the upper half of the curve. This advance, which is so small as to be just perceptible in the oscillograms, amounts to about 1/6000 of a second.

A second, somewhat larger, lead of the rectified cycle ahead of the voltage-phase cycle is at the point of rising from the axis after the rectified current has followed for a half period along the zero axis. The lead here is about 1/1500 second.

A third, very significant, lead of the rectified cycle is at the negative maximum, as is seen in the cases of imperfect rectification, oscillograms Nos. 2 and 3. Here the lead is a large fraction of a half period.

Oscillogram No. 4 — *Carborundum.* — Oscillogram No. 4 was obtained with a carborundum rectifier consisting of a specimen of carborundum, platinized on one side, and held in a clamp under a contact pressure of 3 Kg. When sufficient current was sent through the carborundum to give deflections suitable for the oscillogram, the carborundum was overloaded, and permitted current to pass in the negative direction. The carborundum cycle differs from the molybdenite cycle in the absence of lead at the negative maximum and at the point of rising from the zero axis. This anomaly in the case of the carborundum rectifier is seen later to be the effect of its high resistance.

Oscillogram No. 5 — *Brookite.* — The form of the cycle obtained in this case is intermediate between the carborundum cycle and the cycle of oscillogram No. 3. This is consistent with the value of its resistance.

In order to investigate the meaning of the lead of the rectified cycles in the several cases, a further examination of the oscillograms is made with the aid of the theory of alternating currents.

EXAMINATION OF THE OSCILLOGRAMS WITH THE AID OF THE THEORY OF ALTERNATING CURRENTS.

The so-called "voltage-phase cycle" gives the instantaneous values of the current through the deflecting coils and through a resistance chosen to make the amplitude of this current the same as the amplitude of one loop of the current through the rectifier, under the same applied voltage. Although the current of the voltage-phase cycle lags behind the externally applied voltage by an amount depending on the relation of the self-inductance of the deflecting coils to the resistance of the circuit, the current is nevertheless in phase with the voltage immediately about the substituted resistance; for the voltage about a resistance is *in phase with the current through it.* Now by throwing the switch at T of Figure 6, we put the rectifier in the circuit in the place of the resistance. If the rectifier, when current traverses it, introduces into the circuit electromotive forces out of phase with the current through it, we ought to get a shift of phase of the cycle. We can easily see, for example, that if the rectifier contained capacity or inductance, such a shift would occur. Also, if the action of the rectifier were one of electrolytic polarization, the back e. m. f. of polarization would be approximately determined at any part of the cycle by a time integral of the current, and would introduce a shift of phase resembling that introduced by a capacity.[13]

Also, if the action of the rectifier were due to thermoelectricity, we should expect the thermoelectromotive forces developed to be of the form

$$\pm\, a \int i^2 r\, dt, \tag{1}$$

due to the Joulean heat at the high resistance, and of the form

$$\pm\, b \int i\, dt, \tag{2}$$

due to the Peltier effect at the junctions. To these terms we should have to add also terms taking account of conduction of heat from the junctions. The term for the conduction of heat would be difficult to assign definite values, but it would be functions of the rise of temperature of the junctions, and may be written in the general form

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[13] B. O. Peirce: Newtonian Potential Function, p. 323. Boston, 1902.

$$(3) \qquad F\left(\int i^2 r dt, \int i dt\right).$$

The terms (1), (2), and (3), when put into the differential equation for the current through the circuit and integrated (if possible), would give in the result a shift of phase of the current with respect to the voltage-phase cycle.

Let us, therefore, attempt to determine whether there are any phase differences between the rectified cycle and the voltage-phase cycle that are not accounted for by the conditions existing in the oscillographic apparatus. In doing this we shall make use of the current-voltage characteristic of the molybdenite rectifier, as obtained with the current and voltage in the steady state and recorded in Table III and Figure 5. This table of data was obtained with the same molybdenite rectifier in practically the same adjustment as in the oscillograms Nos. 1 and 2 of the Plate.

Let us derive, first, the numerical equation for the "voltage-phase" curve. In the case of oscillogram No. 1, an ohmic resistance of 400 ohms was in series with the deflecting coils, which had a resistance of 436 ohms, making a total resistance of 836 ohms. Let the inductance of the coils be L. The value of L can be calculated from the voltage and current of the cycle. The R. M. S. voltage impressed on the circuit was 3.54 volts; the maximum voltage was therefore 5.00 volts. The maximum current, taken from oscillogram No. 1, was 4.9×10^{-3} amperes, whence we have

$$4.9 \times 10^{-3} = \frac{5.0}{\sqrt{836^2 + L^2\omega^2}}.$$

Therefore

$$(1) \qquad L\omega = 584,$$

$$(2) \qquad \tan^{-1}\frac{L\omega}{836} = \phi_1 = 35°,$$

and the equation for the current i_1 of the voltage-phase cycle becomes

$$(3) \qquad i_1 = \frac{5.0}{\sqrt{836^2 + 584^2}} \sin(\omega t - 35°).$$

From this equation the values contained in Table V were computed.

TABLE V.

The Voltage-Phase Cycle.

ωt Degrees.	Current in Milliamperes.	ωt Degrees.	Current in Milliamperes.
35	0	135	4.82
55	1.67	155	4.23
75	3.14	175	3.14
95	4.23	195	1.67
115	4.82	215	0
125	4.90		

And from these values three half periods of the voltage-phase cycle are plotted as the sinusoidal curve S of Figure 7.

The computations when the rectifier is put in place of the 400 ohm resistance can be made only approximately. The differential equation for the current i_2 through the circuit in this case is

$$E \sin \omega t - e_r = R_c i_2 + L \frac{di_2}{dt}, \qquad (4)$$

in which e_r is the drop of voltage about the rectifier, E is 5.0 volts, and R_c the resistance of the deflecting coils = 436 ohms. The drop in voltage e_r about the rectifier is a function of the current. This function is the equation of the current-voltage curve of Figure 5. It is difficult to obtain an exact analytical expression for this function. But

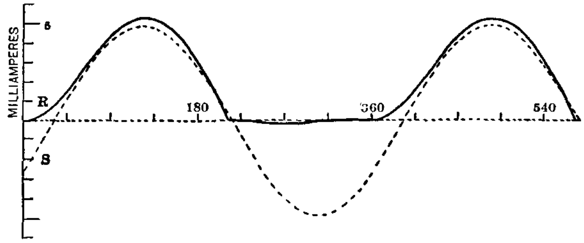

Figure 7. Rectified cycle computed from the current-voltage values of Figure 5.

for values of current between 1 and 6 milliamperes, when the current is from copper to molybdenite, e_r is approximately a linear function of the current, with the equation

(5) $e_r = q + ri$, in which $q = .60$ volts, $r = 183$ ohms.

With this approximation equation (4) becomes

$$(6) \qquad E \sin \omega t - q = (\text{r} + R_c)\, i_2 + L \frac{di_2}{dt}.$$

Integration of this equation gives

$$(7) \qquad i_2 = \frac{E}{\sqrt{(r+R_c)^2 + L^2\omega^2}} \sin\left(\omega t - \tan^{-1}\frac{L\omega}{r+R_c}\right) + ce^{\frac{-(R_c+r)t}{L}} - \frac{q}{R_c + r},$$

TABLE VI.

Computed Values of the Rectified Cycle. Upper Loop.

ωt Degrees.	Current in Milliamperes.	ωt Degrees.	Current in Milliamperes.
0	0	130	5.26
20	0.32	140	5.20
40	1.17	160	4.50
60	2.45	180	3.15
80	3.61	200	1.40
100	4.75	213	0
120	5.21		

in which c is a constant of integration. If we substitute known values in this equation, namely,

$$(8) \qquad r + R_c = 183 + 436 = 619, \quad L\omega = 584, \quad E = 5.0, \quad q = 60,$$

we have

$$(9) \qquad i_2 = 5.87 \times 10^{-3} \sin(\omega t - 43.3^\circ) + ce^{-1.06\omega t} - .97 \times 10^{-3}.$$

For the determination of the constant c, we have the relation $i_2 = 0$, when $E \sin \omega t = q$. This gives $c = 5.1$.

From equation (9) values for the current in the upper loop of the rectified cycle for various values of ωt were computed and are given in Table VI.

The lower loop of the rectified cycle was obtained in a similar manner.

In this case the drop in potential about the rectifier was obtained from the curve of current from molybdenite to copper of Figure 5. The equation to this curve, within the limits employed in the calculations, is approximately

$$(10) \quad e_r = q_1 + r_1 i_3, \text{ in which } q_1 = 3.8 \text{ volts and } r_1 = 6470 \text{ ohms.}$$

These values substituted in an equation of the form of equation (7) gave, since the exponential term was found to be negligible,

$$(11) \qquad -i_3 = .72 \times 10^{-3} \sin(\omega t - 4.8) - .55 \times 10^{-3}.$$

Computations from this equation gave the values of current recorded in Table VII.

TABLE VII.

Computed Values of the Rectified Cycle. Lower Loop.

ωt Degrees.	Current in Milliamperes.	ωt Degrees.	Current in Milliamperes.
220	0	280	.16
240	.07	300	.07
260	.16	320	0
270	.17		

The computed values of Tables VI and VII are plotted as the continuous curve R of Figure 7, along with the voltage-phase curve, which is the dotted sine curve S.

The data used in the computations are entirely independent of the oscillograms, except that the amplitude of the voltage-phase cycle was taken from oscillogram No. 1 or No. 2, and this value was used in determining the self-inductance of the circuit.

The agreement of the diagram of Figure 7 with the oscillograms Nos. 1 and 2 of the Plate is very striking, as regards both the form and the absolute value of the curves. The agreement with oscillogram No. 2 is a little better than with No. 1, and is within the limit of error of the measurement of the photograph. No departure in amplitude or in phase exists between the rectified cycle and the voltage-phase cycle that is not accounted for by the inductance and resistance of the oscillographic apparatus or by the current-voltage curves of the rectifier with steady currents.

This means that if there are any terms contingent upon heating or other effects which involve an integral of a function of the current with respect to the time, this integral attains its final value in a time within the limit of error of measuring the oscillograms, which is about 1/6000 second. This time corresponds to 3.5°, and is about 1 mm. on the original photographs.

It might seem that the approximation made as to the analytical expression for the steady current-voltage curve would not warrant the accuracy here claimed; but if we draw the straight line through the points for which the current is 1 and 6 milliamperes, this line will depart from the observed values only for values of i below 1 milliampere, where the departure will have the following values:

i Milliamperes.	Departure Volts.	Departure in Degrees.
.5	.1	.6
.2	.15	1.7
.1	.3	3.4

In the negative loop of the rectified cycle the departure of the approximation from the observed current-voltage curve is still smaller. However, apart from the specific assumption as to the analytical function representing the current-voltage characteristic of the rectifier under the action of a steady current, the theoretical discussion given above permits a ready qualitative understanding of the lead that occurs in certain parts of the rectified cycle, which may be summarized as follows:

(1) The case of the advance of the rectified cycle on rising from the axis of no current is seen to be due largely to the fact that after a dormant half period the current in the circuit follows the ordinary exponential "building-up" curve for a time before coming into coincidence with the sine curve. This building-up curve starts from the axis with zero lag, and is, therefore, in advance of the sine curve. To this effect is to be added the effect due to an apparently higher resistance of the rectifier for small currents than for large currents. This apparently higher resistance brings the building-up curve a little nearer to the sine curve.

(2) The slightly quicker descent of the rectified cycle on approaching the axis after having traversed the upper half of the curve is also due to this apparently higher resistance of the rectifier when traversed by smaller currents.

(3) The very significant lead of the negative maximum ahead of the corresponding voltage-phase maximum is explicable on the assumption that the rectifier has a much higher resistance in the negative direction than in the positive direction. We have seen above that the angle of lag of the voltage-phase cycle behind the impressed voltage, determined by the inductance and resistance of the circuit, is

$$\tan^{-1}\frac{584}{836} = 35^\circ,$$

while in the negative direction the substituted equivalent resistance should be at least $6470 + 436 = 6906$ ohms, whence the angle of lag in this case would be

$$\tan^{-1}\frac{584}{6906} = 4.8^\circ.$$

Therefore, the angle of lead of the rectified cycle ahead of the voltage-phase cycle, determined as the difference of these two angles of lag, is 30.2°. This value agrees with oscillogram No. 2.

In this connection it is interesting to notice that a lead of this negative maximum in the case of the carborundum oscillograph does not appear. The explanation of this is easily obtained if one substitutes for the resistance values of the molybdenite the corresponding values for the circuit containing the carborundum rectifier. The equivalent resistance of the carborundum in its positive loop is 6000 ohms, so that the angle of lag of the voltage-phase cycle with this resistance in it is only 5.6°, while in the negative direction the equivalent resistance of the carborundum is about 20,000 ohms, giving an angle of lag in the neighborhood of 1°. The difference between these two angles of lag, which would give the phase difference between the carborundum cycle and the corresponding voltage-phase cycle, would be a quantity just perceptible on the oscillogram, as was verified in the original photographs.

In conclusion of this discussion of the oscillograms, I should say that we have not been able to detect in the photographs any evidence of a thermoelectric or other integrative action of the rectifier.

THERMOELECTRIC PROPERTIES OF MOLYBDENITE.

In the present section an account is given of the investigation of the thermoelectromotive force of molybdenite against copper and a determination of the temperature coefficient of resistance of molybdenite. Apart from their possible bearing on the action of the rectifier, the thermoelectric properties of molybdenite are of interest in themselves.

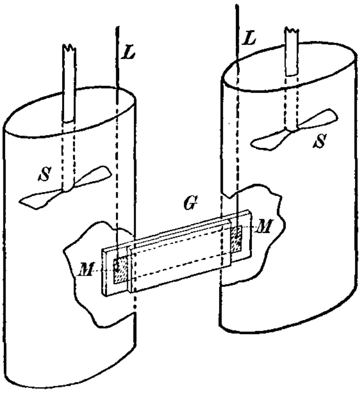

Figure 8. Apparatus for determining thermoelectric force of molybdenite against copper.

Thermoelectromotive Force. — Five specimens were mounted for the study of the thermoelectromotive force of molybdenite against copper. These specimens are referred to as "A," "B," "C," "D," and "E." The method of mounting the specimen E is shown in Figure 8. A thin sheet of molybdenite .1 or .2 mm. thick, 2 cm. wide, and 8 cm. long, was cemented between two glass microscope slides G with a cement made of water-glass and calcium carbonate.[14] The molybdenite was then copper-plated over a small area at each of the exposed ends MM, and to these copper-plated areas were soldered copper wires .2 mm. in diameter, so as to form thermal junctions with the molybdenite. The thermal junctions and the ends of the glass mounting were inserted into two brass vessels for containing the temperature baths of oil. The joints between the brass vessel and the glass mounting were made tight with the cement of water-glass and calcium carbonate. The oil baths were provided with stirrers driven by a motor. One of the baths was kept at 0° C., and the other bath was given various temperatures between 0 and 200° C. The resulting thermoelectromotive force was measured by means of a potentiometer to which the copper wires LL led. The results are recorded in Table VIII and plotted in the curve of Figure 9.

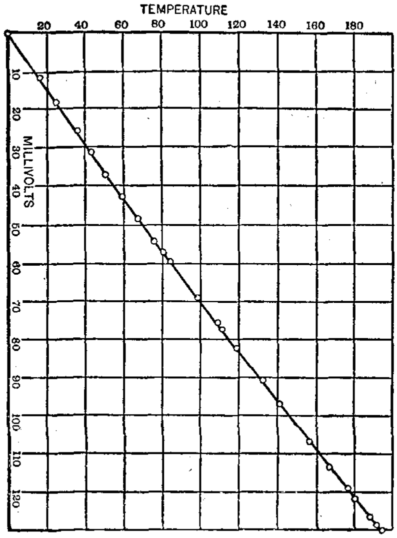

Figure 9. Thermoelectromotive force of copper-molybdenite couple "E," for various temperatures of hot junction. Cold junction at 0° C.

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[14] Otto Reichenheim suggests the use of such a cement in Inaugural Dissertation, Freiburg, 1906.

A slightly different form of mounting was employed for specimens A, B, C, and D. These specimens, which were cut from two different large crystals of molybdenite, were each 1 cm. wide, 5 cm. long, and from .5 to 1 mm. thick, and were mounted in

TABLE VIII.

Thermoelectromotive Force of the Copper-Molybdenite Couple E, the Cold Junction being kept at Zero.

Temperature of Hot Junction.	E. M. F. in Millivolts.	Temperature of Hot Junction.	E. M. F. in Millivolts.
10.1	− 7.5	99.2	− 68.4
14.3	−10.7	109.3	− 75.2
16.2	−11.5	111.6	− 77.2
18.7	−13.8	116.3	− 79.2
21.5	−16.0	118.7	− 83.2
24.1	−17.6	133.2	− 90.7
25.6	−18.5	141.9	− 96.9
33.1	−24.6	156.8	−106.8
36.2	−25.9	166.9	−113.2
41.9	−31.5	176.8	−119.0
51.1	−36.7	179.0	−120.0
59.2	−42.5	180.9	−121.5
67.4	−48.6	188.5	−126.2
70.8	−51.2	192.7	−128.7
76.0	−54.1	195.0	−130.0
80.8	−57.2		

The negative sign before the e. m. f. in the second and fourth columns of Table VIII indicates that this specimen of molybdenite is thermoelectrically *negative* with respect to copper; that is to say, the current at the hot junction flows from the molybdenite to copper.

corks. Each cork, 4.5 cm. long, was split lengthwise, and one of the longitudinal half-corks was grooved out to contain the molybdenite. The two half-corks with the molybdenite between were put together again and cemented with plaster of Paris, so as to leave 2 or 3 mm. of molybdenite protruding from each end of the cork. These small areas were then copper-plated, and copper wires .2 mm. thick were soldered to the copper-plated areas, so as to form thermal junctions. The four corks containing the specimens A, B, C, and D were inserted in round holes in two copper vessels for containing the temperature baths of oil, so that the junction at one end of each specimen should be in the hot bath, while the junction at the other end was in the cold bath. The cold bath was kept at 20° C.; the hot bath was given various temperatures between 20 and 100° C. The thermoelectromotive force of each couple was measured on a potentiometer. The results for A, B, C, and D are contained in Table IX and are plotted in Figure 10. For comparison a part of the curve obtained for E is also plotted in Figure 10.

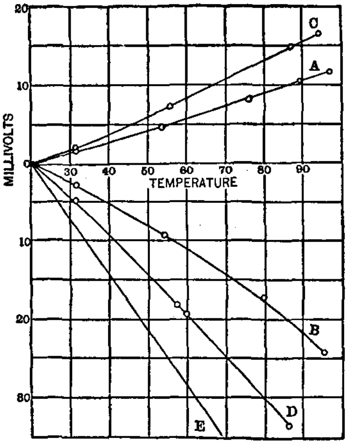

Figure 10. Thermoelectromotive force of five copper-molybdenite couples, for various temperatures of hot junction. Temperature of cold junction, 20° C.

Some of the specimens (B, D, and E) are thermoelectrically negative with respect to copper, while the other specimens (A and C) are thermoelectrically positive with respect to copper. The thermoelectromotive force per degree differs largely with the different specimens, as may be seen by a reference to Table X, which contains the thermoelectromotive force per degree of the different specimens of molybdenite against copper and against lead (obtained from the known value of the lead-copper junction). For comparison Table X also gives the thermoelectromotive power of some other remarkable thermoelectric elements.

The comparison shows that these specimens of molybdenite have very large thermoelectromotive force against copper or against lead. The specimens D and E were found to be at the extreme negative end of the thermoelectric series.

The great variability among the specimens studied may be due to an admixture of small quantities of some other substance with the molybdenite, or it may be due to structural differences from point to point of the crystal. I have not yet investigated the question of the cause of the variability of the phenomenon. The differences in the specimens could not have arisen from the copper-plating or from the heat employed in soldering the junctions, because the specimens A, B, C, and D were tested before the copper-plating and soldering was done, and by means of the preliminary test were classified as positive, negative, positive and negative respectively, which agrees with the determination after soldering.

TABLE IX.

MOLYBDENITE-COPPER JUNCTIONS A, B, C, D. THE COLD JUNCTION WAS AT 20° C. THE HOT JUNCTION WAS AT TEMPERATURE T° C. THE THERMO-ELECTROMOTIVE FORCE V IS IN MILLIVOLTS.

Junction A.		Junction B.		Junction C.		Junction D.	
T.	V.	T.	V.	T.	V.	T.	V.
31.9	1.45	31.6	− 2.70	31.7	2.01	31.6	− 4.81
53.5	4.63	54.1	− 9.21	55.2	7.20	57.5	−17.9
76.6	8.21	80.0	−17.1	. . .	. . .	59.8	−19.4
89.4	10.4	87.4	−20.0	87.2	14.9	86.7	−33.7
97.1	11.5	95.3	−24.2	94.4	16.6		

The preliminary test was made by touching the specimens with two copper wires attached respectively to the two terminals of a galvanometer, one of the wires being slightly warmer than the other. This preliminary test proved very interesting in that it shows that one may find all over many of the pieces cut from a crystal of molybdenite points where the substance is thermoelectrically positive and other points where it is thermoelectrically negative. These positive and negative points sometimes lie so near together that with a fine-pointed exploring electrode attached to a galvanometer and warmed by heat conducted from the hand, one may find the deflections of the galvanometer reversed from large positive values to large negative values on making the slightest possible motion of the pointer over the crystal.

Explorations of this kind failed to show any definite orientation of the thermoelectric quality with respect to the crystallographic axes.

The existence of small thermoelectrically positive and negative patches in a piece of the molybdenite may indicate that the thermoelectromotive force measured by attaching wires to the specimen is too low on account of the inclusion under the electrodes of both positive and negative areas which would partially neutralize the thermoelectric action against another electrode.

TABLE X.

Substance.	Thermoelectromotive Force in Microvolts, per Degree Centigrade, at 20° C.		Authority.
	Against Copper.	Against Lead.	
Molybdenite A . .	110	113	Present experiment
" B . .	−230	−227	"
" C . .	175	178	"
" D . .	−415	−413	"
" E . .	−720	−717	"
Silicon	. . .	−400	Frances G. Wick [1]
Bismuth	. . .	− 89	Matthiessen [2]
Antimony	. . .	26	"
Tellurium	. . .	502	"
Selenium	. . .	807	"

[1] Phys. Rev., **25**, 390.
[2] Everett, Units and Physical Constants.

It may be said in passing that the specimens D and E, with the soldered connections, still showed the phenomenon of rectification when used with alternating currents, even when the two junctions of the copper with the molybdenite were in oil baths at the same temperature as the room and the oil in the baths was vigorously stirred with motor-driven stirrers. The rectification in this case was, however, very imperfect.

Temperature Coefficient of Resistance.— Another interesting thermal property of the molybdenite is its temperature coefficient of resistance. A preliminary report of this coefficient is here given.

Two specimens of the molybdenite were made into the form of resistance thermometers by depositing heavy copper-plated areas near the two ends of thin pieces of the molybdenite and soldering thin copper strips to the copper plate. For insulation a thin strip of mica was placed over the molybdenite, and one of the copper leads was bent back over the mica so that both leads ran away parallel with the mica insulation between. The whole conductor was then placed between two mica strips and inserted in a flattened brass tube. The tube was then mashed tight together so as to clamp securely the molybdenite and its leads. The end of the tube adjacent to the molybdenite was soldered up. The leads were brought out at the other end of the tube and connected to binding posts insulated by a hard rubber head from the tube.

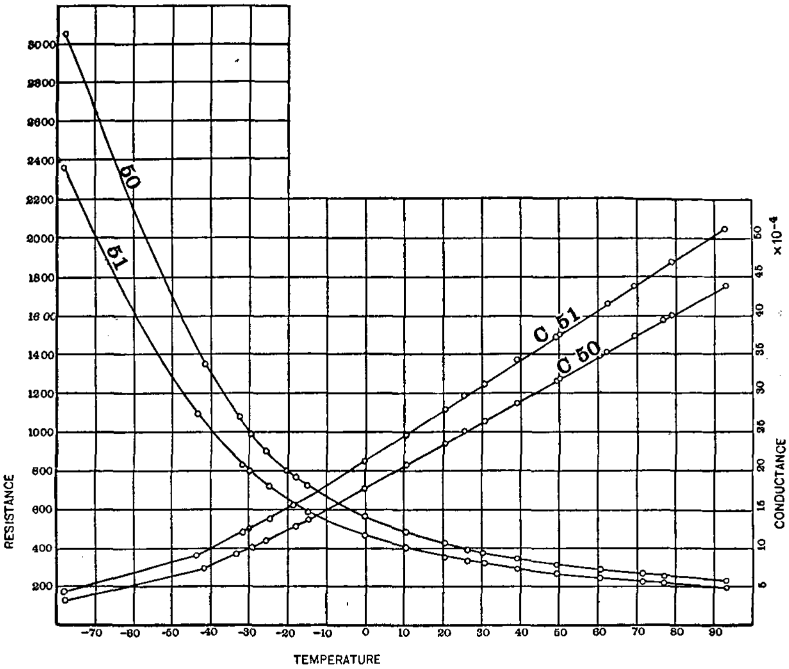

Figure 11. Effect of temperature on electrical resistance of molybdenite.

The two molybdenite resistances thus mounted are called No. 50 and No. 51. The dimensions of the molybdenite used in No. 50 were not recorded. The molybdenite in No. 51 was .65 cm. wide by .7 cm. long; the thickness was about .3 mm.

The resistances of these two conductors were measured at various temperatures with the aid of a Wheatstone bridge. They showed no evidence of rectification. In making the measurements it was necessary to keep the current small so as to avoid electrical heating of the conductors. With successive heatings and coolings the resistance of the molybdenite showed small progressive changes, which, however, after some months almost disappeared. When the resistance of the two specimens of molybdenite had settled down to a practically steady condition, the values plotted in Figure 11 were obtained. The curves marked "50" and "51" give the resistances of No. 50 and No. 51 respectively. The ordinates for these curves are at the left margin of the diagram, and are in ohms. The curves "C 50" and "C 51" are for the reciprocals of the resistance of No. 50 and No. 51 respectively. The ordinates for these curves are at the right-hand margin of the diagram.

Each of the specimens has a large negative temperature coefficient of resistance. With No. 50, for example, the resistance at 93.1° C. is 229 ohms; at 0° C. the resistance is 561 ohms; at —76° the resistance is 3051 ohms; and at the temperature of liquid air the resistance of this specimen was found to be over 6,000,000 ohms. This last value is not plotted on the curves.

It is interesting to note that *between —15° and 93° the temperature-conductance curve of each of the specimens is a straight line.*

At 0° C. the resistance of each of the specimens decreases about 1.53 per cent per degree centigrade increase of temperature; at 20° the decrease of resistance per degree increase of temperature is 1.19 per cent.

A previous determination of the resistance of molybdenite has been made by Otto Reichenheim.[15] He did not solder on his connections, but led the current into the specimen through contact electrodes and found that the resistance depended on the contact pressure. His data are, therefore, not comparable with mine, but I find that one of his specimens,[16] measured parallel to the direction of cleavage, gives the conductance a linear function of the temperature between 19.5° and 92.5° C., with a slope not very different from that obtained in the present experiments.

The large thermoelectromotive force of the molybdenite against the common metals, together with its large negative temperature coefficient of resistance, lends plausibility to the hypothesis that the rectification is due to thermoelectricity. For if we pass an electric current through the rectifier and the current begins to make its way through a small area at the contact, this small area is heated and decreases in resistance, so that the greater part of the current flows through this particular small area, heating it still more, while the portions of the contact through which the current has not started remain cool and continue to offer a high resistance. The effect of this action is to confine the heating to an extremely small area, which is the condition necessary for the extremely rapid and efficient action of the rectifier. That there is, however, strong evidence against this explanation of the phenomenon is, I think, made clear in the succeeding experiments.

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[15] Otto Reichenheim, Inaugural Dissertation, Freiburg, 1906.

[16] Described as *Stab II*, p. 27 of the Dissertation.

Experimental Facts Adverse to the Thermoelectric Explanation of the Phenomenon of Rectification.

Thermoelectric Effect Opposite to the Rectification. — A number of experiments with different specimens of molybdenite were made, in which the rectification and the thermoelectric effect could be simultaneously studied. A diagram of the arrangement of apparatus is given in Figure 12. The specimen of molybdenite is shown at M, and was held down upon a wooden base by a spring clip. One end of each specimen, which was easily interchangeable in the apparatus, was electroplated with copper at S. To this copper-plated area a copper lead was soldered. A copper rod C, supported as in Figure 3, was brought into contact with the part of the molybdenite distant from the soldered junction. The molybdenite and the contact were put in an electric circuit containing a microammeter or galvanometer at A and a source of variable alternating potential at V. The alternating potential V could be applied or omitted by closing or opening the switch at T. A small heating coil was wound on the rod C, and another similar heating coil was wound on a second copper rod E placed immediately below the contact of C with M.

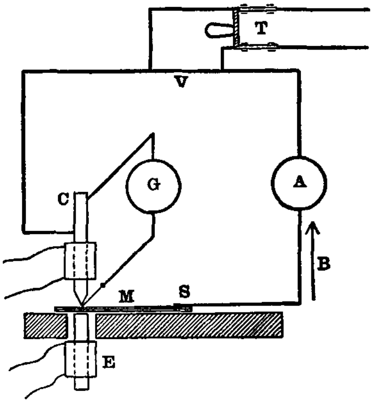

Figure 12. Apparatus for comparison of rectified current with thermal current.

An auxiliary thermal junction formed by a small constantan wire attached to the lower end of the copper rod C was connected to a second galvanometer shown at G, for use in a latter experiment.

TABLE XI.

SIGN OF MOLYBDENITE WHEN HEATED ABOVE OR BELOW AND WHEN SUBJECTED TO ALTERNATING VOLTAGE.

Specimen No.	Heated Above.	Heated Below.	Under Alternating Voltage.
75	+	−	−
81	+	−	−
Turned over	+	−	−
93	−	+	+
Another point	−	−	+
"	−	−	+
Turned over	−	−	+
78	+	+	+
Another point	+	−	−
"	+	+	−
94	−	−	+
Another point	−	+	+
"	−	+	+

The copper rods C or E could be heated by the surrounding coils, and the thermal current in the circuit through the molybdenite or the circuit through the constantan could be read on the galvanometers A or G. Also the rectified current obtained by applying the alternating voltage V could be read on the galvanometer A. When the thermal current or the rectified current through A is in the direction of the arrow B, the molybdenite, following the usage in thermoelectricity, is said to be *positive.* When the current in A is in the direction opposite to the arrow B, the molybdenite is said to be *negative.*

The results obtained with a number of specimens of molybdenite when heat was applied *above*, and when heat was applied *below*, and when the *alternating voltage* was applied are contained in Table XI.

From this table it appears that the thermoelectric voltage *when the junction is heated by heat conducted from above*, in twelve out of the thirteen cases tried, is opposite to the direct voltage obtained when an alternating current is passed through the junction. *When the heat is conducted to the junction from below, through the molybdenite*, the thermoelectromotive force in four cases is opposite to the rectified voltage, and in nine cases is in the same direction as the rectified voltage. In only one case, one point of No. 78, is the rectified voltage in the same direction as the thermal voltage, both when the junction is heated from above and when it is heated from below.

In all of these cases the heat was applied in the neighborhood of the same junction, and there is no opportunity for heat to get to the other junction (copper-plated and soldered) by conduction, on account of the great distance of the other junction from the source of heat. To make this absolutely certain this distant junction was in some cases submerged in an oil bath.

So far as I have been able to learn, this phenomenon of the reversal of the thermoelectromotive force at a thermal junction, conditioned on whether the heat is conducted to the junction through one element of the junction or the other element of the junction, is novel. It may be explained by the assumption of another thermal junction of opposite sign in the molybdenite itself below and in the immediate neighborhood of the copper-molybdenite junction. This assumption is plausible because it has been shown above that the molybdenite with which these experiments are performed is thermoelectrically an extremely hetereogeneous substance. On the other hand the phenomenon may also be explained on the theory that the thermoelectromotive force is determined by the direction of the flow of heat.

Whatever the explanation of the dependence of the sign of the thermoelectromotive force on the manner of applying the heat, it is seen that the thermoelectric effect is usually opposite in sign[17] to the rectified effect.

By applying heat from above and at the same time applying the alternating voltage, one can make the thermal current and the rectified current neutralize each other. This opposition of sign of the rectified current and the thermal current renders the correctness of the thermoelectric explanation of the phenomenon of rectification extremely improbable.

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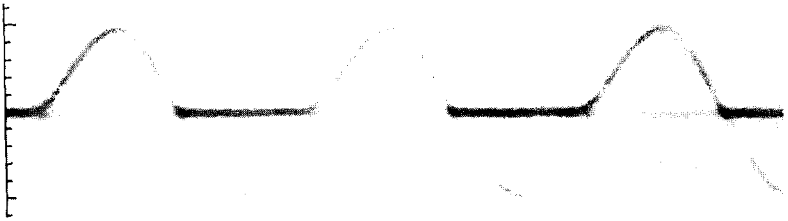
[17] In the case of silicon-steel, carbon-steel, and tellurium-aluminum, L. W. Austin has found that the rectified current generally flows in opposite direction to that produced by heating the junction. In his experiments (Bulletin of the Bureau of Standards, **5**, No. 1, August, 1908) the heat was applied by conduction from above.

Effort to detect Heating of the Contact of the Rectifier. — With the aid of the auxiliary thermal junction of copper-constantan placed at the contact of the copper with the molybdenite, as shown in Figure 12, an effort was made to detect heating of the copper molybdenite junction by the alternating current which was being rectified. When the rectified current was 118 microamperes, the heating shown by the copper-constantan junction did not exceed .01° C. When, on the other hand, as a control experiment, heat was applied to the copper-molybdenite junction from below so as to be conducted through the molybdenite and through the copper-molybdenite junction to the copper-constantan junction, the heating shown by the auxiliary copper-constantan junction was 11.4° C., while the thermal current from the copper-molybdenite junction was only .2 microamperes. In both the case of the rectified current and the case of the application of heat from below the heat had to be conducted from the point of rectification to the auxiliary junction. Therefore, with a rise of temperature of the auxiliary junction 1100 times as great as the rise shown during the rectification, the thermal current in the copper-molybdenite circuit was 1/500 of the rectified current; that is to say, the rectified current, for a rise of temperature of 1/100 of a degree of the auxiliary junction (being approximately a linear function of the temperature) was less than 1/500000 of the rectified current from an alternating current producing the same rise of temperature.

From this experiment, also, it seems to the writer that the hypothesis that the action of the rectifier takes place through the intermediation of thermoelectricity is improbable. Experiments are still in progress.

I have been aided in this investigation by a liberal grant from the Bache Fund of the National Academy, for which I wish to express my hearty thanks.

Jefferson Physical Laboratory,
Harvard University, Cambridge, Mass.,
December 21, 1908.



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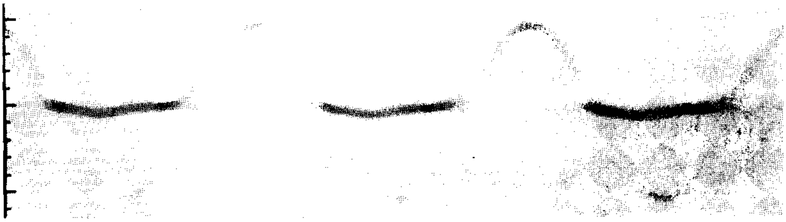

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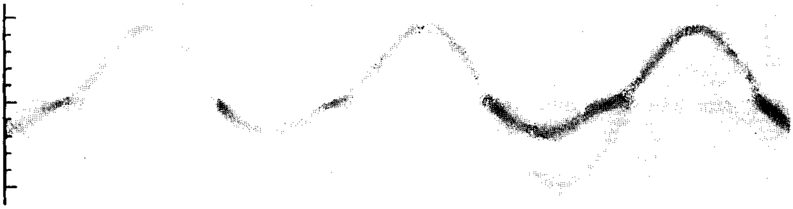

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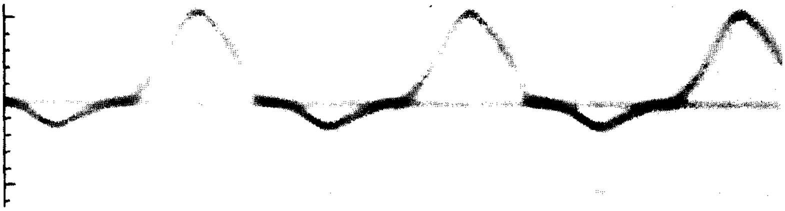

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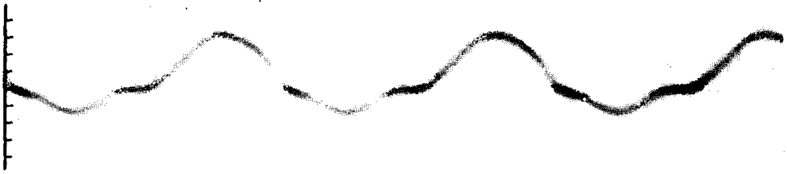

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL
LABORATORY, HARVARD UNIVERSITY.

ON THE MAGNETIC BEHAVIOR OF HARDENED CAST IRON AND OF CERTAIN TOOL STEELS AT HIGH EXCITATIONS.

By B. Osgood Peirce.

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ON THE MAGNETIC BEHAVIOR OF HARDENED CAST IRON AND OF CERTAIN TOOL STEELS AT HIGH EXCITATIONS.

By B. Osgood Peirce.

Presented November 11, 1908. Received December 31, 1908.

During the last few years the use of hardened cast iron for permanent magnets has increased very much, and this material has proved especially useful for such shapes as could not be easily forged from steel without heating the metal red hot a number of times and thus making it magnetically unsatisfactory. Cast-iron magnets are very cheap, and they may be made quite as strong and as permanent as magnets made of the best tool steel, even if in strength, though not in permanence, they fall a little behind magnets made of special "magnet steels." Moreover, and this is sometimes of very great importance, the temperature coefficient[1] of a seasoned cast-iron magnet is usually much smaller than that of a magnet of the same strength made of forged or formed steel. This paper discusses briefly a number of determinations of the permeability of specimens of fairly soft and of glass-hard cast iron of the same kind, under excitations up to about 15,000 gausses,[2] and, for purposes of comparison, considers also some measurements made upon hard and soft Stubs "Polished Drill Rod" and upon hard and soft "Crescent Polished Drill Rod."

The principal apparatus used consisted of a yoke (Figure 1) which weighed about 300 kilograms and was excited by a current (from a storage battery) running through a set of amperemeters in series with

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[1] Peirce, These Proceedings, **38** (1903); **40** (1905).

[2] Rowland, Phil. Mag., **46** (1873); Fromme, Ann. d. Phys., **13** (1881); **33** (1888); Stefan, Ann. d. Phys., **38** (1889); H. E. J. G. DuBois, Ann. d. Phys., **31** (1888); **51** (1894); B. Walter, Ann. d. Phys., **14** (1904); Ewing, Magnetic Induction in Iron and other Metals; DuBois, The Magnetic Circuit in Theory and Practice.

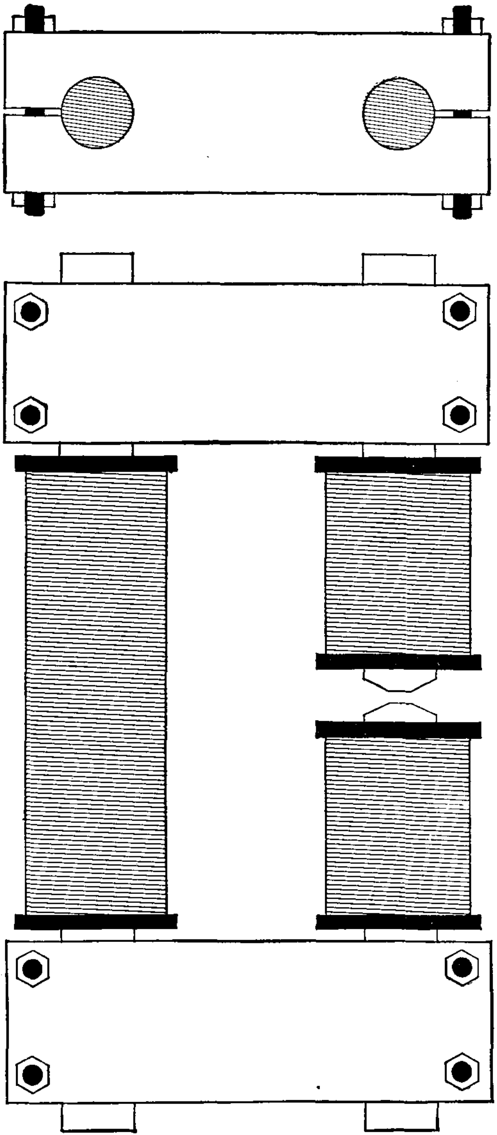

FIGURE 1.

the coil of 2956 turns wound on the spools shown in the diagram. The yoke was furnished with a number of pairs of pole pieces or jaws, to receive specimens of different lengths and shapes. To measure the amount of the induced current in a test coil wound closely upon the piece to be examined, a ballistic galvanometer (V), described in a former paper,[3] was used. The period of this instrument was so long that the throw due to a reversal of the exciting current of the yoke did not appreciably differ from the throw which the same quantity of electricity would have caused if it had been sent instantaneously through the circuit.

The specimens used in the work here described were of two forms. The first form (C, Figure 2) was a cylinder about 1.27 centimeters in diameter and about 15 centimeters long over all, with tapered ends to fit tightly in sockets in the ends of the conical pole pieces of the yoke. The sockets

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[3] These Proceedings, **44** (1909).

(G) were first turned out in the lathe and then finished by a reamer made and ground by the machinery afterwards used to cut the tapers on the ends of the test pieces. Each test piece of the hard cast iron had first to be ground to the form of a true cylinder in a universal grinding machine and then to be tapered off at the ends with the help of a centre grinder, mounted motor and all, in the tool post of an engine lathe. All the work was done by Mr. G. W. Thompson, the mechanician of the Jefferson Physical Laboratory, in the most skilful manner, and the reluctance of the joints must have been relatively very small. When a specimen of this shape was in position between the pole pieces of the yoke, and a steady current of at least two or three amperes was passing through the exciting coil, it was assumed that the value of H within the small cylinder (C) near the middle of its length was the same as the value of H in the air just outside the metal. The ground of this assumption was a series of experiments carried out some months ago. A piece of homogeneous steel rod about half an inch in diameter and about three hundred and fifty diameters long was placed within a solenoid consisting of 20,904 turns of thoroughly insulated wire wound on a straight piece of stout brass tube about an inch in inside diameter and rather more than sixteen feet long. Near the middle of the steel rod a test coil of fine insulated wire was wound closely on it, and then, with its leads, made thoroughly waterproof, so that a current of tap water could be kept running around the rod in the brass tube to hold the temperature of the steel nearly

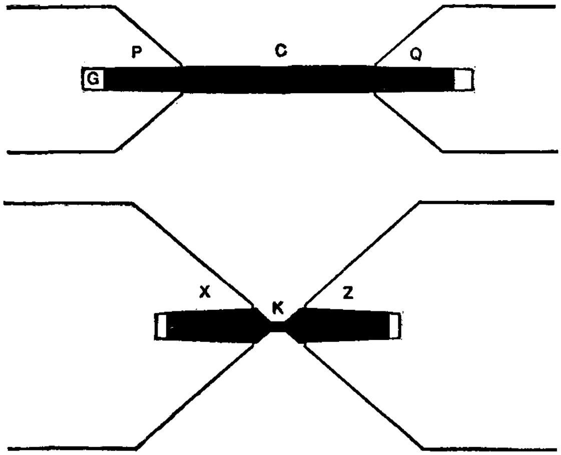

FIGURE 2.

constant when strong currents should be sent through the solenoid. The steel was first demagnetized by means of a long series of currents in the solenoid, alternating in direction and steadily decreasing in intensity, and then a series of steady direct currents of carefully measured intensities, each a little stronger than the last, were sent through the solenoid and reversed many times at each stage to determine the corresponding value of B in the steel. In this manner it was possible to get a satisfactory curve of ascending reversals for the steel up to $H = 400$ and $B = 20{,}500$, nearly. The length of the rod was, relative to its diameter, so great that the demagnetizing factor was very small and the correction for the ends very easily made. The rod was then demagnetized again, and the process described was repeated two or three times until the resulting table of B versus H values seemed to be well determined. After this, short pieces of various lengths, cut from the rod which had been tested, were used in the yoke and were mounted in different ways in the hope of discovering some satisfactory method of studying the permeability of the steel by experiments upon these pieces, which should give the same results up to an induction of about 20,000 as those already obtained by the work with the long solenoid. After long trial, a length of cylinder was found which seemed, in this particular yoke, to make the values of H at the centre of the length of the specimen practically the same as the value in the air just outside the metal. Two different materials were used in stout rod form in the long solenoid, Bessemer Steel and "Compressed Steel," an extremely homogeneous kind of steel prepared for us by the Boston Compressed Shafting Company.

In all the cases tried specimens of the size and shape described above seemed to give the same permeability up to values of the induction as great as 20,000 as the long solenoid did, and, for somewhat higher values of B, to yield results which agreed with those obtained, where it first becomes trustworthy, by the "Isthmus Method."

After the central portion of each of these specimens had been covered with an extremely thin coat of varnish, the diameter was determined under the microscopes of a Zeiss Comparator, reading to the nearest thousandth of a millimeter directly. Then two test coils, each of twenty turns of very fine, well-insulated wire, were wound side by side in a single layer over the varnished metal and extended over perhaps a centimeter at the middle of the rod. These coils were tested against each other when the specimen was in the yoke, to see if they were alike, and if they were, both, in series, formed the inner test coil (L) to be used in the measurements. The second testing coil (M) was wound on a very carefully made spool of boxwood which had been seasoning for many years. This spool kept its diameter practically unchanged during the measurements here recorded, though it shrank very slightly soon after it was first made. The diameter of the wood was about 1.9135 centimeters, and that of the outside of the wire of the coil about 1.9591 centimeters, the last figure in each case being, of course, doubtful.

TABLE I.

Cylinder of Soft Cast Iron.

H.	B.	I.	μ.
114	9950	782	87.3
172	10800	846	62.8
433	13900	1070	32.1
744	15750	1200	21.2
1234	17300	1280	14.0
1820	18170	1300	10.0

TABLE II.

Isthmus of Soft Cast Iron.

H.	B.	I.	μ.
12700	31100	1465	2.5
13550	32100	1475	2.4
13800	32500	1488	2.4
15100	33650	1472	2.2

Hard rubber is so susceptible in a magnetic field as to make it impossible to use a spool of this material to support a testing coil. When the specimen was in place between the jaws of the yoke, it was covered by the shorter spools of the yoke.

The value of H in the air just outside the metal was obtained by reversing the exciting current of the yoke when L and M were opposed to each other in the circuit of the ballistic galvanometer (V) described above. When L alone was used in the galvanometer circuit, and proper corrections for the air lines through L had been made by the use of the H just determined, it was possible to measure the induction flux in the metal.

TABLE III.

CYLINDER OF HARD CAST IRON.

H.	B.	I.	μ.
142	7860	614	55.4
254	9700	752	38.2
339	10850	836	30.6
684	13050	983	19.1
915	14050	1044	15.4
1570	15900	1138	10.1
2020	16800	1176	8.3

TABLE IV.

ISTHMUS OF HARD CAST IRON.

H.	B.	I.	μ.
10900	26540	1245	2.4
13200	28600	1226	2.2
14800	30200	1226	2.0

The second kind of specimen shown approximately by K, Figure 2, was of the shape usually employed in isthmus measurements. Cast iron differs from steel in that it can be heated so hot before it is chilled that it becomes eventually hard throughout its mass, while steel can be hardened only for a little distance from the surface. On the other hand, it is not easy to harden a long slender rod of cast iron without its becoming slightly crooked in the process. An isthmus piece of cast iron has, therefore, to be ground into shape at much labor, from a glass hard

TABLE V.

CYLINDER OF SOFT CRESCENT DRILL ROD.

H.	B.	I.	μ.
122	13060	1030	107.0
209	16730	1315	80.1
272	17190	1351	63.2
486	18400	1425	37.9
783	19150	1462	24.5
1535	20600	1516	13.4
1798	20900	1527	11.6

cylinder. The hardened steel isthmus pieces, on the contrary, were shaped while soft, and were then chilled inside a supporting tube after they had been heated in a gas furnace.

TABLE VI.

ISTHMUS OF SOFT CRESCENT DRILL ROD.

H.	B.	I.	μ.
4860	24600	1570	5.1
7190	27100	1584	3.8
10000	29700	1569	3.0
12020	32500	1629	2.7
13150	33800	1642	2.6

The "Isthmus Method" for determining the permeability of a small piece of magnetic metal at a high excitation rests, of course, upon the assumption that the value of H just without the test piece is equal to the average value of H over the cross section of the metal at the neck. At the surface of a magnet the tangential components of the magnetic force are continuous, while the normal component is discontinuous: it seems desirable, therefore, before one applies the method in any particular case, that one make sure that the lines of the field in the air space to be used are practically straight and parallel to the axis of the specimen. Any person who has had experience in using large yokes at high excitations, where because of the low permeability of the metal the leakage is very great, knows how slight a change in the shape of a specimen

TABLE VII.

CYLINDER OF SOFT STUBS POLISHED DRILL ROD.

H.	B.	I.	μ.
132	14600	1154	110.6
299	16700	1307	55.8
540	18100	1395	33.5
830	19000	1445	22.9
1380	20200	1495	14.6
1780	20800	1514	11.7

TABLE VIII.

ISTHMUS OF SOFT STUBS DRILL ROD.

H.	B.	I.	μ.
7900	26800	1500	3.4
13850	33200	1545	2.4
14900	34400	1554	2.3
15800	36200	1570	2.3
17100	37000	1587	2.2

may alter the field in the neighborhood of the test piece very sensibly. An isthmus piece of steel which had been hardened unequally might warp the field sufficiently to make the observations of the permeability wholly erroneous.

TABLE IX.

Cylinder of Hard Crescent Drill Rod.

H.	B.	I.	H.	B.	I.
114	8600	677	850	14650	1097
175	10050	786	1041	15200	1127
254	11300	879	1337	15950	1162
503	13000	993	1894	17000	1200

After much consideration I have decided not to print the results of my measurements upon isthmus pieces of glass-hard Stubs and Crescent Drill Rod for the reason that the maximum values of I seem to be rather too high. In one case, indeed, the effect of hardening an isthmus piece of steel was to make the ultimate value of I rather greater than before, though for moderate excitations the permeability was less. I hope to try soon the effect upon the uniformity of the field about the isthmus of harder jaws. The results obtained with the hard cast iron seem to be good.

TABLE X.

Cylinder of Hard Stubs Drill Rod.

H.	B.	I.	H.	B.	I.
123	8600	675	564	13750	1049
180	10020	783	982	15350	1143
256	11300	878	1416	16250	1216

The cast iron used for the observations recorded below, which was extremely soft and easy to work, came from the Broadway Iron Foundry of Cambridgeport, Mass., where we have obtained during the last few years a large number of castings of different forms for permanent magnets which proved when made and seasoned to be very strong and to have remarkably small temperature coefficients.

It will be noticed that this iron while soft is rather more permeable than that which was the foundation for the formula for reluctivity in "Ordinary Dynamo Cast Iron" given by Messrs. Houston and Kennelly in their Electro-Dynamic Machinery, but is very similar so far as results are available with the standard "Gray Cast Iron" used for the table given in the pamphlet on the "Magnetic Circuit" of the International Textbook Company. Although I had at command a much larger yoke than the one used, no attempt was made to carry the excitation beyond 15,000 gausses. The ultimate value of I in my hardened cast iron was about the same as that which Ewing gives for "Cast Iron" in "Magnetic Induction in Iron and Other Metals," § 93.

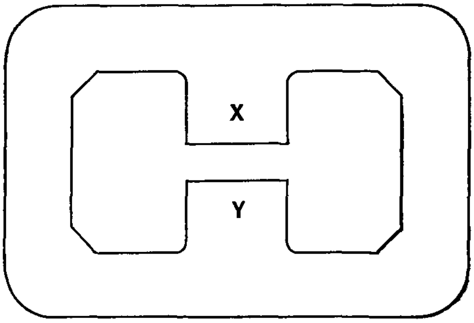

FIGURE 3.

The magnetic effects of hardening upon a mass of cast iron are often very noticeable at comparatively low excitations. The two halves of each of two thick castings, one soft, the other very hard, of the form shown in Figure 3, were wound with 156 turns each of insulated wire, and the two coils on each casting were so connected in series that when a current was sent through the circuit both conspired to make one of the projections (say X) a north pole and the other (Y) a south pole. With each of the castings a rude kind of hysteresis diagram was obtained by measuring for different current strengths the values of the induction flux across a definite area in the air gap between the poles. These fluxes plotted against the corresponding currents gave the diagrams shown in Figure 4. The A curve belongs to the soft casting, the

B curve to the hard one. While it would be difficult to explain the exact meaning of these curves in terms of the permeabilities of the iron, the differences are striking.

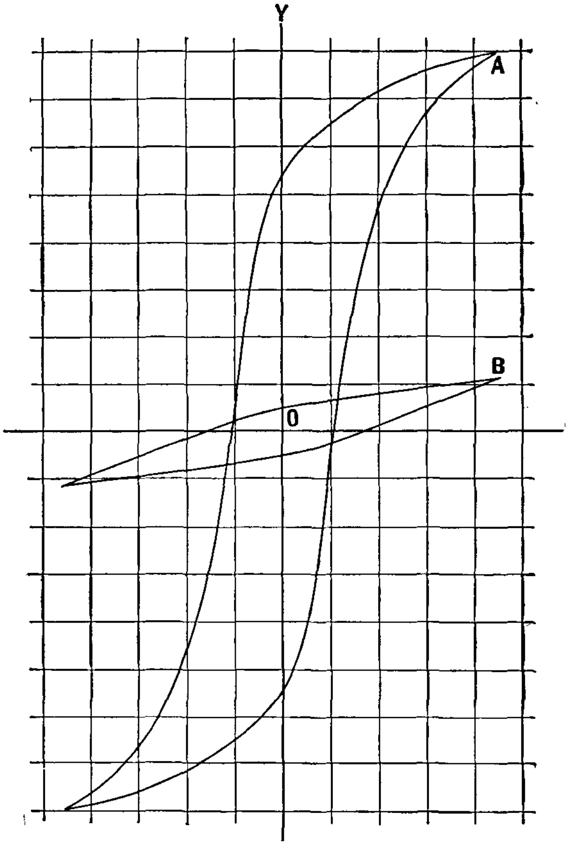

Figure 4.

It appears from the observations of Ewing upon Vicker's Tool Steel that in the case of the specimen he used the value of I was still rising, and at a fairly rapid rate, when H grew to be as great as 14,000. The same tendency, it will be noticed, is shown very clearly in the two kinds of steel which I have studied. These were chosen as being perhaps the best annealed brands of fine tool steel to be had in the market.

The very interesting results given in Table XI were obtained by Mr. John Coulson, who has helped me in all this work, with a standard cylinder, 1.283 centimeters in diameter, made of Jessops Tool Steel. This celebrated brand of steel seems harder under the file than the Stubs or the Crescent Drill Rod, but is remarkably permeable, and has been much used for permanent magnets.

TABLE XI.

CYLINDER OF JESSOPS ROUND TOOL STEEL.

H.	B.	I.	H.	B.	I.
110	15250	1205	960	19950	1510
158	16200	1280	1030	20100	1520
255	17450	1370	1200	20450	1530
500	18850	1460	1680	21100	1545
645	19100	1470	1980	21600	1560
810	19700	1505			

My thanks are due to the Trustees of the Bache Fund of the National Academy of Sciences who have kindly lent me some of the apparatus used in making the observations described in this paper.

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THE PROPERTIES OF AN ALUMINIUM ANODE.

By H. W. Morse and C. L. B. Shuddemagen.

Presented by John Trowbridge, December 9, 1908. Received January 6, 1909.

I. Introduction.

Many of the metals exhibit peculiar properties when used as anode with certain electrolytes in an electrolytic cell. Iron and chromium and, in less degree, nickel and several other elements, assume the so-called "passive state" under these conditions. Some other metals, among them aluminium, magnesium, tantalum, and niobium, show a still more striking change from their usual properties when the same conditions are imposed upon them. If the surface of metallic aluminium is kept free from the protecting film which usually covers it, it is rapidly attacked by the oxygen of the air. It is a familiar lecture experiment to carefully amalgamate a piece of clean aluminium by rubbing it with pure mercury. At the places where the mercury prevents the protecting oxide film from forming, the action of the air is so rapid that a white fibrous mass of oxide, several millimeters in thickness, grows up in a few minutes. While pure aluminium is very sensitive to an attack of reagents, it can under some circumstances act like a noble metal. As long as the film which forms on the surface retains its coherence aluminium is stable in the air, and even when it is used as anode in an electrolytic cell it may, under some circumstances, resist corrosion and solution to a surprising extent. Metals like copper and silver, which lie far down toward the negative end of the electromotive force series of metals, readily go into solution when used as anode in an electrolytic cell, but a plate of aluminium in contact with many electrolytes merely covers itself with a protecting layer and remains otherwise unattacked.

The protecting layer so formed offers a hindrance to the passage of a current through the cell as long as the aluminium plate remains the anode. If the current is reversed, the film no longer opposes the same resistance to its passage. These facts determine the use of aluminium in the rectification of an alternating current. Below a certain critical voltage, which is a function of the electrolyte and the temperature, the film which forms on an aluminium plate is a more or less efficient valve, which permits of the passage of an electric current in one direction and not in the other. The same facts determine the application of an aluminium plate as a condenser.

II. Historical.

Wheatstone (**74**) appears to have been the first to notice the anomalous behavior of an aluminium anode, and he mentions it merely in connection with an investigation on the position of various metals in the voltaic series. Soon afterward Buff (**10**) noticed the remarkable fact that a battery of nine Bunsen cells was insufficient to cause the flow of an appreciable current through a voltaic cell in which aluminium was anode. In 1869 Tait (**72**), using more delicate apparatus, measured the polarization produced at anodes of various metals and found a very high polarization electromotive force to be characteristic of aluminium. During the twenty years following this date a very great number of measurements on galvanic polarization and polarization capacity of electrodes were made all over the world, and the anomalous behavior of an aluminium anode was the subject of frequent notice. The first suggestion that this property might be made use of in the rectification of an alternating current appears to have been offered by Ducretet (**22**), and occasional suggestions of the possibility of using aluminium plates immersed in a proper electrolyte as a substitute for a static condenser are to be found in these earlier papers. The first actual measurement of the apparent capacity of such a cell is perhaps that of Streintz (**69**), who showed that a formed aluminium anode can be used in this way, measuring the capacity of the plate up to 28.8 volts. He assumed, as many others have done, that an aluminium anode acts like a nearly perfect condenser, and that a short time of insulation between charge and discharge introduces no error into the measurement and may therefore be neglected. Oberbeck (**52**) calculates the capacity per square centimeter of anode surface, and from this value, assuming a dielectric constant, he also calculates the thickness of the active insulating film. Application of aluminium plates immersed in an electrolyte as a substitute for an ordinary condenser for practical purposes was suggested by Haagn (**34**) in 1897. Pollak (**55**) had already tested the aluminium rectifier practically, and Graetz (**29**), working quite independently, also showed the possibility of applying the properties of an aluminium anode in the commercial rectičfiation of an alternating current. This was in 1897, and by far the greater part of the scientific and commercial investigations on the aluminium anode which have been made since that date have had direct reference to its application as a rectifier.

III. Polarization Capacity in General.

It has been known for a very long time that the changes produced in an electrolytic cell by the passage of a current resulted in setting up what is called the *counter electromotive force of polarization.* It was also recognized at an early date that a corresponding *polarization capacity* was a property specific for each metal in a given solution under given conditions of current density and temperature. Kohlrausch[1] was the first to offer a formal theory in connection with measurements made on various cells with an alternating current, and he showed that an equation of form

$$iR = E \sin \omega t - L\frac{di}{dt} - p\int idt$$

should hold, p being the counter electromotive force of polarization, which replaced the $\frac{1}{C}$ of the ordinary equation. The integrated form for the resulting wave contained a sine function and two exponentials whose value was negligible under the conditions of the experiment. If Kohlrausch's equation is true, it is evident that the current due to polarization must lead the applied electromotive force by 90°, while the lag due to inductance has the same value. He suggests the possibility of compensating the lag due to inductance by the introduction into the circuit of a polarization cell of the proper size. The current would thus be brought into phase with the applied electromotive force, and the current curve would then have the same form and position as if no inductance were present in the circuit. It has since been shown that Kohlrausch's simple theory does not hold for all the forms of galvanic polarization. It is possible to set up polarization cells in which the phase shift has any value from zero to 90°. The present theory has been given by Wien, Warburg, Elsa Neumann, and Krüger,[2] and the general equation for the polarization e. m. f. is

$$p = \frac{E}{C\omega}\sin\left[\omega t - \left(\frac{\pi}{2} - \theta\right)\right]$$

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[1] Pogg. Ann., **148**, 43 (1872).

[2] Wien, Wied. Ann., **58**, 37 (1896); and Drude's Ann., **8**, 372 (1902); Warburg, Wied. Ann., **67**, 493 (1899); Neumann, Wied. Ann., **67**, 499 (1899); Krüger, Ztsch. f. Phys. Chem., **45**, 1 (1903), and Drude's Ann., **21**, 701 (1906).

in which θ may have any value from zero to 90°. $-\left(\frac{\pi}{2}-\theta\right)$ is the lag of p behind E, and θ depends on the nature of the electrodes and the electrolyte.

This theory has been built up on the basis of Nernst's theory for the single electromotive force of an electrode in a solution containing its ion, and the theory fits the majority of cases very closely indeed. For most metals the maximum of polarization lies below three volts. This means that if we raise the electromotive force applied about the cell beyond three volts, the polarization no longer increases. This is true whether the electrode is a gaseous one, a reversible one (a metal in contact with a solution of its own ion), or any other combination of metal and electrolyte. In all of these cases there enters into the equation of the polarization electromotive force the ratio of concentrations in the ordinary Nernst form

$$E = \frac{RT}{nF}\, ln\, \frac{C_0}{C_1}.$$

It is then a familiar fact that the polarization electromotive force does not rise above three volts in any ordinary electrolytic cell. It is possible to raise the voltage of a cell having an aluminium anode at least as high as 500 volts, and it is possible to raise the voltage about cells having anodes of other metals, tantalum for example, to 1200 or even 1500 volts without reaching a point corresponding to the maximum of polarization as found for ordinary metals. If an aluminium anode has been properly "formed," that is, exposed to an electromotive force which is slowly increased step by step, the cell offers a remarkably complete barrier to the passage of a current. A small residual current flows through the cell under these circumstances, but this falls to a few milliamperes per square centimeter of electrode surface even when the applied electromotive force is measured in hundreds of volts. It seems quite evident that the process which takes place here is not polarization in the ordinary sense of the word. The substitution in the Nernst equation of the value for the "counter electromotive force" of a cell containing an aluminium anode leads to what appear to be absurd values for the ratio of the concentrations or the ion at the electrode and in the electrolyte.

IV. Theories of the Aluminium Anode.

The special characteristic of aluminium and a few other metals appears to be a film which forms on the metal when it is used as anode, and the various theories which have been put forward to explain the behavior of these metals are all connected with the nature of this film. The theories may be summarized as follows :

1. The anode becomes covered with a *thin oxide film* during electrolysis. This oxide film may produce the effects mentioned: (*a*) By opposing an actual ohmic resistance to the passage of the current in one direction : (*b*) By acting as a dielectric pure and simple : (*c*) By acting as a semi-permeable membrane which prevents the passage of the anion and permits the cation to pass freely.

2. The active film is a thin *layer of oxygen gas*. This acts as a dielectric, and the entire system is a true condenser. According to this theory the visible film on the aluminium plate, whatever its chemical composition may be, plays only a secondary part in the process. It serves merely as a support for the gas layer which is produced between it and the plate.

So far in the history of the subject no crucial tests have been found which can decide definitely in favor of one theory or the other. A resistance pure and simple seems insufficient to account for the facts. The resistance in this case must be a variable quantity, decreasing as the current increases, and it must furthermore be of a different order of magnitude in two directions through the cell. Nor do we need a "transition resistance" to explain the facts. There is evidence of the most trustworthy kind that oxygen plays a considerable part in the phenomenon, but it is just as evident that it is not necessarily the only factor. The semi-permeable film theory has much to support it. Membranes have been prepared by precipitating aluminium hydroxide on the surface of a platinum plate, and even in the pores of an earthenware cup, and these membranes are capable of exhibiting all the important peculiarities of an aluminium anode formed in the usual way by electrolysis. It seems evident that neither chemical investigation alone nor the measurement of electrical properties alone can give a satisfactory answer to all the questions which arise concerning the nature of the film and its action in the cell. Chemical investigation has shown that the film consists largely of aluminium oxide or hydroxide, and that oxygen gas is also invariably present in it, and this much we may certainly take as definitely determined.

In the earlier period of research on electrolytic polarization some measurements were made with galvanometers more or less ballistic in nature. Streintz (**71**) called attention to the fact that the discharge from an aluminium plate used as anode consists essentially of two parts, one of which was of the nature of a condenser discharge practically complete within a fraction of a second. The other portion of the discharge, which is superposed over the first portion, takes place more slowly and is therefore difficult to measure by ballistic methods.

When alternating currents came into common use, they were immediately applied in the study of polarization, and the great majority of the measurements which have been made in late years on aluminium anodes have been made by alternating current methods. We have thought it best to return to the older and more difficult method of the ballistic galvanometer, for previous investigations have shown that the film changes very rapidly in properties from the time a current begins to pass through it, and that every change in the electrical condition of the circuit is accompanied by a time change in the film itself. Alternating current measurements cannot give the details of this change, but only an integrated result.

V. Experimental Results.

In beginning these measurements we had clearly in mind the difficulties mentioned by Streintz (**71**). The total discharge from an electrolytic cell having an aluminium anode extends over a considerable time, and it would seem, therefore, at first sight, that a ballistic method would be poorly adapted to the study of it. It was found, however, by using several ballistic galvanometers of different period, that the error due to the slow residual charge could be neglected; by using plates of considerable surface and low resistance ballistic galvanometers of rather long period, it was practically eliminated.

Anodes were formed either from a storage battery or from a dynamo current, and after formation they were charged from a storage battery of small cells capable of giving over 500 volts. The time measurements were made by the apparatus shown in Figure 1. This is merely a simple machine which allows a heavy weight to fall, contacts being made and broken by the weight as it passes. The switches to be opened or closed

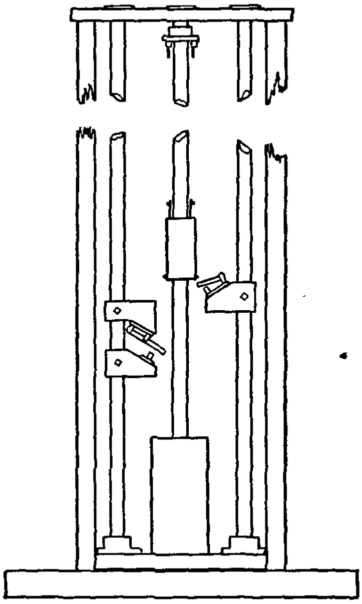

Figure 1.

Apparatus for charging and discharging condensers.

are clamped to the side rods and the times are calculated from the velocities of the falling weight as it meets the switches. The maximum time of charge, discharge, or insulation which can be obtained with this apparatus is about 0.6 seconds. Longer times than this are measured with a stop watch. The minimum time is limited only by the delicacy of the contacts used, as they must always be made strong enough to withstand the heavy blow of the falling weight. The minimum time in most of our experiments is of the order of 0.001 seconds, and this can be measured with considerable accuracy. Three ballistic galvanometers were used in this work. Where a long series of measurements was to be made, involving a large range of capacities, the first readings at higher voltages were made on the least sensitive galvanometer, and as the voltage was decreased until the throw of this galvanometer was no longer sufficient to give the necessary accuracy, connections were thrown over to the second and more sensitive galvanometer and readings continued with its aid. The periods of the galvanometers were 1, 4, and 9 seconds respectively. It has already been mentioned that the discharge

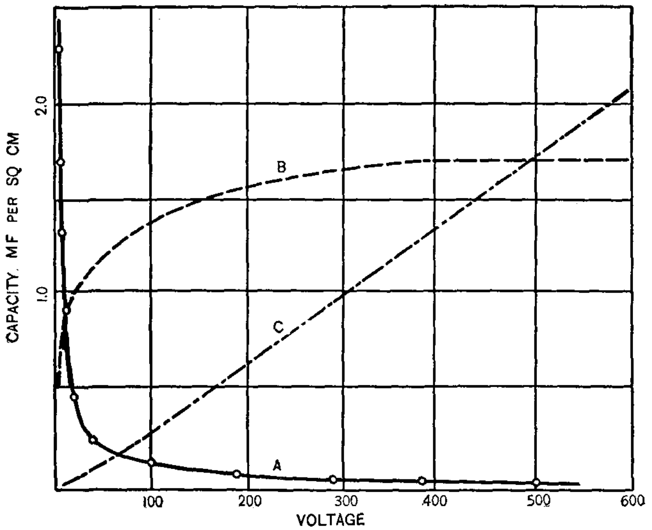

FIGURE 2.

A. Capacity of an aluminium condenser at various formation voltages.
B. Quantity $= C \times V$ from curve A.
C. Energy $= C \times V^2$ from curve A.
Long charge. Long discharge. Insulation time, 0.002 seconds.

from an aluminium film may be considered to consist of two portions, one of which takes place so slowly that part of it passes through the circuit even after the slowest of our galvanometers begins its swing. The results of careful preliminary tests made it probable that this error would be negligible in our measurements, and the experimental results all confirm this assumption. The galvanometers were calibrated against standard mica condensers charged from a storage battery, and the calibration was repeated several times during the progress of the measurements.

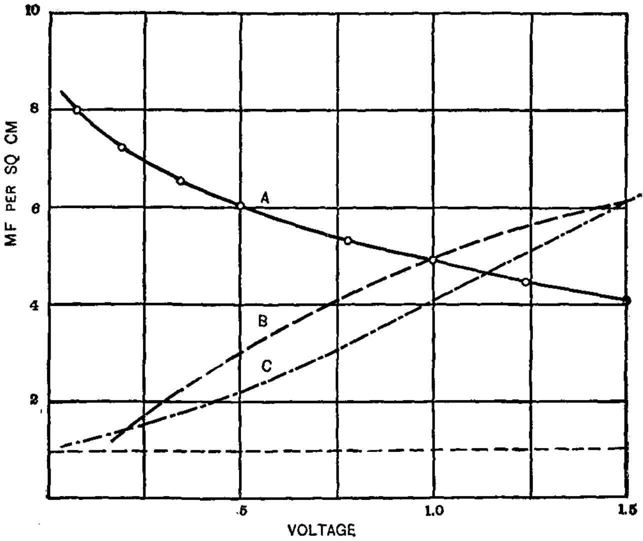

FIGURE 3.

Capacity (A), Quantity (B), and Energy (C) curves for low formation voltages. From tables of Scott (**64**) and our own measurements.
Long charge. Long discharge. Insulation time, 0.002 seconds.

1. *Apparent Capacity and Forming Voltage.* — In Figure 2 the apparent capacity in microfarads per square centimeter of anode surface is plotted in curve A against the forming voltage applied to the cell, the charging voltage being in this case the same as the forming voltage. The following factors are constant throughout this curve: charging time, 1 minute; insulation time, 0.002 sec.; discharge time, complete. The cell was left short-circuited through the galvanometer. It is evident that the curve approaches an hyperbola in its general course, and it has been assumed by Gordon (**27**), Corbino and Maresca (**17**), Schultze

(**58**), and others, that it is an equilateral hyperbola, and that therefore the product of apparent capacity and forming voltage is a constant. A careful examination of the data of the curve shows that this is by no means the case. The curve marked B gives the values for the product capacity × forming voltage (in this case applied voltage also), and this should be of course a straight line parallel to the X axis if the product is to be constant. The third curve, C, of Figure 2 gives the value for the energy per square centimeter stored in an aluminium anode when various voltages are applied to it, and this is very nearly a straight line with only a slight curvature for voltages lower than 100. Figure 3 indicates the characteristics of these curves at very low voltages. The data for this particular curve were taken from the measurements of Scott (**64**), but it is in close agreement with our own results in the same voltage range. It is quite evident that the product of capacity and voltage is not constant, and for these conditions the curvature in the energy curve is also more evident. The values obtained for the capacity of an alu-

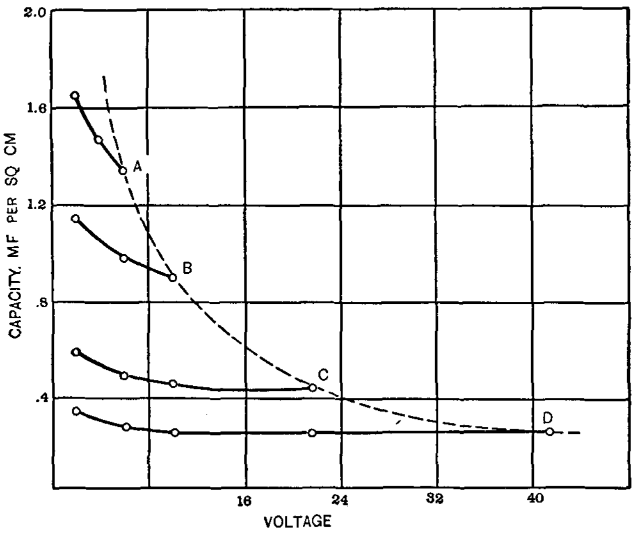

FIGURE 4.

Capacity at less than forming voltage. For the lower range of voltages. Same times of charge, discharge, and insulation as in Figures 2 and 3.

Curve A.	Formed at	6	volts.	Measured at	6, 4, and 2 volts.
B.	"	10	"	"	10, 6, and 2 volts.
C.	"	21	"	"	21,10, 6, and 2 volts.
D.	"	41.6	"	"	41.6, 21, 10, 6, and 2 volts.

minium anode at voltages below two volts are of the same order as polarization capacities found for other metals in electrolytic cells. They are, however, smaller than most of these, the maximum value observed for aluminium being about 8 microfarads per square centimeter, while other metals often show several times this capacity.

An examination of the various measurements we have made on different aluminium anodes shows a remarkable agreement in properties. It is possible to reproduce a capacity with different samples of aluminium, with electrodes of different area, but which have been formed at the same voltage, with an accuracy apparently as great as 2 per cent. Other factors, such as temperature, electrolyte, time of charge, discharge, and insulation, etc., must of course be kept constant, but when these conditions are met, and notwithstanding the complex nature of the film involved, the capacity is a very accurate function of the voltage at which the plate has been formed.

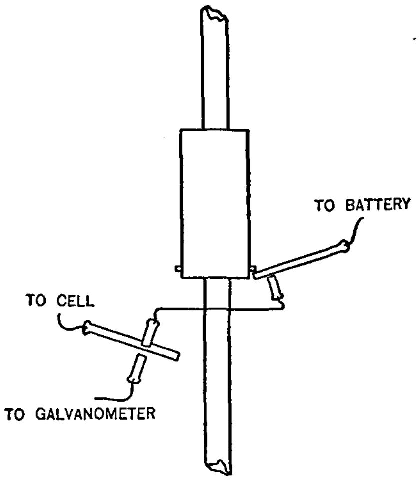

FIGURE 5.

Arrangement of switches for varying short charge. Short insulation and long discharge.

2. *Capacity below Forming Voltage.*—Figure 4 gives the results of a set of measurements of capacity at voltages less than forming voltage, and this family of curves gives an indication of the complexity of the active film. The same times of charge, discharge, and insulation as were used in the previous measurements were maintained in these.

The plate was first formed at 6 volts, and measurements were taken at 6, 4, and 2 volts. The results are plotted in the upper curve. Formation was then continued, and completed at 10 volts, and the results of measurements at 10, 6, and 2 volts are given in the second curve. The other curves give similar results up to a forming voltage of 41.6, measurements being made in each case at the forming voltage and then at several lower voltages. It will be seen from this figure that whatever the nature of the film may be, and whatever the mechanism by which it acts, the capacity is greater at voltages lower than the forming voltage through the range of voltages indicated. The dotted curve is a portion of the capacity curve of Figure 2.

Capacities at various voltages below forming voltage have been measured by other investigators. Corbino and Maresca (**17**) give several tables of data on the point, but all of their results are in contradiction to the ones we have obtained. They find that in every case capacity at voltages below forming voltage is lower than at forming voltage itself. Figure 4 expresses the average of a great many observations, and further confirmation of the correctness of these results will be found in Figures 11 and 12, which give data on apparent capacity below forming voltage after the cell has been left on open circuit for varying lengths of time. The matter is a complex one, and can only be considered as a whole after the other factors involved have been taken up individually. Reference to Figure 17 shows that the capacity is not under all circum-

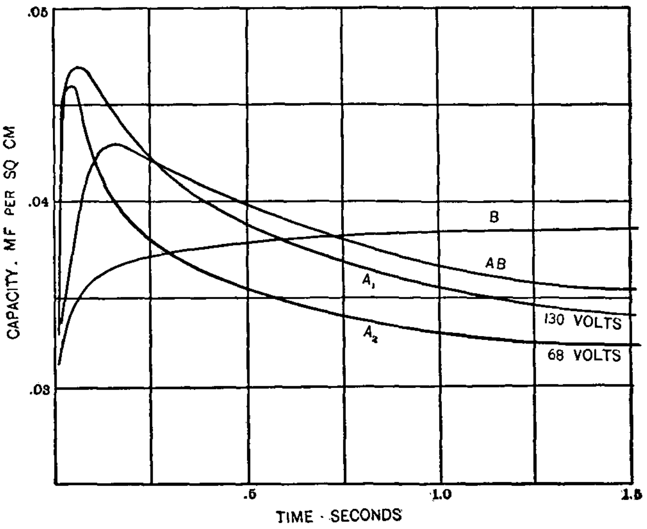

FIGURE 6.

Charging time curves. A, for very fully formed plate. B, ordinary curve for average plate. AB, intermediate condition. Insulation time, 0.002 seconds. Full discharge.

Curve A_1.	Formed at	340 volts.	Measured at	130 volts.
A_2.	"	340 "	"	68 "
B.	"	340 "	"	130 "
AB.	"	340 "	"	130 "

stances higher below forming voltage than at this point. It may in fact be either lower or higher than the capacity at forming voltage. It will be seen from Figure 17 that if one is working with long charge, short insulation time, and long discharge, the capacity is represented by the curve marked A. Under these conditions the apparent capacity of the plate is greater at low voltages and less at intermediate voltages than it is at the forming voltage itself. This matter will be taken up more fully after the other factors have been discussed.

3. *Short Charge.* — Figure 5 shows the arrangement of apparatus for measuring the apparent capacity of an aluminium anode after it has been charged for variable short periods of time. The falling weight closes the upper switch, thus completing the circuit from the storage battery through the cell. Falling further, the weight opens this same circuit, and immediately afterwards closes the circuit from the cell through the galvanometer. The insulation time, which is determined by the distance between the two lower switches, is kept constant at 0.001 second, and the variable charging time is fixed by moving the upper switch up or down on the side rod. Reference to Figure 1 will make this clear.

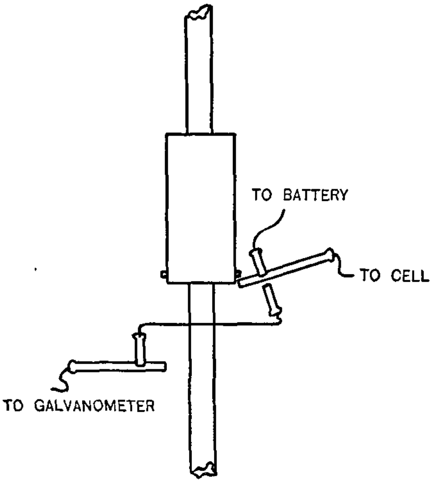

FIGURE 7.

Arrangement of switches for long charge. Short insulation and varying short discharge times.

The results of measurements made in this way show that factors still undetermined play an important rôle. The previous history of the plate becomes of great importance, and wholly different results are obtained from plates which have been formed slowly and carefully and from those which have been hastily formed, or which have been exposed directly to the voltage of the experiment without previous formation.

Figure 6 gives a set of characteristic curves of apparent capacity (ordinates) for various short charging times (abscissas). The two curves marked A_1 and A_2 are characteristic of a plate which has been very carefully and fully formed. This plate was formed at 340 volts, and the two A curves were taken at 130 volts and 68 volts respectively. Under these conditions the shape of the curve is a remarkable one. It evidently takes time for the film to attain its optimum condition, and this was to be expected. But the apparent capacity begins to decrease again after a short time of charge, and this result was an unexpected one.

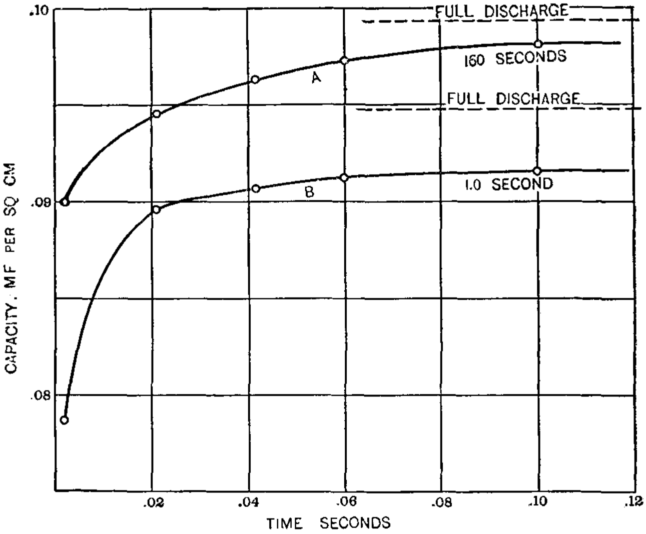

FIGURE 8.

Discharge curves at lower voltages for a very fully formed plate. Plate formed at 140 volts, charged at 67 volts.

We have found similar results for several plates, and there is no reason to doubt that such curves correspond to real physical conditions.

The curve marked B may be taken as representative of another series of measurements on other plates, and this curve we have also found repeatedly. It corresponds to a difference in the previous history of the plate under examination and apparently belongs to incomplete or rapid formation. While the apparent capacity of an "A" plate has its maximum value for a charging time of 0.03 to 0.1 second, that of a "B" plate increases with charging time without passing through a maximum, becoming asymptotic within a few seconds to the value found for a very long time of charge.

We have also found occasionally curves similar to that marked AB. This appears to correspond to a condition of formation intermediate between the two others.

There is evidently a close connection between the data of Figure 6 and the results to be expected from a study of an aluminium condenser under the action of an alternating current. As will be seen from succeeding figures, the relation will be a complicated one, because of the influence of insulation time and discharge time.

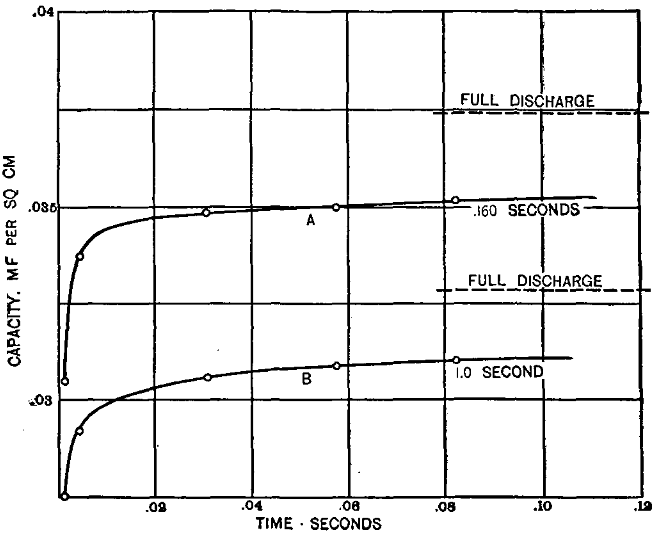

FIGURE 9.

Discharge curves at higher voltages for fully formed plate, as in Figure 8. Plate formed at 340, charged at 195 volts.

4. *Short Discharge.* — Figure 7 shows the arrangement of apparatus for measuring the apparent capacity during a short time of discharge. The insulation time is kept constant at 0.001 second. The charging time, which determines a difference in capacity, as shown by the previous figure, has been given two different values. As shown in the figure, the apparatus is arranged for long charging times, the upper switch being closed and thus connecting the cell with the charging battery. The falling weight opens the charging circuit and closes the discharge circuit after the period of insulation; the weight falling further opens the galvanometer circuit when it strikes the lower switch. The apparatus for measuring the capacity for short

discharge time after a short charging time is different only in the fact that the falling weight closes the charging circuit as it descends, the remainder of the switches being thrown as already indicated.

The data for Figures 8 and 9 was taken on the same plates as were used in obtaining the A curves of Figure 6, and they show again the fact to which attention was called at that point: the apparent capacity is, for all times of discharge, greater for a short time of charge than for a longer one. The dotted lines indicate full discharge. The cell is left short-circuited through the galvanometer to obtain this value.

For a plate similar to that which gave the B curve of Figure 6 the A and B curves of Figures 8 and 9 will merely exchange positions. In this case a longer charge corresponds to a greater apparent capacity for all times of discharge.

Plates having charge-time characteristics like those shown in the AB curve of Figure 6 will show a corresponding set of discharge curves.

The plate of Figure 8 was formed at 140 volts, and both the curves were taken with an applied voltage of 67 volts. The plate of Figure 9 was formed at 340 volts, and the working voltage was 195.

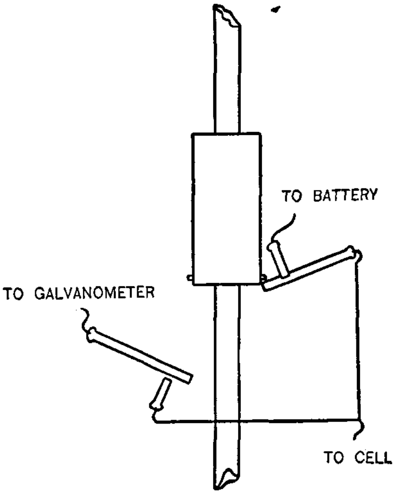

FIGURE 10.

Arrangement of switches for long charge, long discharge, and varying short insulation times.

C. *Insulation Times.* — Figure 10 shows the arrangement of switches for the third of the time factors, variable periods of insulation. As the figure is drawn arrangement is made for long charging times, the upper switch being closed, so that the current passes from the charging battery through the condenser until it is opened by the falling weight. This opens all the circuits, and the cell is then closed through the galvanometer after an insulating time depending upon the distance between the two lower switches. Measurements with short charging times were also made, and for this purpose a third switch is introduced higher up, which is closed first of all by the falling weight. Figures 11 and 12 give the results of these measurements for a constant long charging time (1 minute), complete discharge, and a variable time of insulation. The curves for short charging times are similar in form, but lie a little above or below the curves given. These curves show very clearly the point already mentioned, that such a condenser can under certain conditions act more perfectly at voltages below the voltage of formation.

It is also of interest to know the shape of the leak-curves at the forming voltage itself. Data on this factor is given in Figure 13 for plates formed at 36, 80, 140, and 300 volts. They offer one means of examining the change which takes place in the active film during insulation, but they are complicated by all the other factors involved, and it seems probable that the study of such curves can only lead to a definite solution of the problem when they are examined in connection with the other variables. They do not appear to follow any simple exponential formula.

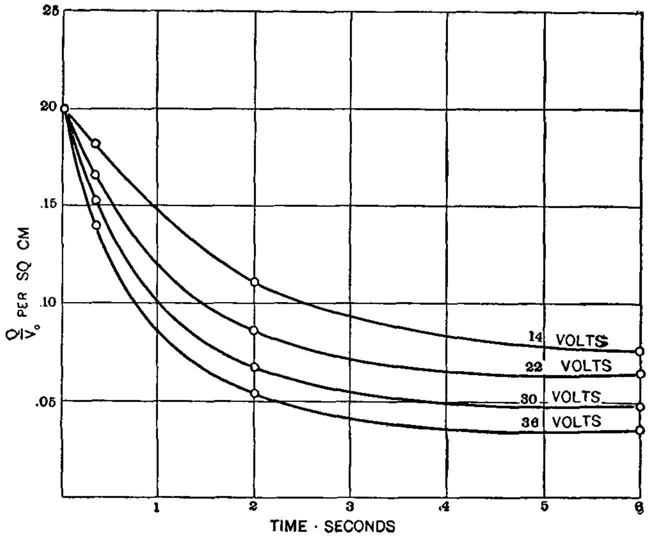

FIGURE 11.

Capacity *vs.* insulation time. Plate formed at 36 volts. Curves for 36, 30, 22, and 14 volts. Long charge. Long discharge.

It should be noted that in these last cases we have not measured a true capacity, but values of Q/V after various times of insulation. The "condenser" is so leaky that even during a very short time of insulation it loses a considerable portion of its charge. The actual capacity could only be found by a method which permitted of the measurement of the voltage about the cell immediately before the discharge through the galvanometer began. It would therefore be better to consider the ordinates in some of our curves as Q/V, rather than *apparent capacity*. This applies to Figures 11, 12, 13, 14, and 15.

In any case our condenser is a very leaky one indeed as compared

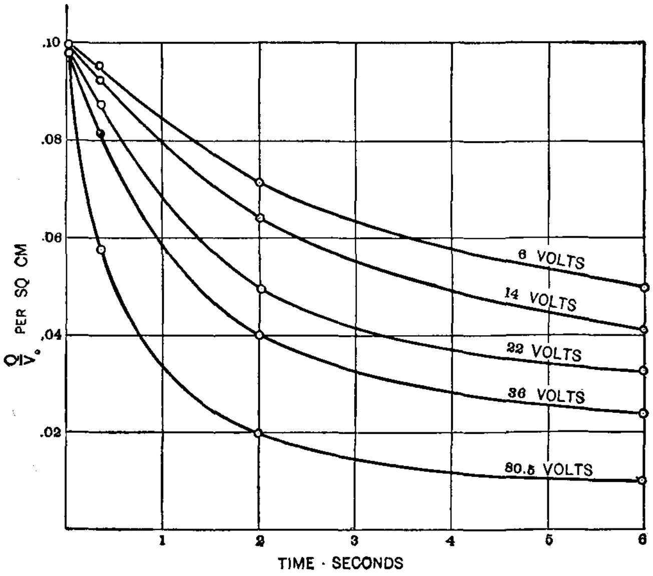

FIGURE 12.

Capacity *vs.* insulation time. Plate formed at 80.5 volts. Curves for 80.5, 36.5, 22, 14, and 6 volts. Long charge. Long discharge.

with a static condenser of even the poorest construction, but the difference in the leakage losses at the forming voltage and at a much lower voltage is very great for considerable insulation times. As the insulation time is made shorter and shorter, the difference in the capacity at various voltages becomes less and less, and for very short insulation times the capacity is practically the same for all voltages below the forming voltage. These differences are clearly shown in Figures 14 and 15. In these two figures capacity is plotted against applied voltage, and the curves represent various insulation times. It will be seen that the curve for short insulation time indicates a practically constant capacity at all voltages below the forming voltage. The curves of Figures 11 and 12 may be regarded as tests showing the approximation to true condenser action which is attained with aluminium electrodes. For an ordinary mica or paper condenser the rate of leak during insulation is of such a form that the charge remaining in the condenser is

$$Q = Q_0 \, e^{-\frac{t}{CR}}.$$

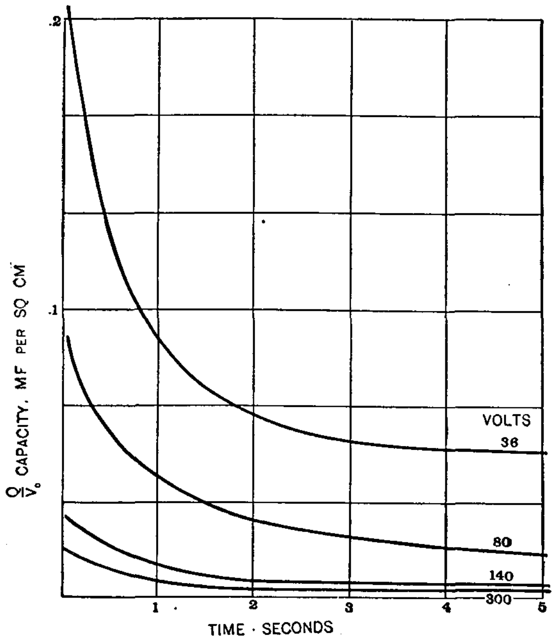

FIGURE 13.

Capacity *vs.* insulation time at various *forming* voltages. Long charge. Long discharge.

If the logarithm of the remaining charge is plotted against insulation time, the resulting curve is a straight line. Figure 16 shows the curves obtained by plotting the data of Figure 12 in this way. It is quite evident from these curves and from the results of time measurement on charge and discharge that we are not dealing with a true condenser. It will be noticed that at voltages far below that of formation the curve of leak follows the logarithmic formula quite closely. In all the curves we have plotted there is, however, a perfectly definite curvature near the beginning of the curve. This point will probably be found of importance in the study of the *efficiency* of aluminium condensers. It is evident from the data at hand that the separation of the effect of capacity from the effect of resistance can probably not be carried out by ballistic measurements on these films. One method which would probably be successful in the separation of these two factors would involve the study of resonance conditions in circuits containing capacity, resistance, and inductance.

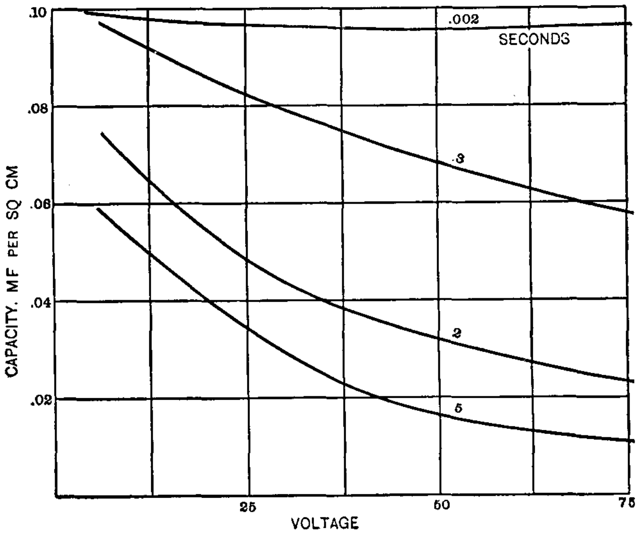

FIGURE 14.

Capacity *vs.* voltage (below forming voltage) for various insulation times. Curves for times .002 seconds, 0.3 seconds, 2.0 seconds, and 5.0 seconds. Lower range of voltages.

6. *Variations in Both Charging Time and Insulation Time.* — We have collected a large mass of data on individual cases in which both charging time and insulation time are varied. This data does not appear at present to be of sufficient value to warrant publication as a whole. The general course of the curves is shown in Figure 17, and the times are indicated below that figure. Several of the facts already mentioned are evident from this figure. The variation in capacity below forming voltage is clearly seen, and the change which takes place as the insulation time is increased is also plain. Similar curves

were found for all voltages, and this set of curves may therefore be considered characteristic.

7. *Three Dimensional Diagrams.* — In the five succeeding figures some of the factors so far studied are plotted in groups of three. It would require a great deal of space and many figures to represent all our data in the usual way, and the conclusions which can be drawn are so far not of a sufficiently quantitative nature to demand great accuracy in the presentation of data. It is easier to grasp the meaning of the data when it is arranged as compactly as possible. We have therefore made use of curves in place of tables of data, and it is hoped that the three-dimensional diagrams will take the place of the large number of curves which they represent.

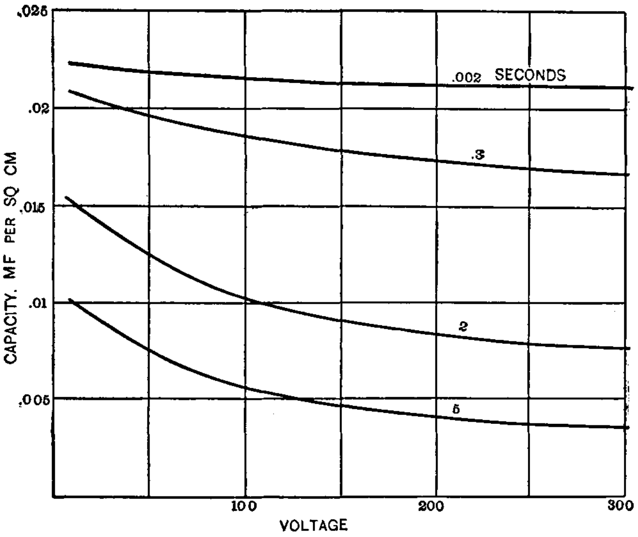

FIGURE 15.

Same as Figure 14 for higher range of voltages.

Figure 18 is a composite figure in which apparent capacity, charging time, and forming voltage are plotted together, the charge being given at the forming voltage.

The diagrams represent the results which we obtained with an aluminium anode which was rather hastily formed for part of the measurements and very carefully and slowly formed later in the series. The low voltage curves therefore show no maximum of charge for a short charging time, while the curves taken at higher voltages after very slow formation show such maxima. The dotted curves are the A curves of Figure 2, and it is evident that these curves may not be the same over the whole sheet which they enclose. This variation, if any exists, we have not yet sifted out from the mass of experimental data.

In Figure 19 apparent capacity, discharge time, and forming voltage are plotted together. At low voltages the discharge curve runs up rather slowly. As the voltage is increased the curve rises more quickly and the turn toward the asymptote (full discharge) is sharper. Here again the dotted lines are A curves of Figure 2, as in Figure 18, and here also it seems very probable that there is variation in the shape of these curves across the sheet which they enclose.

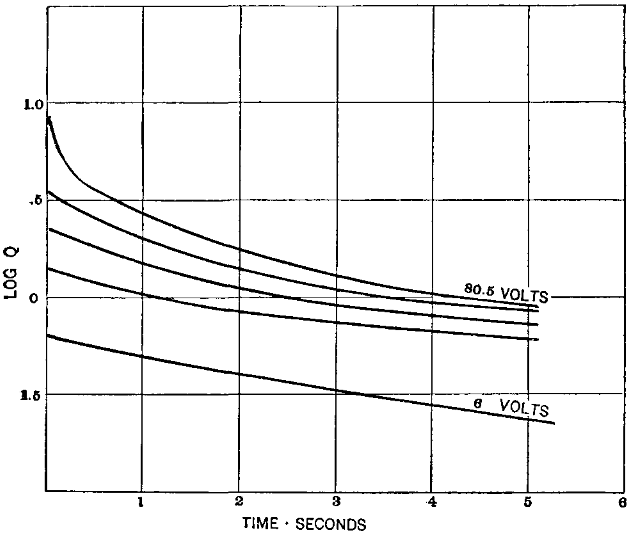

Figure 16.

Test of character of leak and of formula $Q = Q_0 e^{-\frac{t}{CR}}$. Log Q *vs.* insulation time.

In Figure 20 apparent capacity, insulation time, and voltage (below forming voltage) are expressed in one diagram. The full curves are a family similar to that in Figures 11 and 12, and the dotted curves are those of Figures 14 and 15, each a line of constant insulation time. It is probable that these curves turn upward rather sharply at very low voltages, but we have only a few scattered observations on this point.

Figure 21 has apparent capacity, insulation time, and *forming voltage* as its co-ordinates, the charging voltage being that of formation. The heavy curves are similar to those of Figure 13, and the dotted lines are now A curves of Figure 2, since capacity is measured at the voltage of formation.

Finally, in Figure 22 we have plotted the apparent charge of a plate formed at various voltages, and measured at various voltages below that of formation. The full curves are somewhat like those of Figure 4 and the A and B curves of Figure 17. This means that the plate (average formation assumed) is being given a fairly long charge and a short period of insulation. The curves, therefore, will in general rise at rather low working voltage, and the sheet will be somewhat hollow. The projections of these curves are indicated on the plane at the left. These curves have not the same numerical value as those of Figures 4 and 17, but they are somewhat similar in shape, the ordinates being

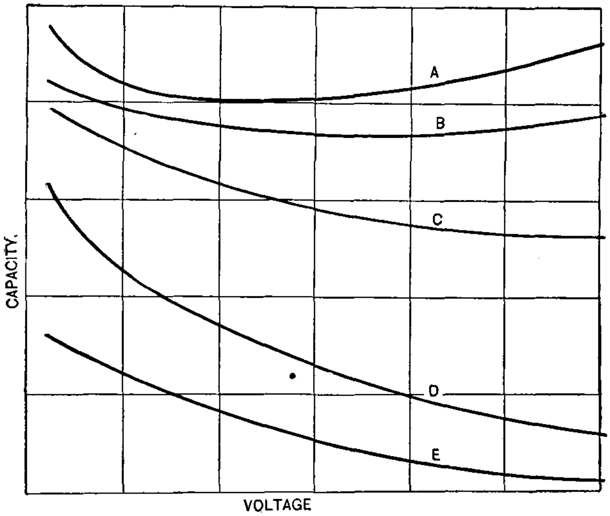

FIGURE 17.

Capacity *vs.* voltage (below formation voltage) for various time combinations.

Curve A.	Charging time,	long.	Insulation time,	.002 sec.	Disch. time, long.
B.	"	0.5 sec.	" "	.002 sec.	" " "
C.	"	0.16 sec.	" "	3.00 sec.	" " "
D.	"	long.	" "	2.0 sec.	" " "
E.	"	long.	" "	5.0 sec.	" " "

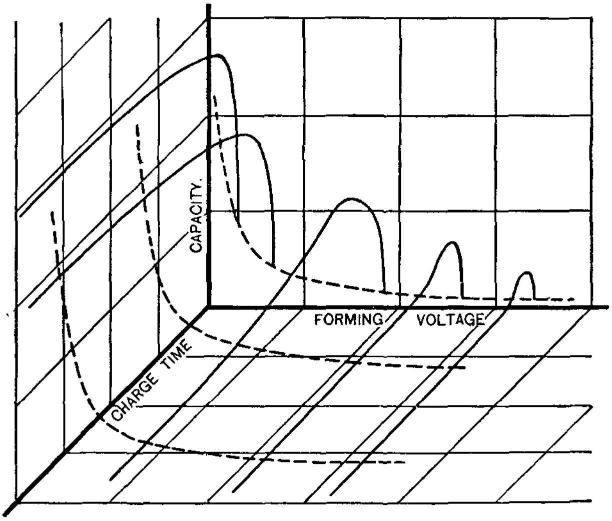

Figure 18.

Capacity *vs.* charging time at various forming voltages. The dotted curves correspond to the A curve of Figure 2.

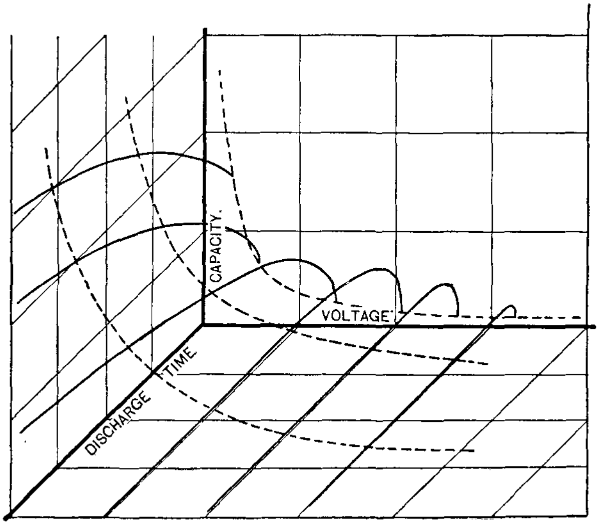

Figure 19.

Capacity *vs.* discharge time at various forming voltages. The dotted lines correspond to the A curve of Figure 2.

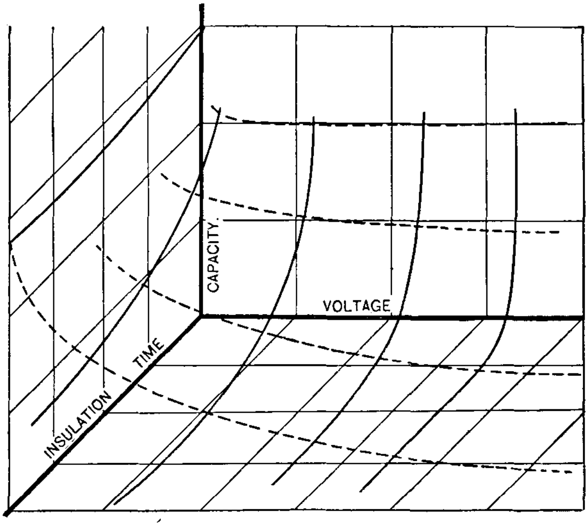

FIGURE 20.

Capacity *vs.* insulation time at various voltages below forming voltage. The dotted lines correspond to the curves of Figures 14 and 15.

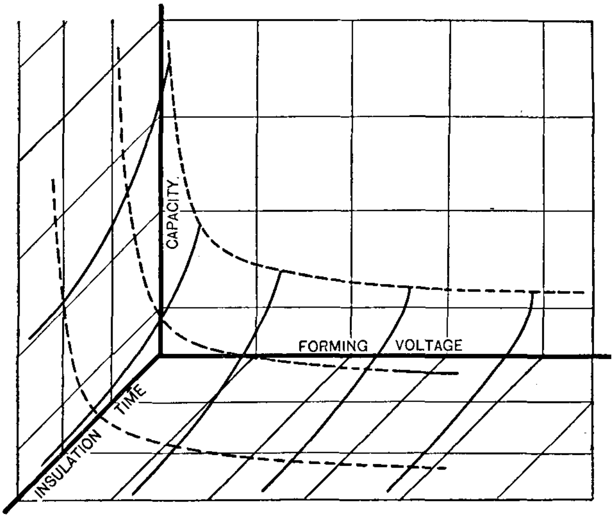

FIGURE 21.

Capacity *vs.* insulation time at various forming voltages. The dotted lines correspond to the A curve of Figure 2.

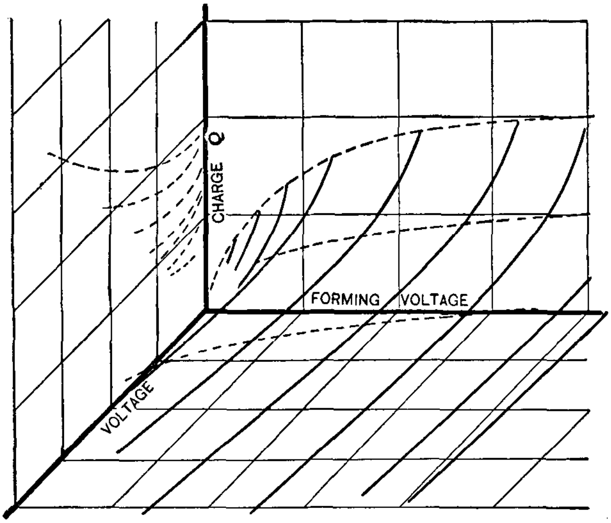

FIGURE 22.

$K = CV$ *vs.* voltages for plates formed at various voltages. The curves parallel to the plane of the paper correspond to the B curve of Figure 2. The dotted lines on the YZ plane are traces of the main curves (full line) on this plane.

obtained by multiplying by a constant (the forming voltage). The dotted lines will then be B curves of Figure 2 as far as they go. They are of course limited by the fact that the plate is only charged at voltages *less* than the formation voltage.

VI. THE FACTORS WHICH DETERMINE CAPACITY.

Summary. — It would appear that the following factors all enter into what we have been calling the *apparent capacity* of an aluminium anode:

1. Formation voltage.
2. Mode of formation (time, voltage-steps, etc.).
3. Applied voltage.
4. Time of charge.
5. Time of insulation.
6. Time of discharge.
7. The electrolyte.
8. Temperature.
9. Electrical constants of the circuit outside the cell.

Some of these factors appear to have a more definite influence, or rather a less complicated influence, than others. Temperature, for example, must be considered a *more general* factor than the others. They are, however, apparently all independent variables within certain limits, and a complete expression for the action of an aluminium anode must include all of them.

It may be of interest to attempt to segregate the effects produced by variation of these factors.

1. Formation voltage may determine —
   (*a*) Thickness of an oxide or hydroxide film.
   (*b*) Density and thickness of a gas film.
   (*c*) The perfection of a semi-permeable membrane.
2. Time of formation (and history of formation in general).
   Same as 1.
3. The applied voltage may determine —
   (1 (*a*) should remain constant for various applied voltages below the voltage of formation unless solution by the electrolyte or other disintegrating action takes place)
   (*a*) Thickness and density of a gas film.
   (*b*) Ionic concentration within the active layer of the film.
4. Time of charge (complete formation assumed) may determine —
   (*a*) The thickness (distributed) of an insulating or other active film.
   (*b*) The ionic concentration within the active layer of the film.
5. Insulation time may determine the rate of return to the uncharged condition —
   (*a*) By disintegration of an insulating solid film.
   (*b*) By gas diffusion.
   (*c*) By ionic diffusion.
6. Time of discharge may determine —
   Factors similar to those in 5, but under conditions varying with the electrical constants of the discharge circuit.
7. The electrolyte may determine the entire activity or non-activity of the anode —
   (*a*) By the ions it furnishes, which may or may not be able to pass the film it forms (semi-permeable film theory).
   (*b*) By its solvent action on the film.
8. Temperature affects all the above.
9. Electrical constants of the circuit can affect 4 and 6 especially.

All of the effects enumerated are quite open to study, and some of them have already been investigated. The authors hope to offer further data on some of these variables in the near future.

It is evident from this summary that alternating current methods of measurement will give much simpler and in some respects more useful results than the ballistic method. If a definite wave-form and a definite frequency are available, we have at once disposed of charging time, insulation time, discharge time, and the constants of the circuit. Making these factors constant is a very great simplification, and the other factors can be approached much more easily than by any ballistic method. But the factors mentioned are of scientific interest, and accurate study of their variations leads to analytical results which could hardly be obtained by the aid of alternating current measurements.

Jefferson Physical Laboratory,
Harvard University.
December 23, 1908.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF CHROMIUM.

FIRST PAPER. — THE ANALYSIS OF SILVER CHROMATE.

By Gregory Paul Baxter, Edward Mueller, and
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Presented January 13, 1909. Received December 11, 1908.

Introduction.

The following table[1] gives the results of investigations upon the atomic weight of chromium from the time of Berzelius, recalculated with the use of recent atomic weight ratios upon the basis of silver (107.88) and oxygen (16.000).[2]

The value chosen by the International Atomic Weight Committee, 52.1, which is based chiefly upon the more recent determinations, seems to be fairly close to the truth, with an uncertainty of one tenth of a unit.

It has been repeatedly shown, especially in this laboratory, that most of the earlier work upon atomic weights has been vitiated by neglect of certain fundamental precautions. The incomplete drying of solids has been responsible for many of the discrepancies and errors which exist. Neglect of the solubility of precipitates, together with the use of too concentrated solutions during precipitation, so that perceptible inclusion and occlusion took place, undoubtedly have influenced many gravimetric processes. Volumetric processes have been affected by inaccurately prepared standard solutions, as well as the difficulty inherent in measuring exactly large volumes of solution.

In discussing in detail the applications of the above causes of constant error to the individual investigations, at the best it is only pos-

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[1] Clarke, A Recalculation of the Atomic Weights, Smith. Misc. Coll., 1897.

[2] The following atomic weights are used in the recalculation of the older values: Ag = 107.88; Cl = 35.457; Pb = 207.09; N = 14.01; Ba = 137.37; S = 32.07; H = 1.008; K = 39.095; As = 74.96; I = 126.92. The values of Rawson and Meineke are reduced to the vacuum standard; the others are not so corrected.

Date.	Investigator.	Ratio Determined.	Atomic Weight.
1818	Berzelius[3]	$Pb(NO_3)_2 : PbCrO_4$	55.95
1844	Peligot[4]	$CrCl_2 : 2AgCl$	52.33
		$2CrCl_2 : Cr_2O_3$	51.58
		$4AgCl : Cr_2O_3$	51.61
1846	Berzelius[5]	$BaCrO_4 : BaSO_4$	54.5
1846	Berlin[6]	$Ag_2CrO_4 : 2AgCl$	52.65
		$2Ag_2CrO_4 : Cr_2O_3$	52.41
		$Cr_2O_3 : 4AgCl$	52.46
		$Ag_2Cr_2O_7 : 2AgCl$	52.11
		$Ag_2Cr_2O_7 : Cr_2O_3$	52.34
1848	Moberg[7]	$Cr_2(SO_4)_3 : Cr_2O_3$	53.42
		$(NH_4)_2Cr_2(SO_4)_4, 24H_2O : Cr_2O_3$	53.46
1850	Lefort[8]	$BaCrO_4 : BaSO_4$	53.04
1853	Wildenstein[9]	$BaCl_2 : BaCrO_4$	53.56
1855	Kessler[10]	$K_2Cr_2O_7 : KClO_3$	52.23
1861	Kessler[11]	$K_2Cr_2O_7 : KClO_3$	52.32
		$2K_2Cr_2O_7 : 3As_2O_3$	51.92
1861	Siewert[12]	$CrCl_3 : 3AgCl$	52.05
		$Ag_2Cr_2O_7 : 2AgCl$	52.14
		$Cr_2O_3 : 2AgCl$	52.04
		$Cr_2O_3 : Ag_2Cr_2O_7$	52.05
1884	Baubigny[13]	$Cr_2(SO_4)_3 : Cr_2O_3$	52.13
1889	Rawson[14]	$(NH_4)_2Cr_2O_7 : Cr_2O_3$	52.09
1890	Meineke[15]	$(NH_4)_2Cr_2O_7 : Cr_2O_3$	52.11
		$2Ag_2CrO_4 : Cr_2O_3$	52.10
		$Ag_2CrO_4 : 2AgCl$	52.03
		$4AgCl : Cr_2O_3$	52.14
		$2Ag_2CrO_4 4NH_3 : Cr_2O_3$	52.27
		$Ag_2CrO_4 4NH_3 : 2AgCl$	51.62
		$4AgCl : Cr_2O_3$	52.14
		$Ag_2CrO_4 : 3I$	52.41
		$Ag_2CrO_4 4NH_3 : 3I$	52.05
		$K_2Cr_2O_7 : KHIO_3$	52.14
		$(NH_4)_2Cr_2O_7 : KHIO_3$	52.13

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[3] Pogg. Annalen, **8**, 22 (1826).
[4] Ann. Chim. Phys., (3), **12**, 530 (1844).
[5] Berzelius' Jahresbericht, **25**, 46 (1846).
[6] J. prakt. Chem., **37**, 509; **38**, 149 (1846).
[7] Ibid., **43**, 114 (1848).
[8] Ibid., **51**, 261 (1850).
[9] Ibid., **59**, 27 (1853).
[10] Pogg. Annalen, **95**, 208 (1855).
[11] Ibid., **113**, 137 (1861).
[12] Zeit. gesammte Naturwissenschaften, **17**, 530 (1861).
[13] Compt. Rend., **98**, 146 (1884).
[14] J. Chem. Soc., **55**, 213 (1889).
[15] Liebig's Annalen, **261**, 339 (1890).

sible merely to indicate the nature of the difficulties; as a rule it is impossible to estimate the magnitude of the error without repetition of the experimental work. Hence in this paper attention is called only to points in the earlier work which have been experimentally investigated. The uncertainty in most of the previous determinations is emphasized by the lack of agreement in the individual analyses in each series, as well as in the different series.

The choice of method for this investigation was influenced by several considerations. In the first place, the substance to be analyzed must be definite in composition and capable of being either fused or heated to a high temperature in order to insure the elimination of moisture. In the second place, in view of the fact that chromium is hard to handle satisfactorily in a quantitative fashion, the analytical operation should involve the determination of some other element. The halogen compounds, which have been employed very successfully many times, especially in this laboratory, for the determination of the atomic weights of metallic elements, are less suited for use in the case of chromium on account of the difficulty in the complete precipitation of the halogens by means of silver nitrate. All things considered, the chromates of silver seemed to offer the most promising possibilities on account of the ease with which their silver content may be determined. It is true, in order to determine the ratio of the atomic weight of chromium to that of either silver or oxygen, this method necessitates a knowledge of the exact ratio of the atomic weights of silver and oxygen, knowledge which is at present lacking. The per cent of silver in the compound being known, however, analytical data may be used at any subsequent time for the calculation of the atomic weight of chromium. Furthermore, since the value for the atomic weight of chromium at present accepted depends very largely upon the analysis of silver chromate, a study of this salt with the application of the most modern methods seemed to promise interesting results, and therefore was first taken up. In a following paper is given a description of the analysis of silver dichromate.

Purification of Materials.

Water. — The laboratory distilled water was twice redistilled, once from alkaline permanganate and once from very dilute sulphuric acid. In both distillations block tin condensers were employed, no cork or rubber connections being necessary.

Silver Nitrate. — The preparation of pure neutral silver nitrate for the precipitation of silver chromate followed the lines laid down in previous researches in this laboratory. A large quantity of heterogeneous silver residues were reduced to metallic silver by means of sticks of pure zinc in slightly acid solution. After the silver had been washed with water until free from halogens, it was dissolved in nitric acid, and the solution was filtered. Silver chloride was precipitated from the diluted nitrate by means of hydrochloric acid, and the precipitate of silver chloride was thoroughly washed. From this silver chloride, metallic silver was again obtained by reduction with cane sugar in strongly alkaline solution. After being washed until free from chloride, the metal was again dissolved in nitric acid in a Jena glass flask. By reduction with ammonic formate (prepared from redistilled formic acid and redistilled ammonia), the silver was once more obtained in the metallic state. The beautiful mass of crystals was then dissolved in the purest nitric acid, and the nitrate, after concentration of the solution, was four times recrystallized from the purest water in platinum until free from acid. In this crystallization, and in all others, centrifugal drainage in a machine employing platinum funnels as baskets[16] was always used, in order to free the crystals entirely from any adhering mother liquor, the mother liquors all being rejected.

Hydrochloric Acid. — Hydrochloric acid was prepared by distilling the commercial chemically pure acid, after dilution with an equal volume of water.

Hydrobromic Acid. — The methods for obtaining pure bromine have been recently tested by one of us,[17] and the processes found suitable for the purpose were employed here. A considerable quantity of hydrobromic acid was prepared by passing a current of pure hydrogen sulphide through a layer of bromine covered with water. The hydrogen sulphide was generated by the action of dilute sulphuric acid on ferrous sulphide, and was thoroughly scrubbed in gas washing bottles and towers containing water. After the precipitated mixture of sulphur bromide and sulphur had been removed by decantation and filtration, the acid was boiled, with the occasional addition of small portions of recrystallized potassium permanganate. This was done to eliminate any iodine which might have been present.

The hydrobromic acid was then heated with the calculated quantity of recrystallized potassium permanganate, the bromine being condensed in a Jena flask cooled with running water. In this way three eighths of the bromine remained behind as potassium and manganous bromides, the remaining five eighths being distilled from the solution of these bromides. The greater part of the chlorine was undoubtedly eliminated by this operation, since the original bromine was fairly pure. In order to be on the safe side, however, the bromine was again reduced to hydrobromic acid, and this in turn was changed to bromine as above. From the product the final hydrobromic acid was prepared with hydrogen sulphide. After filtration and distillation, it was preserved in Jena glass.

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[16] Richards, Jour. Amer. Chem. Soc., **27**, 110 (1905).

[17] Baxter, These proceedings, **42**, 204 (1906).

Chromic Acid. — This was prepared from Merck's "Highest Purity Chromic Acid." The material was dissolved in pure water, and the solution was filtered through a Gooch crucible with a mat of platinum sponge, a quantity of sandy material being thus separated. The solution was then evaporated to saturation and three times systematically recrystallized in platinum dishes with centrifugal draining, each mother liquor being used for the crystallization of three crops of crystals on account of the small temperature coefficient of solubility of chromic acid. The mother liquors from the first crystallization, on testing in the nephelometer, indicated only traces of sulphates and halogens.

Potassic Chromate. — Some of the purest commercial salt, after solution in water, was filtered through a Gooch-Munroe-Neubauer crucible. It was then four times crystallized in platinum, each crop of crystals being centrifugally drained.

Silver Chromate. — The point in the investigation requiring the most attention was the preparation of normal silver chromate free from both basic and acid salts. Since the salt cannot be crystallized, owing to its slight solubility in water, it is necessary so to regulate the conditions during precipitation that neither acid nor basic salts can separate as a distinct solid phase. Even then the *occlusion* of traces of either basic or acid salts is still possible, and it is necessary to form the salt under a fairly wide range of conditions in order to show constancy of composition.

Fortunately data are available which indicate the conditions under which silver dichromate or hydrochromate can exist. Sherrill[18] has recently shown that silver chromate changes into silver dichromate rapidly under a saturated solution in nitric acid more concentrated than 0.075 normal, while silver dichromate changes into silver chromate under a saturated solution in nitric acid less concentrated than 0.06 normal. Some time before, Krüss[19] had shown that silver dichromate is converted into silver chromate by contact with water.

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[18] Jour. Amer. Chem. Soc., **29**, 1673 (1907).

[19] Ber. d. d. Chem. Gesell., **22**, 2050 (1889).

In the light of these facts it is obvious that the solutions of the soluble chromates can safely be employed for the precipitation of silver chromate without the least danger of the precipitation of silver dichromate, and even that the presence of a slight amount of free acid could do no harm.

Owing to the weak nature of the second hydrogen of chromic acid, the first hydrogen dissociating to the same extent as that of hydrochloric acid,[20] but the second hydrogen having the constant 6.0×10^{-7} at 18°,[21] appreciable hydrolysis of solutions of its salts takes place, to a greater extent the weaker the base with which the chromic acid is combined. Sherrill has found, for instance, that ammonium chromate in 0.05 molal solution is 2.7 per cent hydrolyzed. The basicity of the solutions, on the other hand, will be greater the stronger the base. In order to determine whether this hydrolysis is sufficient to produce precipitation or occlusion of basic chromates, precipitates of silver chromate were formed by means of solutions of both ammonium and potassium chromates. The comparison of precipitates formed in this way will show whether the presence of basic salts is to be feared.

Sample I. Ammonic chromate was prepared by adding to a solution of the pure chromic acid a slight deficiency of the purest freshly distilled ammonia. The solution was diluted until about tenth normal, and was slowly poured with constant shaking into a solution of an equivalent quantity of silver nitrate of about the same concentration. The dark red precipitate of silver chromate was washed six times by decantation with large portions of water, centrifugally drained to remove as much water as possible and dried at gradually increasing temperatures in an electric oven, finally at 160° for a long time. The dried lumps were then gently ground to a fine powder in an agate mortar in order to facilitate further drying as well as to insure homogeneity.

During the addition of the chromate to the silver solution, since the chromate solution was slightly deficient in ammonia, acid accumulated in the silver nitrate solution. Hence each succeeding portion of precipitate was formed under conditions of greater acidity, although the concentration of acid in the solution could never have approached that found by Sherrill to be necessary for the existence of the silver dichromate.

Sample II. This preparation was practically identical with Sample I, since part of the precipitate obtained as above was washed by decantation with water eight times more, each wash water being allowed to stand in contact with the precipitate for many hours, and the precipitate being shaken with the wash water very thoroughly at intervals, in order to leach out any accidentally enclosed or adsorbed soluble salts. The prolonged extra washing evidently was unnecessary, since the results are practically the same as those obtained with Sample I.

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[20] Walden, Zeit. physikal. Chem., **2**, 49 (1888).

[21] Sherrill, loc. cit.

Sample III. This sample was prepared from the four times recrystallized potassic chromate. A quantity of this material in about tenth normal solution was precipitated with an equivalent amount of silver nitrate, equally dilute. The precipitation took place in Jena glass, the silver solution being slowly poured into the chromate, in order to accentuate the effect of the hydrolysis if possible. It will be recalled that in the case of Samples I and II prepared with the ammonic salt, the chromate was added to the silver solution. The precipitate was then transferred to platinum and washed seven times with the purest water, the chromate being thoroughly agitated with each washing. After the removal of the greater part of the adhering water by centrifugal settling, this sample was dried in a preliminary fashion at 150° and was pulverized in an agate mortar, as in the case of Samples I and II. The salt was soft and crystalline, and greenish black in color.

Sample IV. A fourth sample also was prepared from recrystallized potassium chromate, which in turn was made from recrystallized chromic acid. In the first place, potassic hydroxide was prepared by the electrolysis of three times recrystallized potassic oxalate, with the use of a mercury cathode and decomposition of the amalgam with pure water in a platinum dish, as in the preparation of potassium hydroxide in an investigation upon the atomic weight of potassium.[22] The solution of the pure hydroxide was added to a solution of three times recrystallized chromic acid, contained in a platinum dish, until the normal chromate had been formed as indicated by the yellow color. From this solution, by three systematic crystallizations, potassium chromate was separated.

The silver chromate was prepared from this material and the purest silver nitrate by slowly adding a six hundredths normal solution of the chromate to a silver nitrate solution of equivalent concentration, this procedure being the reverse of that used in the preparation of Sample III. The dark brownish-red precipitate was allowed to settle in the flask in which precipitation took place. Then, the supernatant solution having been decanted, the silver chromate was transferred to a platinum dish and washed very thoroughly with water. After being freed from water by centrifugal settling, the silver chromate was dried at about 160° in an electric oven, and powdered in an agate mortar.

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[22] Richards and Mueller, Jour. Amer. Chem. Soc., **29**, 645 (1907).

Since in the case of Sample III the silver nitrate was added to the chromate, while in preparing Sample IV precipitation took place in the reverse fashion, a comparison of the two samples would not only throw light upon the effect of hydrolysis, but also show whether the occlusion of potassium chromate or silver nitrate was to be feared.

The Analysis of Silver Chromate.

The fact that salts dried by prolonged heating at 100°, or at even higher temperatures, usually contain appreciable amounts of moisture, owing to included mother liquor, is a point which has been overlooked by most earlier investigators,[23] and the oversight throws doubt on much otherwise very careful work. In exact work the residual water must either be corrected for or entirely avoided. The simplest fashion of drying a substance perfectly is to fuse it in a current of dry gas. In the case of the silver chromate, however, this is not practicable, for even at 300° incipient decomposition sets in. Upon attempting to dissolve in nitric acid samples dried in air at that temperature, a slight insoluble residue was always obtained, while heating in a current of oxygen gave no better results. Since the moisture cannot be entirely expelled from silver chromate by heating at a moderate temperature, it must be determined by the analysis of separate portions of the substance which have been treated in some definite fashion.

Experiments showed that at temperatures below 225° the salt was not appreciably changed, hence this temperature was chosen as a suitable one at which to heat the salt preparatory to analysis. The silver chromate was therefore always heated in a current of pure dry air for two hours at 225°, in order to obtain the separate portions in as nearly as possible the same condition.

The drying apparatus was constructed entirely of glass, rubber connections being especially avoided. A current of air was passed first over red-hot copper oxide to destroy organic matter, then through successive Emmerling washing towers. In the first were beads drenched with silver nitrate solution, in the second with a strong solution of potassic hydroxide containing much potassic manganate, and in the last three with concentrated sulphuric acid. The already very dry air was then passed through a long tube containing resublimed phosphoric anhydride spread over a large surface of glass beads and ignited asbestos. From the drying apparatus the air passed into the tube in which the boat containing the silver chromate was placed.

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[23] Richards, Proc. Am. Phil. Soc., **42**, 28 (1903).

THE DETERMINATION OF SILVER IN SILVER CHROMATE.

During the drying of the silver chromate it was contained in a platinum boat which had been weighed, in a weighing bottle, by substitution for a similar bottle which with its contents displaced the same amount of air as the bottle with the boat. The boat was placed in a hard glass tube connected by a carefully ground joint with a bottling apparatus by means of which the boat could be transferred to the weighing bottle, after being heated, without the slightest exposure to moist air.[24] The tube was heated by means of two solid aluminum blocks which were grooved to contain the tube, by means of which the temperature could be maintained constant within a very few degrees.[25] After two hours' heating at 225° the boat was transferred to the weighing bottle and was allowed to stand in a desiccator near the balance for several hours before being weighed.

Next, the weighed quantity of silver chromate was transferred to a three-liter glass stoppered Jena flask with a carefully ground stopper and, after the boat and bottle had been cleaned with hot dilute nitric acid and water, the rinsings were poured into the flask and the silver chromate dissolved by the application of gentle heat. If the salt had not been heated above 225°, the solution was absolutely clear. Specimens heated above this temperature always showed more or less turbidity.

The chromate was next reduced to the chromic state by the addition of a very slight excess of sulphur dioxide which had been freshly distilled into pure water. The slight excess of sulphurous acid was soon oxidized under the combined influence of heat and nitric acid. In Analyses 1, 2, 3, 12, 13, and 14 the reduction was effected by means of recrystallized hydrazine sulphate, in order to avoid to a large extent the presence of sulphuric acid, for Richards and Jones[26] found that silver chloride occludes silver sulphate very tenaciously. This method of reduction, however, was without effect on the results.

Since in the reduction of the chromate by hydrazine, nitrogen gas is evolved, the flask in which the reduction took place was protected from loss by spattering by means of a long column of bulbs fitting loosely into the neck of the flask. The solution of hydrazine sulphate was added through a funnel with a long fine stem which extended through the column of bulbs nearly to the bottom of the flask. After the addition of the hydrazine, the reaction was allowed to continue slowly, with occasional shaking, and was completed by heating the solution upon a steam bath for a short time. In the presence of acid a dilute solution of hydrazine is without effect upon silver salts.

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[24] Richards and Parker, These proceedings, **32**, 59 (1896).

[25] Baxter and Coffin, These proceedings, **44**, 184 (1909).

[26] Jour. Amer. Chem. Soc., **29**, 831 (1907).

After the solution had been allowed to cool, it was diluted to a volume of one and one half liters, and the silver was precipitated as chloride or bromide by the addition of a very dilute solution of an excess of either hydrochloric or hydrobromic acid. The flask with its contents was shaken thoroughly for a few moments and was then allowed to stand several days, until, the silver bromide having settled, the supernatant solution was perfectly clear.

Since the mother liquor of the silver halide contained both nitric and hydrobromic acids in excess, the use of a Gooch-Munroe-Neubauer crucible seemed to be attended with danger on account of solution of platinum. Such a possibility has already been pointed out,[27] and an actual loss was found to take place in blank experiments carried out at the beginning of this research. Accordingly the ordinary platinum Gooch crucible with an asbestos mat was used. The asbestos had been carefully prepared by ignition, and washing first with nitric acid and then with water. The crucible was prepared for weighing before and after filtration of the silver halide in exactly the same way.

The silver halides were washed many times by decantation with dilute hydrochloric acid in the case of silver chloride, and with very dilute hydrobromic acid in the case of silver bromide. The precipitate was then transferred to the weighed crucible and was dried in an electric oven at 170° for at least sixteen hours.

In order to correct for the small quantity of moisture retained by the silver halides, each precipitate was transferred as completely as possible to a porcelain crucible and fused. From the loss of weight of the portion of silver salt transferred to the crucible, the amount of water in the entire precipitate was calculated.

The small quantity of asbestos, together with a trace of silver bromide which escaped the crucible, was collected by passing the entire filtrate and washings through a small filter. The ash of this filter was treated with nitric and with hydrochloric or hydrobromic acids, then it was reheated and the crucible was weighed. After correction for the ash of the filter, the gain in weight of the crucible was added to the weight of the main mass of silver halide.

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[27] Morse, "Exercises in Quantitative Chemistry," p. 203 (1905).

Another correction was necessary. The filtrate contained dissolved silver salt, even though an excess of halogen acid was used in the precipitation. The larger part of the dissolved halide is due to the marked solubility in solutions of chromic salts, the amount dissolved increasing with increasing concentration of the chromic salts. Berlin overlooked this correction, which was afterwards pointed out by Siewert. Meineke later determined experimentally the quantity of dissolved material, and also proposed the method of separation which was adopted in this work. The entire filtrate of three to four liters was evaporated to small bulk, nearly neutralized with ammonia, and then the silver was precipitated from a hot solution as sulphide. The precipitate was collected upon a filter paper, which was ignited. The residue was converted to the nitrate by digestion with dilute nitric acid, and the solution was then filtered into a graduated flask, in which it was diluted to known volume. By comparison in the nephelometer of this solution with standard solutions of silver the quantity of silver in solution was determined. In using the nephelometer all necessary precautions, as pointed out by Richards,[28] were taken.

That all dissolved silver was recovered in this way was shown by adding an excess of ammonia to the filtrate of the silver sulphide in one analysis, the hydrogen sulphide having been expelled, and after removal of the chromic hydroxide by filtration, testing the acidified filtrate for silver. None could be detected.

THE DETERMINATION OF MOISTURE IN SILVER CHROMATE.

The proportion of moisture in the silver chromate was found by fusing weighed quantities of the salt in a current of pure dry air and collecting the water vapor produced in a weighed phosphorus pentoxide tube. During the fusion of the salt oxygen is evolved, but since the fusing point is low, there is no danger of volatilization of either silver or chromium compounds.

In order to avoid the necessity of removing the fused silver chromate from a platinum boat, boats of copper foil which had been cleaned and ignited were employed.

It was desirable to determine not only whether the proportion of water could be made constant at any one temperature, but also how much the proportion of water is affected by variations in temperature. Experiments were therefore carried out with silver chromate which had been dried for two hours at 200°, 225°, and 300°, in dry air which had been purified as previously described.

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[28] Am. Chem. Jour., **35**, 510 (1906).

After the salt had been dried, a carefully weighed U-tube containing resublimed phosphorus pentoxide was attached to the end of the tube. This U-tube was provided with ground glass stopcocks lubricated with Ramsay desiccator grease. The silver chromate was gradually heated until fusion took place, and a slow current of air was allowed to pass through the system for one half hour in order to make certain that all moisture was carried into the absorption tube. Finally the phosphorus pentoxide tube was reweighed.

Temperature of Heating.	Weight of Silver Chromate.	Weight of Water.	Per Cent of Water.
	grams.	gram.	
200°	4.87	0.00097	0.0199
200°	4.74	0.00098	0.0207
200°	4.43	0.00093	0.0210
Average			0.0205
225°	9.01	0.00136	0.0151
225°	10.85	0.00188	0.0173
225°	10.11	0.00125	0.0124
225°	7.95	0.00105	0.0132
225°	8.23	0.00114	0.0139
Average			0.0144
300°	3.50	0.00034	0.0097

The pentoxide tube was weighed by substitution with the use of a counterpoise of the same size and weight. Before being weighed both tubes were carefully wiped with a damp cloth and were allowed to stand near the balance case for thirty minutes. Care was taken to equalize the pressure inside and outside the tubes by opening one stopcock immediately before hanging on the balance.

In order to test the efficiency of the drying apparatus, blank experiments were carried out by allowing a slow current of air to pass through the apparatus into the weighed pentoxide tube. The variations in the weight of the tube were never much larger than the probable error in weighing the tubes.

As is to be expected, the water content gradually decreases with increasing temperature of heating. The extreme variation with specimens of silver chromate which have been heated at 225° amounts to only five thousandths of a per cent. Evidently the percentage of residual water is as constant as can be reasonably expected, and the mean can safely be assumed to represent with sufficient exactness the average proportion of water in the salt. Hence from every apparent gram of silver chromate 0.000144 gram is subtracted.

DENSITY OF SILVER CHROMATE.

In order to correct the weight of silver chromate to a vacuum standard, a knowledge of its specific gravity is necessary. This has already been determined by Playfair and Joule[29] and Schroeder,[30] who obtained

Weight of Silver Chromate in Vacuum.	Weight of Toluol displaced in Vacuum.	Density of Silver Chromate.
grams.	gram.	25°/4°
5 1584	0.7898	5.628
3 6012	0.5520	5 621
Average		5.625

The following vacuum corrections were applied:

	Specific Gravity.	Vacuum Correction per Gram.
Weights	8.3	
Toluol	0.862	+0.00126
Silver chromate . .	5.625	+0.000069
Silver chloride . .	5.56	+0.000071
Silver bromide . .	6.473	+0.000041

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[29] Mem. Chem. Soc., **2**, 401 (1845).
[30] Lieb. Ann., **173**, 72 (1874).

the values 5.77 and 5.53 respectively. On account of the marked difference between these values, new determinations of the density were made by the displacement of toluol with weighed amounts of salt. The toluol was first dried by stick soda and then distilled. Its specific gravity at 25° referred to water at 4° was found to be 0.86156. Great pains was taken to remove air from the chromate when covered with toluol by placing the pycnometer in an exhausted desiccator before setting.

Balance and Weights.

All weighings were made by substitution upon a nearly new short-armed Troemner balance, easily sensitive to one fiftieth of a milligram with a load of fifty grams.

The gold-plated Sartorius weights were carefully standardized by the method described by Richards,[31] and were used for no other work.

SERIES I.

$2\,AgCl : Ag_2CrO_4$

$$\frac{Ag}{AgCl} = 0.752632\ ^{32}$$

Number of Analysis.	Sample of Ag_2CrO_4.	Corrected Weight of Ag_2CrO_4 in Vacuum.	Weight of AgCl in Vacuum.	Loss on Fusion.	Weight of Asbestos.	Dissolved AgCl from Filtrate.	Corrected Weight of AgCl in Vacuum.	Ratio $2AgCl : Ag_2CrO_4$.
		grams.	grams.	gram.	gram.	gram.	grams.	
1	II	10.30985	8.90835	0.00063	0.00117	0.00019	8.90908	0.864132
2	II	8.26920	7.14327	0.00063	0.00211	0.00017	7.14492	0.864040
3	IV	6.56679	5.67324	0.00039	0.00136	0.00023	5.67444	0.864111
		Average						0.864094
		Per cent of Ag in Ag_2CrO_4 . . . 65.0345						

Discussion of Results.

In comparing the analytical results, it is to be noted first that the compositions of the different samples agree within less than one one hundredth of one per cent, as the following averages show.

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[31] Jour. Amer. Chem. Soc., **22**, 144 (1900).

[32] Richards and Wells, Pub. Car. Inst., No. 28 (1905).

SERIES II.

$2\,AgBr : Ag_2CrO_4$

$$\frac{Ag}{AgBr} = 0.574453$$ [33]

Number of Analysis.	Sample of Ag_2CrO_4.	Corrected Weight of Ag_2CrO_4 in Vacuum. grams.	Weight of AgBr in Vacuum. grams.	Loss on Fusion. gram.	Weight of Asbestos. gram.	Weight of AgBr from Filtrate. gram.	Corrected Weight of AgBr in Vacuum. grams.	Ratio $2AgBr : Ag_2CrO_4$.
4	I	2.63788	2.98579	0.00028	0.00056	0.00014	2.98621	1.13205
5	II	2.82753	3.20018	0.00008	0.00060	0.00014	3.20084	1.13203
6	III	2.33454	2.64054	0.00032	0.00220	0.00026	2.64268	1.13199
7	I	1.77910	2.01304	0.00050	0.00144	0.00004	2.01402	1.13204
8	I	2.33198	2.63988	0.00030	0.00034	0.00002	2.63994	1.13206
9	II	3.10402	3.51311	0.00033	0.00094	0.00018	3.51390	1.13205
10	III	2.92751	3.31411	0.00027	0.00033	0.00010	3.31427	1.13211
11	III	4.21999	4.77677	0.00055	0.00126	0.00014	4.77762	1.13214
12	II	5.24815	5.93939	0.00025	0.00170	0.00020	5.94104	1.13203
13	IV	6.24014	7.06401	0.00039	0.00104	0.00018	7.06484	1.13216
14	IV	7.92313	8.96913	0.00083	0.00129	0.00022	8.96982	1.13211
		Average						1.13207

Per cent of Ag in Ag_2CrO_4 . . . . . . 65.0321

Average per cent of Ag in Ag_2CrO_4 . 65.0333

	$2AgBr : Ag_2CrO_4$		$2AgCl : Ag_2CrO_4$
Sample I	1.13205	Sample II	0.86409
Sample II	1.13204	Sample IV	0.86411
Sample III	1.13208		
Sample IV	1.13214		

If anything, Samples I and II show a somewhat lower percentage of silver than Samples III and IV. These samples were made from ammonium chromate which contained a slight excess of chromic acid. This excess of acid accumulated in the solution during the precipitation of the silver chromate, so that the precipitate formed under distinctly acid conditions, although the acidity was not sufficient to present any danger of the formation of dichromate. Samples III and IV, on the other hand, since they were made from potassium chromate, which is markedly hydrolyzed, were formed under distinctly basic conditions, and the precipitation or occlusion of basic salts is to be feared. Such occluded basic salts would tend to raise the percentage of silver in the chromate. However, Sample IV yielded slightly higher results than Sample III, while on account of the method of precipitation the reverse is to be expected; for Sample III was precipitated by adding the silver nitrate to the chromate, while Sample IV was precipitated by adding the chromate to the silver solution, the mother liquor remaining neutral in both cases. Too much emphasis should not be laid upon the slight apparent difference in the composition of the different samples of salt, since the variations in the experiments with the same sample are as large as the differences between the samples. Hence the average result from the different samples is employed in the final calculations, all the analyses being given equal weight in each series.

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[33] Baxter, These proceedings, **42**, 201 (1906).

In addition to the specimens of silver chromate, the preparation and analysis of which have been described, two other interesting specimens were prepared. One was formed by adding a 0.04 normal silver nitrate solution to a solution of chromic acid of similar concentration. On account of the solubility of silver chromate in nitric acid solutions, precipitation was only partial. The precipitate was washed and dried, and upon analysis was found to contain so little silver that the presence of a small proportion of dichromate was certain, a result which was hardly to be expected in the light of Sherrill's experiments.

The second sample was prepared by heating ammoniacal solutions of silver chromate in platinum vessels, the chromate being gradually precipitated as the ammonia was expelled. This material yielded somewhat irregular results, which on the whole indicated too high percentages of silver, and hence the presence of basic salts, a result which could have been predicted from a consideration of the conditions of preparation.

It is to be noted that Series I and Series II yield percentages of silver differing by less than four thousandths of a per cent, a highly satisfactory agreement, which indicates purity of the halogen acids employed as well as experimental accuracy.

If the percentage of silver in silver chromate is 65.0333, the molecular weight of silver chromate may be calculated from the atomic weight of silver, and from the latter value the atomic weight of chromium by difference. Since the ratio of the atomic weights of silver and oxygen is somewhat uncertain at the present time, these calculations are carried out with various possible assumed values for the atomic weight of silver, oxygen being assumed to have the value 16.000. It is to be noted that the percentage error in the determination of the molecular weight of silver chromate is multiplied six times in the atomic weight of chromium.

If Ag = 107.93	Ag_2CrO_4 = 331.922	and	Cr = 52.062
If Ag = 107.88	Ag_2CrO_4 = 331.768	and	Cr = 52.008
If Ag = 107.85	Ag_2CrO_4 = 331.676	and	Cr = 51.976

Although slightly lower than the previous investigations, these results agree with them as closely as is to be expected, most of the probable errors in earlier work tending to make the results too high.

The more important results of this research may be briefly summed up as follows:

1. Pure silver chromate was prepared.
2. It is shown that silver chromate cannot be completely dried without decomposition.
3. The proportion of residual water was determined in salt dried at definite temperatures.
4. The specific gravity of unfused silver chromate is found to be 5.625 at 25° C. referred to water at 4° C.
5. The per cent of silver in silver chromate is found to be 65.0333 by two closely agreeing methods.
6. With several assumed values for the atomic weight of silver referred to oxygen, the atomic weight of chromium is found to have the following values:

If Ag = 107.93	Cr = 52.06
If Ag = 107.88	Cr = 52.01
If Ag = 107.85	Cr = 51.98

In the following paper the analysis of silver dichromate is described.

We are greatly indebted to the Carnegie Institution of Washington for generous pecuniary assistance in pursuing this investigation; also to the Cyrus M. Warren Fund for Research in Harvard University for many pieces of platinum apparatus.

Cambridge, Mass.,
December 10, 1908.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF CHROMIUM.

SECOND PAPER. — THE ANALYSIS OF SILVER DICHROMATE.

By Gregory Paul Baxter and Richard Henry Jesse, Jr.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
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A REVISION OF THE ATOMIC WEIGHT OF CHROMIUM.

SECOND PAPER. — THE ANALYSIS OF SILVER DICHROMATE.

By Gregory Paul Baxter and Richard Henry Jesse, Jr.

Presented January 13, 1909. Received December 11, 1908.

In the preceding paper[1] is described a successful attempt to prepare pure silver chromate and to determine its silver content, with the object of throwing light upon the atomic weight of chromium, the value found in this way, 52.01, being about one-tenth of a unit lower than the one in common use. The preparation and analysis of silver dichromate was next investigated. Since the proportion of chromium in the dichromate is fifty per cent larger than in the chromate, the effect of experimental uncertainty upon the final result is correspondingly reduced.

Silver dichromate possesses another great advantage over silver chromate for exact work in that it may be readily crystallized from nitric acid solutions, and thus may be freed from impurities included or occluded during precipitation, with the exception of nitric acid and moisture. For, the silver and chromium being present in equivalent proportions during the crystallization, the inclusion of mother liquor could do no harm. If the concentration of the nitric acid is sufficiently high, there is no possibility of the separation of silver chromate as such during this crystallization, since Sherrill[2] has shown that silver chromate changes rapidly into silver dichromate under nitric acid solutions more concentrated than 0.075 normal. This is primarily due to the low value of the dissociation constant of the second hydrogen of chromic acid, which has been found by Sherrill to be 6×10^{-7}, the solubility product of silver chromate being 9×10^{-12}, and that of silver dichromate being 2×10^{-7}. Sherrill has also investigated the part which the hydrochromate ion plays in the equilibrium relations of chromates and dichromates in solution and has found the following equation to hold:

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[1] Baxter, Mueller, and Hines, These Proceedings, **44**, 399–417 (1909).

[2] Jour. Amer. Chem. Soc., **29**, 1641 (1907).

$$\frac{(Cr_2O_7^{=})}{(HCrO_4^{\ -})^2} = 75.$$

Although obviously the concentration of the hydrochromate ion in dichromate solutions (in a 0.1 molal solution of potassic dichromate fifteen per cent of the salt existing as hydrochromate) is always considerable, the precipitation of the solid phase $AgHCrO_4$ seems not to be possible. Sherrill was not able to find any indication of the presence of this salt in the precipitate formed by adding silver nitrate to chromic acid in nitric acid solution. Furthermore, since the water content of our material was carefully investigated, the presence of hydrochromate in traces could do no harm; for the latter substance upon sufficient heating would yield dichromate and water according to the following equation:

$$2\ AgHCrO_4 = Ag_2Cr_2O_7 + H_2O.$$

Although the presence of polychromates other than the dichromate seemed improbable, their absence from our material was shown by crystallizing silver dichromate from nitric acid of different concentrations. Since this variation was without effect, it may be reasonably supposed that more highly acid salts than the dichromates were neither precipitated as solid phases nor occluded.

Purification of Materials.

Only slight changes were made in the methods of purifying the materials used in the various preparations of silver dichromate and in the analyses from those described in the preceding paper.

Nitric Acid. — Nitric acid was freed from chlorine by several distillations through a platinum condenser.

Hydrochloric Acid. — This acid also after dilution was purified by distillation with a quartz condenser.

Hydrobromic Acid. — Hydrobromic acid was prepared from bromine which had been twice distilled from solution in potassium bromide, the bromide in the second distillation being essentially free from chlorine. The hydrobromic acid was synthesized by passing carefully cleansed hydrogen (made from the lead-sodium alloy "hydrone" and water) through the bromine at about 40° and then over hot platinized asbestos, the acid being collected in pure water. Iodine was eliminated from the acid by boiling with free bromine several times. Finally it was redistilled through a quartz condenser three times with rejection of the extreme fractions. The acid, diluted to normal concentration, was kept in a well protected glass bottle.

Silver Nitrate. — Silver nitrate was prepared from silver which had been precipitated once as chloride, and then reduced with invert sugar. The nitric acid solution of the fused product was evaporated to crystallization, and the salt was then three times more crystallized from nitric acid solutions, the crystals being drained centrifugally in a centrifugal machine employing platinum Gooch crucibles as baskets.[3] Heating was carried out over electric stoves in order to avoid contamination by the combustion products of illuminating gas, both in this and in all other preparations in this research.

Potassium Dichromate. — The best commercial material was crystallized four times, once from aqueous solution in Jena glass, and three times in platinum vessels.

Chromic Acid. — This substance was three times recrystallized in platinum vessels as described in the preceding paper.

Silver Dichromate. — Silver dichromate was prepared by combining either potassium dichromate or chromic acid with silver nitrate in nitric acid solution in platinum vessels. Precipitation was carried out in fairly concentrated solution, since in the subsequent crystallization of the silver salt from nitric acid solution any included substance was sure to be eliminated. Although the inclusion of nitric acid during the crystallization was to be feared, and was actually found to have taken place, a method was devised for the determination of this nitric acid, together with the moisture retained by the solid.

Sample I. Silver nitrate and potassium dichromate were dissolved in equivalent proportions in 3 normal nitric acid, the concentration of each salt being about 0.7 normal. The cold silver nitrate solution was added very slowly, with constant vigorous stirring, to the dichromate solution. After the precipitate had been allowed to settle, the mother liquor was decanted, and the precipitate was centrifugally drained, and rinsed in the centrifugal machine with 3 normal nitric acid.

The salt was then five times recrystallized from solution in 3 normal nitric acid with centrifugal drainage after each crystallization. Owing to the small solubility of silver dichromate in nitric acid solutions the following scheme of crystallization was adopted. The dichromate was heated with the nitric acid solution upon the electric stove until the acid was saturated with silver dichromate. Then the hot solution was decanted into a dish through a platinum Gooch crucible without a mat of any sort but with small holes, in order to remove particles of silver dichromate either suspended in the solution or floating on the surface. These particles were always of considerable size, so that the resulting solution was clear. After the saturated solution had cooled and had deposited the greater part of its charge of salt, the mother liquor was continuously used to dissolve fresh portions of salt. About one liter of acid was used for the crystallization of about fifty grams of dichromate. Although by this method the impurities in the original salt accumulate in the mother liquor, on account of the relatively large volume of the mother liquor, there was little danger of these impurities being carried into the second crop of crystals. It was shown, for instance, that the mother liquor from the third crystallization was free from potassium. This mother liquor was evaporated to small bulk, neutralized with ammonia, and reduced and precipitated with hydrogen sulphide. The filtrate after evaporation and expulsion of the ammonium salts gave no spectroscopic flame test for potassium.

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[3] Jour. Amer. Chem. Soc., **30**, 286 (1908).

The silver dichromate was not allowed to come in contact with water or any solution except the 3 normal nitric acid solution.

All of the above operations were carried out in platinum vessels.

Sample II. This sample was made exactly as in the case of Sample I, except that chromic acid was employed instead of potassium dichromate, and that both precipitation and crystallization took place from 0.8 normal nitric acid. The silver dichromate was crystallized five times.

Sample III. The most dilute nitric acid which was used in the preparation of the silver dichromate was about 0.16 normal, solutions of this concentration being employed in the precipitation and crystallization of Sample III. This sample was made from chromic acid and silver nitrate, and was six times crystallized from 0.16 normal nitric acid.

The chief difference in the purification of the three specimens, aside from the concentration of acid used in their preparation, lies in the fact that Sample I was prepared from recrystallized potassium dichromate and Samples II and III from chromic acid. All three samples were crystallized many times as silver dichromate.

After the final drainage in the centrifugal apparatus, the crystals were dried in an electric oven at 150° for several hours. Then they were powdered gently in an agate mortar and kept in platinum vessels.

THE DETERMINATION OF SILVER IN SILVER DICHROMATE.

In preparing the silver dichromate for analysis, the complete elimination of moisture by fusion of the salt was impossible, owing to the ease with which silver dichromate decomposes. Even at the comparatively low temperature of the melting point of the dichromate, about 400°, oxygen is given off rapidly, while at temperatures considerably below this point, 300°, and to a very slight extent at 250°, there seemed to be evidence of decomposition, since salt heated to these temperatures did not give an absolutely clear solution in dilute nitric acid. In order to be on the safe side, the drying of the salt took place at 200° C.

The heating of the dichromate was effected much as described in the preceding paper in the case of silver chromate. The salt, contained in a weighed platinum boat, was heated in a current of pure dry air in a hard glass tube for four hours at 200° C., the air being purified and dried by passing over hot copper oxide, solid potassic hydroxide, concentrated sulphuric acid containing dichromate, and resublimed phosphorus pentoxide successively. An oven composed of solid aluminum blocks [4] was used, by means of which the temperature could be maintained constant within two degrees.

After the boat had been allowed to cool in the tube, it was transferred to the weighing bottle by means of a "bottling apparatus," [5] and was reweighed. Then the dichromate was transferred to a flask and was dissolved in hot 0.8 normal nitric acid, the boat and the weighing bottle being carefully cleansed with nitric acid and the rinsings being added to the main solution. The solution, which was always perfectly clear, was quantitatively transferred to the 3-liter glass stoppered precipitating flask, and at a dilution of about one liter was reduced by the addition of a very slight excess of sulphur dioxide. When the solution was cold, a slight excess of hydrobromic acid was diluted to about 800 c.c. and then was slowly added to the silver solution with continual agitation. The flask was stoppered and vigorously shaken. After twenty-four hours' standing the flask was again shaken, and then was allowed to stand two days or more, until the supernatant solution was clear.

Next the silver bromide was washed at least eight times by decantation with pure water and collected upon a weighed Gooch crucible. Then it was dried in an electric oven, first at 100° for two hours, then at 175° for about eighteen hours. After cooling in a desiccator near the balance for several hours, the weight of the silver bromide was determined.

The use of an asbestos mat in the Gooch crucible made it necessary to collect and determine the fibres detached during the filtration. This was done by passing the entire filtrate and wash waters through a small filter paper. The paper was ignited in a weighed porcelain crucible, and the ash was treated with nitric acid and then hydrobromic acid to convert a trace of reduced silver to the state of bromide. In order to avoid any danger from adsorption of chromic salts by the filter paper, at the end of the filtration the paper was rinsed with hot dilute hydrobromic acid. The correction for asbestos could have been avoided if it had been possible to employ a Gooch-Munroe-Neubauer crucible with a mat of platinum sponge. It has already been shown, however, in the preceding paper,[6] that such crucibles lose markedly in weight when exposed to the action even of the dilute aqua regia of the mother liquors of these analyses.

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[4] Baxter and Coffin, These proceedings, **44**, 184 (1909).

[5] Richards and Parker, These proceedings, **32**, 59 (1896).

The moisture retained by the silver bromide was found by fusing the dried salt in a porcelain crucible, the loss in weight on fusion being determined. The fused silver bromide was always light yellow and gave every indication of purity.

As in the preceding research a small quantity of silver bromide dissolved in the filtrate and wash waters was found by evaporating the combined filtrate and wash waters until nearly all the excess of acid had been expelled, and then, after slight dilution, precipitating the silver as sulphide. The sulphide was collected on a small paper, the ash of which, after ignition, was treated with nitric acid. The amount of silver thus obtained was found by comparison in a nephelometer of precipitates of silver bromide produced in this solution and in very dilute standard solutions of silver.

In Analysis 9 the silver was precipitated as silver chloride, the only other difference in the procedure being that the precipitate was washed with dilute hydrochloric acid instead of pure water.

THE DETERMINATION OF MOISTURE AND NITRIC ACID IN SILVER DICHROMATE.

Silver dichromate which has been crystallized from nitric acid, after being dried at 200°, contains traces of both nitric acid and water. Both of these substances can be expelled from the salt by fusion, although slight decomposition of the salt takes place simultaneously. Since the only readily volatile substance which can be formed by the decomposition of the salt is oxygen gas, the problem of the determination of the moisture and nitric acid consisted in that of absorbing in a quantitative fashion the water, nitric acid, and nitric peroxide formed by decomposition of the nitric acid. This was effected by passing the current of air containing the moisture and nitrogen compounds through two weighed U-tubes, one containing a concentrated solution of potassium hydroxide and solid potassium hydroxide and the other resublimed phosphorus pentoxide. The air current passed first through the potassium hydroxide tube in order that moisture vaporized from the hydroxide might be retained by the pentoxide tube. That the absorption of oxides of nitrogen was complete was shown by the fact that no test for nitric acid could be obtained beyond the phosphorus pentoxide tube either with moist litmus paper or with diphenylamine.

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[6] Baxter, Mueller, and Hines, loc. cit.

Since the three samples of silver dichromate were crystallized from nitric acid of different concentrations, it was necessary to make separate determinations of the moisture and nitric acid content with each sample. Extreme purity of material was unnecessary, and, as rather large quantities of salt were desired, three samples were prepared from ordinary silver nitrate and potassium dichromate and then were crystallized from nitric acid of the concentrations 3 normal, 0.8 normal, and 0.16 normal, respectively, glass vessels being employed throughout.

Weighed portions of the silver dichromate were heated for four hours at 200° in a current of pure dry air exactly as in preparing the salt for the silver analyses. Then the weighed potassium hydroxide and phosphorus pentoxide tubes were attached to the hard glass tube, with a protection tube containing phosphorus pentoxide at the end. The silver dichromate was gradually heated to complete fusion, and the air current was allowed to pass through the system for one half hour in order to make certain that all the vapors expelled from the dichromate were carried into the absorbing tubes. The absorption tubes were then reweighed.

Before the tubes were weighed, they were carefully wiped with a clean damp cloth and were allowed to stand near the balance case for one hour. The tubes were provided with ground glass stopcocks lubricated with Ramsay desiccator grease. During the weighing one stopcock in each tube was open to equalize the air pressure within and without the tubes. In order to lessen the error in weighing, as well as to save time and labor, the tubes were not weighed separately, but together as one system. Counterpoise tubes of the same shape and size were always employed. Blank determinations showed that the air current and manipulation of the tubes caused an increase in weight of 0.00010 gram in one half hour. This quantity is applied as a correction in every case.

In place of a platinum boat a superficially oxidized copper boat was used in these experiments. At the low temperature of fusion of silver dichromate there is little danger of decomposition of nitric acid or oxides of nitrogen by the oxidized copper. It is to be noted that if the nitric acid is decomposed during the experiment according to the following equation :

$$2HNO_3 = H_2O + 2NO_2 + O,$$

and is absorbed by the potassium hydroxide as NO_2, there is a slight loss of oxygen. The proportion of nitric acid present being very small, however, this error could have no appreciable effect on the results.

Sample.	Weight of $Ag_2Cr_2O_7$.	Gain in Weight of Absorption Tubes.	$\frac{\text{Gain}}{\text{Weight of } Ag_2Cr_2O_7}$.
I	22.52	0.00448	0.000194
I	20.74	0.00378	0.000177
I	12.25	0.00235	0.000184
		Average	0.000186
II	13.13	0.00309	0.000235
II	15.91	0.00317	0.000193
II	21.35	0.00391	0.000178
II	19.60	0.00373	0.000185
Average, rejecting the first determination, 0.000186			
III	20.89	0.00353	0.000164
III	19.94	0.00348	0.000169
		Average	0.000167

It is somewhat surprising that Samples I and II contain the same proportion of volatile matter. This agrees with the result of the silver determinations, however, the samples proving to be otherwise very similar. As is to be expected, Sample III contains less impurity than either of the other two.

The negative corrections as found above are applied to all the final weights of silver dichromate given in the table of analyses.

THE SPECIFIC GRAVITY OF SILVER DICHROMATE.

The specific gravity of silver dichromate has been found by Schröder[7] to be 4.669, but on account of the uncertainty of most of the older specific gravity determinations this constant was very kindly redetermined for us by Mr. Victor Cobb. The silver dichromate was precipitated from dilute nitric acid solution and once recrystallized from normal nitric acid. Then it was dried at 200° for many hours. The determination was effected by displacement of toluol of specific gravity 0.86218. Care was taken to extract entangled air from the crystals by exhausting the air from the pycnometer in a vacuum desiccator.

Weight of $Ag_2Cr_2O_7$ in Vacuum.	Weight of Toluol displaced in Vacuum.	Specific Gravity of $Ag_2Cr_2O_7$.
grams.	grams.	25° /4°
29.308	5.299	4.769
25.330	4.578	4.770

The following corrections were applied:

	Specific Gravity.	Vacuum Correction.
Weights	8.3	
Toluol	0.862	+0.00126
Silver Dichromate . .	4.770	+0.000107
Silver Bromide . . .	6.473	+0.000041
Silver Chloride . . .	5.56	+0.000071

A No. 10 Troemner balance easily sensitive to one fiftieth of a milligram was used in all the weighings. The gold-plated weights were carefully standardized to hundredths of a milligram by the method described by Richards.[8]

Weighing was always carried out by substitution, with the use of a counterpoise as nearly as possible like the object weighed, both in material, shape, and volume.

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[7] Liebig's Jahresb., 1879, 31.

[8] Jour. Amer. Chem. Soc., **22**, 144 (1900).

SERIES III.

$2\ AgBr : Ag_2Cr_2O_7$

$\frac{Ag}{AgBr} = 0.574453$[9]

Number of Analysis.	Sample of $Ag_2Cr_2O_7$.	Corrected Weight of $Ag_2Cr_2O_7$ in Vacuum.	Weight of AgBr in Vacuum.	Weight of Asbestos.	Dissolved AgBr from Filtrate.	Loss on Fusion.	Corrected Weight of AgBr in Vacuum.	Ratio $2AgBr : Ag_2Cr_2O_7$.
		grams.	grams.	gram.	gram.	gram.	grams.	
1	II	5.71554	4.97107	0.00024	0.00025	0.00007	4.97149	0.869820
2	II	4.87301	4.23870	0.00019	0.00003	0.00004	4.23888	0.869869
3	II	7.45476	6.48380	0.00034	0.00019	0.00008	6.48425	0.869813
4	III	4.75269	4.13409	0.00020	0.00003	0.00012	4.13420	0.869865
5	III	8.15615	7.09477	0.00022	0.00005	0.00009	7.09495	0.869890
6	III	6.15412	5.35306	0.00007	0.00007	0.00011	5.35309	0.869839
7	I	6.83662	5.94656	0.00030	0.00009	0.00017	5.94678	0.869842
8	I	5.39883	4.69610	0.00027	0.00007	0.00013	4.69631	0.869876
9	III	6.26657	4.16034[10]	0.00018	0.00040	0.00016	4.16076	0.869903[11]
			Average					0.869857
Total .		55.60829	. . . .				48.37126	0.869854

Average from Sample I 0.869859
Average from Sample II 0.869834
Average from Sample III 0.869874
Average 0.869856

Per cent of Ag in $Ag_2Cr_2O_7$, if $2AgBr : Ag_2Cr_2O_7$
$= 0.869857 : 1.000000$ 49.9692

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[9] Baxter, These proceedings, **42**, 201 (1906).

[10] AgCl.

[11] Calculated from the ratio AgBr : AgCl = 131.0171 : 100.0000. Baxter, loc. cit. 4.16076 grams AgCl ⇔ 5.45131 grams AgBr.

The preceding table gives the results of all the final experiments in the order in which they were carried out. The preliminary analyses, which were defective in various ways, are not recorded.

The results of the foregoing experiments are as concordant as one can reasonably expect, since the insoluble silver salts are in general difficult to obtain definite in composition.[12] The extreme values differ by only one one hundredth of a per cent, while the averages of the different samples show an extreme difference of less than five thousandths of a per cent. The composition of the dichromate is evidently not affected by the concentration of the nitric acid from which it is crystallized, since the averages from the different samples do not vary regularly with the concentration of the nitric acid, the average result obtained from Sample II being lower than that of either Sample I or Sample III.

If the per cent of silver in silver dichromate is 49.9692, the molecular weight of silver chromate may be calculated from the atomic weight of silver, and from the molecular weight of the chromate the atomic weight of chromium by difference. Since the ratio of the atomic weights of silver and oxygen is somewhat uncertain at the present time, these calculations have been made with various possible assumed values for the atomic weight of silver, oxygen being assumed to have the value 16.000. It is to be noted that the percentage error in the determination of the molecular weight of silver chromate is multiplied four times in the atomic weight of chromium.

If Ag = 107.930 $Ag_2Cr_7O_7$ = 431.986 and Cr = 52.063
If Ag = 107.880 $Ag_2Cr_2O_7$ = 431.786 and Cr = 52.013
If Ag = 107.850 $Ag_2Cr_2O_7$ = 431.666 and Cr = 51.983

In the following table are given the results of the preceding research upon silver chromate by Baxter, Mueller, and Hines, together with the average of their values and those presented in this paper :

	Baxter, Mueller, and Hines.	Average.
If Ag = 107.930	Cr = 52.062	52.063
If Ag = 107.880	Cr = 52.008	52.011
If Ag = 107.850	Cr = 51.976	52.980

The agreement of the two independently determined values is highly satisfactory, no matter which value for the atomic weight of silver is assumed, although the higher values for silver give slightly better agreement.

The atomic weights of both chromium and silver may be calculated independently of any assumption except the atomic weight of oxygen from the following equations :

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[12] Baxter and Coffin, These proceedings, **44**, 184 (1909); Baxter, Mueller, and Hines, loc. cit.

$$\frac{2\mathrm{Ag}}{2\mathrm{Ag} + \mathrm{Cr} + 64} = 0.650333$$

$$\frac{2\mathrm{Ag}}{2\mathrm{Ag} + 2\mathrm{Cr} + 112} = 0.499692$$

to be 52.074 and 107.941 respectively. However interesting these results may be, they have little real significance, since an error of five thousandths of a per cent in either ratio causes an error of over one tenth of a unit in the atomic weights of both silver and chromium.

The most important results of this research are as follows :

1. Pure silver dichromate was prepared.

2. It is shown that silver dichromate cannot be completely dried without decomposition.

3. It is shown that silver dichromate when crystallized from nitric acid retains traces of the nitric acid.

4. The proportion of moisture and nitric acid in silver dichromate treated in definite fashions was determined.

5. The specific gravity of silver dichromate is found to be 4.770 at 25° C. referred to water at 4° C.

6. The per cent of silver in silver dichromate is found to be 49.9692.

7. With several assumed values for the atomic weight of silver referred to oxygen 16.000, the atomic weight of chromium is found to have the following values :

If Ag = 107.93 Cr = 52.06
If Ag = 107.88 Cr = 52.01
If Ag = 107.85 Cr = 51.98

8. If these results are averaged with those previously found by Baxter, Mueller, and Hines, the atomic weight of chromium is found to be as follows :

If Ag = 107.93 Cr = 52.06
If Ag = 107.88 Cr = 52.01
If Ag = 107.85 Cr = 51.98

We are greatly indebted to the Carnegie Institution at Washington for generous pecuniary assistance in pursuing this investigation ; also to the Cyrus M. Warren Fund for Research in Harvard University for many pieces of platinum apparatus.

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CONTRIBUTIONS FROM THE HARVARD MINERALOGICAL MUSEUM. — XIII.

NOTES ON THE CRYSTALLOGRAPHY OF LEADHILLITE.

I. *LEADHILLITE FROM UTAH.*

By C. Palache and L. La Forge.

II. *LEADHILLITE FROM NEVADA.*

By C. Palache.

With Three Plates.

CONTRIBUTIONS FROM THE HARVARD MINERALOGICAL MUSEUM. — XIII.

NOTES ON THE CRYSTALLOGRAPHY OF LEADHILLITE.

By C. Palache and L. La Forge.

I. LEADHILLITE FROM UTAH.

Presented December 9, 1908. Received January 14, 1909.

The crystals of leadhillite described in this paper were found and sent to the Harvard Mineralogical Museum for identification and study by A. F. Holden, then of Salt Lake City, in 1897. The writers desire to express here their thanks to Mr. Holden for so generously placing this rare material in their hands for investigation.

The leadhillite was found in the Eureka Hill Mine, Tintic Mining District, Utah, at a depth of 500 feet. It occurred in a few cavities in massive galena which are coated with quartz and anglesite, upon which the leadhillite is implanted. Of its occurrence Mr. Holden writes that it seems to appear only where the galena is impure, anglesite being the sole alteration product where the galena is free from impurities. The anglesite is both massive and in small clear colorless crystals, elongated parallel to the *b* axis and showing the forms c (001), b (010), m (110), l (104), o (011), and y (122), the latter form dominant.

So far as known to us the material sent us is all that was found.[1] It consists of several loose crystals of rhombohedral appearance and dull lustre, semitransparent, and of several pieces of massive galena with leadhillite crystals still attached to the walls. The latter crystals are transparent, of a faintly yellowish white color and adamantine lustre. They are mostly tabular, half an inch or less across, and upwards of an eighth of an inch thick. The most prominent characteristic by which they may certainly be distinguished from the accompanying anglesite is the highly perfect basal cleavage parallel to which the lustre is pearly. The crystals detached for measurement are with one exception minute fragments removed from aggregates or larger crystals; the cleavage develops so readily that it is exceedingly difficult to remove a crystal entire. These fragments are in nearly all cases, therefore, bounded by cleavage above and below, with edges more or less completely faceted with faces of pyramids, domes, and prisms. Their complex character may be judged by one crystal (Table II, No. 14, p. 439), a fragment about 2 mm. in diameter, on which were measured seventy faces belonging to thirty-five forms. On this crystal and some others, faces of both positive and negative forms occur on the upper end of the crystal; in others the forms are clustered about the end of the a axis, so that the positive forms are on the upper part and negative ones on the lower part, requiring two adjustments on the goniometer for measurement. With added complications due to twinning, described in another place, the adjustment of the crystals, their orientation, and the interpretation of the forms, were problems of some difficulty, which could hardly have been solved without the use of the two-circle goniometer and of the graphical method in gnomonic projection. The method followed was generally as follows. The basal cleavage, always present, is so nearly in polar position ($\beta = 89° 30'$), that an approximate adjustment was made by its means. The prism zone was then sought by turning the horizontal circle of the goniometer 90° from polar position, and this zone if present gave a final adjustment. In some cases it was necessary to make a rough determination of some of the forms with the first approximate adjustment by the base, and then to readjust to the calculated angles of these forms, a somewhat laborious but entirely accurate process.

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[1] In "Utah Minerals and Localities," Maynard Bixby, Salt Lake City, 1904, the occurrence of leadhillite in the Tintic District is described as follows: "Leadhillite has been observed rarely, but the crystals seen were of good quality, nearly colorless, and averaged possibly more than a half inch across." This is the only published reference to this occurrence.

Once adjusted, the clinodome zone could generally be recognized by its striated character, but in general no attempt to identify the forms was made until a projection had been constructed from the measurements. Here the principal zones at once appeared, and the positive and negative forms could be separated and forms in twin position sifted out. Cases were very rare where by these means the orientation of the crystal could not be made with entire certainty.

Some twenty crystals were measured, and of these fifteen yielded measurements that could be used in the computation of the elements. Sixty-three forms were observed, as shown in Table I, in which is given for each the computed angles ϕ and ρ, the arithmetic mean of the observed values of ϕ and ρ, the deviation in minutes of the extreme observations for each from the computed value, and the number and quality of the observations.

TABLE I.

Letter.	Symbol — Gdt.	Symbol — Miller.	No. Times.	Computed. φ	Computed. ρ	Measured. φ	Measured. ρ	Variation. φ +	Variation. φ −	Variation. ρ +	Variation. ρ −	Average Quality.	Occurs in Twinned Position.
				° ′	° ′	° ′	° ′	′	′	′	′		
c	0	001	39	90 00	0 30	. . .	0 26	. .	. .	29	27	good	comm'ly
a	$\infty 0$	100	9	90 00	90 00	90 00	90 00	. .	. .	. .	. .	good	
b	0∞	010	9	0 00	90 00	. . .	. . .	. .	. .	. .	. .	poor	
d	2∞	210	10	66 23	90 00	66 16	. . .	2	26	. .	. .	fair	once
l	∞	110	8	48 50	90 00	48 49	. . .	7	9	. .	. .	fair	once
L	$\frac{2}{3}\infty$	230	9	37 20	90 00	37 20	. . .	8	7	. .	. .	fair	twice
m	$\infty 2$	120	17	29 46	90 00	29 48	. . .	48	12	. .	. .	fair	
ν[1]	$0\frac{1}{4}$	014	6	1 46	15 33	1 09	15 21	. .	48	60	90	bad	no
χ[1]	$0\frac{1}{3}$	013	5[2]	1 20	20 21	0 54	19 46	. .	36	9	60	bad	no
α	$0\frac{1}{2}$	012	10	0 53	29 05	0 44	29 14	13	24	50	6	fair	once
η[1]	$0\frac{2}{3}$	023	2	0 40	36 33	0 43	36 42	4	. .	22	3	good	no
Γ[1]	$0\frac{3}{4}$	034	3[2]	0 35	39 50	0 34	39 15	. .	2	36	75	bad	once
[1]	$0\frac{5}{6}$	056	1[2]	0 32	42 50	0 09	43 42	. .	23	52	. .	poor	no
g	01	011	7	0 27	48 02	0 21	48 02	. .	12	24	16	good	twice
h	$0\frac{3}{2}$	032	4	0 18	59 04	0 13	59 24	. .	5	43	4	fair	twice
π[1]	$0\frac{5}{3}$	053	1	0 16	61 39	0 23	61 38	7	. .	. .	1	good	no
φ[1]	02	021	9	0 13	65 48	0 23	65 54	45	13	46	12	fair	once
Λ[1]	03	031	1	0 09	73 19	1 32	73 13	83	. .	. .	6	good	no
ψ[1]	$0\frac{5}{2}$	052	2	0 11	70 13	0 10	70 01	. .	1	. .	25	fair	no
y	40	401	7	90 00	78 54	90 00	79 07	. .	. .	39	6	poor	no
u	20	201	9	"	68 37	"	68 41	. .	. .	20	3	fair	once
z	$\frac{3}{2}0$	302	1	"	62 27	"	61 37	. .	. .	. .	50	bad	no
w	10	101	9	"	52 01	"	52 12	. .	. .	12	48	poor	no
[1]	$\frac{3}{4}0$	304	1[2]	"	43 55	"	44 34	. .	. .	39	. .	fair	no
i	$\frac{2}{3}0$	203	2	"	40 35	"	40 09	. .	. .	. .	47	poor	no
D	$\frac{1}{2}0$	102	1	"	32 48	"	31 30	. .	. .	. .	78	poor	once
Δ[1]	$-\frac{1}{2}0$	$\bar{1}02$	1[2]	−90 00	32 06	"	31 57	. .	. .	. .	9	bad	no
E[1]	$-\frac{2}{3}0$	$\bar{2}03$	5	"	40 01	"	40 13	. .	. .	44	5	fair	once
f	−10	$\bar{1}01$	3	"	51 39	"	51 35	. .	. .	38	22	poor	no
e	−20	$\bar{2}01$	9	"	68 29	"	68 32	. .	. .	30	9	good	no
k	1	111	7	49 02	59 29	48 56	59 19	32	29	1	33	fine	twice
s	$1\frac{1}{2}$	212	11	66 32	54 23	66 24	54 15	. .	23	0	42	fine	twice
θ	$1\frac{3}{2}$	232	6	37 31	64 34	37 23	64 34	37	44	21	14	fine	once
x	12	121	14	29 56	68 43	29 49	68 34	10	40	14	39	fair	twice
q	$-1\frac{1}{2}$	$\bar{2}12$	10	−66 15	54 05	−66 22	54 08	10	14	50	10	good	twice
p	−1	$\bar{1}11$	15	−48 39	59 17	−48 43	59 20	12	7	51	8	fine	twice
o	$-1\frac{3}{2}$	$\bar{2}32$	8	−37 09	64 28	−37 04	64 46	9	4	64	27	fair	twice
r	−12	$\bar{1}21$	14	−29 36	68 39	−29 39	68 50	12	4	46	7	good	no
A[1]	$-1\frac{5}{2}$	$\bar{2}52$	6	−24 26	71 52	−24 43	71 48	43	2	23	30	good	once
G[1]	−13	$\bar{1}31$	1	−20 45	74 21	−20 49	74 19	4	. .	. .	2	good	no
ζ	21	211	4	66 28	70 15	66 24	69 43	. .	5	32	43	poor	comm'ly
ρ	$-2\frac{1}{2}$	$\bar{4}12$	6	−77 38	68 55	−77 42	68 53	20	8	16	22	fair	no

[1] New forms.

[2] Forms needing confirmation.

TABLE I. — *Continued.*

Letter.	Symbol — Gdt.	Symbol — Miller.	No. Times.	Computed.		Measured.		Variation.				Average Quality.	Occurs in Twinned Position.
				ϕ	ρ	ϕ	ρ	ϕ		ρ			
								+	−	+	−		
				° ′	° ′	° ′	° ′	′	′	′	′		
Y[1]	-21	$\bar{2}11$	2	−66 19	70 09	−66 13	70 41	6	18	66	. .	poor	no
W[1]	$-2\frac{3}{2}$	$\bar{4}32$	2[2]	−56 40	71 46	−56 33	71 28	2	2	32	32	bad	no
M[1]	$-2\frac{5}{2}$	$\bar{4}52$	1	−42 22	75 07	−42 01	75 21	. .	21	14	. .	fair	no
R[1]	-24	$\bar{2}41$	12	−29 41	78 57	−29 48	78 59	52	23	51	36	fair	once
J[1]	$\frac{1}{3}$	113	5	49 24	29 40	49 16	29 39	0	25	43	46	good	no
β	$\frac{1}{3}\ \frac{2}{3}$	$\bar{1}23$	4	30 16	40 39	29 34	40 22	. .	64	1	38	fair	no
B[1]	$-\frac{1}{3}\ \frac{2}{3}$	$\bar{1}23$	2	−29 16	40 22	−30 21	39 01	65	. .	. .	81	good	no
λ	$-\frac{1}{3}\ \frac{1}{6}$	216	8	−65 57	24 28	−66 05	24 32	25	. .	10	1	good	twice
δ	$\frac{1}{2}\ \frac{1}{4}$	214	6	66 40	35 04	66 19	34 51	. .	31	2	40	poor	twice
	$\frac{1}{2}$	112	4	49 13	40 25	48 55	40 20	11	49	1	33	fine	once
t	$\frac{1}{2}1$	122	8	30 06	52 07	29 45	52 36	. .	54	52	. .	poor	no
N[1]	$\frac{1}{2}\ \frac{5}{8}$	458	3	24 53	56 52	24 46	56 44	. .	11	1	14	poor	once
μ	$-\frac{1}{2}\ \frac{1}{4}$	$\bar{2}14$	21	−66 06	34 28	−66 05	34 26	30	54	11	13	good	twice
P[1]	$-\frac{1}{2}$	$\bar{1}12$	2	−48 27	39 59	−48 32	40 05	8	. .	8	. .	fine	no
Q[1]	$-\frac{1}{2}\ \frac{3}{4}$	$\bar{2}34$	20	−36 57	46 14	−36 53	46 17	49	38	38	32	good	thrice
v	$-\frac{1}{2}1$	$\bar{1}22$	11	−29 26	51 56	−29 37	51 46	77	15	29	44	good	twice
T[1]	$-\frac{1}{2}\ \frac{5}{4}$	$\bar{2}54$	2	−24 18	56 45	−24 27	56 45	9	. .	3	3	good	once
σ	$-\frac{2}{3}1$	$\bar{2}33$	1	−37 03	54 20	−37 15	54 22	12	. .	2	. .	fine	no
U[1]	$\frac{1}{3}\ \frac{1}{2}$	$\bar{2}36$	3	37 53	35 10	37 32	34 42	. .	21	31	32	bad	once
[1]	$-1\frac{1}{4}$	414	3[2]	−77 35	52 18	−77 10	52 03	. .	40	. .	77	bad	no
H[1]	2	221	4[2]	48 56	73 33	48 52	73 04	. .	4	. .	90	poor	once

[1] New forms. [2] Forms needing confirmation.

Of the observed forms thirty-six were previously known and twenty-seven are new, seventeen of these being well established and ten requiring confirmation. But five of the forms previously known for the mineral were not present, namely, F, n, ω, γ, and τ.

The combinations observed are shown in Table II. The prevailing habit is strikingly hexagonal and of two types; (1) tabular, with hexagonal outline (Figures 2 and 3), the prism angle m $\wedge$ m being 120° 28′; (2) rhombohedral through the combination of a positive orthodome with a negative pyramid of about the same inclination to the vertical, there being three groups of forms that produce this effect, namely, w (101) with v ($\bar{1}22$); u (201) with r ($\bar{1}21$); and y (401) with R ($\bar{2}41$). Figures 1 and 9 show the first pair of forms in pseudo-rhombohedral combination. The apparent rhombohedral character is enhanced by the fact that the angle β is very nearly 90°, so that the basal pinacoid,

TABLE II.

Utah Leadhillite.

Crystal No.	c	a	b	d	l	L	m	ν	χ	*a*	η	Γ	$0\frac{5}{6}$	g	h	π	φ	ψ	y	u	z	w	$\frac{3}{4}0$	i	D	Δ	E	f	e	k	s	θ
1	×	×	×	×	×	×	×				×	×		×		×			×	×		×							×	×	×	
2 Fig. 3	×	×	×				×							×			×	×	×	×									×			
2*a* " 4	×		×	×		×	×																						×			
3	×	×					×				×	×								×							×		×			
4	×																			×		×								×	×	×
5 Fig. 5	×		×					×	×	×							×															
6	×	×		×	×	×	×	×	×	×		×		×			×		×	×		×						×	×		×	
7 Fig. 6	×	×					×	×		×				×													×		×			
8 Fig. 8	×	×		×	×		×	×		×			×				×		×	×		×								×	×	×
9	×		×				×		×					×	×		×		×	×		×					×	×	×	×	×	
10 Fig. 2	×	×																		×		×				×			×			
11	×	×	×	×	×	×	×													×							×		×	×	×	×
12 Fig. 7	×		×	×	×		×	×		×				×	×		×															
13	×		×	×	×	×	×			×									×	×	×	×		×						×	×	×
14	×	×	×	×	×	×	×			×			×	×			×	×	×			×	×	×			×			×	×	×
15	×	×		×	×		×		×	×			×			×			×	×		×		×	×						×	

Crystal No.	x	i	p	o	r	A	G	ζ	ρ	Y	W	M	R	J	β	B	λ	δ	ε	t	N	μ	P	Q	v	T	σ	U	−1¼	H
1	×	×		×	×	×		×					×					×		×				×	×					
2 Fig. 3	×		×	×	×								×									×		×	×					
2*a* " 4		×	×		×	×							×									×		×	×		×			
3		×	×	×	×				×					×		×		×				×		×						
4	×													×				×												
5 Fig. 5	×		×														×					×		×	×					
6	×	×	×	×	×				×		×		×				×					×		×	×				×	
7 Fig. 6			×		×					×		×					×					×	×	×						
8 Fig. 8	×				×			×					×	×	×			×		×	×	×		×	×	×		×		×
9	×		×		×									×	×			×	×	×					×					
10 Fig. 2		×																				×		×	×					
11	×	×	×	×	×	×		×	×				×									×		×						
12 Fig. 7		×	×		×	×	×	×	×	×	×		×				×					×		×	×	×		×	×	
13	×			×	×	×							×	×	×			×	×	×				×						×
14	×	×	×	×	×					×			×	×	×		×			×	×	×		×	×					
15		×			×				×	×			×		×					×										

generally present as face or cleavage, truncates the summit of the pseudo-rhombohedron with entire symmetry. As before stated, most of the crystals measured were but fragments, and the table of combinations does not therefore give an entirely correct idea of the relative frequency of occurrence of the various forms.

The forms c, a, m, u, and r are present on nearly every crystal. Of the new forms Q alone is conspicuous by its frequent occurrence. b, d, l, w, e, s, x, q, p, R (a new form), μ, and v are also of frequent occurrence, being found on from one half to two thirds of the measured crystals. The remaining forms are of minor importance, many of them found on but one or two crystals.

The new forms are established upon the following data :

E, $-\frac{2}{3}0$ ($\bar{2}03$). A narrow but distinct face in the orthodome zone on five crystals, giving fair reflections (Figure 6).

		ϕ		ρ		
Crystal	3	−90°	00′	39°	58′	fair.
“	7	−90	00	40	13	poor.
“	9	−89	05	40	45	fine.
“	11	−90	00	40	00	poor.
“	14	−89	42	40	40	good (in twin position).
Calculated		−90	00	40	01	

Clinodome Zone. — This zone is usually largely developed and is apt to be deeply and closely striated parallel to the zonal axis, often with a curved surface. The reflection of the signal from these curved surfaces is a band of light with occasional brighter portions and numerous more or less distinct images of the signal. Most of the latter are in positions corresponding to simple symbols, but only in the cases of those images which were also observed as given by distinct faces has the form been accepted as confirmed.

ν, $0\frac{1}{4}$ (014). Observed repeatedly as a signal in the striated clinodome zone, twice found as a distinct face (Figures 5 and 6).

		ϕ		ρ		
Crystal	5	1°	11′	14°	00′	
“	6	1	04	15	45	
“	7	1	17	15	42	perfect.
“	7	0	53	16	45	poor (in twin position).
“	8	0	00	15	00	
“	8	0	00	17	45	
“	12	1	04	16	21	
“	13	1	10	15	09	
“	13	0	58	15	56	
Calculated		1	46	15	33	

η, $0\frac{2}{3}$ (023).

		ϕ		ρ		
Crystal	1	0°	45′	36°	30′	poor.
“	3	0	41	36	55	good.
Calculated		0	40	36	33	

π, 0$\frac{5}{3}$ (053).

		ϕ	ρ	
Crystal	1	0° 23′	61° 38′	good.
"	15	1 33	61 54	poor.
"	16	0 46	60 34	fair.
Calculated		0 16	61 39	

ϕ, 02 (021). Figures 5, 7, and 8.

		ϕ	ρ	
Crystal	2	0° 00′	66° 10′	poor.
"	5	0 40	65 40	good.
"	6	0 00	66 34	poor.
"	6	0 01	66 00	"
"	8	0 19	65 36	"
"	9	0 00	66 04	"
"	9	0 06	65 43	perfect.
"	12	0 06	65 50	good.
"	14	0 14	65 43	"
Calculated		0 13	65 48	

ψ, 0$\frac{5}{2}$ (052).

		ϕ	ρ	
Crystal	2	0° 00′	69° 48′	fair.
"	14	0 10	70 13	poor.
Calculated		0 11	70 13	

A, −1$\frac{5}{2}$ ($\bar{2}$52). On five crystals, usually with large and distinct faces, of high lustre, giving good reflection (Figures 4 and 7).

		ϕ	ρ	
Crystal	1	−24° 54′	72° 15′	bad.
"	2a	−24 57	71 22	poor.
"	11	−24 24	71 41	good.
"	11	−25 13	71 49	good (in twin position).
"	12	−24 32	71 51	perfect.
"	12	−24 34	71 50	" (in twin position).
"	13	−24 34	71 46	fair.
Calculated		−24 26	71 52	

G, −13 ($\bar{1}$31). Seen but once as a large, distinct, lustrous face with good reflection (Figure 7).

		ϕ	ρ	
Crystal	12	−20° 49′	74° 19′	good.
Calculated		−20 45	74 21	

Y, −21 ($\bar{2}$11). On two crystals, small, not lustrous, and with poor reflections, but certainly a face (Figures 6 and 7).

		ϕ	ρ	
Crystal	7	—66° 01′	71° 15′	bad.
“	12	—66 25	70 12	poor.
“	14	—67 13	70 08	poor (in twin position).
Calculated		—66 19	70 09	

M, $-2\frac{5}{2}$ ($\bar{4}52$). Observed but once as a distinct face with fair reflection (Figure 6).

		ϕ	ρ	
Crystal	7	—42° 01′	75° 21′	fair.
Calculated		—42 22	75 07	

R, —24 ($\bar{2}41$). An important form, found fourteen times on nine crystals, faces distinct and often large, not very lustrous, and reflections often confused with that of (401) in twin position (Figures 3, 4, 7, and 8).

		ϕ	ρ	
Crystal	1	—29° 52′	79° 01′	perfect.
“	1	—30 40	78 15	good (in twin position).
“	2	—29 39	79 10	poor.
“	2a	—29 29	78 53	perfect.
“	3	—30 32	79 00	poor.
“	6	—29 54	78 30	“
“	11	—29 41	78 58	good.
“	“	—29 44	79 48	poor.
“	“	—28 59	79 26	“
“	“	—29 37	78 49	“
“	“	—29 33	79 48	“
“	12	—29 18	78 29	“
“	13	—29 43	79 04	“
“	14	—30 56	78 56	fair.
Calculated		—29 41	78 57	

J, $\frac{1}{3}$ (113). Small bright faces with good reflections on six crystals (Figure 8).

		ϕ	ρ	
Crystal	1	49° 24′	29° 35′	good.
“	3	48 49	29 05	poor (in twin position).
“	4	49 03	30 23	good.
“	8	49 57	29 29	fair (in twin position).
“	13	48 59	29 43	good.
“	14	49 18	29 38	“
Calculated		49 24	29 40	

B, $-\frac{1}{3}\ \frac{2}{3}$, $(\bar{1}23)$. A poor face on two crystals giving a fair reflection. Not an entirely satisfactory form.

		ϕ	ρ	
Crystal	3	−30° 19′	38° 59′	fair.
"	15	−30 43	40 14	"
Calculated		−29 16	40 22	

N, $\frac{1}{2}\ \frac{5}{4}$ (254). On three crystals with distinct smooth faces, small and of slight lustre (Figure 8).

		ϕ	ρ	
Crystal	1	20° 42′	56° 38′	poor.
"	8	24 22	56 50	good.
"	14	24 51	56 53	fair.
"	14	24 31	54 30	poor.
Calculated		24 53	56 52	

P, $-\frac{1}{2}$ $(\bar{1}12)$. Two small faces on the same crystal, very bright, with fine reflection (Figure 6).

		ϕ	ρ	
Crystal	7	−48° 30′	40° 07′	perfect.
"	7	−48 35	40 04	good.
Calculated		−48 27	39 59	

Q, $-\frac{1}{2}\ \frac{3}{4}$ $(\bar{2}34)$. Observed on every crystal not broken away in the part where it should occur. Faces often large and generally of high lustre, giving good reflections. A characteristic form for the locality (Figures 2, 3, 4, 5, 6, 7, 8, and 9).

		ϕ	ρ	
Crystal	1	−36° 15′	45° 45′	poor.
"	1	−36 50	46 55	fair (in twin position).
"	2	−37 00	46 04	perfect.
"	2	−36 41	46 35	"
"	2a	−37 00	46 13	good.
"	3	−36 57	46 13	perfect.
"	3	−36 52	46 16	"
"	5	−36 49	46 10	fair.
"	6	−36 42	46 24	poor.
"	6	−37 15	46 04	good.
"	7	−37 07	46 29	poor.
"	8	−36 40	46 50	good.
"	10	−36 45	46 06	perfect.
"	10	−36 32	46 14	good.
"	11	−36 51	48 08	"
"	12	−37 42	46 15	poor.

Crystal	13	—37 10	46 24	poor.
"	14	—36 51	46 17	perfect.
"	14	—37 53	46 16	good.
"	14	—35 59	46 15	poor.
Calculated		—36 57	46 14	

T, $-\frac{1}{2}\,\frac{5}{4}$ $(\bar{2}54)$. On two crystals, with small distinct faces, bright, and giving good reflections (Figures 7 and 8).

		ϕ	ρ	
Crystal	8	—24° 41′	56° 48′	good.
"	12	—24 27	56 42	"
Calculated		—24 18	56 45	

U, $\frac{1}{3}\,\frac{1}{2}$ (236). With three faces on two crystals. Faces distinct, but small, and reflections indistinct (Figure 8).

		ϕ	ρ	
Crystal	8	37° 18′	34° 42′	poor.
"	8	38 16	34 52	" (in twin position).
"	13	37 32	35 13	"
Calculated		37 53	35 10	

The following forms have been observed once or more as faces or reflections, but owing to their poor quality, or to the too great discrepancy between observations and calculated values, or for other reasons, they are considered as requiring confirmation :

χ, $0\frac{1}{3}$ (013). Not observed as a distinct face (Figure 5).

		ϕ	ρ	
Crystal	15	1° 11′	20° 00′	
"	6	0 44	20 18	
"	12	0 53	19 12	
"	12	1 08	19 53	(in twin position).
"	13	0 44	20 30	
"	13	0 58	19 20	
"	14	0 54	21 23	
"	15	1 19	20 41	
Calculated		1 20	20 21	

Γ, $0\frac{3}{4}$ (034). Observed three times, not as a definite face.

		ϕ	ρ	
Crystal	1	0° 38′	38° 48′	(in twin position).
"	3	0 45	40 25	" " "
"	6	0 34	38 45	
Calculated		0 35	39 50	

$0\frac{5}{6}$ (056). Same remarks as (034).

		ϕ	ρ	
Crystal	8	1° 11′	43° 00′	(in twin position).
"	8	0 09	43 42	
"	14	0 30	43 49	
"	15	0 29	42 50	
Calculated		0 32	42 50	

Λ, 03 (031). Observed but once on a crystal with a rich clinodome zone.

		ϕ	ρ
Crystal	6	1° 32′	73° 13′
Calculated		0 09	73 19

Δ, $-\frac{1}{2}0$ ($\bar{1}02$). Seen but once as a narrow line face truncating the edge between $\bar{2}14$ and $\bar{2}\bar{1}4$. Is probably to be counted with the certain forms (Figure 2).

		ϕ	ρ	
Crystal	10	−90° 00′	31° 57′	poor.
Calculated		−90 00	32 06	

$\frac{3}{4}0$ (304). Seen but once — a very doubtful form.

		ϕ	ρ
Crystal	1	89° 22′	44° 34
Calculated		90 00	43 55

W, $-2\frac{3}{2}$ ($\bar{4}32$). Seen but twice, faces of very doubtful quality (Figure 7).

		ϕ	ρ
Crystal	6	−56° 31′	72° 00′
"	12	−56 35	70 56
Calculated		−56 40	71 46

H, 2 (221). Observed on two crystals as a narrow line face between 111 and 110. A likely form, but needing better observations to establish it (Figure 8).

		ϕ	ρ	
Crystal	8	48° 52′	72° 45′	poor.
"	13	48 19	73 04	bad.
Calculated		48 56	73 33	

$-1\frac{1}{4}$ ($\bar{4}14$). Observed three times on two crystals, but variations in position too great to permit of its acceptance (Figure 7).

			ρ
Crystal	6	−77° 36′	52° 00′ bad.
"	6	−78 00	54 00 "
"	12	−76 55	51 01 "
Calculated		−77 35	52 18

Computation of the Elements. — Since the monoclinic character of leadhillite has been generally accepted, the elements commonly used have been those of Laspeyres[2] and of Artini,[3] determined on crystals from Sardinia.

Laspeyres, $a : b : c = 1.7476 : 1 : 2.2154.$ $\beta = 89° 47' 38''$
Artini, $a : b : c = 1.7515 : 1 : 2.2261.$ $\beta = 89° 31' 55''$

The result of our computation of elements, based on the measurements of 112 best faces of 15 crystals of the Utah leadhillite is intermediate between these values:

$$a : b : c = 1.7485 : 1 : 2.2244. \quad \beta = 89° 30' 28''$$

We have followed Goldschmidt, however, in halving the values of a and c, these elements giving on the whole simpler symbols for the form series, and the elements used by us, therefore, read as follows:

$$a : b : c = 0.8742 : 1 : 1.1122. \quad \beta = 90° 29' 32,''$$

which are derived from the polar elements, whose computation follows, by the relations,

$$\beta = 180° - \mu, \qquad a = \frac{q_0}{p_0 \sin \mu}, \qquad c = \frac{q_0}{\sin \mu}.$$

Believing that this axial ratio is more thoroughly established than those earlier deduced, we have calculated a new table of angles based upon it to replace that found in Goldschmidt, Winkeltabellen, p. 217 (Table V. p. 460).

In order to test the angles yielded by the new axial ratio as compared with those calculated from Laspeyres' elements as given in Goldschmidt, Winkeltabellen, the following measurements are recorded, made on a very perfect untwinned crystal of leadhillite from Sardinia, under conditions similar to those used in the study of the Utah crystals. Although the differences are of course slight, the agreement is in almost every case better with the new angles.

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[2] Zeit. für Kryst., **1**, 193 (1877).
[3] Giorn. Min., **1** 1, (1890).

Form.	Observed.				Calc. P. & LaF.				Calc. Gold.			
	ϕ		ρ		ϕ		ρ		ϕ		ρ	
	°	′	°	′	°	′	°	′	°	′	°	′
001	90	00	00	29	90	00	00	30	90	00	00	12
120	29	42	90	00	29	46	90	00	29	47	90	00
101	89	53	51	59	90	00	52	01	90	00	51	49
401	89	57	78	50	90	00	78	54	90	00	78	51
011	00	22	48	07	00	27	48	02	00	11	47	55
111	49	13	59	30	49	02	59	29	48	56	59	20
121	29	53	68	43	29	56	68	43	29	51	68	37
212	66	32	54	23	66	32	54	23	66	27	54	12
$\bar{1}22$	−29	28	52	00	−29	26	51	56	−29	38	51	53
$\bar{2}14$	−66	15	34	25	−66	06	34	28	−66	17	34	33

The calculation of the elements proceeded according to the method of Goldschmidt[4] as follows. For each of the best faces measured the two quantities,

$$x' = \sin \phi \tan \rho$$
$$y' = \cos \phi \tan \rho$$

were calculated, ϕ and ρ being the measured angles for each face and x′ and y′ the rectangular coördinates of the projection point of the face in gnomonic projection.

Now in the monoclinic system the following relations hold:

$$\left.\begin{aligned} x' &= p\,p_0 + e \\ -x' &= -p\,p_0 + e \end{aligned}\right\}\ \mathrm{I}$$
$$y' = q\,q_0 \qquad \mathrm{II}$$

where p and q are rational multiples of the elements p_0 and q_0 (coördinates of the unit form) and e = cot μ.

Since μ could not be measured directly on our crystals, it was necessary to calculate both e and p_0 in equations I and q_0 in equation II, these three quantities being the elements of the mineral which it was desired to determine.

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[4] Ueber Lorandit von Allchar in Macedonien, Zeit. für. Kryst., **30**, 281 (1898).

Ten equations were formed by substituting in equations I the various values of p and the averages of all corresponding values of x′ as follows:

(A)	$\frac{1}{3} p_0 + e =$	.4311	based on	2	values of	x′
(B)	$-\frac{1}{3} p_0 + e = -$	.4155	"	3	"	x′
(C)	$\frac{1}{2} p_0 + e =$	.6442	"	6	"	x′
(D)	$-\frac{1}{2} p_0 + e = -$	.6272	"	21	"	x′
(E)	$-\frac{4}{3} p_0 + e = -$	.8392	"	5	"	x′
(F)	$p_0 + e =$	1.2808	"	10	"	x′
(G)	$- p_0 + e =$	−1.2635	"	21	"	x′
(H)	$2 p_0 + e =$	2.5556	"	5	"	x′
(I)	$-2 p_0 + e =$	−2.5359	"	11	"	x′
(J)	$4 p_0 + e =$	5.0953	"	4	"	x′

and these equations were solved in pairs for e and p_0 (D), based on the largest number of the best values of x′ being combined with each of the others for this purpose. The following nine values for e and p_0 were thus obtained, weighted in accordance with their relative importance, and combined in a final average. It is the close accordance of these values which seems to attest the reliability of the elements here determined.

D and A	$e = .0078$	$p_0 = 1.2700$
D and B	$e = .0079$	$p_0 = 1.2702$
D and C	$e = .0085$	$p_0 = 1.2714$
D and E	$e = .0088$	$p_0 = 1.2720$
D and F	$e = .0088$	$p_0 = 1.2720$
D and G	$e = .0091$	$p_0 = 1.2726$
D and H	$e = .0094$	$p_0 = 1.2731$
D and I	$e = .0090$	$p_0 = 1.2725$
D and J	$e = .0086$	$p_0 = 1.2717$

Weighted mean, $\cot \mu = e = .0086$ $\quad p_0 = 1.2722$

$$\mu = 89° \, 30' \, 28''.$$

In like manner the value of q_0 was found by subsituting in equation II various values of q and the averages of corresponding values of y′, and then weighting and averaging the results.

(A)	$\frac{1}{6} q_0 = 0.1852$	3	values of	y′	$q_0 = 1.1112$
(B)	$\frac{1}{4} q_0 = 0.2779$	7	"	y′	$q_0 = 1.1116$
(C)	$\frac{1}{3} q_0 = 0.3702$	2	"	y′	$q_0 = 1.1106$
(D)	$\frac{1}{2} q_0 = 0.5563$	14	"	y′	$q_0 = 1.1126$
(E)	$\frac{2}{3} q_0 = 0.7421$	4	"	y′	$q_0 = 1.1131$

(F)	$\frac{3}{4} q_0 = 0.8343$	6	values of	y′	$q_0 = 1.1124$
(G)	$1 q_0 = 1.1116$	15	“	y′	$q_0 = 1.1116$
(H)	$\frac{5}{4} q_0 = 1.3909$	1	“	y′	$q_0 = 1.1127$
(I)	$\frac{3}{2} q_0 = 1.6705$	5	“	y′	$q_0 = 1.1136$
(J)	$2 q_0 = 2.2231$	7	“	y′	$q_0 = 1.1115$
(K)	$\frac{5}{2} q_0 = 2.7793$	4	“	y′	$q_0 = 1.1117$
(L)	$3 q_0 = 3.3297$	1	“	y′	$q_0 = 1.1099$
(M)	$4 q_0 = 4.4371$	3	“	y′	$q_0 = 1.1093$
			Weighted mean,		$q_0 = 1.1122$

Twinning.— The crystals are often twinned, the twinning plane being regarded as the prism m (120) according to the usual twinning law of the species. Three types of twins may be recognized: (1) contact twins of the aragonite type with a face of the twinning plane m as composition plane, seen chiefly in cleavage flakes under the microscope; (2) contact or lamellar twins, the composition face parallel to a face of v ($\bar{1}22$), (see Figures 8 and 9); (3) interpenetration twins in which the faces in normal position and those in twin position are mingled without any apparent system and can only be distinguished by measurement and projection.

The gnomonic projection is particularly useful in the study of such complex twin crystals of this general type where the twin plane is normal to the plane of projection. The projection points of a face and its twin then lie symmetrically on either side of the trace of the twin plane, that is, equidistant from the trace and on a perpendicular to it. This test can be quickly and easily applied in the projection to any face concerning which there is doubt as to whether it is in normal or twin position, and the rule was adopted, after much study in the special case of these crystals, that the position of a face should be accepted as correct, which, tested in this way, gave the simplest indices.

It was noted in applying this test that the prism F (320) is almost at right angles to m ($320 \wedge \bar{1}20 = 89° 32'$), and this relation leads to a certain amount of ambiguity in the interpretation of the twinning. The prism F has been recorded as the twin plane of lamellar twins of leadhillite due to elevation of temperature, but it is not found in the form series of the mineral. Since their planes are so nearly at right angles, twinning on m and on F will produce closely similar effects, and the decision in favor of the former law is somewhat arbitrary, as may be judged from the following statement of the respective relations.

The most striking effect of twinning by either law is the practical superposition of certain faces lying in radial zones. If the twinning be on (120), the radial zone containing the forms v, r, and R is, in twin position, almost coincident in direction with the positive orthodome zone, and the three forms named correspond in position to the domes w, u, and y.

Twinned on m (120),

		φ	ρ
w	(101)	90° 00′	52° 01′
v	$(\bar{1}22)$ twin	88 58	51 56
u	(201)	90 00	68 37
r	$(\bar{1}21)$ twin	89 08	68 39
y	(401)	90 00	78 54
R	$(\bar{2}41)$ twin	89 13	78 57

If on the other hand the twinning is on (320), the above-named pyramid zone occupies in twin position nearly the same direction as before, but the forms correspond to the negative domes f and e.

Twinned on F (320),

		φ	ρ
f	$(\bar{1}01)$	−90° 00′	51° 39′
v	$(\bar{1}22)$ twin	−90 06	51 56
e	$(\bar{2}01)$	−90 00	68 29
r	$(\bar{1}21)$ twin	−89 56	68 39

The same relation exists for twinning on (120) between the pyramids t (122) and x (121) and the domes f and e: and for twinning on (320) between t and x and the domes w and u. Hence in twinned crystals any of these pairs of faces usually appears as a single face, which, however, reflects a double or (owing to vicinals) a multiple signal. The face can, however, sometimes be seen to be made up of two very slightly inclined portions separated by an oblique line, the trace of the composition face v (Figures 8 and 9).

The measurements obtained on twinned crystals were too variable to decide between the two laws where the angular differences were so slight; but it was found that the pyramid series v, r, and R occurred repeatedly in twin position with the dome series w, u, and y, and since the negative dome corresponding to y and the positive pyramid corresponding to R were not found on our crystals and are not known for the mineral, it seems necessary to conclude that the twinning is on the first law or m (120).

A second case of approximate superposition of zones by twinning is in the case of the radial zone containing the pyramids ζ, s, δ, μ, q, and Y, which in twin position by either law lies about six degrees from the direction of the clinodome zone. Here, however, the polar distances of the faces in the two zones are different, and the result of the twinning is generally the formation of wedge-shaped faces dovetailing irregularly into one another (Figure 8).

It will be seen from what has been said that the twinning does not in any way obscure, but rather tends to increase the pseudo-rhombohedral appearance of the crystals. Figure 9 is intended to bring out this striking habit.

Cleavage plates examined under the microscope in polarized light are usually found to be twins of the second kind mentioned, but in thin plates the lamellae appear to be united on the prism m. When a sufficiently thick plate is examined, the lamellae are seen to be oblique to the cleavage, and the composition face was found to be parallel to v ($\bar{1}22$). Twins of the third kind, in polarized light, usually show three sets of axial figures inclined to each other at 60° and they do not give complete extinction in any position.

No chemical analysis was made of this leadhillite, and the optical characters have been only partially determined. The axial angle of a cleavage plate was measured in air and in cedar oil with the following results:

$2E_{Na} = 19° \; 54'$ $2E_{Li} = 19° \; 14'$ Temp. 23° C.
$2H_{Na} = 13° \; 24'$ $2H_{Li} = 12° \; 38'$ (in cedar oil) " "

The axial angle was observed to grow smaller with increase of temperature, but no successful measurement of the rate of change, nor of the temperature at which it becomes uniaxial, was obtained.

This study was begun at the time of the receipt of the leadhillite, by Palache, but the crystals proved so complex that it was thought best to put the matter aside in the hope that more material would be found for study without breaking up any of the original lot. Several years elapsed, and the investigation was renewed by La Forge, when, by using a part of the finer specimens, material was obtained which sufficed to unravel the complexities of the crystallization. The work was again interrupted by the illness of the last named, and again a long period passed before the results obtained could be put into shape for publication. In its present form the paper has been prepared by Palache, but the observations in large part, and all of the calculations involved, as well as the drawings, are the work of La Forge.

II. LEADHILLITE FROM NEVADA.

By C. Palache.

The results of the investigation of leadhillite from Utah are confirmed and extended in an interesting manner by the study of another occurrence of the mineral recently brought to light by Dr. T. A. Jaggar. In the course of an examination of the Quartette Gold Mine, at Searchlight, Lincoln County, Nevada, Dr. Jaggar collected specimens of the ores which were submitted to the writer for determination of some of the constituent minerals. Much of the ore at present worked is massive cerussite; imbedded in this substance glistening cleavage plates of a pale green mineral were noted which proved to be leadhillite. Careful search revealed a single cavity in the cerussite, lined and partly filled by interlaced tabular crystals of the mineral, which though very small and for the most part fragmentary, proved to be very well adapted to measurement and yielded a surprisingly rich form series.

The other minerals of the ores of this mine are, first and most important, free gold, which occurs in visible particles in a quartz vein-stuff brilliantly stained with blue chrysocolla. Wulfenite is also found implanted on quartz in crystals of two types, one pale yellow with cubical habit showing the forms m (110) μ (430), n (111), e (101), and c (001); the other in deep red tabular crystals showing the forms l (740, e (101), u (102), n (111) and s (113). In a few cavities in massive gray cerussite were crystals of cerussite with the forms b (010), c (001), m (110), x (120), γ (013), i (021), z (041), y (102), and e (101). Many ore surfaces are covered with a drusy black coating, greenish when rubbed, which proves to be cuprodescloizite in crystals too minute to be interpreted. Calcite, malachite, and hematite are abundant in crevices of the brecciated vein material and wall-rock. Sulphide ores, except minute amounts of galena, have not yet been met with in the mine.

The crystals of leadhillite are always tabular, and most of those measured had one or both of the basal planes as crystal faces rather than as cleavage. The tiny tables, rarely more than a millimeter across, were attached to the cavity wall by an edge and projected freely, so that faces were present in both upper and lower octants, requiring two adjustments on the goniometer for complete measurement. Some seventeen crystals were measured, and yielded the forms shown in Table III. The crystals proved to be largely free from twinning, and when twinned the two individuals were in contact rather than interpenetrating, so that the interpretation of the results of measure-

TABLE III.

Letter.	Symbol — Gdt.	Symbol — Miller.	Calculated.		Observed Mean.		Differences in Minutes.				Quality of Reflection.	No. of Faces.	No. of Crystals.
			ϕ	ρ	ϕ	ρ	ϕ		ρ				
							+	−	+	−			
			° ′	° ′	° ′	° ′	′	′	′	′			
c	0	001	90 00	00 30	90 00	00 30	..	..	14	10	perfect	19	14
b	0∞	010	00 00	90 00	00 00	90 00	..	..	..	..	good	11	10
a	$\infty 0$	100	90 00	90 00	90 00	90 00	02	10	..	..	perfect	12	11
j[1]	4∞	410	77 40	90 00	77 32	90 00	02	27	..	..	good	6	6
d	2∞	210	66 23	90 00	66 18	90 00	06	29	..	..	fair	8	7
l	∞	110	48 50	90 00	48 49	90 00	16	20	..	..	good	11	9
L	$\frac{2}{3}\infty$	230	37 20	90 00	37 20	90 00	14	7	..	..	good	6	5
m	$\infty 2$	120	29 46	90 00	29 46	90 00	12	15	..	..	good	17	12
ν	$0\frac{1}{4}$	014	1 46	15 33	00	15 34	..	..	..	..	..	1	1
χ	$0\frac{1}{3}$	013	1 20	20 21	39	20 22	..	41	19	17	poor	2	2
α	$0\frac{1}{2}$	012	53	29 05	39	28 54	..	14	..	11	poor	1	1
Γ	$0\frac{3}{4}$	034	35	39 50	20	39 50	4	35	6	9	poor	3	3
h	$0\frac{3}{2}$	032	18	59 04	15	59 04	7	15	7	10	fair	5	5
g	01	011	27	48 02	18	48 06	7	27	13	9	good	6	6
ϕ	02	021	13	65 48	06	65 56	2	13	21	16	fair	11	8
Λ	03	031	09	73 19	05	73 20	8	4	6	2	fair	3	2
y	40	401	90 00	78 54	90 00	78 39	..	..	6	54	poor	3	3
u	20	201	90 00	68 37	90 00	68 38	3	..	21	2	poor	6	6
z	$\frac{3}{2}0$	302	90 00	62 27	90 00	61 45	..	..	..	42	poor	1	1
C[1]	$\frac{4}{3}0$	403	90 00	59 36	90 00	59 45	..	..	9	..	fair	1	1
w	10	101	90 00	52 01	90 00	52 02	3	..	8	7	fair	7	8
i	$\frac{2}{3}0$	203	90 00	40 35	90 00	40 50	..	..	41	..	poor	3	3
D	$\frac{1}{2}0$	102	90 00	32 48	90 15	32 30	15	..	..	18	good	1	1
f	−10	$\bar{1}01$	−90 00	51 39	−90 00	51 47	..	..	31	4	poor	4	4
e	−20	$\bar{2}01$	−90 00	68 29	−90 00	68 32	..	9	19	4	fair	4	4
k	1	111	49 02	59 29	49 01	59 33	13	14	18	9	good	9	8
s	$1\frac{1}{2}$	212	66 32	54 23	66 32	54 26	9	16	4	15	good	7	6
θ	$1\frac{3}{2}$	232	37 31	64 34	37 29	64 38	17	14	19	6	good	7	6
x	12	121	29 56	68 43	29 57	68 41	5	7	17	10	perfect	8	7
I[1]	$1\frac{5}{2}$	252	24 44	71 54	24 49	71 55	5	..	1	..	fair	1	1
K[1]	13	131	21 00	74 22	20 57	74 27	..	7	11	..	good	3	2
q	$-1\frac{1}{2}$	$\bar{2}12$	−66 15	54 05	−66 12	54 03	18	10	14	12	good	9	8
p	−11	$\bar{1}11$	−48 39	59 17	−48 34	59 17	10	31	7	8	good	9	7
o	$-1\frac{3}{2}$	$\bar{2}32$	−37 09	64 28	−37 11	64 26	15	9	7	13	good	7	6
r	−12	$\bar{1}21$	−29 30	68 39	−29 35	68 38	12	21	14	17	good	11	10
A	$-1\frac{5}{2}$	$\bar{2}52$	−24 23	71 52	−24 29	71 48	17	2	5	22	perfect	5	4
G	−13	$\bar{1}31$	−20 45	74 21	−20 39	74 31	..	12	23	..	fair	3	3
n	$-1\frac{7}{2}$	$\bar{2}72$	−17 59	76 16	−18 06	76 31	19	3	28	6	fair	5	5
S[1]	−14	$\bar{1}41$	−15 51	77 48	−15 47	77 57	7	12	12	..	fair	3	3
V[1]	$-1\frac{9}{2}$	$\bar{2}92$	−14 10	79 02	−14 07	79 14	..	..	3	12	..	1	1
ω	$2\frac{1}{2}$	412	77 43	69 03	77 41	69 09	11	7	12	..	good	5	5
ζ	21	211	66 28	70 15	66 31	70 08	8	0	13	2	fair	6	5

[1] New forms.

TABLE III. — *Continued.*

Letter.	Symbol. — Gdt.	Symbol — Miller.	Calculated.		Observed Mean.		Differences in Minutes.				Quality of Reflection.	No. of Faces.	No. of Crystals.
			ϕ	ρ	ϕ	ρ	ϕ		ρ				
							+	−	+	−			
							′	′	′	′			
γ	31	$\underline{3}11$	73 47	75 55	73 47	75 56	4	7	20	15	good	6	5
ρ	$-2\frac{1}{2}$	$\underline{4}12$	−77 38	68 55	−77 32	68 55	4	20	8	9	fair	6	6
Y	−21	$\underline{2}11$	−66 19	70 09	−66 08	70 11	4	25	5	1	poor	2	2
W	$-2\frac{3}{2}$	$\underline{4}32$	−56 40	71 46	−56 35	71 47	4	15	10	2	fair	4	4
X[1]	−22	$\underline{2}21$	−48 45	73 29	−48 39	73 31	6	25	6	8	poor	5	5
M	$-2\frac{5}{2}$	$\underline{4}52$	−42 22	75 07	−42 23	75 17	9	8	27	4	poor	4	3
Z[1]	−23	$\underline{2}31$	−37 14	76 35	−37 04	76 37	. .	10	2	. .	fair	1	1
R	−24	$\underline{2}41$	−29 41	78 57	−29 44	79 00	3	. .	3	. .	fair	2	2
Σ[1]	$-\frac{3}{2}\ \frac{1}{4}$	614	−81 40	62 29	−81 35	62 39	7	16	18	. .	**poor**	3	2
β	$\frac{1}{3}\ \frac{2}{3}$	123	30 16	40 39	30 11	40 42	. .	5	3	. .	perfect	1	1
Φ[1]	$\frac{1}{3}\ \frac{5}{6}$	$\underline{2}56$	25 01	45 39	25 05	45 39	4	. .	. .	. .	fair	1	1
λ	$-\frac{1}{3}\ \frac{1}{6}$	$\underline{2}16$	−65 57	24 28	−65 34	24 30	. .	44	13	9	fair	2	1
B	$-\frac{1}{3}\ \frac{2}{3}$	123	−29 16	40 22	−29 29	40 18	21	. .	5	10	poor	3	2
δ	$\frac{1}{2}\ \frac{1}{4}$	214	66 40	35 04	66 49	34 56	20	1	. .	13	poor	2	2
ϵ	$\frac{1}{2}\ \frac{1}{2}$	112	49 13	40 25	49 17	40 20	7	. .	1	10	good	2	2
Ψ[1]	$\frac{1}{2}\ \frac{3}{4}$	234	37 42	46 30	37 30	46 27	. .	12	. .	3	perfect	1	1
t	$\frac{1}{2}\ 1$	122	30 06	52 07	30 14	52 03	31	8	5	13	good	4	3
N	$\frac{1}{2}\ \frac{5}{4}$	254	24 53	56 52	24 53	57 03	. .	. .	11	. .	poor	1	1
Ω[1]	$\frac{1}{2}\ \frac{3}{2}$	$\underline{1}32$	21 08	60 47	21 13	60 54	9	8	9	. .	poor	3	2
μ	$-\frac{1}{2}\ \frac{1}{4}$	$\underline{2}14$	−66 06	34 28	−66 14	34 20	16	. .	. .	10	fair	2	1
P	$-\frac{1}{2}\ \frac{1}{2}$	$\underline{1}12$	−48 27	39 59	−48 37	39 59	20	5	18	9	poor	3	2
Q	$-\frac{1}{2}\ \frac{3}{4}$	$\underline{2}34$	−36 57	46 14	−37 18	46 17	21	. .	3	. .	good	1	1
v	$-\frac{1}{2}\ 1$	$\underline{1}22$	−29 26	51 56	−29 27	51 51	22	10	7	32	good	9	7
Θ[1]	$-\frac{2}{3}\ \frac{1}{2}$	$\underline{4}36$	−56 29	45 12	−56 49	45 02	20	. .	. .	10	fair	1	1
O[1]	$-\frac{7}{8}\ \frac{3}{4}$	768	−52 56	54 09	−52 57	54 16	1	. .	7	. .	perfect	1	1

[1] New forms.

ment was much less difficult than in the case of the Utah leadhillite. But the crystals were so fragmentary and so complex, and there was such an entire lack of features by which the forms could be identified on inspection, that it was only by means of the graphic treatment of the measurements in gnomonic projection that they could be clearly understood. Adjustment on the goniometer was always made approximately by means of the base and accurately by the never-failing prism zone.

Of the sixty-seven forms observed, fourteen were new, bringing the total forms known for the mineral to seventy-seven. Of equal interest with the new forms, however, was the observation on this material of many of the forms first found on the Utah leadhillite, and particularly of the best established ones. Ten of the Utah forms regarded as certain and five of those considered doubtful were found on the Nevada material, furnishing a welcome confirmation of the results recorded in the preceding paper. Moreover, the thirteen Utah forms not observed here were with one exception weak or uncertain forms.

Only two of the forms known on leadhillite previous to this investigation were not observed. The first of these, σ ($\bar{2}33$), was first found by Artini as a minute face; he could obtain no measurements and regarded it as doubtful. One face was found on a crystal from Utah near this position, and the form is probably to be regarded as established.

The second form, τ ($\bar{4}$. 14. 7), with complex symbol and abnormal position in the form system of leadhillite, is a dubious form, probably to be replaced by the simpler form ($\bar{1}42$), which is not far removed. This possibility was, however, considered by Artini and rejected. He observed a single face of the form, the observed zonal relations and angles of which seemed to him to preclude its interpretation as ($\bar{1}42$).

The combinations observed are shown in Table IV. As was the case with the Utah crystals, the forms most frequently found are c, a, m, and r, which are present on nearly every crystal. b, d, l, g, φ, u, w, k, x, q, and v are present on at least half the crystals. Of the remaining forms the new prism, j, and the pyramids A, n, γ, and ρ are the most important, all others being of very rare occurrence.

The new forms on the leadhillite from Nevada, with which will be included the five uncertain Utah forms here confirmed, are based on the following data:

j, 4 ∞ (410). A prism, well established by frequent occurrence with distinct faces, often of good quality.

		φ	ρ	
Crystal	3	77° 30′	90° 00′	poor.
“	7	77 42	“	perfect.
“	9	77 13	“	“
“	10	77 33	“	poor.
“	12	77 35	“	very poor.
“	14	77 40	“	fair.
Calculated		77 40	90 00	

χ, 0⅓ (013). Seen twice as a distinct face in the clinodome zone. Reflections poor. Found also on the Utah leadhillite, and hence regarded as assured.

TABLE IV.

NEVADA LEADHILLITE.

Cryst. No.	c	b	a	j	d	l	L	m	ν	χ	α	Γ	g	h	φ	Λ	y	u	z	C	w	i	D	f	e	k	s	θ	x	I	K	q	p
1	×	×							×	×		×	×																				
2	×		×			×		×																		×	×	×	×				
3	×		×	×				×										×	×		×	×			×	×			×			×	
4	×	×	×					×							×														×				
5	×		×		×	×	×											×			×					×	×						
6	×		×					×																×									
7	×		×	×	×	×	×	×																	×							×	×
8	×	×	×		×	×							×	×	×			×			×					×	×	×	×			×	
9	×	×	×	×	×	×		×				×					×	×			×				×	×		×	×			×	×
10	×	×	×	×	×	×	×	×					×	×	×	×	×	×			×					×	×	×	×		×		
11	×	×						×		×	×	×	×		×	×					×								×	×	×		
12	×		×	×	×	×	×	×									×									×		×	×			×	×
13	×	×	×										×	×	×			×			×	×		×								×	×
14	×	×	×	×	×	×		×					×	×	×		×	×		×	×	×		×	×	×	×	×	×		×	×	×
15	×	×	×			×	×	×					×	×	×						×								×		×	×	×
16	×	×											×	×	×																		
17	×		×	×	×	×	×	×									×	×			×	×	×	×	×	×	×		×			×	×

	o	r	A	G	n	S	V	ω	ζ	γ	ρ	Y	W	X	M	Z	R	Σ	β	Φ	λ	B	δ	ε	Ψ	t	N	Ω	μ	P	Q	v	Θ	O
1																																		
2								×		×																								
3		×						×																		×								
4																																		
5								×	×	×																								
6		×																																
7	×	×									×	×	×																			×		
8								×		×																								
9		×	×	×	×				×	×	×	×	×	×			×						×				×					×		
10		×	×	×	×	×	×		×		×		×			×	×											×				×		×
11		×			×	×											×									×	×	×				×		
12	×	×							×	×	×			×	×																			
13	×	×	×	×	×						×											×								×	×	×		
14	×	×	×	×	×			×	×	×	×			×	×			×	×	×				×	×	×	×					×		
15	×	×	×	×	×	×								×								×								×	×	×	×	
16	×	×	×		×																											×		
17	×	×	×						×	×	×		×	×	×			×	×		×	×	×	×		×			×	×	×	×		

		φ	ρ	
Crystal	1	00° 00′	20° 04′	fair.
"	11	00 39	20 40	poor.
Calculated		1 20	20 21	

Γ, 0$\frac{3}{4}$ (034). On three crystals as narrow faces with poor reflections but in good position. Also observed on Utah material.

	φ	ρ	
Crystal 1	00° 00′	39° 52′	poor.
" 9	00 00	39 56	very poor.
" 11	00 39	39 41	fair.
Calculated	00 35	39 50	

Λ, 03 (031). Seen on two crystals with three faces, two of them large and distinct, giving good reflections, in excellent agreement with calculated position. Since it was also observed once on Utah crystals, the form is well established.

	φ	ρ	
Crystal 10	00° 05′	73° 25′	perfect.
" 10	00 17	72 42	very poor.
" 11	00 09	73 17	fair.
Calculated	00 09	73 19	

C, $\frac{4}{3}$0 (403). This dome was seen but once as a distinct though narrow face in the orthodome zone. Although the reflection was poor, it is in good position and the form is regarded as established.

	φ	ρ	
Crystal 14	90° 00′	59° 45′	poor.
Calculated	90 00	59 36	

I, 1$\frac{5}{2}$ (252). Observed but once as a distinct face in an important zone in excellent position.

	φ	ρ	
Crystal 11	24° 49′	71° 55′	fair.
Calculated	24 44	71 54	

K, 13 (131). Observed twice on one crystal and once on a second with excellent faces. It is in the same zone with the foregoing and in excellent position.

	φ	ρ	
Crystal 10	21° 00′	74° 24′	good.
" 10	20 59	74 26	"
" 11	20 58	74 25	"
Calculated	21 00	74 22	

S, $\bar{1}4$ ($\bar{1}41$). Observed on two crystals as a narrow line face in an important zone and on a third as a larger face with excellent reflection in good position.

	ϕ	ρ	
Crystal 10	−15° 44′	78° 00′	poor.
" 11	−15 46	77 52	perfect.
" 15	−15 58	77 56	fair.
Calculated	−15 51	77 48	

V, $\bar{1}\frac{9}{2}$ ($\bar{2}92$). Observed but once as a distinct facet in the same zone with the last and established by its good position.

	ϕ	ρ
Crystal 10	−14° 10′	79° 09′
Calculated	−14 10	79 02

W, $\bar{2}\frac{3}{2}$ ($\bar{4}32$). This form, which was observed twice on Utah crystals but could not be established, was found on four crystals with distinct faces in good position. With the two following forms it is in an important zone.

	ϕ	ρ	
Crystal 7	−56° 37′	71° 44′	fair.
" 9	−56 25	71 45	perfect.
" 10	−56 36	71 45	fair.
" 17	−56 44	71 56	poor.
Calculated	−56 40	71 46	

X, $\bar{2}$, ($\bar{2}21$). Observed on five crystals and in good position despite the poor quality of the reflections.

	ϕ	ρ	
Crystal 9	−48° 20′	73° 21′	poor.
" 12	−48 40	73 35	fair.
" 14	−48 40	73 35	"
" 15	−48 43	73 35	very poor.
" 17	−48 51	73 28	poor.
Calculated	−48 45	73 29	

Z, $\bar{2}3$ ($\bar{2}31$). Observed once as a distinct facet in the zone [$\bar{1}21 \wedge \bar{1}10$] and in good position.

	ϕ	ρ	
Crystal 10	−37° 08′	76° 30′	fair.
Calculated	−37 14	76 35	

Σ, $\frac{\bar{3}}{2}\frac{1}{4}$ ($\bar{6}14$). Observed three times on two crystals as distinct facets. Accepted despite the poor quality of faces and somewhat variable position because of its simple position in the zone [$\bar{2}01 \wedge 011$].

	ϕ	ρ	
Crystal 14	−81° 43′	62° 40′	poor.
" 14	−80 07	62 30	fair.
" 17	−81 34	62 36	"
Calculated	−81 40	62 29	

Φ, $\frac{1}{3}\ \frac{5}{6}$ (256). Observed as a distinct face with good reflection on a single crystal, in the zone $[012 \wedge 122]$. Position good.

	ϕ	ρ	
Crystal 14	25° 00′	45° 35′	fair.
Calculated	25 01	45 39	

Ψ, $\frac{1}{2}\ \frac{3}{4}$ (234). Observed as a distinct face with good reflection on the same crystal as the last, in the zone $[111 \wedge 123]$. Confirmed by its good position.

	ϕ	ρ	
Crystal 14	37° 33′	46° 24′	good.
Calculated	37 42	46 30	

Ω, $\frac{1}{2}\ \frac{3}{2}$ (132). Observed with two faces on one crystal and one on a second, small and with poor reflections. Accepted, however, because of its good position and place in an important zone.

	ϕ	ρ	
Crystal 10	21° 17′	60° 50′	poor.
" 10	21 09	61 00	fair.
" 11	21 12	60 53	poor.
Calculated	21 08	60 47	

Θ, $\frac{\bar{2}}{3}$, $\frac{1}{2}$ $(\bar{4}36)$. Observed but once as a distinct face with fine reflection. The position is not wholly satisfactory.

	ϕ	ρ	
Crystal 15	−56° 49′	45° 02′	fair.
Calculated	−56 29	45 12	

O, $\frac{\bar{7}}{8}\ \frac{3}{4}$ $(\bar{7}68.)$ Observed but once as a large distinct face with perfect reflection. The position of the face is extremely close to that of the common form q $(\bar{2}12)$ in twin position; but as the crystal on which it occurs shows no other indications of twinning, as the form lies in the important zone $[\bar{2}01 \wedge \bar{1}22]$, and as the measured angles agree more closely with the calculated position of this form than with those of q in twin position, the form is regarded as assured despite its somewhat complex symbol.

	ϕ	ρ	
Crystal 10	−52° 57′	54° 16′	perfect.
Calculated, twin of q	−53 17	54 05	
Calculated $(\bar{7}68)$	−52 56	54 09	

TABLE V.

LEADHILLITE. *Monoclinic.*

$a = 0.8742$	$\log a = 9.94165$	$\log a_0 = 9.89546$	$\log p_0 = 0.10454$	$a_0 = 0.7861$	$p_0 = 1.2721$
$c = 1.1122$	$\log c = 0.04618$	$\log b_0 = 9.95382$	$\log q_0 = 0.04618$	$b_0 = 0.8992$	$q_0 = 1.1122$
$\left.\begin{matrix}\mu = \\ 180° - \beta\end{matrix}\right\} 89° 30\frac{1}{2}'$	$\left.\begin{matrix}\log h = \\ \log \sin \mu\end{matrix}\right\} 9.99998$	$\left.\begin{matrix}\log e = \\ \log \cos \mu\end{matrix}\right\} 7.93398$	$\log \frac{p_0}{q_0} = 0.05836$	$h = 0.99996$	$e = 0.0086$

Number.	Letter.	Symbol—Gdt.	Symbol—Miller.	ϕ	ρ	ξ_0	η_0	ξ	η	x′	y′	d′ = tan ρ
				° ′	° ′	° ′	° ′	° ′	° ′			
1	c	0	001	90 00	0 30	0 30	0 00	0 30	0 00	0.0086	0	0.0086
2	b	0∞	010	0 00	90 00	0 00	90 00	0 00	90 00	0	∞	∞
3	a	∞0	100	90 00	"	90 00	0 00	90 00	0 00	∞	0	"
4	j	4∞	410	77 40	"	"	90 00	77 40	12 20	4.5753[1]	∞	"
5	d	2∞	210	66 23	"	"	"	66 23	23 37	2.2877[1]	"	"
6	F	$\frac{3}{2}\infty$	320	59 46	"	"	"	59 46	30 14	1.7158[1]	"	"
7	l	∞	110	48 50	"	"	"	48 50	41 10	1.1439[1]	"	"
8	L	$\frac{2}{3}\infty$	230	37 20	"	"	"	37 20	52 40	0.7626[1]	"	"
9	m	∞2	120	29 46	"	"	"	29 46	60 14	0.5719[1]	"	"
10	ν	$0\frac{1}{4}$	014	1 46	15 33	0 30	15 32	0 28	15 32	0.0086	0.2780	0.2782
11	χ	$0\frac{1}{3}$	013	1 20	20 21	"	20 21	0 28	20 20	"	0.3707	0.3708
12	α	$0\frac{1}{2}$	012	0 53	29 05	"	29 05	0 26	29 05	"	0.5561	0.5561
13	η	$0\frac{2}{3}$	023	0 40	36 33	"	36 33	0 24	36 33	"	0.7414	0.7414
14	Γ	$0\frac{3}{4}$	034	0 35	39 50	"	39 50	0 23	39 50	"	0.8341	0.8341
15	g	01	011	0 27	48 2	"	48 02	0 20	48 02	"	1.1121	1.1121
16	h	$0\frac{3}{2}$	032	0 18	59 04	"	59 04	0 15	59 04	"	1.6682	1.6682
17	π	$0\frac{5}{3}$	053	0 16	61 39	"	61 39	0 14	61 39	"	1.8535	1.8535
18	ϕ	02	021	0 13	65 48	"	65 48	0 12	65 48	"	2.2242	2.2242
19	ψ	$0\frac{5}{2}$	052	0 11	70 13	"	70 13	0 10	70 13	"	2.7803	2.7803
20	Λ	03	031	0 09	73 19	"	73 19	0 08	73 19	"	3.3363	3.3363
21	y	$\overset{+}{4}0$	401	90 00	78 54	78 54	0 00	78 54	0 00	5.0974	0	5.0972

[1] $\frac{x'}{y'}$.

TABLE V — *Continued.*

Number.	Letter.	Symbol—Gdt.	Symbol—Miller.	φ	ρ	ξ_0	η_0	ξ	η	x′	y′	d′ = tan ρ
				° ′	° ′	° ′	° ′	° ′	° ′			
22	u	$\overset{+}{20}$	201	"	68 37	68 37	"	68 37	"	2.5529	"	2.5529
23	z	$\overset{+}{\frac{3}{2}0}$	302	"	62 27	62 27	"	62 27	"	1.9168	"	1.9168
24	C	$\overset{+}{\frac{4}{3}0}$	403	"	59 36	59 36	"	59 36	"	1.7045	"	1.7045
25	w	$\overset{+}{10}$	101	"	52 01	52 01	"	52 01	"	1.2808	"	1.2808
26	i	$\overset{+}{\frac{2}{3}0}$	203	"	40 35	40 35	"	40 35	"	0.8567	"	0.8567
27	D	$\overset{+}{\frac{1}{2}0}$	102	"	32 48	32 48	"	32 48	"	0.6446	"	0.6446
28	E	$\overset{-}{\frac{2}{3}0}$	$\bar{2}03$	−90 00	40 01	−40 01	"	−40 01	"	−0.8395	"	0.8395
29	f	$\overset{-}{10}$	$\bar{1}01$	"	51 39	−51 39	"	−51 39	"	−1.2636	"	1.2636
30	e	$\overset{-}{20}$	$\bar{2}01$	"	68 29	−68 29	"	−68 29	"	−2.5357	"	2.5357
31	k	$\overset{+}{1}$	111	49 02	59 29	52 01	48 02	40 35	34 23	1.2808	1.1121	1.6962
32	s	$\overset{+}{1\frac{1}{2}}$	212	66 32	54 23	"	29 05	48 13	18 54	"	0.5561	1.3963
33	θ	$\overset{+}{1\frac{3}{2}}$	232	37 31	64 34	"	59 04	33 22	45 45	"	1.6682	2.1032
34	x	$\overset{+}{12}$	121	29 56	68 43	"	65 48	27 43	53 51	"	2.2242	2.5666
35	I	$\overset{+}{1\frac{5}{2}}$	252	24 44	71 54	"	70 13	23 26	59 42	"	2.7800	3.0610
36	K	$\overset{+}{13}$	131	21 00	74 22	"	73 19	20 11	64 02	"	3.3363	3.5740
37	q	$\overset{-}{1\frac{1}{2}}$	$\bar{2}12$	−66 15	54 05	−51 39	29 05	−47 50	19 02	−1.2636	0.5561	1.3805
38	p	$\overset{-}{11}$	$\bar{1}11$	−48 39	59 17	"	48 02	−40 12	34 37	"	1.1121	1.6833
39	o	$\overset{-}{1\frac{3}{2}}$	$\bar{2}32$	−37 09	64 28	"	59 04	−33 01	46 00	"	1.6682	2.0927
40	r	$\overset{-}{12}$	$\bar{1}21$	−29 36	68 39	"	65 48	−27 23	54 05	"	2.2242	2.5581
41	A	$\overset{-}{1\frac{5}{2}}$	$\bar{2}52$	−24 26	71 52	"	70 13	−23 09	59 54	"	2.7803	3.0539
42	G	$\overset{-}{13}$	$\bar{1}31$	−20 45	74 21	"	73 19	−19 56	64 13	"	3.3363	3.5676
43	n	$\overset{-}{1\frac{7}{2}}$	$\bar{2}72$	−17 59	76 16	"	75 36	−17 27	67 31	"	3.8924	4.0923
44	S	$\overset{-}{14}$	$\bar{1}41$	−15 51	77 48	"	77 20	−15 29	70 05	"	4.4484	4.6240
45	V	$\overset{-}{1\frac{9}{2}}$	$\bar{2}92$	−14 10	79 02	"	78 42	−13 54	72 09	"	5.0045	5.1620
46	ω	$\overset{+}{2\frac{1}{2}}$	412	77 43	69 03	68 37	29 05	65 52	11 28	2.5529	0.5561	2.6128
47	ζ	$\overset{+}{21}$	211	66 28	70 15	"	48 02	59 38	22 05	"	1.1121	2.7847
48	γ	$\overset{+}{31}$	311	73 47	75 55	75 21	48 02	68 39	15 43	3.8251	"	3.9836
49	ρ	$\overset{-}{2\frac{1}{2}}$	$\bar{4}12$	−77 38	68 55	−68 29	29 05	−65 43	11 32	−2.5357	0.5561	2.5960
50	Y	$\overset{-}{21}$	$\bar{2}11$	−66 19	70 09	"	48 02	−59 28	22 12	"	1.1121	2.7689
51	W	$\overset{-}{2\frac{3}{2}}$	$\bar{4}32$	−56 40	71 46	"	59 04	−52 31	31 28	"	1.6682	3.0352
52	X	$\overset{-}{22}$	$\bar{2}21$	−48 45	73 29	"	65 47	−46 07	39 12	"	2.2242	3.3730
53	M	$\overset{-}{2\frac{5}{2}}$	$\bar{4}52$	−42 22	75 07	"	70 13	−40 38	45 34	"	2.7803	3.7630

TABLE V — *Continued.*

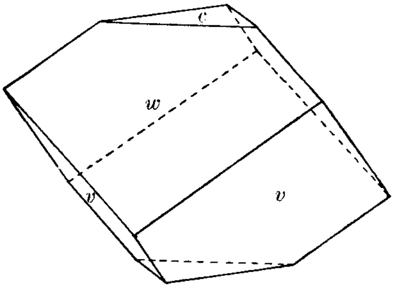
Number.	Letter.	Symbol—Gdt.	Symbol—Miller.	ϕ	ρ	ξ_0	η_0	ξ	η	x′	y′	d′ = tan ρ
				° ′	° ′	° ′	° ′	° ′	° ′			
54	Z	$\bar{2}3$	$\bar{2}31$	−37 14	76 35	"	73 19	−36 03	50 45	"	3.3363	4.1910
55	R	$\bar{2}4$	$\bar{2}41$	−29 41	78 57	"	77 20	−29 05	58 30	"	4.4485	5.1205
56	Σ	$\bar{\frac{3}{2}}\,\frac{1}{4}$	$\bar{6}14$	−81 40	62 29	−62 14	15 32	−61 20	7 23	−1.8995	0.2780	1.9199
57	J	$\overset{+}{\frac{1}{3}}$	113	49 24	29 40	23 24	20 20	22 05	18 47	0.4326	0.3707	0.5697
58	U	$\overset{+}{\frac{1}{3}}\,\frac{1}{2}$	236	37 53	35 10	23 24	29 05	20 43	27 02	"	0.5561	0.7045
59	β	$\overset{+}{\frac{1}{3}}\,\frac{2}{3}$	123	30 16	40 39	"	36 33	19 10	34 14	"	0.7414	0.8584
60	Φ	$\overset{+}{\frac{1}{3}}\,\frac{5}{6}$	256	25 01	45 39	"	42 49	17 36	40 22	"	0.9268	1.0227
61	λ	$\bar{\frac{1}{3}}\,\frac{1}{6}$	$\bar{2}16$	−65 57	24 28	−22 34	10 30	−22 13	9 43	−0.4154	0.1854	0.4549
62	B	$\bar{\frac{1}{3}}\,\frac{2}{3}$	$\bar{1}23$	−29 16	40 22	"	36 33	−18 27	34 24	"	0.7414	0.8499
63	δ	$\overset{+}{\frac{1}{2}}\,\frac{1}{4}$	214	66 40	35 04	32 48	15 32	31 51	13 09	0.6446	0.2780	0.7020
64	ϵ	$\overset{+}{\frac{1}{2}}$	112	49 13	40 25	"	29 05	29 24	25 03	"	0.5561	0.8513
65	Ψ	$\overset{+}{\frac{1}{2}}\,\frac{3}{4}$	234	37 42	46 30	"	39 50	26 20	35 02	"	0.8341	1.0541
66	t	$\overset{+}{\frac{1}{2}}\,1$	122	30 06	52 07	"	48 02	23 19	43 04	"	1.1121	1.2854
67	N	$\overset{+}{\frac{1}{2}}\,\frac{5}{8}$	458	24 53	56 52	"	54 16	20 38	49 27	"	1.3901	1.5323
68	Ω	$\overset{+}{\frac{1}{2}}\,\frac{3}{2}$	132	21 08	60 47	"	59 04	18 20	54 30	"	1.6682	1.7881
69	μ	$\bar{\frac{1}{2}}\,\frac{1}{4}$	$\bar{2}14$	−66 06	34 28	−32 06	15 32	−31 09	13 15	−0.6274	0.2780	0.6863
70	P	$\bar{\frac{1}{2}}\,\frac{1}{2}$	$\bar{1}12$	−48 27	39 59	"	29 05	−28 44	25 13	"	0.5561	0.8384
71	Q	$\bar{\frac{1}{2}}\,\frac{3}{4}$	$\bar{2}34$	−36 57	46 14	"	39 50	−25 44	35 15	"	0.8341	1.0437
72	v	$\bar{\frac{1}{2}}1$	$\bar{1}22$	−29 26	51 56	"	48 02	−22 46	43 17	"	1.1121	1.2769
73	T	$\bar{\frac{1}{2}}\,\frac{5}{4}$	$\bar{2}54$	−24 18	56 45	"	54 16	−20 07	49 40	"	1.3901	1.5251
74	τ	$\bar{\frac{4}{7}}2$	$\bar{4}.14.7$	−17 54	66 50	−35 42	65 48	−16 25	61 02	−0.7183	2.2242	2.3373
75	Θ	$\bar{\frac{2}{3}}\,\frac{1}{2}$	$\bar{4}36$	−56 29	45 12	−40 01	29 05	−36 16	23 04	−0.8395	0.5561	1.0070
76	σ	$\bar{\frac{2}{3}}1$	$\bar{2}33$	−37 03	54 20	"	48 02	−29 19	40 25	"	1.1121	1.3934
77	O	$\bar{\frac{7}{8}}\,\frac{3}{4}$	$\bar{7}68$	−52 56	54 09	−47 50	39 50	−40 18	29 14	−1.1045	0.8341	1.3841
78	[1]	$0\frac{5}{6}$	056	0 32	42 50	0 30	42 49	0 22	42 49	0.0086	0.9268	0.9268
79	[1]	$\overset{+}{\frac{3}{4}}0$	304	90 00	43 55	43 55	0 00	43 55	0 00	0.9627	0.0	0.9627
80	Δ[1]	$\bar{\frac{1}{2}}0$	$\bar{1}02$	−90 00	32 06	−32 06	0 00	−32 06	0 00	−0.6274	0.0	0.6274
81	[1]	$\bar{\frac{2}{3}}\,\frac{2}{3}$	$\bar{2}23$	−29 31	59 36	−40 01	56 00	−25 09	48 38	−0.8395	1.4829	1.7040
82	[1]	$\bar{1}\frac{1}{8}$	$\bar{8}18$	−77 35	52 18	−51 39	15 32	−50 36	9 43	−1.2636	0.2780	1.2938
83	H[1]	$\overset{+}{2}$	221	48 56	73 33	68 37	65 48	46 19	39 03	2.5529	2.2242	3.3860

[1] Uncertain forms.

The combination shown in Figure 10 does not exactly correspond to any of the measured crystals, although the forms present differ but little from those observed on one crystal (Table IV, p. 456, no. 13), which is, however, even more complex. It reproduces approximately the more complex type of combination prevailing among the Nevada crystals and illustrates the relations of most of the new forms.

The amount of leadhillite present in Dr. Jaggar's specimens from the Quartette Gold Mine was so small as to preclude the possibility of obtaining sufficient material for chemical analysis or for physical investigation. The hope that more material would be found suitable for such studies has not, however, been fulfilled after the lapse of two years or more.

The table of angles (Table V), calculated according to Goldschmidt (Winkeltabellen, 1897, p. 19 a) for the new axial ratio derived from the Utah crystals and here adopted, includes all the observed forms of leadhillite, which are also shown in the gnomonic projection (Plate **3**).



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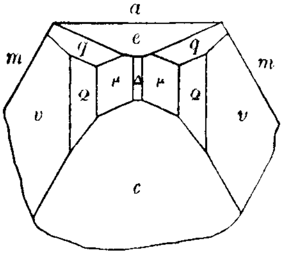
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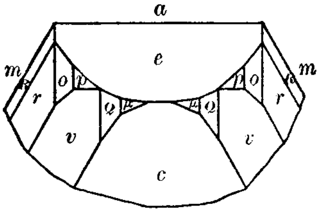
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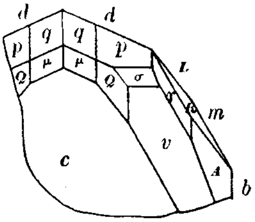
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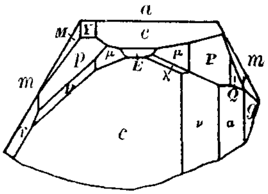
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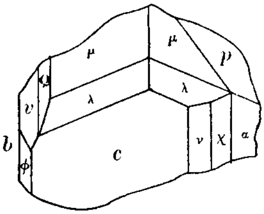
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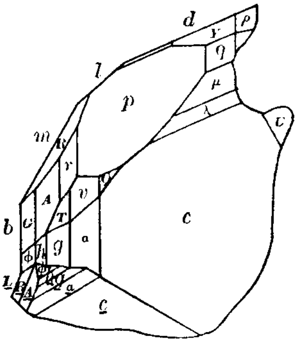
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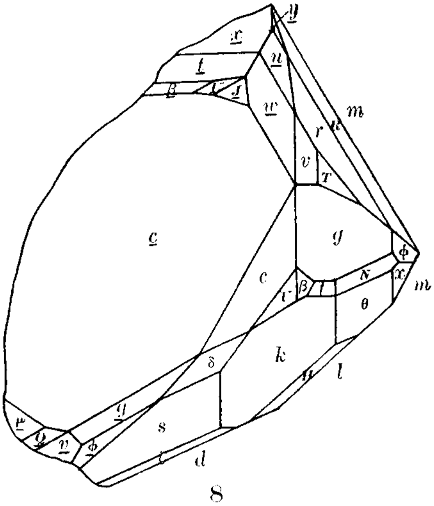
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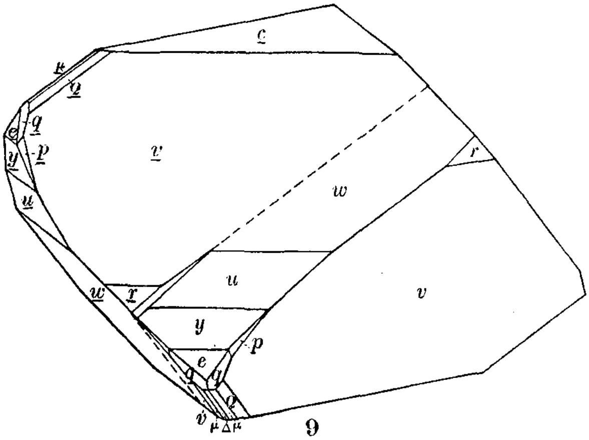
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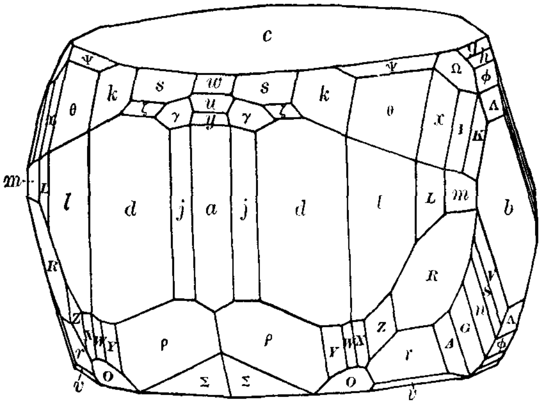
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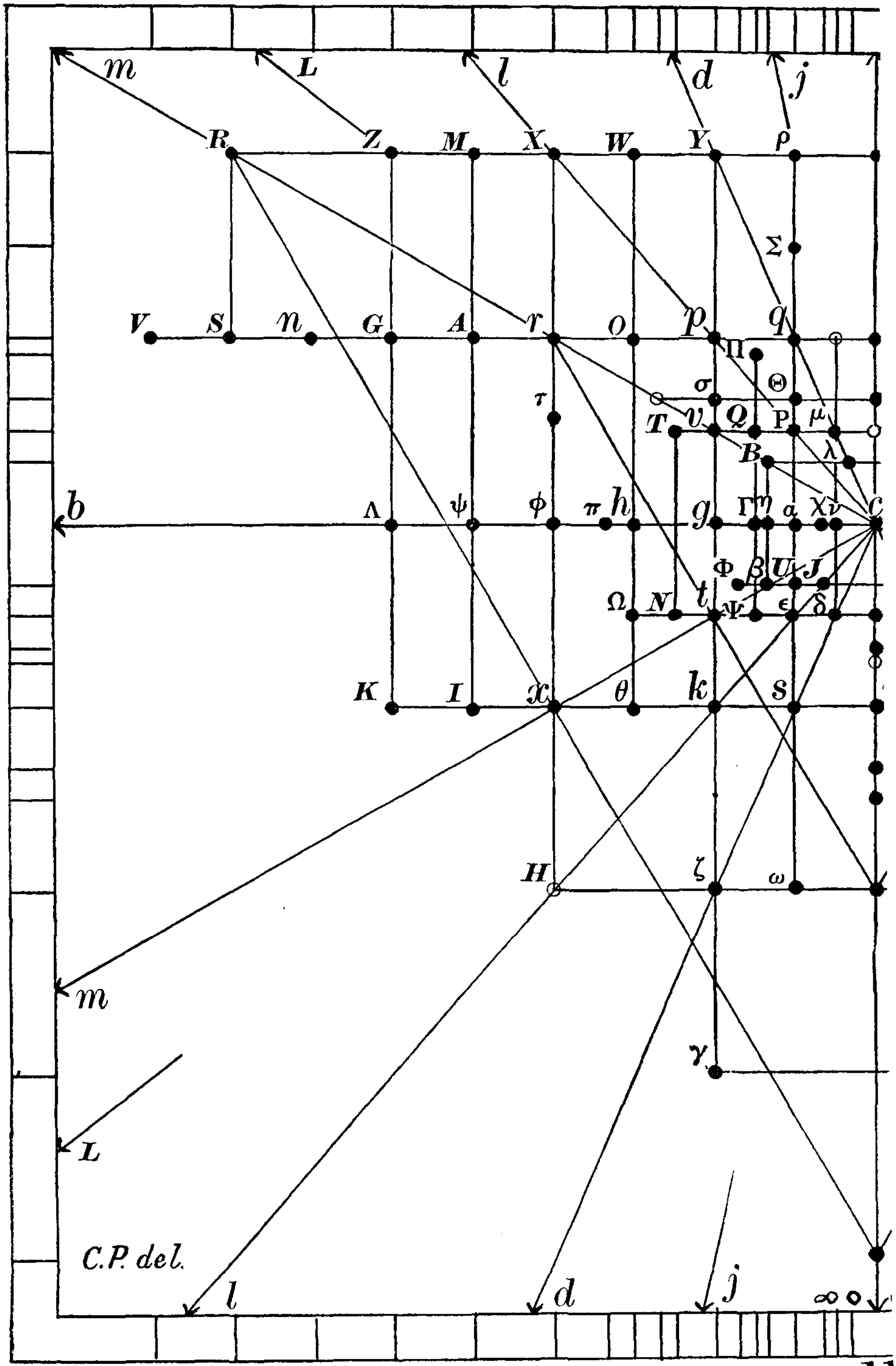
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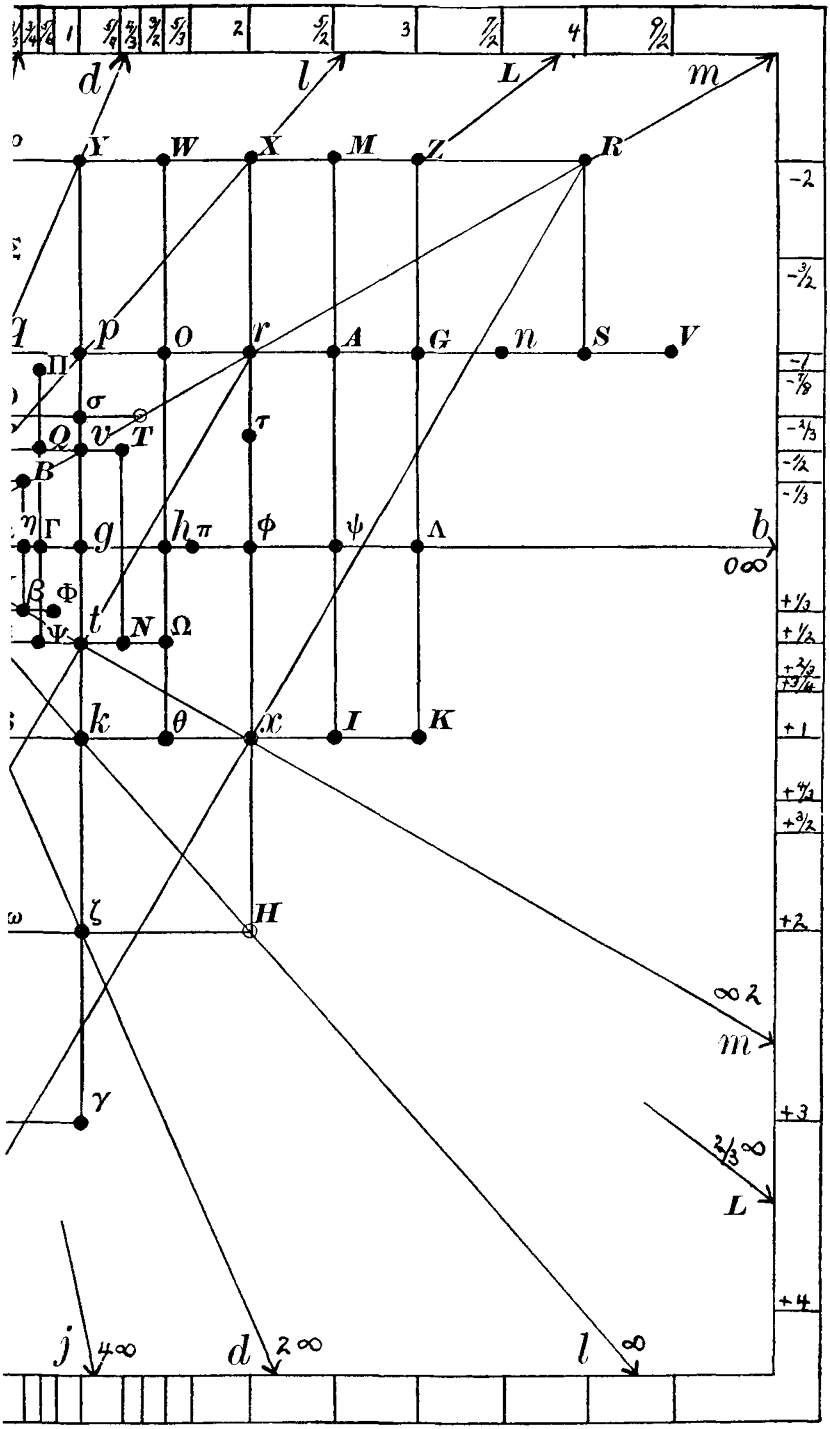
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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL
LABORATORY, HARVARD UNIVERSITY.

RESIDUAL CHARGES IN DIELECTRICS.

By C. L. B. Shuddemagen.

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RESIDUAL CHARGES IN DIELECTRICS.

By C. L. B. Shuddemagen.

Presented by B. O. Peirce, November 11, 1908. Received January 21, 1909.

Introduction.

The curious phenomenon of the residual charge which appears after a discharge by a momentary short circuit in a condenser which has a solid dielectric was observed as early as 1768 by Franklin, in the case of a glass "Franklin's plate"; but systematic research into the laws governing the formation and liberation of residual charge did not begin until about 1854, when R. Kohlrausch published the first important article on the subject. Up to that date it was the common belief that electric charge actually penetrated from the armatures of a charged condenser into the dielectric substance, from which it slowly returned to the armatures after each momentary discharge. The results of Kohlrausch showed, however, when viewed in the light of the theory of electric potential, that the penetration hypothesis was unsound, and that the true explanation was to be looked for in a polarized state of the molecules in the dielectric, in accordance with Maxwell's theory. Kohlrausch laid down the following two fundamental laws governing residual charge formation:

1. *The actual charge which can be drawn instantaneously from a charged condenser is at all times proportional to the potential difference of the condenser terminals.*

2. *In the same condenser the residual charges formed during equal times after charging are proportional to the initial charges, or the charging potentials.*

If the penetration hypothesis were correct, then during a momentary short circuit of a charged condenser charges of opposite sign should flow on to the condenser armatures in order to neutralize the potential of the charges which penetrated a short distance into the dielectric; while according to Kohlrausch's views the polarization of the molecules in the dielectric has the effect of neutralizing the potentials of a part of the initial charge, "binding" it, as it were, so that it cannot take part in the discharge, and only becomes free gradually as the polarization decays. A simple but crucial test as to which theory must certainly be wrong is therefore to remove the armature plates of a condenser immediately after an instantaneous discharge and test the sign of their charges. This was done by Wüllner, and the results conclusively disproved the older theory.

Wüllner observed the decreasing potential of charged condensers made of the same kind of glass but of varying thicknesses, and the results established a third law, which had been overlooked by Kohlrausch :

3. *In condensers of the same dielectric but of different thicknesses and shape the rate of fall of potential after equal times is the same.*

Still another law, of great importance, seems to have been first discovered by Thomson, and may be stated thus :

4. *Residual charges come out of a condenser in the inverse order to that in which they went in. Or, the rate of decay of residual charge during a long-continued short circuit is the same as its rate of formation during a long-continued charging.*

The second and third laws are ordinarily put together into a single one, called the law of superposition. The first three may be generalized and briefly put into mathematical form :

For condensers made of the same dielectric, the following equations hold, provided we neglect losses to the air and those due to internal conductivity :

$$V_t = V_0 \cdot f(t) \qquad Q_t = Q_0 \cdot f(t) \qquad R_t = Q_0 - Q_t ;$$

where V_0 = charging potential,
V_t = potential t seconds after charging,
Q_0 = initial instantaneous charge,
Q_t = charge which may be drawn from condenser in an instantaneous short circuit after t seconds of insulation,
R_t = residual charge formed after t seconds of insulation.

Thus the function $f(t)$ is one which is characteristic of any given kind of dielectric, as paraffin or mica.

Later researches have in general confirmed the law just given, but have not added any others, unless we are willing to accept Hopkinson's generalization of the law of superposition to include with instantaneous forces forces acting at different times, and this has hardly been conclusively proved.

The theories attempting to account for the cause of formation of residual charges have in the main followed one of two fundamentally different lines of thought. One holds that the heterogeneity of the dielectric is the cause of residual charge, and this theory has been developed by Maxwell and Rowland. The second ascribes the greatest importance to the elastic properties of the dielectric in the formation of residual charges. Hopkinson developed a theory of residual charge analogous to Boltzmann's theory of elastic after-effects, but this is too general to be of practical use. Of the many other later theories which take account of the elasticity of the dielectric, the one formulated by Houllevigue[1] seems to be the most promising. He gets a fairly simple solution of his differential equation for the current flowing into a condenser during a continuous charging. This current is made up by superposing the ordinary, practically instantaneous, charging current upon the slower residual forming current, which lasts for an appreciable interval of time. This latter current is considered to be due to a slow displacement of a part of the ether, being conditioned by the molecules of the dielectric.

In recent years the questions of "viscous dielectric hysteresis" or "lagging polarization," and of "energy losses" in the dielectric, have claimed much attention among physicists, and for a considerable time the problem of residual charge was completely overshadowed by these later questions. Some energy is undoubtedly lost in the form of heat in the dielectric, when the electric force is continually varied, as in an alternating current or a rotating electrostatic field. It is still an open question whether this loss of energy is chiefly to be associated with Joulean heat production in the dielectric, or with a viscous lag of the dielectric polarization behind the polarizing force. Each side of the question has found numerous and able supporters. It is greatly to be desired that a conclusive answer be obtained as soon as possible, for the subject is not only of immense practical importance in all telegraphy, telephony, and electrical engineering practice, but has undoubtedly very close relations to the problem of the ultimate constitution of matter. In fact the question of dielectric viscosity, or energy losses in dielectrics, seems to be an important part of electric dispersion, a subject which is just now receiving considerable attention.

The latest development of these very interesting questions of dielectric viscosity and energy losses seems to be a reopening of the older problem of residual charge formation. Indeed some of the most recent writers on the subject, especially E. R. von Schweidler,[2] apparently consider that both "viscous hysteresis" and "energy losses" are nothing more than results of the older phenomenon of residual charge formation, and are most satisfactorily explained in terms of it. Residual charge had been considered to be only a slow after effect of dielectric polarization, and almost every one who dealt with the subject tacitly assumed that the residual forming current is negligibly small during the charging of the condenser, so that no residual charge worth mentioning forms, say, in one thousandth, or even one hundredth, of a second after the charging voltage is applied. This assumption explains why nearly all investigators of residual charge, except some of very recent years, thought it unnecessary to make their charging times and short-circuit times extremely short, or even to measure or to estimate them. Even the wording of the "laws" which have been stated is very indefinite, as they speak of "instantaneous charges" and "instantaneous short circuits" if they attempt to define these time-intervals at all.

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[1] Ann. de l'Univ. de Lyons, **32** (1897); J. de Phys., **6**, 113–120, 120–126 (1897).

[2] Ann. der Phys., **24**, 711 (1907). This paper gives an excellent bibliography of the subject.

The present research started out with an attempt to test for the presence of an appreciable lag of polarization in paraffin paper condensers. The effect observed was, however, found to be due to a residual charge formation occurring in less than one tenth of a second, and I was led to an extensive investigation of the rate of residual charge formation at times as near to the instant of beginning the charging of a condenser as it was possible to obtain with the apparatus employed.

Neglecting for the moment various results of secondary importance, I wish to describe in detail in this paper three things which I hope will prove to be of some interest and value as contributions to the scientific study of dielectrics:

First, a method of studying the rate of formation of residual charge during very short charging intervals. This is a differential, or second order, method, and is capable of a very high degree of accuracy. Its great advantage is that it measures *all* the residual charge formed, no charge being lost in the process of short-circuiting the condenser.

Secondly, the best results of many observations on various dielectrics embodied in a series of curves, which although only first approximations, give correctly the general character and magnitude of the residual forming current for the time interval 0.00007 to 0.00170 of a second during which the charging voltage has been applied. These results show that the residual charge formed in this very short time is considerable in condensers made of paraffined paper and glass, and appreciable even in mica condensers.

Thirdly, a process for preparing with the greatest ease sheets of pure paraffin of almost any desired thinness, to be used in building up condensers of considerable capacity. Three condensers thus built up showed practically no residual charge, even when tested by the sensitive method used in this investigation.

PRELIMINARY EXPERIMENTS WITH ELECTROSTATIC VOLTAGE CYCLES.

The results of some experiments conducted in the fall of 1907 with the view of testing for a possible lag of polarization were of value to the writer only because they led him to investigate the rate of formation of residual charge for very short times after the charging. However, a brief description of the method employed may not be without some interest.

By means of two wooden arms, which swept contact brushes over two rows of copper plugs connected to sections of a storage battery of fairly high voltage (say 800), two condensers of very nearly equal capacities were simultaneously charged to the same final potential, then by an electromagnetic device immediately discharged against each other, and the charge left over was then sent through a ballistic galvanometer and measured. In this process both condensers were charged by increasing the voltage by steps of 30 or 60 volts, but one was charged to the final voltage by stopping its arm over any desired plug, while the other was charged up to say 420 volts, then decreased by steps until the voltage was again equal to that of the first condenser. I thought that the polarization corresponding to the highest voltage might not have time to decay before the two condensers were connected together. The wooden arms were flung over the copper plugs by hand, however, so that the time interval of decreasing the potential of one condenser was of the order of 1/20 second. This is probably too long a time for a perceptible lag effect to continue; the throws obtained were, however, considerable. But the charges behaved in every way just like residual charges, taking an appreciable time to come out of the condenser, although they had been formed in a very short time.

The principle of the method of mixtures which was here used was carried over into the later work with great advantage. In these new experiments the condenser to be tested was opposed to a standard air condenser, in which no residual charge formation was supposed to occur. Thus comparisons were rendered simple, as no variable effects due to one condenser had to be eliminated.

Description of Apparatus used in Later Experiments.

The Storage Battery.

The storage battery which charged my condensers is of the same type as the large 40,000 volt battery used by Professor Trowbridge for discharge experiments in tubes of high vacua, although it has a total voltage of only about 900 volts. The cells are test-tubes with lead strips dipping in a sulphuric acid solution; they are placed in racks of paraffined whitewood, each rack holding two rows of 30 cells each. Such a storage battery cannot yield large steady currents for any considerable time, but for furnishing a constant electromotive force and for charging condensers it is extremely useful. An hour or two of charging the battery early in the morning is usually sufficient to give it a fairly constant voltage for the whole day.

The Air Condenser.

The preliminary experiments briefly described above, although quantitatively almost worthless, showed clearly two things: first, that residual charges can form in considerable amounts in a very short time interval, say in a tenth of a second; and, secondly, that if the neutralizing two-condenser method was to yield the best results, in fact if it was to yield results of any quantitative value at all, it would be necessary to construct a standard condenser which should be free from residual charge formation, or which should show this effect only to a negligible degree. I therefore decided to build an air condenser of such capacity that its charge might give ballistic throws of large amplitudes, so that the "difference effect," when used against a test condenser in the manner already described, might still be of measurable magnitude. An air condenser was desirable because gaseous dielectrics, if they form residual charges at all, do so only in exceedingly minute quantities.

I selected, therefore, twelve large sheets of very flat plate-glass from the stock of the Boston Plate and Window Glass Co. in South Boston. Of these, seven were of dimensions 63.5 by 66 cms., and the other five were 61 by 66 cms. Their thicknesses varied considerably, being from 0.8 to 1.0 cm., but this did not make any difference for my purpose. The plates were carefully cleaned, and then on both sides of each plate tinfoil sheets were pasted with Higgins' Photo-Mounter paste considerably softened with water. It was found that the best results could be got when a squeegee roller, continually dipped in water, was used to roll out the tinfoil sheets and to force out all the surplus pasty liquid from under the tinfoil. Care had to be exercised in order not to tear the tinfoil, which was very thin, — about 0.004 of a centimeter. It was bought in the form of a continuous roll, 30.5 cms. in width; thus only two sheets were necessary to cover each side of a plate, a free margin of about 4.5 cms. being left around three edges, while the tinfoil itself projected over the fourth edge about 1 cm. The reason for using the paste instead of shellac was that paste is a conducting material, and thin films of it, which might possibly be left over the tinfoil, would not cause any residual charge, while if the dielectric shellac had been used, these thin films might perhaps have given rise to small but troublesome residual charges, which were especially to be avoided. The tinfoils on the two sides of a plate projected over the same edge of the plate, and were pressed down with thicker paste over a fine strip of copper foil all along this edge. The copper wire terminals of the condenser were soldered on to these strips with wax flux.

To separate the tinfoil-coated glass plates, which must be done by some very good insulator, it was decided to use thin glass disks, provided they could be found of the proper thickness, rather than disks of hard rubber, because this latter substance changes its surface condition in time. Fortunately a pane of glass was found of just the desired thickness, 0.076 cm., and a great number of disks 1.1 cms. in diameter were cut out of it and ground smooth at their edges.

Ten of these were placed between every two successive plates of glass, seven around the marginal space, and only three in the tinfoiled region. For these three circular pieces of tinfoil, 2.5 cms. in diameter were removed, and the paste below them carefully cleaned off. The disks were pressed down onto the glass plate with a very small drop of liquid shellac in between. Small weights were then placed on top for a day or two, so that the shellac might have time to harden under pressure. Then, for the sake of better insulation, a little melted paraffin was guided around the under edge of each glass disk with a hot iron wire.

In the air condenser built up of these plates there were eleven layers of air, each about three quarters of a millimeter thick. This condenser, which was mounted in a large oak case made for the purpose, has a capacity of 0.0428 microfarads and an insulation resistance of 35,000 megohms.

The Falling Weight Machine.

In studying the rate of formation of residual charges for very short charging intervals, Professor B. O. Peirce's large falling weight machine was found to be of the greatest use. A massive oak frame 244 centimeters high inside, 45 centimeters wide, and 22 centimeters deep (Figure 1), serves to support three vertical rods or columns made of straight round steel shafting 3.8 centimeters in diameter held at top and bottom in iron castings. On the middle column slides smoothly a cylindrical iron weight **Q** which can be caught and held at any convenient height by a latch **K** which can be slipped from a distance by a string. The weight as it falls can be made to trip in succession a number of switches supported on the other columns, and thus to open or close a a series of circuits at definite intervals. A dash pot **N** at the bottom of the middle column catches the falling weight.

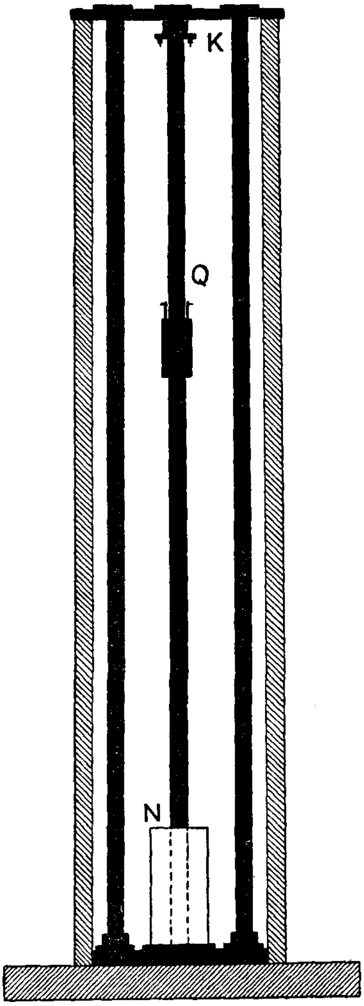

Figure 1.

In the early experiments made with this apparatus the falling weight was used to close in succession three keys. The first completed the charging circuit so that both condensers were charged to the same potential, usually 64 volts, the second discharged the condensers against each other, and the third put both condensers, still opposed, in circuit with a d'Arsonval galvanometer. For most of the work, however, the falling weight was equipped with six knife edges at the ends of short horizontal steel rods projecting, two towards the north, two towards the south, and two towards the front (east) of the apparatus. The last pair were insulated from the iron. These knives ploughed furrows in type metal pieces held in elaborate brass clamps mounted on the outer columns of the machine, but the south furrows were less than a millimeter long, while the east and the north furrows were 19 millimeters and 22 millimeters long respectively.

The Condensers used in the Tests.

Of the many condensers used in the work here described, some were built up of tinfoil and sheets of the best linen ledger paper saturated with paraffin wax of high grade. These were about twelve centimeters long and six and a half centimeters wide. After the paper had been soaked in the wax, the paper and tinfoil were built up into a pile and ironed together with a small flatiron moderately hot; the pile was then clamped permanently in large malleable iron holders made for the purpose. In the cases of two condensers, known as "Par. KA" and "Par. KB," the flatiron was not used. In "Par. B" and "Par. C" the paper was saturated with paraffin at a temperature near that of boiling water. In "Par. A," "Par. AA," "Par. BB," and "Par. CC" the paraffin was very hot, and the paper was kept in it until all the air bubbles in the paper had apparently been expelled. "Mica A_0" and "Mica B_0" were built up at room temperatures of tinfoil and single sheets of mica: after these condensers had been baked and waxed over to keep moisture out they were known as "Mica A" and "Mica B." Besides these a glass condenser, and three to be described later on in which the dielectric was clean, thin paraffin sheets were used.

Early Experiments with the Falling Weight Machine.

In these experiments, as has been said above, the falling weight first closed a switch which caused the two condensers to be charged to the same potential of 64 volts, then the relay broke the charging circuits and discharged the condensers against each other, and finally the last switch discharged the compound condenser through a galvanometer of such sensitiveness that the air condenser charged to 1 volt caused a throw of 0.732 centimeters. The sliding weight always dropped through a distance of 57.7 centimeters before it closed the first key, and a total distance of 130 centimeters before it closed the last key. The relay key could be set at any convenient height on its column, but if raised too high there would be no charging of the condensers. Experiment of this kind showed that there was a time lag of 0.0212 seconds in the relay circuit, and this had to be allowed for in all the computation. The voltage of the battery was determined by a Weston voltmeter.

When the relay key was placed as high as it could be without preventing the charging of the condensers, the fall of the weight caused a small throw of the galvanometer coil. This throw was due, just as it would be even if the time interval of charging were longer, to two factors:

first, the difference in the capacities of the air condenser and the test condenser; and, second, to the residual charge which had time to form during the charging interval. The test condensers "Mica A_0," "Mica B_0, " and "Par. A" were adjusted to give very small throws when the charging interval was thus cut down as far as possible. But it is important to notice that this small throw does not necessarily measure the difference in the capacities of the condensers. For although the charging interval is indeed small, yet if it were reduced still further, the air condenser might gain in apparent capacity on the test condenser, and the small throw, after perhaps first passing through the zero value, if it was at first in favor of the test condenser, might finally increase and keep on increasing. In other words, it is only when the small throw is in favor of the air condenser, that is, in the direction in which a throw coming from the air condenser by itself would read, that we can assert that the capacity of the air condenser is greater than that of the test condenser, for if the throw is in favor of the test condenser, we do not know whether the residual charge formed is less, equal to, or greater than, this charge causing the small throw. In fact, we see

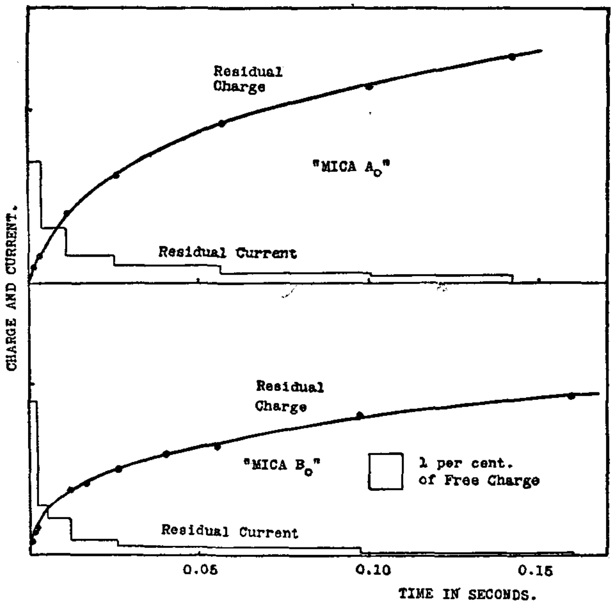

FIGURE 2. (Tables I and II.)

that there may be considerable difficulty in defining the so-called "free charge capacity" of the test condenser. It seems to me that this term can only be safely used when it can clearly be shown that the charge from a condenser, with constant charging voltage, approaches a definite limit as the charging time is continually decreased toward zero, or, rather, as close to zero as the conditions for complete charging will allow. Considerable light will be thrown on this question, I hope, by the later experiments in this work. For the purpose of constructing Tables I, II, and III and the curves of Figure 2 the simplifying assumption is in general made in this work that *no residual charge is formed*

TABLE I. (Figure 2.)

"MICA B_0" *vs.* AIR.

V = 64 volts. Total Throw = 46.5 cms.

Charging Time in Seconds.	Ballistic Throw in Centimeters.	Throw expressed in Percentage of Total Throw (corrected).
0	−0.11	0
0.0006	−0.28	0.36
0.0016	−0.42	0.66
0.0022	−0.47	0.76
0.0053	−0.68	1.20
0.0124	−1.03	2.00
0.0170	−1.10	2.10
0.0121	−1.01	1.90
0.0265	−1.30	2.50
0.0410	−1.52	3.00
0.0560	−1.61	3.20
0.0980	−2.06	4.20
0.1600	−2.30	4.70
(4 min.)	−4.13	8.60
(23 min.)	−4.23	8.80

in the shortest charging interval secured in the experiment. In other words, we shall assume that the small throws obtained after this shortest charging interval are due *wholly* to the difference in "free charge capacity" of the two condensers. After all, since we find it so difficult to know the actual amount of the residual charge, we must temporarily content ourselves with the *differences* in residual charge formed for varying charging intervals. When ballistic throws are in favor of the air condenser, they will be regarded as positive; when the test condenser's charge prevails, we shall record the throws as negative. With these explanations we may now tabulate the results. (Tables I, II, III.)

TABLE II. (Figure 2.)

"Mica A_0" *vs.* Air.

V = 64 volts. Total Throw = 46.5 cms.

Charging Time in Seconds.	Ballistic Throw in Centimeters.	Throw expressed in Percentage of Total Throw (corrected).
0.1600	−3.50	6.90
0+	−0.27	0
0.0013	−0.47	0.42
0.0038	−0.62	0.74
0.0115	−1.18	1.90
0.0260	−1.70	3.00
0.0570	−2.41	4.60
0.1010	−2.90	5.60
0.1430	−3.30	6.50
1.0000	−6.70	13.70
(1 min.)	−8.50	17.60
(12 min.)	−8.50	17.60

If the principle of superposition, or in this case the simple proportionality of residual charge to the electromotive force applied to the condenser, held true for the range of potential used in this experiment, then the numbers in the last columns should be constant for each charging interval. This is not true, however, for the higher voltages show a much greater percentage of residual formation than the lower ones, as will be seen from the data of Table III.

TABLE III.

“Mica A_0” *vs.* Air Condenser.

Total Throw = V. (0.73).

Charging Time in Seconds.	Charging Voltage.	Ballistic Throw.	Actual Throw expressed in Percentage of Total Throw.
0+	128	− 0.80	0.86
“	64	− 0.47	1.00
“	192	− 1.22	0.87
“	256	− 1.82	1.00
0.0044	256	− 3.92	2.11
“	192	− 2.78	1.99
“	128	− 1.78	1.91
“	64	− 0.88	1.88
“	32	− 0.38	1.63
0.0155	32	− 0.70	3.00
“	64	− 1.52	3.25
“	128	− 3.30	3.54
“	192	− 5.30	3.78
“	256	− 7.40	3.98
“	318	−10.30	4.50
“	383	−13.22	4.80
0.0740	318	−19.60	8.40
“	256	−14.60	7.90
“	192	−10.43	7.50
“	128	− 6.58	7.00
“	64	− 3.00	6.40
“	32	− 1.42	6.10
0.1430	32	− 1.80	7.70
“	64	− 3.82	8.20
“	128	− 8.30	8.90
“	192	−13.3	9.50
“	256	−18.70	10.00
“	318	−25.00	10.70

The residual throws from the condenser “Par. A” are expressed in Table IV, for purposes of comparison, in terms of the total throw which the charging voltage would have caused in the air condenser.

TABLE IV.

"PAR. A." *vs.* AIR.

Volts.	Throw in Cms.	Time of Charge.	Percentage of Residual.	Volts.	Throw in Cms.	Time of Charge.	Percentage of Residual.
34	0.6	0+.	2.4	452	5.38	0.145	2.06
66	1.2	"	2.5	388	4.70	"	2.10
131	2.4	"	2.5	324	3.90	"	2.09
196	3.85	"	2.72	259	3.12	"	2.08
262	5.2	"	2.71	194	2.34	"	2.09
327	6.6	"	2.76	129	1.57	"	2.11
393	7.9	"	2.74	66	0.78	"	2.05
196	3.95	"	2.75	33	0.39	"	2.06
33	0.6	0.0062	2.48	332	0.19	5 sec.	1.02
66	1.2	"	2.47	65	0.40	"	1.06
131	2.4	"	2.5	128	0.79	"	1.07
....	....	......	....	193	1.22	"	1.09
195	3.2	"	2.84	255	1.70	"	1.15
131	2.1	"	2.78				
33	0.48	0.0331	2.53	381	−14.2	Charged 1 min. Discharged 15 sec. Then discharged through galvanometer	−6.47
66	0.90	"	2.38	440	−15.90		−6.27
129	1.88	"	2.53	315	−12.25		−6.75
194	2.80	"	2.50	250	−9.8		−6.80
259	3.80	"	2.53	190	−7.8		−7.11
324	4.76	"	2.55	128	−5.3		−7.17
388	5.78	"	2.58	64	−3.05		−8.25
452	6.70	"	2.57	32	−1.60		−8.66

This tests the principle of superposition by the constancy of the percentage of residual for a given charging interval. The air condenser gave at first a ballistic throw of 0.732 cms. per volt; after the line of dots an accident changed this to 0.580 cms. per volt; this sensitiveness was kept nearly constant thereafter. The air condenser has a larger capacity than the paraffin condenser. This explains the change of sign, with increasing time of charging in the ballistic throws. To get true percentages of residual formed in any interval we may, in this case, subtract the percentage values for the longer time from those of the shorter.

The principle of superposition may be here tested again, if we see whether the percentage values of residual throw are constant for every different charging interval. This condition is seen to be fairly well satisfied, perhaps as well as experimental errors allow, though, in the last block of observations there is a continual numerical decrease in the numbers as we go from lower to higher voltages. The conditions here were, however, somewhat different from those in the other cases. The condensers were charged for a minute, then discharged against each other and left in that connection 15 seconds, then discharged through the galvanometer.

Experiments with the Falling Weight Machine on the Residual Charges after Short-Circuiting.

As we see from the results obtained for residual charges formed in different charging intervals, as exhibited in the broken curves of Figure 2 which indicate the mean relative values of the residual-forming current during various increments of the charging time, this current is very much greater during the earlier than during the later stages of the charging. To investigate this matter for much shorter charging times the sliding weight armed with the six knife points in the manner described above was used. The first experiments were made after the manner shown in Figure 3, but without the use of the air condenser. The lead strips are shown below the diagram at k in the relative positions as seen by an observer in front of the machine, it being here assumed that the knife edges are all on the same horizontal level. It will be seen that the charging takes place through one of the right-hand or north knives, and through one of the east knives during the time necessary for the latter to plough across the surface of its lead strip.

The residual-forming current flows into the dielectric not only for this length of time, but also for the time necessary for the south knife to reach the edge of its lead strip. While it cuts through this, the north knife still ploughing across its surface of lead, the potential difference of the test condenser is made zero. This short-circuiting lasts for, perhaps, 0.001 of a second or more, if a knife edge notches the whole edge of a lead strip, but may be as short as 0.00007 of a second, when a knife point barely notches the sharp edge of a lead strip which has been filed down to a narrow V-point. After the iron weight has been dropped from its trigger device and has thus charged and short-circuited the test condenser momentarily, a brief time is allowed the condenser for the residual charge to become "free," and then it is discharged through the d'Arsonval galvanometer.

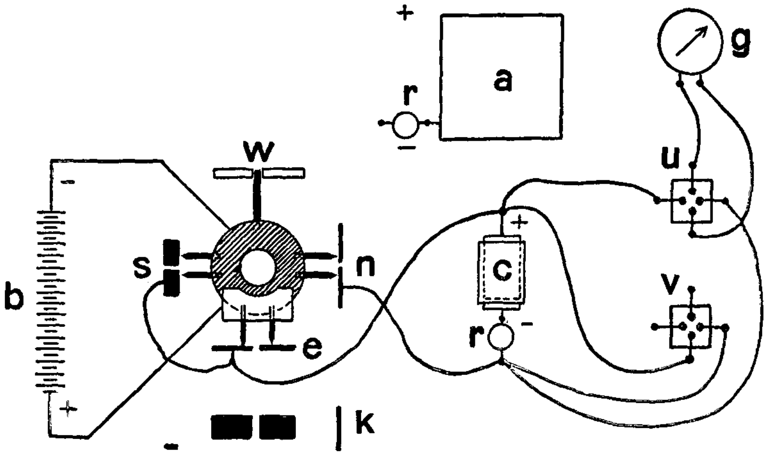

FIGURE 3.

The results obtained by these experiments are not of much quantitative value; for there is no way of knowing how much of the residual charge discharges during the short circuit along with the "free charge." What residual charge can form in 0.0032 of a second, which is the usual charging time in these experiments, is necessarily of a very mobile character, and perhaps a large part of it discharges in a short circuit even as brief as 0.00007 of a second. There is thus no reason to expect a number of measurements, taken under apparently the same conditions, to agree very closely; for a very slight difference in the time of short-circuiting may, perhaps, cause a large difference in the residual charge remaining behind.

As remarked above, the usual charging time in these experiments, or, more accurately, the time in which the test condenser is under the

TABLE V.

“Par. A.”

	Volts.	Throw.	Charging Time.	Throw / Volts.
Jan. 31. Knife Edge Short Circuit.	128	0.42	0.0032	0.0033
	“	0.55	“	0.0043
	“	0.42	“	0.0033
	“	0.42	“	0.0033
	124	0.40	0.0016	0.0032
	“	0.28	“	0.0023 *
	“	0.37	“	0.0030
	“	0.33	“	0.0027
Feb. 6. Knife Edge Short Circuit.	122	1.76	0.57	0.0144
	“	1.90	“	0.0156
	“	1.28	0.111	0.0103
	“	1.92	0.57	0.0158
	“	1.98	“	0.0162
Feb. 7. Knife Point Short Circuit.	63	0.40	0.0032	0.0062
	123	0.88	“	0.0071
	“	0.78	“	0.0063
	“	0.69	0.0060	0.0056
	“	1.63	0.111	0.0133
	122	0.72	0.0032	0.0059
	“	0.73	0.0060	0.0060
	“	0.78	0.0032	0.0064
	121	0.57	“	0.0047 *
	“	0.78	“	0.0064
	“	0.91	veloc.	0.0075
	“	0.42	0.0032	0.0035 **

charging voltage, is 0.0032 of a second. But, by using a narrower strip of lead for the north knife to plough over, this time can be shortened. Again, two extra pairs of the lead strip holders were mounted higher up on the north rod, so that the charging voltage could be applied for longer times. This accounts for the residual-forming intervals of 0.111 second and 0.57 second. For convenience in comparing results, values of the ballistic throws divided by the voltage are given so as to show the residual charge left in the condenser after short circuit, expressed in centimeters of throw per charging volt. The ballistic sensitiveness of the d'Arsonval galvanometer was such as

TABLE VI.

“Mica B_0”.

	Volts.	Throw.	Charging Time.	Throw/Volts.
Feb. 7. Knife Point Short Circuit.	120	0.62	0.0032	0.0052
	“	2.22	0.1110	0.0185
	“	3.77	0.5700	0.0314
	“	0.62	0.0032	0.0052
	“	2.22	0.1110	0.0185
	“	3.67	0.5700	0.0306
	“	3.60	“	0.0300
	“	0.50	veloc.	0.0042

TABLE VII.

“Par. B.”

	Volts.	Throw.	Charging Time.	Throw/Volts.
Feb. 8. Knife Point Short Circuit.	46	12.72	0.111	0.280
	28	6.80	“	0.240
	90	5.04	0.0032	0.056

TABLE VIII.

“ Par. C.”

	Volts.	Ballistic Throw.	Charging Time.	Throw / Volts.
Feb. 8. Knife Point Short Circuit.	94	7.32 + 0.65 + 0.18 = 8.15	0.0032	0.087
	“	6.10 + 0.60 + 0.32 = 7.02	“	0.075
	“	6.30 + 0.53 + 0.20 = 7.03	“	0.075
	93	6.00 + 0.50 + 0.12 = 6.62	“	0.071
	“	6.20 + 0.59 + 0.09 = 6.88	“	0.074
	180	10.40 + 1.10 + 0.11 = 11.61	“	0.065
	“	10.90 + 0.88 + 0.12 = 11.90	“	0.066
	264	14.90 + 1.30 + 0.12 = 16.32	“	0.062
	“	14.10 + 1.55 + 0.20 = 15.85	“	0.060
	420	23.20 + 2.07 + 0.30 = 25.57	“	0.061
	46	14.50 + 1.30 + 0.12 = 15.92	0.111	0.346
	28	14.80 + 1.47 + 0.40 = 16.67	0.570	0.559

TABLE IX.

“ Par. A.”

	Volts.	Throw.	Charging Time.	Throw/Volts.
Feb. 12. Knife Point Short Circuit.	27.5	0.20	0.0032	0.0073
	113	0.78	“	0.0069
	220	1.34	“	0.0061
	“	3.14	0.111	0.0143
	“	5.75	0.570	0.0261
	“	5.58	“	0.0254
	“	3.40	0.111	0.0154
	“	1.38	0.0032	0.0063

to give a throw of 13.7 cms. per micro-coulomb of charge. The "free charge" capacities of the condensers are approximately as follows:

Air	0.0428 mf.
"Par. A"	0.041 "
"Par. B"	0.040 "
"Par. C"	0.047 "
"Mica B_0"	0.043 "

TABLE X.

"Par. A."		"Par. A."		"Mica B_0."	
Knife Edge. Volts : 122–128.		Knife Point. Volts : 65–123.		Knife Point. Volts : 120.	
Charging Time.	Throw/Volts.	Charging Time.	Throw/Volts.	Charging Time.	Throw/Volts.
0.0016	0.0028	0.0032	0.0064	0.0032	0.0052
0.0032	0.0034	0.111	0.0133	0.111	0.0185
0.111	0.0105			0.57	0.0307
0.57	0.0155				

"Par. A."		"Par. B."		"Par. C."		
Knife Point. Volts : 113–220.		Knife Point. Volts : 28–90.		Knife Point. Volts : 28–420.		
Charging Time.	Throw/Volts.	Charging Time.	Throw/Volts.	Volts.	Charging Time.	Thr./Volts.
0.0032	0.0065	0.0032	0.056	94	0.0032	0.076
0.111	0.0149	0.111	0.260	180	"	0.065
0.57	0.0258			264	"	0.061
				420	"	0.061
				46	0.111	0.35
				28	0.57	0.56

Table V gives the detail of the observations taken under these conditions.

The next to the last observation of Table V was taken under the same conditions as for a time of 0.0032 seconds, save that the weight was given an acceleration by hand. This shortened both the times of charging voltage and short circuit in much the same proportion, but the larger throw indicates that the change of time of short circuit was of greater influence. For the starred observations, the short circuits were longer than for the others.

The residual charges in "Par. B" and "Par. C" of Tables VII and VIII had to be short-circuited several times through the galvanometer, since the first discharge did not take away all of the residual formed.

Table X contains a summary of mean results.

Experiments with the Falling Weight Machine, using the "Test Condenser versus Air Condenser" Method.

I now decided to make observations on the actual quantities of residual charge formed in various short charging intervals by using the air condenser to neutralize approximately the whole of the "free charge" of the test condenser, and then measuring the remainder ballistically. The method used from now on till the end of the work was quite similar to the former one in which the knife switches were used and the relay lever changed circuits so that the charge of the air condenser neutralized nearly all the charge of the test condenser. But the relay was now discarded, since its use made the time of charging impossible to control when very short, and it was found best to let the falling weight machine do the charging merely, while the neutralization of the charges was effected by lowering a commutating key by hand immediately after. Then after a short pause, which varied according to the quickness with which the residual charge reappeared, the remaining charge was sent through the galvanometer by lowering another commutating key.

The arrangement of the apparatus and connections is shown in the accompanying diagram (Figure 4). One of the north knives is no longer necessary. The chief points of difference from the short-circuiting method of experimenting are: (*a*) the addition of the air condenser **a**, and (*b*) the slight raising of the block on which the south lead strip holders **s** are mounted as indicated in the relative positions at **k**. The new arrangement changes the former short-circuiting action over into a charging action. The air condenser was as a rule uniformly charged by means of a knife edge cutting the edge of a lead strip clamped horizontally by one of the south holders (**s**, upper one), while at the same time one of the high voltage east knives ploughed over the surface of its lead strips, shown at **e**. The test condenser **c** could be charged either by means of the north knife, which gives from one to two centimeters of ploughing contact, or by means of lead strips placed in the other south lead clamp. The time of charging could here be varied by letting a knife point notch the edge of one, two, or three thicknesses of lead strips (**s**, lower strip), placed together with their edges all even, or by letting the sharp knife point barely dent the sharpened edge of a single lead strip, as in the short-circuiting experiments. By the use of very fine knife points and very sharp edges of the lead strip it was estimated that charging times as short as 0.00005 of a second could be obtained, if the lead strip was carefully adjusted so that the knife point would just slightly notch the sharpened edge. More often the time would be about 0.00007 of a second, and this number is usually taken in reducing the observations. Each thickness of lead strip adds 0.00012 second to the charging time, but the number 0.00020 has been adopted as the charging interval when the knife point notches the whole edge (0.7 mm.) of a single lead strip, because in this case the strip was not adjusted to be notched on quite so narrow a margin. The height above these lead strips, from which the iron weight with the knives was usually dropped and for which the

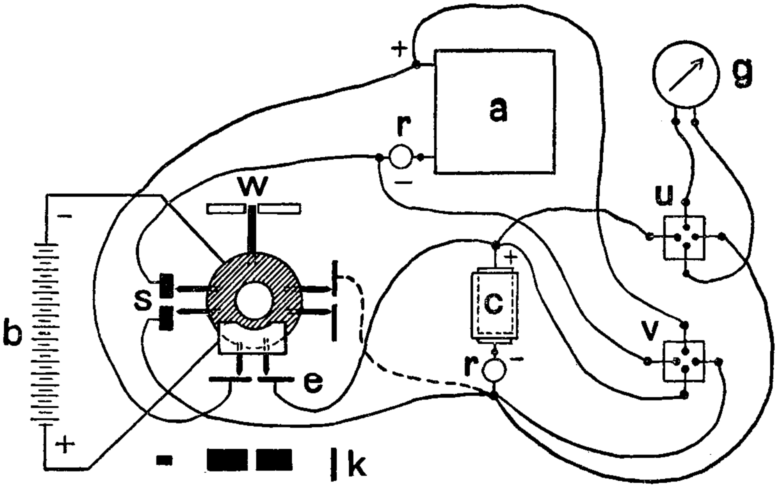

FIGURE 4.

figures have been given, is about 185 cms.; this was the highest drop obtainable on the machine.

The method of procedure was as follows: after a test condenser of capacity very nearly equal to that of the air condenser had been connected up as shown in the diagram, while the battery circuit was still open, the iron weight was raised a little above the lead strips, and these were clamped after having been properly adjusted, so that the knife edges should plough furrows of moderate depth on the surfaces of the lead. Then the iron weight was pushed up into its trap (**k**, Figure 1), and the commutating key **v** of the condensers, which had thus far kept both condensers short-circuited, was lifted from the mercury wells. The battery circuit was now closed, thus keeping the brass plate of the east knives at high potential, and the iron weight with the north and south knives at low potential. The observer now brought the coil of the d'Arsonval galvanometer **g** to rest, pulled with the right hand the string which released the iron weight, and at the moment when the iron weight was heard to strike into the dash pot he dropped the commutator key **v** into its mercury wells in the neutralizing position, connecting the positive terminal of each condenser with the negative plate of the other. The condensers destroyed each others' charges approximately, leaving a remainder which was then sent through the galvanometer by dropping the galvanometer key **u** into its mercury wells. The ballistic throw was read and recorded, together with the voltage of the battery and the conditions controlling the charging interval. Then, if there were no secondary residual charges, the condensers were short-circuited by their commutating key, the galvanometer coil was brought to rest by short-circuiting its terminals, the key of the storage battery was opened so as to protect the battery from a possible short circuit while the lead strips were loosened and drawn aside, and the iron disk in the dash pot was pulled up to its normal position. Then operations were repeated.

The experiments just described were begun on February 10, and carried on until March 27, 1908. The earlier results were not of the high accuracy which characterizes nearly all the observations taken on and after March 10. It was at one time suspected that the storage battery could not respond fully to demands in the very short charging intervals. But the real cause of occasional disagreements in the ballistic throws obtained was later found to lie in imperfect contacts of the storage battery leads on the switch-board. I shall merely summarize below the results obtained in the earlier part of the work on various test condensers, by giving mean values of several observations, and their reduction to the final values of residual charge expressed in percentage of total "free charge," without giving all the individual observations. The meaning of the positive and negative ballistic throws and the method of making the reductions is fully described on page 500, in connection with the results of March 10 and 11.

It should be noted here that various resistance coils, from 5 to 85 ohms and higher, were used in the condenser circuits, connected directly to one of their terminals as indicated by small circles (*rr*) in the diagram. Usually, however, the air condenser had a 10 ohm coil, and the test condenser a 5 ohm coil, connected to it. The exact value of the resistance is not important; the object of the resistance is merely to prevent too great an initial rush of charge.

All the pieces of apparatus, the storage battery, the falling weight machine, the condensers, the commutating keys, and the galvanometer, were carefully insulated by means of large porcelain knobs or blocks of paraffin. These were often cleaned and scraped and, so far as could be ascertained, none of the troubles experienced were due to leakage of any kind. It will be noticed later that the air condenser and most of the test condensers have a small internal conductivity, but as the operation of neutralizing the charges takes place immediately after the charging, this conductivity could not result in a measurable loss of charge from either condenser.

On February 26 a condenser made up of 12 separate commercial paraffined paper condensers, giving a total capacity of about 50 microfarads, was connected across the terminals of the storage battery. This was done to avoid a possible source of trouble in that the battery might not be able to furnish complete charges for the test condenser in the very short charging intervals. It was found to be useful, but the

TABLE XI. (Figure 7.)

"PAR. A" *vs.* AIR. FEBRUARY 11.

No. Obs.	Volts.	Throw.	Charging Time.	Throw/Volts.	Residual Throw.	Percentage Residual.
3	124	4.17	0.00010	0.0336	(0)	(0)
6	124	3.84	0.00015	0.0310	+0.0026	0.47
5	124	3.70	0.00020	0.0299	+0.0037	0.67
1	124.5	3.56	0.00032	0.0286	+0.0050	0.90
2	124.5	3.45	0.00044	0.0277	+0.0059	1.07

TABLE XII. (Figure 8.)

"PAR. B" *vs.* AIR. FEBRUARY 11.

No. Obs.	Volts.	Throw.	Charging Time.	Throw/Volts.	Residual Throw.	Percentage Residual.
2	126	5.16	0.00010	0.0410	(0)	(0)
2	125.5	3.85	0.00020	0.0302	+0.0108	2.00
2	125	2.95	0.00032	0.0236	+0.0174	3.21
2	125	2.62	0.00044	0.0210	+0.0200	3.50

TABLE XIII.

"SHELLAC-MICA" *vs.* AIR. FEBRUARY 20.

No. Obs.	Volts.	Throw.	Charging Time.	Throw/Volts.	Residual Throw.	Percentage Residual.
2	240	1.14	0.00007	0.0048	(0)	(0)
1	58	0.17	"	0.0030		
1	124	−1.94	60 secs.	−0.0156	+0.0215	2.7
1	122.5	−2.76	3 min.	−0.0225	+0.0274	4.7

TABLE XIV. (Figure 5.)

"MICA B" *vs.* AIR. FEBRUARY 21.

No. Obs.	Volts.	Throw.	Charging Time.	Throw/Volts.	Residual Throw.	Percentage Residual.
2	129	− 1.51	0.00007	0.0117	(0)	(0)
2	130	− 1.99	0.00020	0.0153	+0.0036	0.6
2	115	− 2.33	0.0025	0.0202	+0.0085	1.4
2	115	− 3.73	0.111	0.0325	+0.0208	3.4
1	117	− 5.20	0.57	0.0445	+0.0328	5.4
1	115	− 9.0	20 secs.	0.0781	+0.0664	10.8
1	115	−15.9	2 min.	0.138	+0.1260	20.6

TABLE XV. (Figure 5.)

"MICA A" *vs.* AIR. FEBRUARY 21.

No. Obs.	Volts.	Throw.	Charging Time.	Throw/Volts.	Residual Throw.	Percentage Residual.
2	130	−0.40	0.00020	0.0031	(0)	(0)
2	130	−0.66	0.0025	0.0051	+0.0020	0.34
2	130	−1.70	0.111	0.0131	+0.0100	1.7
1	129	−2.50	0.57	0.0194	+0.0163	2.7
1	123	−4.59	5 secs.	0.0373	+0.0342	5.7
1	123	−7.26	40 secs.	0.0590	+0.0560	9.4
1	114	−8.39	2 min.	0.0735	+0.0700	11.7

TABLE XVI. (Figure 5.)

"MICA B" *vs.* AIR. FEBRUARY 22.

No. Obs.	Volts.	Throw.	Charging Time.	Throw/Volts.	Residual Throw.	Percentage Residual.
3	122	−1.61	0.00007	0.0132	(0)	(0)
2	119	−1.73	0.00020	0.0145	+0.0013	0.21
1	122	−2.50	0.0025	0.0205	+0.0073	1.2
1	122	−3.66	0.111	0.0300	+0.0168	2.8
2	112	−1.61	0.00007	0.0143	(0)	(0)
1	112	−1.81	0.00020	0.0162	+0.0019	0.31

TABLE XVII. (Figure 5.)

"MICA A" *vs.* AIR. FEBRUARY 22.

No. Obs.	Volts.	Throw.	Charging Time.	Throw/Volts.	Residual Throw.	Percentage Residual.
2	119	−0.35	0.00020	0.0029	(0)	(0)
3	118	−0.50	0.0032	0.0042	+0.0013	0.22
3	116	−1.64	0.111	0.0141	−0.0112	1.9

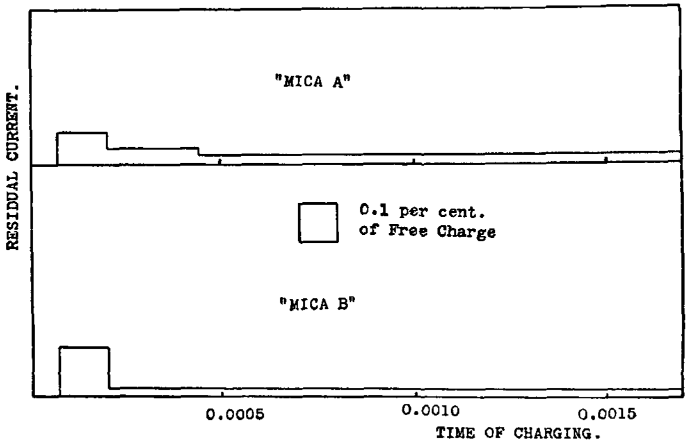

FIGURE 5.

(Tables XXVII, XXIX, XXXII, XXXIII, XXXVII, XXXVIII.)

TABLE XVIII. (Figure 6.)

" PAR. BB " *vs.* AIR. FEBRUARY 29.

No. Obs.	Volts.	Throw.	Charging Time.	Throw/Volts.	Residual Throw.	Percentage Residual.
2	133.5	−3.71	0.0025	0.0277	(0)	(0)
3	132	−4.80	0.111	0.0363	+0.0086	1.5
2	132	−5.22	0.57	0.0394	+0.0117	2.04

TABLE XIX. (Figure 5.)

" MICA B " *vs.* AIR. FEBRUARY 29.

No. Obs.	Volts.	Throw.	Charging Time.	Throw/Volts.	Residual Throw.	Percentage Residual.
4	125	−2.88	0.00007	0.0230	(0)	(0)
2	128	−3.03	0.00007	0.0237		
3	128	−3.36	0.0025	0.0262	+0.0032	0.52

50 microfarad condenser should not have much internal leakage, as this would run down the voltage of the storage battery too fast. The test-tube cells of the battery naturally have not a large current capacity, but they are excellent for giving a steady difference of potential and small charges such as are required for these experiments. In the case of each condenser the first residual throw is assumed to be zero.

The mean results reduced for the experiments up to March 10 are shown in the preceding nine tables.

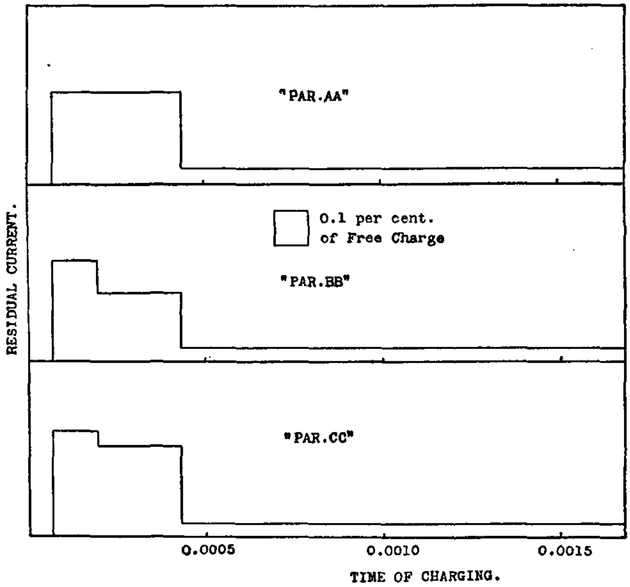

Figure 6. (Tables XXVIII, XXX, XXXI, XXXIII.)

By comparing these summarized results of Tables XI–XIX with those which are to follow, we see that they do not all agree very well. But there is a substantial similarity in the behavior of the various condensers, and some condensers, as "Par. A," "Par. B," "Par. AA," and "Par. BB," show very close agreement with results as determined more accurately later. The results from the mica condensers are not so good.

It will be seen that the condensers of plain mica sheets show a very much greater residual capacity for long charges than the condenser made of shellacked mica sheets. This is hardly what we should have expected, according to Maxwell's heterogeneity theory.

After the preliminary experiments had been made the whole network of conductors was overhauled, and many of the joints were soldered with the help of white pitch as a flux. Sometimes in the later work the 50 microfarad condenser was connected across the poles of the charging battery but seemed not to be necessary. Local conditions made it difficult to bring the coil of the d'Arsonval galvanometer quite to rest and some of the throws had to be made when the coil was swinging over a double amplitude of half a millimeter.

In the tables given below the charging intervals are expressed in terms of the amount of the lead cut through by the knife point. It was calculated that

1 centimeter	means	0.0017	seconds	of	charge
3 lead widths	"	0.00044	"	"	"
2 lead widths	"	0.00032	"	"	"
1 lead width	"	0.00020	"	"	"
very short	"	0.00007	"	"	"
extra short	"	0.00005	"	"	"

Observations, March 10.

TABLE XX. (Figure 7.)

" PAR. A " *vs.* AIR.

Volts.	Throw.	Charging Time.
132	4	very short
"	3.98	" "
"	3.80	1 width
128	3.8	"
"	3.78	"
"	4.3	very short
"	4.32	extr. short
"	4.30	" "

TABLE XXI. (Figure 6.)

" PAR. BB " *vs.* AIR.

Volts.	Throw.	Charging Time.
128	2.29	extr. short
"	2.20	" "
"	2.29	" "
"	2.02	1 width
128	2.01	"
124	1.96	"

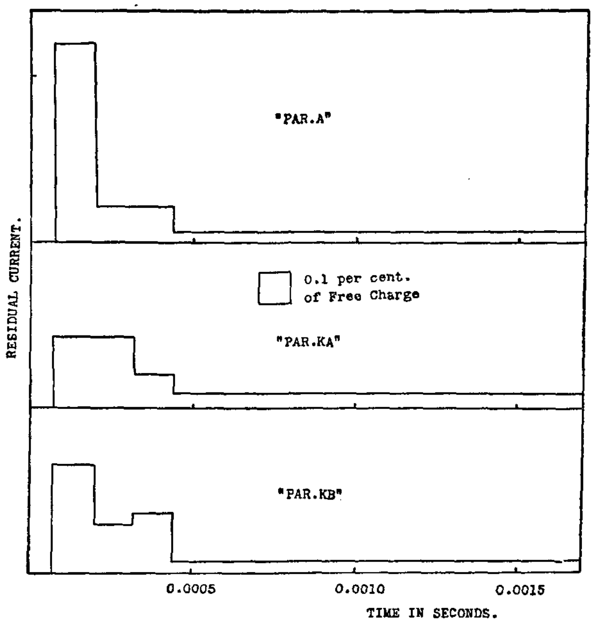

FIGURE 7. (Tables XXVI, XXXIV–XXXVI, XXXVIII.)

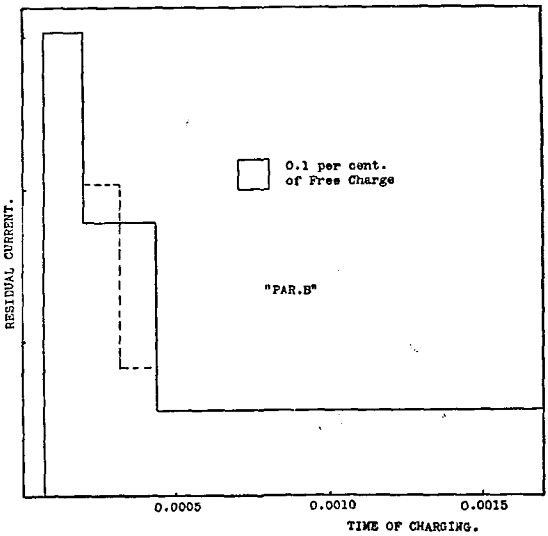

FIGURE 8. (Tables XLII and XLIII.)

TABLE XXII. (Figure 5.)

“Mica B” *vs.* Air.

Volts.	Throw.	Charging Time.
124	−3.19	2 widths
“	−3.15	“
“	−3.13	“
“	−3.05	very short
123.5	−3.02	“ “
123	−3.07	short
“	−3.13	1 width
122.5	“	“
120	−3.09	2 widths
“	−3.10	“

TABLE XXIII. (Figure 5.)

“Mica A” *vs.* Air.

Volts.	Throw.	Charging Time.
132	−1.51	very short
133	−1.53	“ “
“	−1.61	1 width
132	−1.57	2 widths
“	−1.63	“
“	−1.58	“
“	−1.50	very short
131.5	−1.52	“ “
“	−1.53	“ “

TABLE XXIV. (Figure 6.)

“Par. AA” *vs.* Air.

Volts.	Throw.	Charging Time.
131.5	1.39	very short
“	1.30	“ “
“	1.35	“ “
“	1.42	“ “
131	0.99	2 widths
“	0.98	“
“	1.19	1 width
131−	1.21	“
130.5	1.30	“
127	1.26	“

TABLE XXV. (Figure 6.)

“Par. CC” *vs.* Air.

Volts.	Throw.	Charging Time.
127	1.88	1 width
“	2.10	very short
125	2.13	extr. short
“	1.64	2 widths
“	1.81	“
“	1.73	“
“	1.79	“

Observations, March 11.

TABLE XXVI. (Figure 7.)

"PAR. A" *vs.* AIR.

	Volts.	Throw.	Charging Time.	Temp.
A. M.	130	3.50	2 widths	22°.0
	"	3.49	"	
	"	"	"	
	"	4.30	extr. short	
	"	4.01	very short	
	"	4.21	" "	
	"	4.13	" "	
	"	3.72	1 width	
	"	3.74	"	
	129.5	3.63	"	
	"	3.67	"	
	125	3.57	3 widths	
	124	3.54	"	
	123	3.50	"	
	"	3.52	"	
	"	3.31	1 cm.	
	121	3.28	"	
	120	3.22	"	
	119	3.17	"	
	128	3.18	"	
	"	"	"	
	"	3.40	"	
	"	3.21	"	
P. M.	132	2.90	1 cm.	
	"	2.92	"	
	"	3.23	3 widths	
	"	3.14	"	
	"	3.31	"	
	"	3.20	"	

TABLE XXVII. (Figure 5.)

"MICA B" *vs.* AIR.

Volts.	Throw.	Charging Time.	Temp.
132	−3.32	3 widths	
"	−3.28	"	
"	−3.32	"	21°.8
"	−3.61	1 cm.	
"	−3.63	"	

TABLE XXVIII. (Figure 6.)

"PAR. BB" *vs.* AIR.

Volts.	Throw.	Charging Time.	Temp.
132	1.39	1 cm.	
"	"	"	
"	1.64	3 widths	21°.8
"	1.84	"	
"	1.70	"	

TABLE XXIX. (Figure 5.)

"MICA A" *vs.* AIR.

Volts.	Throw.	Charging Time.
132	−1.67	3 widths
"	−1.68	"
"	−1.89	1 cm.
"	−1.88	"

TABLE XXX. (Figure 6.)

"PAR. CC" *vs.* AIR.

Volts.	Throw.	Charging Time.
131+	1.11	1 cm.
132	1.13	"
"	1.48	3 widths
"	"	"

TABLE XXXI. (Figure 6.)

"PAR. AA" *vs.* AIR.

Volts.	Throw.	Charging Time.	Temp.
132 −	0.62	3 widths	22°.7
131	0.58	"	
131 +	0.59	"	
"	0.20	1 cm.	
131	0.21	"	
"	1.30	extr. short	
131 −	1.08	very short	
"	1.33	extr. short	

TABLE XXXII. (Figure 5.)

"MICA B" *vs.* AIR.

Volts.	Throw.	Charging Time.	Temp.
131	−3.22	very short	23°.0
"	−3.32	" "	
"	−3.23	" "	
"	−3.28	" "	

In working up the data here printed to derive the results shown in Table XXXIII, below, the following method was used:

I first determined from the observations the ratios (R) of the throw obtained to the charging voltage and set the R's opposite the corresponding charging intervals. Then I found mean values of the R's for the various charging intervals. Then that R which I believed to correspond to the shortest charging interval secured was taken as a standard of comparison and the unknown residual charge in centimeters of throw per volt which had already been formed in the condenser in this shortest obtainable charging interval was called x. By taking the difference between this standard R and the R corresponding to any other charging interval and calling this difference d, I got $(x + d)$ for the residual charge which formed in the other interval. I then divided all the numbers $(x + d)$ by the total charge per volt which went into the test condenser in the shortest charging time. This gave numbers which are independent of the apparent capacity of the test condenser used. When multiplied by 100, these give the residual charges formed in the given charging times, expressed in percentage of the total charge formed in the shortest time.

Thus in Table XXVI we have for the condenser "Par. A" a capacity of 0.0404 mf., or 0.554 cm. when expressed in ballistic throw per volt. Then for "Par. A," let $x(100)/0.554 = y$. Then for "1 width" of charge $(x + 0.0044)(100)/0.554 = y + 0.80$; and the number $(y + 0.80)$ will be the residual charge which forms in the charging time of 0.00020 second, expressed in percentage of the total charge formed in the condenser "Par. A" in the charging time 0.00005 second. We shall express the results obtained in the $(y + d)$ form for all the test condensers, but must remember that the y is in general widely different for the different condensers. We thus obtain the following table :

TABLE XXXIII. (Figures 5, 6, and 7.)

RESIDUAL CHARGE IN PERCENTAGE OF TOTAL CHARGE.

March 10, 11, 1908. Temp. = 22°–23°.

Condenser.	Time of Charge in Seconds.				
	0.00005	0.00020	0.00032	0.00044	0.00170
"Par. A"	y	$y + 0.80$	$(y + 1.14)$?	$y + 1.06$	$y + 1.48$
"Par. AA"	y	$y + 0.22$	$y + 0.51$	$y + 0.95$	$y + 1.47$
"Par. BB"	y	$y + 0.37$	....	$y + 0.83$	$y + 1.30$
"Par. CC"	y	$y + 0.39$	$y + 0.56$	$y + 1.02$	$y + 1.49$
"Mica A"	y	$y + 0.105$	$y + 0.102$	$y + 0.20$	$y + 0.48$
"Mica B"	y	$y + 0.17$	$y + 0.20$	$(y + 0.11)$?	$y + 0.48$

A great difference will be immediately observed between the paraffin condensers and the mica condensers. The variation is large in the paraffin, while in mica there is almost no variation in the region of charging intervals considered. And if we examine the original throws observed, we find that for the very short charging times the throws vary greatly in case of paraffin, while for the mica they are practically constant. All the paraffin condensers show close agreement in their behavior, and so do the two mica condensers. (See figures 5, 6, and 7.)

Observations of March 12 *and* 13.

The following tables (XXXIV–XXXVIII) give mean values of ballistic throws observed and reductions. The condensers "Par. KA" and "Par. KB" are built of the same paraffined paper as the others, but the sheets were merely piled together without the use of the hot flat-iron. Thus we have layers of air as well as the paper sheets as the dielectrics.

TABLE XXXIV. (Figure 7.)

"Par. KA" *vs.* Air.

No. Obs.	Volts.	Ballistic Throw.	Charging Time.	Throw/Volts.	Residual Charge in cms./Volts.	Temp.
1	133	−0.91	0.00007	0.0068	x	
1	"	−1.86	0.0017	0.0140	$x + 0.0072$	
1	132	−1.50	0.00005	0.0114	x	
1	133	−1.78	0.00007	0.0134	....	
6	"	−2.36	0.0017	0.0177	$x + 0.0063$	
1	"	−1.60	0.00007	0.0120	....	
2	132	−2.55	0.0017	0.0193	$x + 0.0073$	
2	131.5	−2.505	0.00044	0.0190	$x + 0.0039$	
2	"	−2.95	0.0017	0.0224	$x + 0.0073$	21°.2
2	"	−2.41	0.00032	0.0183	$x + 0.0032$	

TABLE XXXV. (Figure 7.)

“ Par. KB ” *vs.* Air.

No. Obs.	Volts.	Ballistic Throw.	Charging Time.	Throw / Volts.	Residual Charge in cms./Volts.	Temp.
4	132	−3.40	0.00020	0.0258	$x + 0.0026$	20°.6
2	“	−3.55	0.00032	0.0269	$x + 0.0037$	
2	“	−3.73	0.00044	0.02825	$x + 0.00505$	
2	“	−4.12	0.0017	0.0312	$x + 0.0080$	
2	“	−3.06	0.00007	0.0232	x	
1	123	−1.61	0.0017	0.0131	$x + 0.0080$	
1	“	−0.63	0.00007	0.0051	x	

TABLE XXXVI. (Figure 7.)

“Par. A ” *vs.* Air.

No. Obs.	Volts.	Ballistic Throw.	Charging Time.	Throw/Volts.	Residual Ch. in cms./Volts.	Temp.
3	132	3.11	0.0017	0.0236	$x + 0.0053$	20°.6
1	“	3.81	0.00007	0.0289	x	
1	“	3.46	0.00020	0.0262	$x + 0.0027$	

TABLE XXXVII. (Figure 5.)

“ Mica A ” *vs.* Air.

No. Obs.	Volts.	Ballistic Throw.	Charging Time.	Throw/Volts.	Residual Ch. in cms./Volts.	Temp.
2	132	−1.41	0.0017	0.0107	$x + 0.0006$	20°.6
1	“	−1.33	0.00007	0.0101	x	

From these we get

TABLE XXXVIII. (Figures 5 and 7.)

RESIDUAL CHARGE IN PERCENTAGE OF TOTAL CHARGE.

March 12, 13, 1908. Temp. = 20°–22°.

Condenser.	Time of Charge in Seconds.				
	0.00007	0.00020	0.00032	0.00044	0.00170
" Par. KA "	y	. . .	$y + 0.53$	$y + 0.65$	$y + 1.22$
" Par. KB "	y	$y + 0.43$	$y + 0.61$	$y + 0.83$	$y + 1.31$
" Par. A "	y	$y + 0.48$	. . .	. . .	$y + 0.95$
" Mica A "	y	. . .	. . .	. . .	$y + 0.10$

We see from these results that the layers of air in the condensers "Par. KA" and "Par. KB" apparently make little, if any, difference in the amount of residual charge.

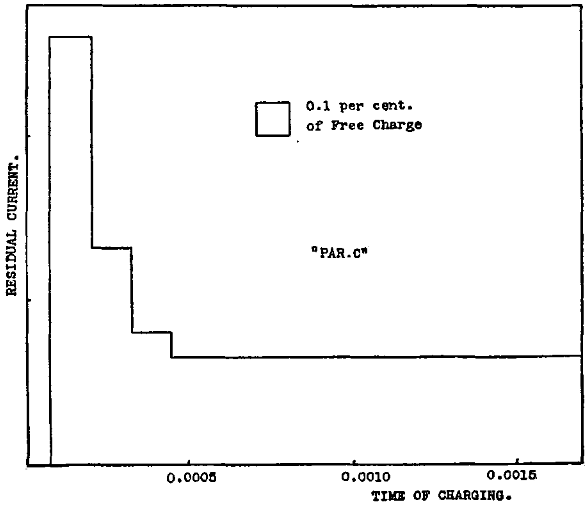

FIGURE 9. (Tables XLI and XLIII.)

Observations of March 17, 1908.

TABLE XXXIX. (Figure 11.)

Glass Condenser *vs.* Air.

Volts.	Throw.	Charging Time.	$\frac{\text{Throw}}{\text{Volts}}$	Residual Charge in cms. per volt.	Temp.
104	0.68	0.0017	0.00654	$x + 0.0087$	
103	0.58	“	0.00563	$x + 0.0096$	
“	0.52	“	0.00505	$x + 0.0101$	
102	1.48	0.00007	0.0145	...	
100	1.51	“	0.0151	...	21°.1
“	1.52	“	0.0152	x	
98	1.15	0.00020	0.01175	$x + 0.0036$	
97	1.11	“	0.01145	...	
132.5	1.39	0.00020	0.01010	$x + 0.0044$	
133	1.44	“	0.01075	$x + 0.0038$	
“	0.77	0.0017	0.00579	$x + 0.0088$	
“	0.68	“	0.00507	$x + 0.0095$	15°.0
“	1.67	“	...	...	
“	1.53	0.00020	0.0116	$x + 0.0030$	
“	1.55	“	...	...	
132.5	1.37	0.00032	0.0104	$x + 0.00415$	
“	1.39	“	...	...	
“	1.41	0.00044	...	...	
“	1.33	“	0.0102	$x + 0.00435$	
132	1.31	“	...	...	
“	0.85	0.0017	...	...	
“	0.84	“	0.00647	$x + 0.0081$	15°.5
“	0.87	“	...	...	
“	1.37	0.00044	0.01022	$x + 0.00433$	
“	1.33	“	...	...	
“	1.69	0.00007	0.0128	...	
“	1.92	“	0.01455	x	

To equal approximately the capacity of the glass condenser, only the six uppermost air layers were used; the capacity of this new air condenser was 0.0207 mf., and that of the glass condenser for very short charges about 0.0196 mf. We get —

TABLE XL. (Figure 11).

Residual Charge in Percentage of Free Charge. Glass.

Charging Time.	Residual.	Temp.	Charging Time.	Residual.	Temp.
0.00020	$y + 1.34$	21 C.	0.00020	$y + 1.30$	15° C.
0.00170	$y + 3.54$		0.00032	$y + 1.55$	
			0.00044	$y + 1.62$	
			0.00170	$y + 3.25$	

Observations of March 18, 1908.

TABLE XLI. (Figure 9).

" Par. C " *vs.* Air.

Volts.	Throw.	Charging Time.	$\frac{\text{Throw}}{\text{Volts.}}$	Residual Charging.	Temp.
132	−12.00	0.0017	0.0910	. . .	
"	−12.53	"	0.0950	$x + 0.044$	
133	− 7.54	0.00007	0.0567	. . .	
132.5	− 6.53	"	0.0493	x	15°.0
"	− 6.83	"	0.0515	. . .	
"	−12.41	0.0017	0.0942	$x + 0.045$	
131.5	− 7.95	0.00020	0.0605	. . .	
131	− 7.80	"	0.0598	$x + 0.0108$	
117	− 8.55	0.00044	0.0731	. . .	
"	− 8.67	"	0.0741	$x + 0.0181$	
116	− 8.12	0.00032	0.0700	. . .	
114	− 8.00	"	0.0702	$x + 0.0151$	
113	−11.14	0.0017	0.0986	. . .	15°.3
"	−11.19	"	0.0992	$x + 0.044$	

TABLE XLII. (Figure 8.)

"Par. B" *vs.* Air.

Volts.	Throw.	Charging Time.	Throw / Volts.	Residual Charging.	Temp.
131	3.91	0.00020	0.0299	$x + 0.0115$	
"	4.12	"	0.0314	$x + 0.0100$	
130	5.38	0.00007	0.0414	x	
128	5.21	"	0.0407	...	
130	4.10	"	0.0316	...	15°.0
130+	4.35	"	0.0334	...	
130	4.54	"	0.0349	...	
"	+0.25	0.0017	0.0019	...	
128	(−0.42)	"	(−0.0033)	$(x + 0.042)$	
"	(0)	"	(0)	...	
120	2.93	0.00032	0.0244	$x + 0.0162$	
119	3.09	"	0.0260	...	
117	2.20	0.00044	0.0188	...	15°.3
"	2.29	"	0.0196	$x + 0.0224$	
"	2.18	"	0.0186	...	

These condensers, "Par. C" and "Par. C_1," are the ones which were made with paraffined paper soaked in wax only moderately warm, so that the air bubbles of the paper were not expelled. They show enormous residual capacity; in fact, to get the throws for "Par. C" four residual charges had to be read by the galvanometer, besides the first throw directly after neutralization of the charges of the air condenser and the test condenser.

The bracketed values in the last set of observations were obtained by reversing the terminals of the "Par. B" condenser. After these two readings the terminals were changed back again. The bracketed figures, when compared with those immediately preceding and following, show a curious "set" in the polarization. This point deserves further study, and it is hoped that it may sometime be taken up at length.

We derive the following values for the

TABLE XLIII. (Figures 8 and 9.)

Residual Charge in Percentage of Total Charge.

Condenser.	Time of Charge in Seconds.				
	0.00007	0.00020	0.00032	0.00044	0.00170
" Par. C "	y	$y + 1.70$	$y + 2.37$	$y + 2.85$	$y + 6.92$
" Par. B "	y	$y + 1.97$	$y + 2.97$	$y + 4.12$	$y + 7.72$

It will readily be seen that these two condensers show very close family resemblance.

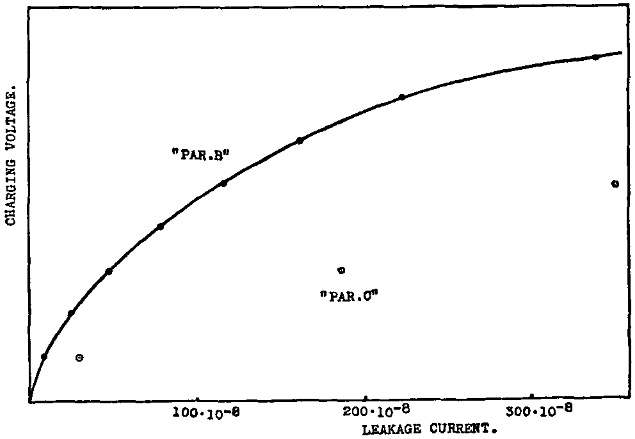

Figure 10. (Table XLIV.)

“Insulation Resistances” of the Condensers.

The following observations were taken on March 19 and March 27. The pure paraffin condensers and the air condenser were measured for leakage on the later date.

TABLE XLIV. (Figure 10.)

Condenser.	Volts.	Steady Deflections in cms.	Current in amp.
“Par. A”	325	0.13	1.43×10^{-8}
“Par. AA”	328	0.02	0.22 “
“Par. BB”	“	0.03	0.33 “
“Par. CC”	“	“	“ “
“Par. KA”	“	0.09	0.99 “
“Par. KB”	“	0.10	1.10 “
“Mica A”	“	0.02	0.22 “
“Mica B”	“	0.04	0.44 “
“Par. C”	65	2.75	30.30 “
“	328	32.00	352.00 “
“	195	17.00	187.00 “
“Par. B”	66	0.86	9.46 “
“	131	2.33	25.60 “
“	195	4.40	48.40 “
“	262	7.17	78.90 “
“	328	10.57	116.30 “
“	390	14.72	161.90 “
“	457	20.40	224.40 “
“	517	11.00	341.00 “
“P”	518	0	0 “
“Q”	“	0.03	0.33 “
“R”	“	0.10	1.10 “
Air	510	0.13	1.43 “

The sensitiveness of the d'Arsonval galvanometer in measuring steady currents is 9.1 × 10^6 cms. per ampere of current.

The leakage currents given are, as a rule, near the maximum values, for most of the currents were slowly decreasing with the time when they were taken. But "Par. B" showed a marked *increase* of the current with the time.

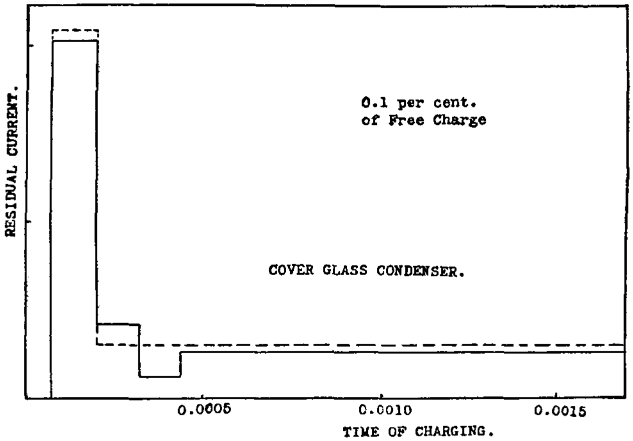

Figure 11. (Tables XXXIX and XL.)

The Condensers of Pure Paraffin Sheets.

The preparation of thin slabs of pure paraffin for use as the dielectric of a parallel plate condenser for experimental purposes has always been a difficult task. Boltzmann recommends that the melted paraffin be poured between two plates of plate glass whose inner surfaces have been coated over with a thin film of oil, in order that the slab may be readily separated from the plates after it has cooled and become hard. This leaves thin films of oil over both surfaces of the paraffin slab, and these should be scraped off before the slab can be used. The writer made some fairly thin slabs three years ago by pouring hot paraffin into a square frame of wood placed on a single plate of glass lying horizontally and having a film of oil to allow the later separation of the paraffin. But he has never yet seen any paraffin so formed which was free from air bubbles or small cavities. It is possible, and perhaps after all the easiest way, to saw off slabs of paraffin from a large block and then to plane down the surfaces, but this will not give very thin slabs.

It occurred to the writer that perhaps thin sheets of paraffin might be made by the same method which is used to obtain thin sheets of beeswax, such as are manufactured into "comb foundation" for use in modern apiaries. A trial experiment on a small scale proved completely successful. Smooth sheets of paraffin were obtained as thin as sheets of paper and apparently quite homogeneous. Then the necessary apparatus was secured to make the sheets of parraffin larger and in great numbers. Two tanks were constructed (Figure 12) by a plumber, according to the following specifications: The material used was galvanized sheet iron (copper sheeting, however, would be more durable). One tank, which was to hold hot paraffin, was to hang inside the other one, in which water was to be kept heated to the proper temperature by means of Bunsen burners. The dimensions were: inner tank: height, 61 cms., base, 30.5 by 5.1 cms.; outer tank: height, 63.5 cms., base 35.6 cms. by 10.5 cms. The inner tank had three projecting strips of galvanized iron, bent down near their ends,

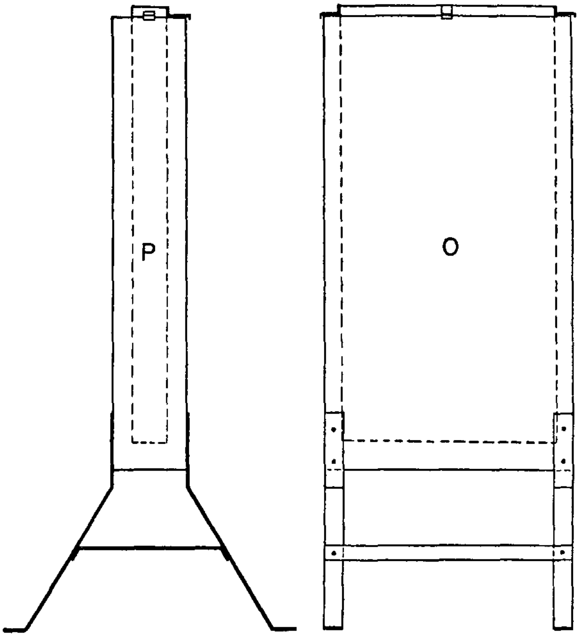

FIGURE 12.

Side Views of Dipping Tanks.

which just reached over the top rim of the outer tank, and held the inner one at such a height that its top was about 1.8 cms. higher than the top of the larger tank. This was done to keep the water from getting from the outer tank into the inner one.

Meanwhile two "dipping boards" were obtained. These were made of light pine (whitewood may be better) and are 61 cms. long, 25 cms. wide, and about 0.5 cms. thick. They are bevelled down to a narrow V-shape along both of the long edges and one end. Near the other end a hole is bored through, so that each board can be hung from a hook. These boards are carefully planed, and then sand-papered until they have a very smooth surface and are free from loose fibres of wood.

A day or two before the paraffin sheets are to be "dipped," the two dipping boards are wholly immersed in water and left there until needed. Before this immersion strips of wood should be tied across the boards so as to keep them from warping when they become thoroughly water-soaked. This must also be done after the work of making the paraffin sheets is finished, and the boards are to be allowed to dry; otherwise they will surely warp in drying. When the boards have become thoroughly water-soaked, the paraffin is melted in some convenient large vessel, placed in another one containing water to which the heat is applied; meanwhile the larger tank is filled about half full of water, and this is heated by Bunsen burners placed under the tank. Before the water has reached its boiling point the burners are taken away or turned down very low, then the smaller tank is placed in the larger one and the melted paraffin is filtered into it through some clean piece of cloth, preferably linen. The smaller tank is filled to such a depth with paraffin that, when one of the dipping boards is lowered all the way down into it, the paraffin will rise nearly to the top of the tank but not run over. Now one of the dipping boards is flushed with water under a faucet, and when this has been allowed to drain off until the water falls by drops, the board is quickly pushed down into the paraffin and as quickly withdrawn, being held at one end by both hands. This will result in a thin layer of paraffin quickly cooling all over the two sides of the board, and if the conditions are just right very little paraffin will drip from it. When after about a minute the surface of the paraffin has become firm, cold water is again flushed all over the board, but only for a very short time. This makes the paraffin layer so firm that the board can be hung from a hook and the paraffin peeled off in two layers. These sheets, about 25 by 52 cms., are piled up one on top of the other on flat board, just as they are peeled off the dipping board, and can usually be left in that way for a day or two in a moderately cool room, since the water still on the sheets will prevent them from sticking to each other. As soon as the two sheets have been removed from the board and laid away, the dipping board is again flushed with cold water and the process is repeated.

The reason for having the dipping boards bevelled sharp at the side and bottom edges is that this causes a break in the paraffin layer there, and so allows the two sheets to be peeled off separately. The reason for having more than one dipping board is that one is liable to give poor results; for the paraffin may begin to stick to its surface, and when it once starts to do so, the trouble is hard to correct in any other way than by letting the board get dry once more and vigorously sandpapering the troublesome place. Using two or three boards, one may pick out after trial the one which gives the best sheets.

By varying the conditions somewhat, one may obtain smooth paraffin sheets of almost any thinness desired. They are very easily, and perhaps most conveniently, made about half a millimeter thick. If the water bath is made hotter, the sheets of paraffin will turn out thinner, until finally there comes a time when the paraffin film will split and full-size sheets cannot be obtained. Another means of controlling results is found in the temperature of the wet dipping board. The warmer it is allowed to become the thinner will be the sheets. With a little practice and judgment one can get sheets of good, smooth surface. A good deal depends on keeping the surface of the melted paraffin in the smaller tank free from air bubbles. Usually the sheets obtained will be thinner near the top end of the dipping board and thicker near the lower end, but this makes little difference if one cuts condenser sheets out of the middle portion.

About two hundred good sheets were made by the writer in a few hours one afternoon, of the grade of paraffin melting at 47°.5. The water adhering to these sheets was allowed to evaporate over night by laying the paraffin sheets singly on large sheets of rough paper, such as is used for mimeograph work. The water in evaporating is likely to leave a conducting film over the paraffin, and moreover there are some slight unevennesses in the surfaces of sheets. It was found that the thin blades of steel used in the Gillette safety razor were admirably adapted for use in lightly scraping the paraffin, and in this way the conducting films were removed and very smooth sheets resulted. For this operation, and in fact to be handled at all, the paraffin should be kept in a room at a temperature of about 23° or 25°. The sheets are then sufficiently yielding and plastic, so that they may be scraped without danger of cracking.

When a sheet had been thus scraped smooth, it was immediately used to build up the condenser. A smoothly scraped surface of a sheet would be placed on a tinfoil sheet, and then the other surface scraped down somewhat, this process serving to press the paraffin into close contact with the tinfoil. Then another sheet of tinfoil would be placed on top and pressed down smoothly by a small plate of soft rubber, and the process continued as before. The paraffin sheets were 20.5 cms. by 31.0 cms., and a margin of about 1.5 cms. was left outside the tinfoil sheets. About 18 or 20 dielectric sheets of paraffin sufficed to give a

Observations on March 25, 1908.

TABLE XLV.

"PURE PAR. P" *vs.* AIR.

Volts.	Throw.	Charging Time.	Temp.
136	2.30	0.0017	
136	"	"	(cold)
136	2.49	0.00007	
	After	10 A. M.	
131	4.29	0.00007	
"	"	"	
"	4.39	0.0017	
"	"	"	
"	4.38	"	25°.0
"	"	0.00007	
"	4.47	0.00005	
"	4.43	"	
"	4.39	0.0017	
"	4.17	30 secs.	
126	4.22	" "	
131	4.42	0.0017	25°.2
"	4.40	0.00007	

capacity approximately equal to that of the air condenser. These sheets were placed on a wooden base of the same size, but nothing was put on top, and no pressure was applied. The tinfoil ends were, as usual, soldered together with low melting point solder and were furnished with copper wire terminals. Finally the edges of the pile of paraffin sheets were melted together, and melted paraffin was run all over the tinfoil ends so as to insulate the whole from the air.

Three such condensers were built up. The first one was made with great care, only the most perfect sheets being used for it. The other two were not so carefully prepared and their sheets were considerably thinner. The first one, "Pure Par. P," showed no leakage current whatever under 520 volts on the d'Arsonval galvanometer, while the other two leaked very slightly. But the most pleasing observation was that each of these three condensers showed almost no residual charge formation. In fact, in the region of small charging intervals, where the mica condensers still show a considerable residual forming current, none whatever could be observed in the three pure paraffin condensers. Nor do the throws obtained for the shortest charging time bear any evidence of a probable increase of residual forming current for still shorter charging times.

The observations taken are shown in Table XLV (520 volts across charged condenser and d'Arsonval give no deflection).

It will be seen from these figures of Table XLV that no certain evidence for a measurable residual charge exists, in the region of charging times up to .0017 second. With the combination of condensers here used the ballistic throws for the shortest charging intervals should, if

Observations on March 27, 1908.

TABLE XLVI.

"Pure Par. Q" *vs.* Air.

Volts.	Throw.	Charging Time.	Temp.
133	−7.32	0.0017 sec.	
"	(−7.61, −0.25, −0.06)	2 min.	
132.5	(−7.62, −0.28, −0.05)	" "	20°.0
132	(−7.30, −0)	0.0017 sec.	
"	−7.30	0.00007 sec.	

there were any considerable residual charge, be larger than for those with 0.0017 second of charge, and this relation is found in scarcely more than half the cases. On the other hand, there is a continual increase of the throws, due probably to a temperature influence. Of course any effects due to a temperature-coefficient of capacity will be highly magnified in measuring differential throws, as is done here.

In the last three measurements, recorded above and in all the following ones, the 50 microfarad condenser was connected across the storage battery.

TABLE XLVII.

"PURE PAR. R" *vs.* AIR.

Volts.	Throw.	Charging Time.	Temp.
132	−4.37, −0	0.00007 sec.	
"	−4.39, −0	0.0017 sec.	
"	(−4.61, −0.10, −0.01)	2 min.	
131.5	(−4.60, −0.16, −0.01)	" "	
"	−4.32	0.00007 sec.	20°.0
"	−4.31	0.0017 sec.	
"	(−4.57, −0.15, −0.01)	2 min.	

TABLE XLVIII.

"PURE PAR. P" *vs.* AIR.

Volts.	Throw.	Charging Time.	Temp.
131	−3.59	0.00007 sec.	
"	(−3.87, −0.19, −0.06)	2 min.	20°.0
"	−3.60	0.0017 sec.	

TABLE XLIX.

Tests for Leakage through Condenser.

Condenser.	Capacity.	Volts.	Deflection.	Current in amp.
P	0.0448	518	0	0
Q	0.0468	"	0.03	0.33×10^{-8}
R	0.0152	"	0.10	1.10 "
Air	0.0428	510	0.13	1.43 "

From these results we derive—

TABLE L.

Residual Charge Formation in Pure Paraffin.

Condenser.	Time of Charge.	Percentage Residual.
P	2 min.	$y + 0.67$
Q	" "	$y + 0.69$
R	" "	$y + 0.50$

Explanation of the Current Curves.

(Figures 2–9, 11.)

We have thus found experimentally a number of values of the residual charge which is formed in various charging times in various condensers. To get a rough but fairly correct insight into the behavior of a condenser during the short charging intervals which have been used, we may proceed as follows: Taking the experimental results in the form of the residual charge, as expressed in percentage of "free charge," we find the increments of residual charge corresponding to the increments of charging time. We measure off the various charging times used in the experiment as abscissas, on any convenient scale. Then we divide all the residual charge increments by their corresponding charging time increments, thus getting several quotients. Straight lines are now drawn parallel to the axis of charging times, of length equal to the various charging increments and at distances from the axis which are proportional to the quotients obtained for the corresponding time-increments, and finally the ends of these straight lines are joined by lines parallel to the ordinate axis. No line is drawn above the time-interval 0–0.00007 second, since the amount of residual charge formed in this interval is unknown, being represented by y per cent of the "free charge."

After having thus constructed a broken curve for a certain test condenser, we see that the area under each horizontal part of the broken curve represents the residual charge which was formed in the time interval corresponding to this part. Furthermore, the distance of any horizontal line from the time-axis, or the ordinate of this line, will represent the strength of the *average* residual forming current which flowed into the condenser during the corresponding time interval. If we had accurately determined the residual charges formed for a very large number of charging times, spaced closely together on the time-axis, the broken curve, constructed as just described, would give an extremely close approximation to the actual residual forming current. As we have data for only three or four charging increments, our broken curves are necessarily very rough; nevertheless, they give us a correct idea of the general behavior of the current and its high value near the instant of beginning a charge.

The curves of residual current (Figures 2, 5–11) have been constructed as just described. In Figure 2 curves are given of both residual charge and residual forming current. A centimeter of abscissa represents 0.01 second of charging.[3] A centimeter of ordinate means for the charge curve a residual charge of 1 per cent of the "free charge" of the condenser, and for the current curve a residual forming current which in one second would charge the condenser with a residual charge equal to its "free charge." Accordingly a square centimeter of area under the broken curve is equivalent to 1 per cent of "free charge." All the other figures which show these broken line residual forming currents are on the scale of 1 cm. abscissa for 0.0001 second of charging and 1 cm. ordinate for a residual forming current which would give a residual charge of ten times the "free charge," if it continued to flow uniformly for 1 second after the charging begins. Thus, a square centimeter of area under these curves represents one tenth of one per cent of the "free charge" of the condenser.

---

[3] These dimensions have been changed in reproduction, but the square corresponding to 1 or 0.1 per cent of free charge is given with each figure.

Several of the current curves, if plotted out for each interval of the charging time, would show a depression such as has been shown in the case of the cover glass condenser (Figure 12). It is barely possible that this peculiar result may be genuine and indicate a "backward surge" of the extra dielectric polarization which is conditioned by the molecules of the dielectric. But it is more likely that it is due to experimental error in the estimation of the charging time and perhaps in the reading of the ballistic throws. The peculiarity occurs sometimes in the charging interval corresponding to the second thickness of the type metal strip and sometimes in that of the third thickness. In either case the experiment is extremely delicate and one would expect a slight shifting to occur.

CONCLUSION.

The results of this research, as shown graphically in the "current curves of the Figures," prove clearly that the current which forms residual charge, or, in other words, the "absorption current," is far from negligible when the charging interval is very small. Not only is the current very large, but the residual charge which it forms within 0.0017 of a second after charging begins, is of the order of several per cent of the "free charge." Glass and paraffined paper condensers show the greatest residual charge formation for short charging times. In each of the two mica condensers which were tested the residual charge which is formed in 0.0017 seconds is only one-half of 1 per cent of the "free charge." But, on the other hand, the mica condensers exhibit an absorption current which decreases but slowly with the time, so that for long-continued charging they may take up much more residual charge than the paraffined paper condensers, whose absorption current is very large at first but decreases much more rapidly as the time increases. The glass condenser shows both a high residual forming current immediately after the beginning of the charge and a rather slow decrease as the time increases. To give a striking example of its high initial value, we may note that during the charging interval from 0.00007 to 0.00020 seconds its average value is such that if it continued uniformly for one second, the condenser would get a total residual charge equal to one hundred times the total "free charge."

It thus appears that the conception of "free charge" is not a very convenient one, for various investigators have shown that the law of superposition holds true, at least to a very close approximation, and this law gives the corollary that if a condenser has been charged for a long time with a constant potential difference and is then discharged, the residual charge will be liberated at precisely the rate which characterized the residual forming current on its entrance into the condenser during the long-continued charge. No experiments have been made in the present work in which the rate of liberation of residual charge was observed, but the law, if closely tested, will probably be found verified fairly well, and, if this is so, we may conclude that the so-called "free charge" of condensers such as glass and paraffined paper contains an appreciable quantity of very mobile residual charge.

Many investigators have noticed that the capacities of most condensers vary considerably with the frequency of the alternating current, when determined by one of the bridge methods, the capacities invariably decreasing as the frequency is increased to high values. Now if the results of the present research can be applied to chargings by means of an alternating electromotive force, and we see no reason why they should not apply, then it follows that the variation in the capacity of a condenser is not primarily due to the increased frequency, or decreased period, but to the decreased charging interval, or time of contact of the vibrating tongue with the condenser terminal. In fact, it seems that the measured capacity should *increase* with increasing frequency of alternation, provided the contact time of the vibrating tongue is made longer at the same time. Of course this condition can be realized for a certain range of frequency only.

The fact that a considerable part of the residual charge is very mobile is well illustrated by some observations on one of the condensers made of pure paraffin sheets. As shown by the results tabulated above, no satisfactory evidence was obtained of a measureable quantity of residual charge formed in such condensers within 0.0017 of a second after the beginning of charging. When this condenser was charged for two minutes, it was found to have formed 0.7 per cent of residual charge, as measured by the air condenser neutralization method, in which no residual charge is lost. But when the same condenser had been charged for many minutes and then discharged by momentary short circuit, only 0.1 per cent of residual charge was obtained, all the rest having apparently disappeared along with the main discharge. Yet this momentary short circuit forms an essential feature of the experiments carried out by many investigators, who have studied, by means of the quadrant electrometer, the reappearance of residual charge after a momentary short circuit.

As to the cause of residual charge, the results of the work cannot give much information. It seems likely, however, that air bubbles in the dielectric medium play a very important rôle in absorption of charge. I hope to be able to carry on further investigations with even shorter charging intervals, and I should not be at all surprised if by these means the " free charge capacity " of a good condenser of paraffined paper sheets without the air bubbles could be decreased considerably toward the capacity of a condenser of like dimensions using pure paraffin.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

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RUMFORD PREMIUM.

In conformity with the terms of the gift of Benjamin, Count Rumford, granting a certain fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of said fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a premium to the author of any important discovery or useful improvement in light or in heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference being always given to such discoveries as shall, in the opinion of the Academy, tend most to promote the good of mankind; and to add to such medals, as a further premium for such discovery and improvement, if the Academy see fit so to do, a sum of money not exceeding three hundred dollars.

STANDING VOTES.

1. Communications of which notice has been given to the Secretary shall take precedence of those not so notified.

2. Associate Fellows, Foreign Honorary Members, and Resident Fellows, who have paid all fees and dues chargeable to them, are entitled to receive one copy of each volume or article printed by the Academy on application to the Librarian personally or by written order within two years of the date of publication. Exceptions to this rule may be made in special cases by vote of the Academy.

3. The Committee of Publication shall fix from time to time the price at which the publications of the Academy may be sold. But members may be supplied at half this price with volumes which they are not entitled to receive free, and which are needed to complete their sets.

4. Two hundred extra copies of each paper accepted for publication in the Memoirs or Proceedings of the Academy shall be placed at the disposal of the author, free of charge.

5. Resident Fellows may borrow and have out from the Library six volumes at any one time, and may retain the same for three months, and no longer.

6. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

7. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy, except by special leave of the Librarian.

8. Books, publications, or apparatus shall be procured from the income of the Rumford Fund only on the certificate of the Rumford Committee that they, in their opinion, will best facilitate and encourage the making of discoveries and improvements which may merit the Rumford Premium; and the approval of a bill incurred for such purposes by the Chairman shall be accepted by the Treasurer as proof that such certificate has been given.

9. A meeting for receiving and discussing scientific communications may be held on the second Wednesday of each month not appointed for stated meetings, excepting July, August, and September.

10. No report of any paper presented at a meeting of the Academy shall be published by any member without the consent of the author, and no report shall in any case be published by any member in a newspaper as an account of the proceedings of the Academy.

enactment a majority of two-thirds of the members present, and at least eighteen affirmative votes.

2. Standing votes may be passed, amended, or rescinded at a stated meeting, or a special meeting called for the purpose by a majority of two-thirds of the members present. They may be suspended by a unanimous vote.

CHAPTER XII.

Of Literary Performances.

1. The Academy will not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its publications.

candidate for election, unless he shall have received the approval of at least five members of the Council present at a meeting. All nominations thus approved shall be read to the Academy at any meeting, and shall then stand on the nomination list until the next stated meeting, and until the balloting. No person shall be elected a Resident Fellow, unless he shall have been resident in this Commonwealth one year next preceding his election. If any person elected a Resident Fellow shall neglect for one year to pay his admission fee, his election shall be void; and if any Resident Fellow shall neglect to pay his annual assessments for two years, provided that his attention shall have been called to this article, he shall be deemed to have abandoned his Fellowship; but it shall be in the power of the Treasurer, with the consent of the Council, to dispense (*sub silentio*) with the payment both of the admission fee and of the assessments, whenever in any special instance he shall think it advisable so to do. In the case of officers of the Army or Navy who are out of the state on duty, payment of the annual assessment may be waived during such absence if continued during the whole official year and if notification of such absence be sent to the Treasurer.

3. The nomination and election of Associate Fellows hall take place in the manner prescribed in reference to Resident Fellows.

4. The nomination and election of Foreign Honorary Members shall take place in the manner prescribed for Resident Fellows, except that the nomination papers shall be signed by at least seven members of the Council before being presented to the Academy.

5. Three-fourths of the ballots cast must be affirmative, and the number of affirmative ballots must amount to eleven to effect an election of Fellows or Foreign Honorary Members.

6. If, in the opinion of a majority of the entire Council, any Fellow — Resident or Associate — shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his Fellowship; and provided that a majority of two-thirds of the Fellows at a stated meeting, consisting of not less than fifty Fellows, shall adopt this recommendation, his name shall be stricken off the roll of Fellows.

CHAPTER XI.

OF AMENDMENTS OF THE STATUTES.

1. All proposed alterations of the Statutes, or additions to them, shall be referred to a committee, and, on their report at a subsequent stated meeting or a special meeting called for the purpose, shall require for

7. The Librarian shall have custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

CHAPTER IX.

OF MEETINGS.

1. There shall be annually four stated meetings of the Academy; namely, on the second Wednesday in May (the Annual Meeting), on the second Wednesday in October, on the second Wednesday in January, and on the second Wednesday in March. At these meetings, only, or at meetings adjourned from these and regularly notified, or at special meetings called for the purpose, shall appropriations of money be made, or alterations of the statutes or standing votes of the Academy be effected.

Special meetings shall be called by the Recording Secretary at the request of the President or of a Vice-President or of five Fellows. Notifications of the special meetings shall contain a statement of the purpose for which the meeting is called.

2. Fifteen Resident Fellows shall constitute a quorum for the transaction of business at a stated or special meeting. Seven Fellows shall be sufficient to constitute a meeting for scientific communications and discussions.

3. The Recording Secretary shall notify the meetings of the Academy to each Resident Fellow; and he may cause the meetings to be advertised, whenever he deems such further notice to be needful.

CHAPTER X.

OF THE ELECTION OF FELLOWS AND HONORARY MEMBERS.

1. Elections shall be made by ballot, and only at stated meetings.

2. Candidates for election as Resident Fellows must be proposed by two Resident Fellows of the section to which the proposal is made, in a recommendation signed by them; and this recommendation shall be transmitted to the Corresponding Secretary, and by him referred to the Council. No person recommended shall be reported by the Council as a

Committee, and shall report the same at the expiration of his term of office or whenever called on so to do by the Academy or Council.

3. The Treasurer shall keep separate accounts of the income and appropriation of the Rumford Fund and of other special funds, and report the same annually.

4. The Treasurer may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, the Treasurer shall be responsible; or the Treasurer may employ any Trust Company, doing business in Boston, as agent to perform his duties, the compensation of such Assistant Treasurer or agent to be paid from the funds of the Academy.

CHAPTER VIII.

OF THE LIBRARIAN AND LIBRARY.

1. It shall be the duty of the Librarian to take charge of the books, to keep a correct catalogue of them, to provide for the delivery of books from the Library, and to appoint such agents for these purposes as he may think necessary. He shall make an annual report on the condition of the Library.

2. The Librarian, in conjunction with the Committee on the Library, shall have authority to expend such sums as may be appropriated, either from the General, Rumford, or other special Funds of the Academy, for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

3. To all books in the Library procured from the income of the Rumford Fund, or other special funds, the Librarian shall cause a stamp or label to be affixed, expressing the fact that they were so procured.

4. Every person who takes a book from the Library shall give a receipt for the same to the Librarian or his assistant.

5. Every book shall be returned in good order, regard being had to the necessary wear of the book with good usage. If any book shall be lost or injured, the person to whom it stands charged shall replace it by a new volume or set, if it belongs to a set, or pay the current price of the volume or set to the Librarian; and thereupon the remainder of the set, if the volume belonged to a set, shall be delivered to the person so paying for the same.

6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

meeting he shall present the letters which have been addressed to the Academy since the last meeting. Under the direction of the Council, he shall keep a list of the Resident Fellows, Associate Fellows, and Foreign Honorary Members, arranged in their Classes and in Sections in respect to the special sciences in which they are severally proficient; and he shall act as secretary to the Council.

2. The Recording Secretary shall have charge of the Charter and Statute-book, journals, and all literary papers belonging to the Academy. He shall record the proceedings of the Academy at its meetings; and after each meeting is duly opened, he shall read the record of the preceding meeting. He shall notify the meetings of the Academy, apprise officers and committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy. He shall post up in the Hall a list of the persons nominated for election into the Academy; and when any individual is chosen, he shall insert in the record the names of the Fellows by whom he was nominated.

3. The two Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them calculated to promote its interests.

4. Every person taking any books, papers, or documents belonging to the Academy and in the custody of the Recording Secretary, shall give a receipt for the same to the Recording Secretary.

CHAPTER VII.

OF THE TREASURER.

1. The Treasurer shall give such security for the trust reposed in him as the Academy shall require.

2. He shall receive all moneys due or payable to the Academy and all bequests and donations made to the Academy. He shall pay all bills due by the Academy, when approved by the proper officers (except those of the Treasurer's office, which may be paid without such approval). He shall sign all leases of real estate in the name of the Academy. All transfers of stocks, bonds, and other securities belonging to the Academy shall be made by the Treasurer with the written consent of one member of the Committee of Finance. He shall keep an account of all receipts and expenditures, shall submit his accounts annually to the Auditing

and proper execution of the trust. All bills incurred on account of the C. M. Warren Fund, within the limits of the appropriations made by the Academy, shall be approved by the Chairman of the C. M. Warren Committee.

5. The Committee on Publication, to consist of three Fellows, one from each class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and Memoirs shall be entrusted. This Committee shall report to the Council in March of each year on the appropriations needed for the coming year. All bills incurred on account of publications, within the limits of the appropriations made by the Academy, shall be approved by the Chairman of the Committee on Publication.

6. The Committee on the Library, to consist of the Librarian *ex officio*, and three other Fellows, one from each class, who shall examine the Library and make an annual report on its condition and management. This Committee, through the Librarian, shall report to the Council in March of each year, on the appropriations needed for the Library for the coming year. All bills incurred on account of the Library, within the limits of the appropriations made by the Academy, shall be approved by the Librarian.

7. The House Committee to consist of three Fellows. This Committee shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to other Committees. This Committee shall report to the Council in March in each year on the appropriations needed for their expenses for the coming year. All bills incurred by this Committee within the limits of the appropriations made by the Academy shall be approved by the Chairman of the House Committee.

8. An auditing Committee, to consist of two Fellows, for auditing the accounts of the Treasurer, with power to employ an expert and to approve his bill.

9. In the absence of the Chairman of any Committee, bills may be approved by a member of the Committee designated by the Chairman for the purpose.

CHAPTER VI.

Of the Secretaries.

1. The Corresponding Secretary shall conduct the correspondence of the Academy, recording or making an entry of all letters written in its name, and preserving on file all letters which are received; and at each

CHAPTER IV.

OF THE PRESIDENT.

1. It shall be the duty of the President, and, in his absence, of the senior Vice-President present, or next officer in order as above enumerated, to preside at the meetings of the Academy; to direct the Recording Secretary to call special meetings; and to execute or to see to the execution of the Statutes of the Academy. Length of continuous membership in the Academy shall determine the seniority of the Vice-Presidents.

2. The President, or, in his absence, the next officer as above enumerated, shall nominate members to serve on the different committees of the Academy which are not chosen by ballot.

3. Any deed or writing to which the common seal is to be affixed shall be signed and sealed by the President, when thereto authorized by the Academy.

CHAPTER V.

OF STANDING COMMITTEES.

1. At the Annual Meeting there shall be chosen the following Standing Committees, to serve for the year ensuing, viz.: —

2. The Committee on Finance to consist of three Fellows to be chosen by ballot, who shall have, through the Treasurer, full control and management of the funds and trusts of the Academy, with the power of investing and of changing the investment of the same at their discretion.

3. The Rumford Committee, to consist of seven Fellows to be chosen by ballot, who shall consider and report to the Academy on all applications and claims for the Rumford premium. They shall also report to the Council in March of each year on all appropriations of the income of the Rumford Fund needed for the coming year, and shall generally see to the due and proper execution of the trust. All bills incurred on account of the Rumford Fund, within the limits of the appropriation made by the Academy, shall be approved by the Chairman of the Rumford Committee.

4. The C. M. Warren Committee, to consist of seven Fellows to be chosen by ballot, who shall consider and report to the Council in March of each year on all applications for appropriations from the income of the C. M. Warren Fund for the coming year, and shall generally see to the due

ing Secretary, and the Treasurer, proposing the appropriations for their work during the year beginning the following May. The Treasurer at the same meeting shall report on the income which will probably be received on account of the various Funds during the same year.

At the Annual Meeting, the Council shall submit to the Academy, for its action, a report recommending the appropriations which in the opinion of the Council should be made for the various purposes of the Academy.

4. If any office shall become vacant during the year, the vacancy shall be filled by a new election, at the next stated meeting, or at a meeting called for this purpose.

CHAPTER III.

OF NOMINATIONS OF OFFICERS.

1. At the stated meeting in March, the President shall appoint a Nominating Committee of three Resident Fellows, one for each Class.

2. It shall be the duty of this Nominating Committee to prepare a list of candidates for the offices of President, Vice-Presidents, Corresponding Secretary, Recording Secretary, Treasurer, Librarian, Councillors, and the Standing Committees which are chosen by ballot; and to cause this list to be sent by mail to all the Resident Fellows of the Academy not later than four weeks before the Annual Meeting.

3. Independent nominations for any office, signed by at least five Resident Fellows, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call for the Annual Meeting, which shall then be issued not later than one week before that meeting.

4. The Recording Secretary shall prepare for use, in voting at the Annual Meeting, a ballot containing the names of all persons nominated for office under the conditions given above.

5. When an office is to be filled at any other time than at the Annual Meeting, the President shall appoint a Nominating Committee in accordance with the provisions of Section 1, which shall announce its nomination in the manner prescribed in Section 2 at least two weeks before the time of election. Independent nominations, signed by at least five Resident Fellows and received by the Recording Secretary not later than one week before the meeting for election, shall be inserted in the call for that meeting.

the Commonwealth. Each Resident Fellow shall pay an admission fee of ten dollars and such annual assessment, not exceeding ten dollars, as shall be voted by the Academy at each annual meeting. Resident Fellows only may vote at the meetings of the Academy.

3. The number of Associate Fellows shall not exceed one hundred, of whom there shall not be more than forty in either of the three classes of the Academy. Associate Fellows shall be chosen from persons residing outside of the Commonwealth of Massachusetts. They shall not be liable to the payment of any fees or annual dues, but on removing within the Commonwealth they may be transferred by the Council to resident fellowship as vacancies there occur.

4. The number of Foreign Honorary Members shall not exceed seventy-five; and they shall be chosen from among persons most eminent in foreign countries for their discoveries and attainments in either of the three departments of knowledge above enumerated. There shall not be more than thirty Foreign Members in either of these departments.

CHAPTER II.

Of Officers.

1. There shall be a President, three Vice-Presidents, one for each Class, a Corresponding Secretary, a Recording Secretary, a Treasurer, and a Librarian, which officers shall be annually elected, by ballot, at the annual meeting, on the second Wednesday in May.

2. There shall be nine Councillors, chosen from the Resident Fellows. At each annual meeting, three Councillors shall be chosen, by ballot, one from each Class, to serve for three years; but the same Fellow shall not be eligible for two successive terms. The nine Councillors, with the President, the three Vice-Presidents, the two Secretaries, the Treasurer, and the Librarian, shall constitute the Council. Five members shall constitute a quorum. It shall be the duty of this Council to exercise a discreet supervision over all nominations and elections. With the consent of the Fellow interested, they shall have power to make transfers between the several sections of the same Class, reporting their action to the Academy.

3. The Council shall at its March Meeting receive reports from the Rumford Committee, the C. M. Warren Committee, the Committee on Publication, the Committee on the Library, the President and Record-

STATUTES AND STANDING VOTES.

◆

STATUTES.

Adopted May 30, 1854: *amended September* 8, 1857, *November* 12, 1862, *May* 24, 1864, *November* 9, 1870, *May* 27, 1873, *January* 26, 1876, *June* 16, 1886, *October* 8, 1890, *January* 11, *and May* 10, 1893, *May* 9, *and October* 10, 1894, *March* 13, *April* 10, *and May* 8, 1895, *May* 8, 1901, *January* 8, 1902, *May* 10, 1905, *February* 14 *and March* 14, 1906, *January* 13, 1909.

CHAPTER I.

OF FELLOWS AND FOREIGN HONORARY MEMBERS.

1. The Academy consists of Resident Fellows, Associate Fellows, and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, viz.: Class I. The Mathematical and Physical Sciences; — Class II. The Natural and Physiological Sciences; — Class III. The Moral and Political Sciences. Each Class is divided into four Sections, viz.: Class I., Section 1. Mathematics and Astronomy; — Section 2. Physics; — Section 3. Chemistry; — Section 4. Technology and Engineering. Class II., Section 1. Geology, Mineralogy, and Physics of the Globe; — Section 2. Botany; Section 3. Zoölogy and Physiology; — Section 4. Medicine and Surgery. Class III., Section 1. Theology, Philosophy, and Jurisprudence; — Section 2. Philology and Archæology; — Section 3. Political Economy and History; — Section 4. Literature and the Fine Arts.

2. The number of Resident Fellows residing in the Commonwealth of Massachusetts shall not exceed two hundred, of whom there shall not be more than eighty in any one of the three classes. Only residents in the Commonwealth of Massachusetts shall be eligible to election as Resident Fellows, but resident fellowship may be retained after removal from

SECTION III. — *Political Economy and History.* — 5.

James Bryce	London
Adolf Harnack	Berlin
John Morley, Viscount Morley of Blackburn	London
Sir George Otto Trevelyan, Bart.	London
Pasquale Villari	Florence

SECTION IV. — *Literature and the Fine Arts.* — 5.

Georg Brandes	Copenhagen
Samuel Henry Butcher	London
Frederick James Furnivall	London
Jean Léon Gérôme	Paris
Rudyard Kipling	Burwash

Section II. — *Botany.* — 6.

Jean Baptiste Edouard Bornet	Paris
Adolf Engler	Berlin
Sir Joseph Dalton Hooker	Sunningdale
Wilhelm Pfeffer	Leipsic
Hermann, Graf zu Solms-Laubach	Strassburg
Eduard Strasburger	Bonn

Section III. — *Zoölogy and Physiology.* — 5.

Ludimar Hermann	Königsberg
Hugo Kronecker	Bern
Sir Edwin Ray Lankester	London
Elias Metschnikoff	Paris
Magnus Gustav Retzius	Stockholm

Section IV. — *Medicine and Surgery.* — 7.

Emil von Behring	Marburg
Sir Thomas Lauder Brunton, Bart.	London
Angelo Celli	Rome
Sir Victor Alexander Haden Horsley	London
Robert Koch	Berlin
Joseph Lister, Baron Lister	London
Friedrich von Recklinghausen	Strassburg

Class III. — *Moral and Political Sciences.* — 22.

Section I. — *Philosophy and Jurisprudence.* — 5.

Arthur James Balfour	Prestonkirk
Heinrich Brunner	Berlin
Albert Venn Dicey	Oxford
Frederic William Maitland	Cambridge
Sir Frederick Pollock, Bart.	London

Section II. — *Philology and Archæology.* — 7.

Ingram Bywater	Oxford
Friedrich Delitzsch	Berlin
Hermann Diels	Berlin
Wilhelm Dörpfeld	Athens
Henry Jackson	Cambridge
Hermann Georg Jacobi	Bonn
Gaston Camille Charles Maspero	Paris

FOREIGN HONORARY MEMBERS.—63.

(Number limited to seventy-five.)

Class I.—*Mathematical and Physical Sciences.*—19.

Section I.—*Mathematics and Astronomy.*—6.

Arthur Auwers	Berlin
Sir George Howard Darwin	Cambridge
Sir William Huggins	London
Felix Klein	Göttingen
Émile Picard	Paris
Jules Henri Poincaré	Paris

Section II.—*Physics.*—5.

Oliver Heaviside	Torquay
Wilhelm Friedrich Kohlrausch	Marburg
Joseph Larmor	Cambridge
John William Strutt, Baron Rayleigh	Witham
Sir Joseph John Thomson	Cambridge

Section III.—*Chemistry.*—5.

Adolf, Ritter von Baeyer	Munich
Emil Fischer	Berlin
Jacobus Henricus van't Hoff	Berlin
Wilhelm Ostwald	Leipsic
Sir Henry Enfield Roscoe	London

Section IV.—*Technology and Engineering.*—3.

Maurice Lévy	Paris
Heinrich Müller-Breslau	Berlin
William Cawthorne Unwin	London

Class II.—*Natural and Physiological Sciences.*—22.

Section I.—*Geology, Mineralogy, and Physics of the Globe.*—4.

Sir Archibald Geikie	London
Julius Hann	Vienna
Albert Heim	Zurich
Sir John Murray	Edinburgh

Section IV. — *Medicine and Surgery.* — 8.

John Shaw Billings	New York
William Stewart Halsted	Baltimore
Abraham Jacobi	New York
William Williams Keen	Philadelphia
William Osler	Oxford
Theophil Mitchell Prudden	New York
William Hughes Welch	Baltimore
Horatio Curtis Wood	Philadelphia

Class III. — *Moral and Political Sciences.* — 21.

Section I. — *Philosophy and Jurisprudence.* — 5.

Joseph Hodges Choate	New York
Melville Weston Fuller	Washington
William Wirt Howe	New Orleans
Charles Sanders Peirce	Milford, Pa.
George Wharton Pepper	Philadelphia

Section II. — *Philology and Archæology.* — 5.

Timothy Dwight	New Haven
Basil Lanneau Gildersleeve	Baltimore
Thomas Raynesford Lounsbury	New Haven
Rufus Byam Richardson	New York
Andrew Dickson White	Ithaca

Section III. — *Political Economy and History.* — 7.

Henry Adams	Washington
George Park Fisher	New Haven
Arthur Twining Hadley	New Haven
Henry Charles Lea	Philadelphia
Alfred Thayer Mahan	New York
Henry Morse Stephens	Berkeley
William Graham Sumner	New Haven

Section IV. — *Literature and the Fine Arts.* — 4.

James Burrill Angell	Ann Arbor
Horace Howard Furness	Wallingford, Pa.
Herbert Putnam	Washington
John Singer Sargent	London

Section IV. — *Technology and Engineering.* — 7.

Henry Larcom Abbot	Cambridge
Cyrus Ballou Comstock	New York
William Price Craighill	Charlestown, W. Va.
John Fritz	Bethlehem, Pa.
Frederick Remsen Hutton	New York
William Sellers	Edge Moor, Del.
Robert Simpson Woodward	New York

Class II. — *Natural and Physiological Sciences.* — 31.

Section I. — *Geology, Mineralogy, and Physics of the Globe.* — 9.

Cleveland Abbe	Washington
George Jarvis Brush	New Haven
Thomas Chrowder Chamberlin	Chicago
Edward Salisbury Dana	New Haven
Walter Gould Davis	Cordova, Arg.
Samuel Franklin Emmons	Washington
Grove Karl Gilbert	Washington
Raphael Pumpelly	Newport
Charles Doolittle Walcott	Washington

Section II. — *Botany.* — 6.

Liberty Hyde Bailey	Ithaca
Douglas Houghton Campbell	Palo Alto
John Merle Coulter	Chicago
Cyrus Guernsey Pringle	Charlotte, Vt.
John Donnell Smith	Baltimore
William Trelease	St. Louis

Section III. — *Zoölogy and Physiology.* — 8.

Joel Asaph Allen	New York
Charles Benedict Davenport	Cold Spring Harbor, N. Y.
Franklin Paine Mall	Baltimore
Silas Weir Mitchell	Philadelphia
Henry Fairfield Osborn	New York
Addison Emory Verrill	New Haven
Charles Otis Whitman	Chicago
Eugene Benjamin Wilson	New York

ASSOCIATE FELLOWS. — 87.

(Number limited to one hundred.)

Class I. — *Mathematical and Physical Sciences.* — 35.

Section I. — *Mathematics and Astronomy.* — 13.

Edward Emerson Barnard	Williams Bay, Wis.
Sherburne Wesley Burnham	Williams Bay, Wis.
George Davidson	San Francisco
Fabian Franklin	Baltimore
George William Hill	West Nyack, N. Y.
Edward Singleton Holden	West Point
Emory McClintock	Morristown, N. J.
Eliakim Hastings Moore	Chicago
* Simon Newcomb	Washington
Charles Lane Poor	New York
George Mary Searle	Washington
Vesto Melvin Slipher	Flagstaff, Ariz.
John Nelson Stockwell	Cleveland

Section II. — *Physics.* — 6.

Carl Barus	Providence
George Ellery Hale	Williams Bay, Wis.
Thomas Corwin Mendenhall	Worcester
Albert Abraham Michelson	Chicago
Edward Leamington Nichols	Ithaca
Michael Idvorsky Pupin	New York

Section III. — *Chemistry.* — 9.

Frank Austin Gooch	New Haven
Eugene Waldemar Hilgard	Berkeley
Samuel William Johnson	New Haven
John William Mallet	Charlottesville, Va.
Edward Williams Morley	West Hartford, Conn.
Charles Edward Munroe	Washington
John Ulric Nef	Chicago
† John Morse Ordway	New Orleans
Ira Remsen	Baltimore

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* Died July 11, 1909.

† Died July 4, 1909.

Section III. — *Political Economy and History.* — 10.

Charles Francis Adams	Lincoln
Thomas Nixon Carver	Cambridge
Andrew McFarland Davis	Cambridge
Ephraim Emerton	Cambridge
Abner Cheney Goodell	Salem
Henry Cabot Lodge	Nahant
Abbott Lawrence Lowell	Cambridge
James Ford Rhodes	Boston
Charles Card Smith	Boston
Frank William Taussig	Cambridge

Section IV. — *Literature and the Fine Arts.* — 16.

Francis Bartlett	Boston
Arlo Bates	Boston
Le Baron Russell Briggs	Cambridge
Henry Herbert Edes	Cambridge
William Wallace Fenn	Cambridge
Kuno Francke	Cambridge
Edward Henry Hall	Cambridge
Thomas Wentworth Higginson	Cambridge
George Lyman Kittredge	Cambridge
Gardiner Martin Lane	Boston
William Coolidge Lane	Cambridge
James Hardy Ropes	Cambridge
Denman Waldo Ross	Cambridge
William Robert Ware	Milton
Herbert Langford Warren	Cambridge
Barrett Wendell	Boston

Reginald Heber Fitz	Boston
Samuel Jason Mixter	Boston
William Lambert Richardson	Boston
Theobald Smith	Jamaica Plain
Oliver Fairfield Wadsworth	Boston
Henry Pickering Walcott	Cambridge
John Collins Warren	Boston
Francis Henry Williams	Boston

CLASS III. — *Moral and Political Sciences.* — 51.

SECTION I. — *Philosophy and Jurisprudence.* — 8.

James Barr Ames	Cambridge
Joseph Henry Beale	Cambridge
John Chipman Gray	Boston
Francis Cabot Lowell	Boston
Hugo Münsterberg	Cambridge
Josiah Royce	Cambridge
Frederic Jesup Stimson	Dedham
Samuel Williston	Belmont

SECTION II. — *Philology and Archæology.* — 17.

Charles Pickering Bowditch	Jamaica Plain
Lucien Carr	Cambridge
Franklin Carter	New Haven
Jesse Walter Fewkes	Washington
William Watson Goodwin	Cambridge
Henry Williamson Haynes	Boston
Albert Andrew Howard	Cambridge
Charles Rockwell Lanman	Cambridge
David Gordon Lyon	Cambridge
George Foot Moore	Cambridge
Morris Hicky Morgan	Cambridge
Frederick Ward Putnam	Cambridge
Edward Robinson	New York
Edward Stevens Sheldon	Cambridge
Herbert Weir Smyth	Cambridge
Franklin Bache Stephenson	Boston
John Williams White	Cambridge

SECTION II. — *Botany.* — 11.

Frank Shipley Collins	Malden
William Gilson Farlow	Cambridge
Charles Edward Faxon	Jamaica Plain
Merritt Lyndon Fernald	Cambridge
George Lincoln Goodale	Cambridge
John George Jack	Jamaica Plain
Edward Charles Jeffrey	Cambridge
Benjamin Lincoln Robinson	Cambridge
Charles Sprague Sargent	Brookline
Arthur Bliss Seymour	Cambridge
Roland Thaxter	Cambridge

SECTION III. — *Zoölogy and Physiology.* — 24.

Alexander Agassiz	Cambridge
Robert Amory	Boston
Francis Gano Benedict	Boston
Henry Pickering Bowditch	Jamaica Plain
William Brewster	Cambridge
Louis Cabot	Brookline
Walter Bradford Cannon	Cambridge
William Ernest Castle	Cambridge
Samuel Fessenden Clarke	Williamstown
William Thomas Councilman	Boston
Harold Clarence Ernst	Jamaica Plain
Samuel Henshaw	Cambridge
Edward Laurens Mark	Cambridge
Charles Sedgwick Minot	Milton
Edward Sylvester Morse	Salem
George Howard Parker	Cambridge
James Jackson Putnam	Boston
Herbert Wilbur Rand	Cambridge
Samuel Hubbard Scudder	Cambridge
William Thompson Sedgwick	Boston
William Morton Wheeler	Boston
James Clarke White	Boston
Harris Hawthorne Wilder	Northampton
William McMichael Woodworth	Cambridge

SECTION IV. — *Medicine and Surgery.*—10.

Edward Hickling Bradford	Boston
Arthur Tracy Cabot	Boston

SECTION IV. — *Technology and Engineering.* — 18.

Comfort Avery Adams	Cambridge
Alfred Edgar Burton	Boston
Eliot Channing Clarke	Boston
Heinrich Oscar Hofman	Jamaica Plain
Ira Nelson Hollis	Cambridge
Lewis Jerome Johnson	Cambridge
Arthur Edwin Kennelly	Cambridge
Gaetano Lanza	Boston
Erasmus Darwin Leavitt	Cambridge
William Roscoe Livermore	New York
Hiram Francis Mills	Lowell
Cecil Hobert Peabody	Brookline
Andrew Howland Russell	Paris
Albert Sauveur	Cambridge
Peter Schwamb	Arlington
Henry Lloyd Smyth	Cambridge
George Fillmore Swain	Boston
William Watson	Boston

CLASS II. — *Natural and Physiological Sciences.* — 62.

SECTION I. — *Geology, Mineralogy, and Physics of the Globe.* — 17.

Henry Helm Clayton	Milton
Algernon Coolidge	Boston
William Otis Crosby	Jamaica Plain
Reginald Aldworth Daly	Cambridge
William Morris Davis	Cambridge
Benjamin Kendall Emerson	Amherst
Oliver Whipple Huntington	Newport
Robert Tracy Jackson	Cambridge
Thomas Augustus Jaggar, Jr.	Brookline
Douglas Wilson Johnson	Cambridge
William Harmon Niles	Cambridge
Charles Palache	Cambridge
John Elliott Pillsbury	Washington
Robert DeCourcy Ward	Cambridge
Charles Hyde Warren	Auburndale
John Eliot Wolff	Cambridge
Jay Backus Woodworth	Cambridge

Alexander Wilmer Duff	Worcester
Arthur Woolsey Ewell	Worcester
Harry Manley Goodwin	Roxbury
Edwin Herbert Hall	Cambridge
Hammond Vinton Hayes	Cambridge
William Leslie Hooper	Somerville
William White Jacques	Newton
Frank Arthur Laws	Boston
Henry Lefavour	Boston
Theodore Lyman	Brookline
Charles Ladd Norton	Boston
Benjamin Osgood Peirce	Cambridge
George Washington Pierce	Cambridge
Abbott Lawrence Rotch	Boston
Wallace Clement Sabine	Boston
John Stone Stone	Boston
Elihu Thomson	Swampscott
John Trowbridge	Cambridge
Arthur Gordon Webster	Worcester
Robert Wheeler Willson	Cambridge

SECTION III. — *Chemistry*. — 21.

Gregory Paul Baxter	Cambridge
Arthur Messinger Comey	Chester, Pa.
James Mason Crafts	Boston
Charles William Eliot	Cambridge
Henry Fay	Boston
Charles Loring Jackson	Cambridge
Walter Louis Jennings	Worcester
Leonard Parker Kinnicutt	Worcester
Gilbert Newton Lewis	Boston
Charles Frederic Mabery	Cleveland
George Dunning Moore	Worcester
James Flack Norris	Boston
Arthur Amos Noyes	Boston
Robert Hallowell Richards	Jamaica Plain
Theodore William Richards	Cambridge
Charles Robert Sanger	Cambridge
Stephen Paschall Sharples	Cambridge
Francis Humphreys Storer	Boston
Henry Paul Talbot	Newton
William Hultz Walker	Newton
Charles Hallet Wing	Boston

LIST

OF THE

FELLOWS AND FOREIGN HONORARY MEMBERS.

(Corrected to June 1, 1909.)

RESIDENT FELLOWS. — 193.

(Number limited to two hundred.)

CLASS I. — *Mathematical and Physical Sciences.* — 80.

SECTION I. — *Mathematics and Astronomy.* — 13.

Solon Irving Bailey	Cambridge
William Elwood Byerly	Cambridge
Seth Carlo Chandler	Wellesley Hills
Percival Lowell	Boston
Edward Charles Pickering	Cambridge
William Henry Pickering	Cambridge
John Ritchie, Jr.	Dorchester
Arthur Searle	Cambridge
William Edward Story	Worcester
Henry Taber	Worcester
Harry Walter Tyler	Boston
Oliver Clinton Wendell	Cambridge
Paul Sebastian Yendell	Dorchester

SECTION II. — *Physics.* — 28.

Alexander Graham Bell	Washington
Louis Bell	Boston
Clarence John Blake	Boston
Francis Blake	Weston
George Ashley Campbell	New York
Harry Ellsworth Clifford	Newton
Charles Robert Cross	Brookline
Louis Derr	Brookline

American Academy of Arts and Sciences

OFFICERS AND COMMITTEES FOR 1909-10.

PRESIDENT.

John Trowbridge.

VICE-PRESIDENTS.

Class I.	Class II.	Class III.
Elihu Thomson,	Henry P. Walcott,	John C. Gray.

CORRESPONDING SECRETARY.

Edwin H. Hall.

RECORDING SECRETARY.

William Watson.

TREASURER.

Charles P. Bowditch.

LIBRARIAN.

A. Lawrence Rotch.

COUNCILLORS.

Class I.	Class II.	Class III.
Henry P. Talbot,	John E. Wolff, *Terms expire* 1910.	George L. Kittredge.
William L. Hooper,	Harold C. Ernst, *Terms expire* 1911.	Frederic J. Stimson.
William R. Livermore,	Theobald Smith, *Terms expire* 1912.	Charles R. Lanman.

COMMITTEE OF FINANCE.

John Trowbridge, Eliot C. Clarke, Francis Bartlett.

RUMFORD COMMITTEE.

Charles R. Cross, *Chairman*,
Erasmus D. Leavitt, Edward C. Pickering, Elihu Thomson,
Arthur G. Webster, Theodore W. Richards, Louis Bell.

C. M. WARREN COMMITTEE.

Leonard P. Kinnicutt, *Chairman*,
Henry P. Talbot, Theodore W. Richards, George D. Moore,
Charles R. Sanger, Arthur A. Noyes, James F. Norris.

COMMITTEE OF PUBLICATION.

Charles R. Sanger, of Class I, *Chairman*,
Walter B. Cannon, of Class II, Morris H. Morgan, of Class III.

COMMITTEE ON THE LIBRARY.

A. Lawrence Rotch, *Chairman*,
Harry M. Goodwin, of Class I, Samuel Henshaw, of Class II,
Henry W. Haynes, of Class III.

AUDITING COMMITTEE.

A. Lawrence Lowell, Frederic J. Stimson.

HOUSE COMMITTEE.

William R. Ware, *Chairman*.
A. Lawrence Rotch, Louis Derr.

Six Lectures on School Hygiene and the Relation of our Public Schools to the Disorders of the Nervous System. 1886.
Mental Diseases.
Amer. System of Medicine, Vol. V. Reprinted Oct. 25, 1886.
Cases of Multiple Neuritis. *Ibid.*, May 19, 1887.
The Early Stages of General Paralysis.
Bost. Med. and Surg. Jour., Oct. 3, 1889.
Treatment in Typhoid Fever. *Ibid.*, Dec. 5, 1889.
Disorders of Sleep, Insomnia. *Ibid.*, July 3, 1890.
Some Points Regarding General Paralysis. *Ibid.*, Sept. 3, 1891.
Address at the Opening of Johns Hopkins Medical School for Women. 1891.
Henry Ingersoll Bowditch.
Amer. Acad. Arts and Sci., 1892. Vol. XXVIII.
Cases of Traumatic Headache. *Ibid.*, June 28, 1894.
The Prevalence and Fatality of Pneumonia. *Ibid.*, July 16, 1896.
Report of the Committee to Investigate the Public Charitable and Reformatory Interests and institutions of the State. Feb., 1897.
Address Harvard Medical Alumni Association. Oct., 1903.

SOCIETIES OF WHICH DR. FOLSOM WAS A MEMBER BESIDES THOSE MENTIONED IN THE TEXT.

Association of American Physicians. Original Member; later, Hon. Member.
American Medical Society.
Massachusetts Medical Society.
Massachusetts Medico-Legal Society.
Suffolk District Medical Society.
Society of Psychiatry and Neurology.
Boston Society of Medical Improvement.
American Academy of Arts And Sciences.
American Statistical Association.
American Social Science Association.
American Association for the Advancement of Science.
National Geographical Society.
Boston Society of Natural History.
Reading Masters Society.
St. Botolph Club.

Five Resident Fellows have resigned.

Nine Resident Fellows have been elected.

The roll of the Academy now includes 188 Resident Fellows, 88 Associate Fellows, and 61 Foreign Honorary Members.*

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* By the election of new members at the annual meeting of May 12, 1909, and the deaths of two Associate Fellows, not previously noted, the roll stands at date of publication, 193 Resident Fellows, 87 Associate Fellows, and 63 Foreign Honorary Members.

editorial upon Dr. Folsom which concludes as follows: "But it was not as an authority on public health and on mental and nervous diseases or as a College officer that his former patients and colleagues have sought to perpetuate his name in an institution which he loved so well. It was as a friend, perhaps as a host to whom entertaining was a fine art, that they knew him. Wise, firm, kind, and indefatigable, he rarely departed from a sick-room without leaving his patient stronger in mind, if not in body. His constant thoughtfulness of his charges, in health as in illness, was unending, and many a patient owes a sound mind and a sound body to Charles Folsom's sagacity, skill, and loving care. Indeed, it may be said of him more truly than of many physicians and of most men that he was like "rivers of water in a dry place and the shadow of a great rock in a weary land."

James J. Putnam.

PRINCIPAL PUBLICATIONS

A Scotch Insane Asylum. Boston Med. and Surg. Jour., Aug. 12, 1875.

The Treatment of Insanity in England and America. *Ibid.*, Dec. 9, 1875.

Report by a Commission on the Sewerage of Boston. 1876.

The Present Aspect of the Sewerage Question. 1877.

Diseases of the Mind and other Papers. State Board of Health, 1877.

Causes of Typhoid Fever. Bost. Med. and Surg. Jour., March 4, 1880.

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The Relation of the State to the Insane. *Ibid.*, Aug. 4, 1881.

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The Crime at Washington and its Lesson. Editorial *Ibid.*, July 14, 1881.

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The Case of Guiteau. *Ibid.*, Feb. 16, 1882.

Some Obscure Mental Symptoms of Disease. *Ibid.*, Aug. 17, 1882.

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40th Report to the Legislature of Massachusetts relating to the Registry and Return of Births, Marriages, and Deaths for the Year ending Dec. 31, 1881.

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General Paralysis in the Prodromeal Period. *Ibid.*, Nov. 5, 1885.

certainly he once said that he knew he was approaching his end and that "the clock had struck twelve;" but this may be taken rather as a temperamental note of acquiescence than as a conclusion based on evidence. He died at last quietly and without pain.

The examination showed that he had been suffering from an ulcerative, infective endocarditis, with embolisms, to which it was thought his old valvular heart-disease had rendered him susceptible.

It would be easy to multiply testimonials to the character and ability of Dr. Folsom from the words — spoken, written, or printed — of his colleagues and his friends. Perhaps, however, the most fitting close to this brief sketch is given in the final paragraphs of a private letter from Mr. Gannett, who was the oldest and probably the closest of Dr. Folsom's friends. After referring to the fact that at each new meeting following a long interval of separation he found him always "hard at work, the same loyal friend, simple, modest, gentle, high-minded, lovable . . . yet growing in power and in service, . . ." Mr. Gannett goes on to say, "It is strange how well one can know a man's self while knowing so little of his works and days. The reason, no *doubt, lies in the same loyalty,* — *he was loyal to himself; through* his growth and success he remained the same man I knew in our youth. I was always grateful for his holding on to me, and counted it an honor. And it seems so easy to hold on to *him* now for the same reason, — now when his greeting no longer waits me in Boston. I happened yesterday to be looking up something about George William Curtis, and came across what Mr. Roosevelt — not yet even Governor — said of him at some club in New York City, not long after his death. He spoke of the serene purity and goodness of character which impressed every one who came in contact with Curtis, — and then said, 'I have used the adjective *serene*, it is a beautiful adjective, and it is the only adjective I know of which is sufficiently beautiful to describe his beautiful character.' I think of Folsom in that way, — the adjective and the noun, and the whole expression apply well to him."

A testimonial of another form deserves especial mention. A large number, nearly seventy, of his friends and patients, "who wished in this way to express their grateful appreciation of Dr. Folsom's unfailing care and skill as a physician, and their admiration for him as a man" (Harvard Bulletin, March 4, 1908), presented Harvard University with a fund of ten thousand dollars for the establishment in the Harvard Medical School of "The Charles Follen Folsom Teaching Fellowship," in Hygiene or in Mental and Nervous Diseases. The issue of the Bulletin in which this gift was announced contains also an visit to Europe had been to withdraw from unnecessary labors, not on account of obvious ill health, and surely not from indolence, but from prudence. In 1899 his horse fell with him, and this accident cost him a broken rib and an attack of pleurisy, and marks the period subsequent to which his strength and power of work were never quite what they had been before. In 1901 he writes to Mr. Gannett: "I am sorry that I do not write to —— oftener and to you and to —— and that I do not do a lot of extra things in the way of work of all kinds and of social duties and pleasures. But I discovered some time ago that there was not enough of me to go around. Starting in debt and having something to do for others all the time, one has to be economical of his strength if he is going to practise medicine."

Many men would have met this need of economy of strength by longer and more frequent holidays than he took. But, fond as he was of the country, of travel, of new friends, his habit of long years had been to husband his strength by careful living, and not to separate himself far or for long from his patients and his desk. Perhaps he knew himself better than his advisers knew him when he chose this mode of life, or accepted it as a satisfactory one when it seemed forced upon him by his duties. His recreation lay in friendly intercourse, in horseback riding, and, of late years, in absences of short duration at Little Boar's Head, New Hampshire, where he and his wife, with several friends, spent a number of consecutive summers. The final visit to Europe, which at best was to have been of but two months duration, was looked forward to by both his wife and himself with the greater pleasure for the fact that it had been so long postponed. He was pretty well tired before starting, but in essential ways had seemed as well and as serene as common. Perhaps, in fact, he felt less well than he admitted. At any rate, even on the passage outward he seemed poorly, and when in England a constant though slight fever set in and he was unable to obtain the expected pleasure from the visits and excursions that he made. While in London he consulted physicians, among them Sir Lauder Brunton and Sir Almroth Wright, but without avail. During the voyage homeward his fever increased to a high point and he became delirious. On arriving in New York he was taken to the Roosevelt Hospital and carefully tended by Dr. Walter B. James. Here he lay for several weeks, at times improving slightly, at times worse again, but on the whole gradually losing ground. Much of the time his mind wandered a little, but it was striking to note how fully he retained his characteristic patience and his unmurmuring readiness to accept results, whatever they might be. Perhaps he felt sure from the first that he should not get well, and the education of nieces and nephews. That it was a joy to him to do this, as it had been to contribute to the comfort of his parents' declining years, is shown by the following extract from a letter written in 1901: "Just now I am sending two nieces to school and a nephew to college, and hiring an outside man for my brother, who is ill. Many of the other things I do not care for, it is such a pleasure and such a privilege to do these." His sister writes: "What he was to us all as counsellor could n't well be told — it includes a much wider family circle of cousins and broadens into the same service for patients and friends."

Dr. Folsom's public services did not cease with his resignation from the State Board. In 1891 he was chosen overseer of Harvard College, and to this important post he was repeatedly re-elected, until he had served twelve years. In the spring of 1896 he was one of the commission appointed by the Governor and Council "to investigate the public charitable and reformatory interests and institutions of the Commonwealth; to inquire into the expediency of revising the system of administering the same, and of revising all existing laws in regard to pauperism and insanity, including all laws relating to pauper settlements," etc. The other members of this commission were Mr. William F. Wharton and Professor Davis R. Dewey. Their report, covering a hundred printed pages, was submitted in February, 1897. In 1901 he was offered — so his letters show — the chairmanship of the State Board of Lunacy, but decided to decline this tempting offer. "Think," he writes, "of following in Dr. Howe's footsteps with twice as big a field." In 1903 he was selected as president of the Harvard Medical School Alumni Association. Truly, a rare list of honors and opportunities for service.

As early as 1898 Dr. Folsom resigned his position as visiting physician to the Boston City Hospital,* "long before his usefulness to the institution began to wane," a colleague writes,† and although he was chosen consulting physician in 1901, this appointment was one rather of honor than of active service. The fact was, as many of his friends observed, that Dr. Folsom's policy for several years before his last at work, the observation of those who fall, he writes, "increases one's admiration for those many persons in all stations of life who lead lives of purity and nobleness, and to whom trial and temptation only give added purity and strength. If people will only place their ideals high enough, they easily or with a fight may make them real. —— does not believe this, but I know it."

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* The whole period of Dr. Folsom's active work in connection with the City Hospital, not including his service as assistant, was from December, 1881, to the time of his resignation in 1898. He was first appointed Physician to Out-Patients (December, 1881), then Physician to Out-Patients with Diseases of the Nervous System (November, 1882), then Visiting Physician to Patients with Diseases of the Nervous System (September, 1885), and finally member of the general visiting staff (December, 1886). After his resignation in 1898, he was appointed Consulting Physician in 1901.

† Editorial, Boston Medical and Surgical Journal, August 29, 1907.

In the spring of 1886 Dr. Folsom was married to Martha Tucker Washburn, sister of his classmate William T. Washburn, and this fortunate event filled with happiness and serenity the whole remainder of his life. Domestic, affectionate, home-loving, and hospitable, his marriage brought to him as much fulness of satisfaction as any of his friends could have desired. It gave new scope, too, to his hospitality and his strong social instincts, for these traits were eminently characteristic of his wife also, and their table became well known as one where good talk, good fellowship, and good humor in the best sense were to be found. Dr. Folsom had had a wide experience with men, with books, and with affairs; he had a good memory, a good sense of humor, a fondness for a good story and the capacity to tell one, and these characteristics, combined with his real love for his fellow-men, made him a highly acceptable companion.

For a number of years he had been very busy in his private practice and his marriage only increased his zeal in this respect and his opportunities for conducting his work as he desired. To an unusual degree he treated his patients as his friends and made them welcome visitors at his house. This tendency, which was instinctive with him and formed a part of his desire to lead a life which should bring him into close contact "with individuals needing help," was thoroughly sympathized in and actively forwarded by his wife, and materially increased his power for good.

As a diagnostician and practitioner Dr. Folsom was a careful, accurate observer, sound and conservative in judgment and resourceful in meeting practical needs, and it was these qualities rather than an ability and instinct for scientific investigation that brought him his success. His contributions to what might be called pure science were in fact not numerous, and became less so as time went on. It was always the vision of "the individuals needing help" that led him on. The worrying habit might readily have developed itself in him, but he systematically discouraged this tendency and opposed to it a simple and gentle philosophy of living which methodical, well-ordered habits aided to make effective. Generosity was a constant trait throughout his life and for nearly twenty years he contributed substantially to the support of a brother who was ill, and even to the very last to and Dr. S. G. Webber had been the first physicians. This ward had been devoted partly to nervous and partly to renal diseases, but even thus it was the first neurological ward to be established in Boston, and would stand, if it still existed, as the only department in a public institution of this city, with the exception of the Long Island Hospital, where disorders of the nervous system could be systematically and adequately taught and studied under expert supervision. In the year following Dr. Folsom's appointment this ward was given over, to the great sorrow of onlooking neurologists, to the general purposes of the hospital. At the same time Dr. Folsom became a member of the regular visiting staff, and at about the same period made a strong and indeed successful effort to change the character of his private and consulting practice to that of an "internist" or general practitioner.

In 1882 Dr. Folsom was appointed consulting physician to the Adams Nervine Asylum.

In 1886, while still especially interested in nervous diseases, he delivered six lectures on school hygiene,* one of which, "On the Relation of our Public Schools to the Disorders of the Nervous System," was reprinted for distribution. This sort of task, in which his two-fold instincts and training, as a hygienist and as a neurologist, were to be enlisted in the practical service of a concrete set of public needs, was a congenial one to him and was always well performed.

In the next year (1887) he took part in the discussion of another topic of public interest, namely, whether the State should establish a hospital for dipsomaniacs. To this plan he was opposed.

This is perhaps the proper place to mention that Dr. Folsom had been warmly interested for many years in the question of the proper treatment of prostitution. He studied this subject diligently, at home and abroad, and wrote his views upon it at length to Mr. Gannett. Unfortunately he did not publish them, and it would perhaps be unjust to consider them as final. They are, however, of interest as an example of his habitual generosity of sentiment. Like the majority of cultivated men, and especially those who have labored practically in the harness of organized progress, Dr. Folsom was conservative and inclined to see two sides to every proposition. On the other hand, he was by inheritance and by temperament a reformer, a hater of injustice, of oppression, and of immorality. These sometimes conflicting tendencies were all drawn upon in his studies into the question of prostitution. Whatever is to be said of the varied influences and motives importance to have the system adopted by the State Board, its value recognized, and its work established on a larger scale.

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* Given before the teachers in the public schools, under the auspices of the Massachusetts Emergency and Hygiene Association.

Besides serving on the State Board Dr. Folsom gave much time during the early eighties to the Danvers Lunatic Hospital, in the establishment of which he had been greatly interested and of which he had been made trustee. In 1881 he read an excellent paper entitled "The Management of the Insane," before the Hospital Trustees Association, discussing and forecasting the conditions needed to make a hospital fulfil its possibilities of efficiency. As usual, practical good sense, thorough information and earnest desire for reform inspire its pages, on one of which he refers to his studies made during five visits in different years to Great Britain. Another paper, on "The Relation of the State to the Insane," was read at the American Medical Association this same year.

In the following year, 1882, occurred the trial of Guiteau for the assassination of President Garfield, followed by his condemnation and execution, notwithstanding the protest of a large number of the best physicians of the country. Dr. Folsom took part in the public discussion of the merits of this case, and in so doing revived an interest in medical jurisprudence which had expressed itself, even in 1875, in a paper entitled "Limited Responsibility: a Discussion of the Pomeroy Case," in 1877 by an article on "Medical Jurisprudence in New York," and in 1880 by an account of "Cases of Insanity and of Fanaticism," devoted mainly to the remarkably interesting case of Freeman, the religious fanatic of the quiet village of Pocasset on Cape Cod who had killed a favorite child under a supposed Divine command. The study of such borderland cases, involving questions of moral and of legal responsibility, continued, indeed, to interest him throughout his life, and it is well known to his friends that he analyzed with extreme care, through several years, the data in the noted case of Jane Toppan. Pomeroy and Jane Toppan he believed to be essentially criminals, Guiteau insane. Freeman he rightly judged a crank of the fanatic type, a product of his environment, and only technically insane. He kept close watch of Freeman from the beginning onward, was instrumental in securing his release on probation from the asylum in which he was confined, and rejoiced at the continued reports of his subsequent good behavior, which have continued to come in even to the present day.

In 1881 Dr. Folsom was appointed physician to out-patients at the Boston City Hospital, and in 1886 he took charge, as visiting physician, of the ward for nervous and renal diseases, which had been established in 1877 at the request of Dr. R. T. Edes, and of which Dr. Edes formed the occasion for the establishment of the National Board of Health, and of this Dr. Billings and Dr. H. I. Bowditch were appointed members. There were thus several ties that bound Dr. Folsom's interest to the work of this important Board, and it was only natural that on Dr. Bowditch's retirement, in 1882, Dr. Folsom should be chosen his successor. The work of the Board by that time, to be sure, was already waning under the inanition treatment to which it was subjected by the government at Washington, and in the few remaining years of its life it did but little active work. Nevertheless, it served to cement still closer the bond of friendship between Dr. Folsom and Dr. Billings, and also brought the former into wider notice among public men.

The absorption of the Board of Health into the combined Board of Health, Lunacy and Charity, was a matter of profound regret to Dr. Folsom as to Dr. Bowditch, and to all their colleagues. They felt that the co-operative effectiveness of the small group of men who had learned to work so well together was likely to be impaired, and with no compensating benefit. Dr. Bowditch who was appointed on the new Board, but resigned almost at once, partly to gain more time for other labors, partly as a means of expressing his disapproval. Dr. Folsom was made secretary of the new Board, at first with special duties relative to the health department, but resigned in January, 1881, just a year after Dr. Bowditch. He had identified himself with many of the important measures that were adopted by the Board during his brief term of service, and lent his aid to carry into effect a scheme which then, perhaps, seemed to most onlookers to be of much less consequence than it later proved. This was the appointment by the State Board of carefully selected women, from the different towns throughout the State, to act as "Auxiliary Visitors" to the State Board of Health, Lunacy, and Charity, in looking after the girls from the State Primary School at Monson, and the State Industrial School at Lancaster, as well as those committed to the custody of the board itself and placed out with relatives or in other families, while still remaining wards of the State. The appointment of these visitors increased very materially the value of the Board's work in that direction. Similar work had been going on for some years, on a small scale, as an informal outgrowth of the efforts of a few women who had been assisting Colonel Gardiner Tufts, Superintendent of the State Visiting Agency, but it was of great

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citizen, giving time and skilled labor to public interests without a thought of personal benefit — a skilled physician, beloved by his patients, and a gentleman in all the best senses of that word. I am proud of the fact that he was my friend."

Dr. Folsom's part and a visit by Dr. Clouston to America. Several of Dr. Folsom's patients spent some time at the pleasant institution of Morningside, under Dr. Clouston's care.

It was within a year after the publication of this paper that Dr. Folsom was offered and declined the superintendency of Danvers Hospital, as above described.

The work of the State Board of Health, extensive as it was, did not prevent him, at this period, from giving a certain amount of time to private practice, especially among the insane, nor from lecturing at the Harvard Medical School. His connection with this school began in 1877 and continued until 1888. He served first as lecturer on hygiene, then gave instruction in both hygiene and mental diseases, and finally became assistant professor of Mental Diseases. His resignation was prompted partly by the lack of proper clinical facilities for teaching, partly by the fact that he had finally decided to withdraw from the exclusive study of diseases of the mind and to devote himself to the work of a general practitioner and consultant. But this is to anticipate, as we still have several interesting years of public work to chronicle.

I have sketched the principal features of his labors as secretary of the State Board of Health as far as 1879. In that year two events of importance for him occurred, namely, the appointment of the Yellow Fever Commission, of which he was made a member, and the submerging of the Board of Health in the combined Board of Health, Lunacy, and Charity, of which he was appointed secretary and of which he was made a member in the following year.

The yellow fever epidemic of 1879–1880 ravaged several of the Southern States, especially those bordering on the Mississippi River, and the National Advisory Commission was appointed to inspect the infected districts and consult with local authorities and officers of public health. As a member of this commission Dr. Folsom visited a number of Southern cities, especially Memphis and New Orleans, and left behind him a pleasant impression of tact, judgment, and good breeding, of which Dr. H. P. Walcott, Dr. Folsom's successor on the Board of Health, still found traces on the occasion of a visit, many years later, to the same localities. The most important result of the trip for Dr. Folsom himself was, however, that it brought him into close contact with Dr. John S. Billings, and laid the basis for one of those enduring friendships in which he was so rich.* This same outbreak of yellow fever other trip to Europe on Dr. Folsom's part (in 1875), during which the material was collected which was published as an appendix to the report of the commission. The plan recommended in this report was, as is well known, the building of the great system of the Metropolitan intercepting-sewer for that portion of the city lying on the south side of the Charles River, with pumping stations at Moon Island, discharging on ebb-tide into the bay. Dr. Folsom afterwards appeared before the Joint Committee on Improved Sewerage and presented an elaborate defence and explanation of this plan, contrasting it with that offered by the Superintendent of Sewers, which he admitted to be cheaper but believed to represent a false economy. The plan advised by the commission was finally adopted, and was carried out, and has proved, in many ways, remarkably successful. The same principle was applied later to the north side. The preliminary investigation had been thorough, the reasoning based on it was convincing, and the conclusions were conservative and sound. Besides contributing to the able and impressive reports made by this commission and by the State Board of Health, with all their many maps and tables, Dr. Folsom read a paper before the American Statistical Association, in April, 1877, in which the sewage-farm question in particular was discussed, on the basis of a remarkable amount of knowledge and of judgment. Other communications on this and kindred subjects had appeared in the Boston Medical and Surgical Journal in the form of letters written during his trip abroad.

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* In a recent letter Dr. Billings writes: "From my first acquaintance with him I had the greatest respect for his judgment, and the frank honesty of the way he gave it, and as we became intimately associated the friendship grew into a warm affection which continued to the end. He was a model

As soon as the work of the board with reference to water-supply and drainage began to relax, Dr. Folsom turned his attention again to the duties of the State with relation to insanity and to the general question of the treatment of the insane. In 1877 he published the long article on this subject entitled Diseases of the Mind, which was republished in book form. This excellent monograph reviews the history of the treatment of insane patients from the earliest times, and describes with accuracy what was being done and what was being planned in all the great institutions of Europe and America. It tells a striking and highly interesting story. The materials for this work had been collected partly during his visit to Europe in 1875, when he had industriously visited asylums and formed the acquaintance of several prominent alienists, especially in England. With him acquaintance was more than apt to ripen into friendship, and such was the case as regards his relationship to Dr. T. S. Clouston of Edinburgh, perhaps the leading alienist of Great Britain at that day, and a man of warm and fine personal qualities which attracted Dr. Folsom strongly. The friendship between them was strengthened by subsequent visits to Edinburgh on various towns and villages of the State, in the interests of sanitary reform. It was after one of these trips, in November, 1877, that the North Adams Transcript published a long editorial, impressive with figures and with facts, the opening paragraphs of which here follow.

"As stated in a previous issue, Dr. Charles F. Folsom, Secretary of the State Board of Health, recently visited our village for the purpose of making a thorough investigation into its sanitary condition. For the limited time which he spent here, his work was been remarkably thorough, and the results of his examination, which we publish in full, are of a nature calculated to startle our citizens and awaken a profound interest in an important and heretofore neglected subject."

The investigations with which Dr. Folsom became especially identified (besides the question of meat-supply, above referred to) in the five years that followed his appointment, related to water-supply and the disposal of sewage, vital statistics, and his old love, — diseases of the mind. On these vast problems he made himself an expert, so far as this could be done without actual laboratory work. For this he was not trained, but what he did and what his mental constitution admirably fitted him to do was to scrutinize and estimate and contrast and afterward to summarize the work of other men, in Europe and at home, and then intelligently to form a plan suited for Massachusetts and for Boston. One reason why the work of the State Board at the period of Dr. Folsom's service was so largely given up to questions of water-supply and drainage and the disposal of sewage was that these subjects had begun to attract the public interest in a high degree. This led to legislation by the State authorities and permission to employ experts, the results of whose investigations are given in the successive annual reports. In these inquiries the City of Boston took an active part, and the problem of its sewerage was studied in 1875–1876 by a special commission, consisting of E. S. Cheesborough and Moses Lane as representing the department of civil engineering, and Dr. Folsom as standing for the interests of the public health. This commission was appointed by the city government in February, 1875, only a few months after the nomination of Dr. Folsom to the position of Secretary to the State Board of Health, and the choice of him as a member may therefore be considered as a recognition of his merits. The commission was called on to consider, one by one, a series of important practical problems relating to the sewerage system of the city and the modes by which it could be bettered. One portion of the investigation consisted in a study of the methods of dealing with the sewage-waste adopted in other cities of America and Europe and the experiments in utilizing it through irrigation-farms. The investigation of these matters necessitated an-on September 12 of the same year, the gap of four months having been filled by Dr. F. W. Draper. The members of the Board at this time, besides Dr. Bowditch, were J. C. Hoadley, C.E., David L. Webster, Richard Frothingham, Robert T. Davis, M.D., and T. B. Newhall. These same members served until 1879, when the departments of health, lunacy and charity were combined and Dr. Folsom was chosen secretary of the united Board.

Dr. Folsom believed that in accepting the appointment as secretary of the State Board of Health he was shaping his life-work, and in the letter to Mr. Gannett, above cited, he continues: "Of course, you can never appreciate the disappointment it cost me to give up the practice of medicine. It seemed like having in my palm something for which I had bent every energy for a dozen years, and then calmly throwing it away, and the silly *hankering* took shape in Danvers as the only practicable form; but that is now gone, like all my other buried hopes at which I can now smile and joke."

The occupations of the conscientious secretary of such a board as this, certainly of this board, are but faintly indicated in his title. His duties cannot all be specified in detail and he does much that passes unrecorded. Besides his labors as recording and executive officer, nothing goes on that does not pass his judgment, feel his touch, receive his contribution. He is the nucleus of the busy cell. The reports are in great part his work, and it is a striking tribute to Dr. Folsom's industry and ability that the volume which was issued on the first of January, 1875, only three months after his appointment, was not only ready at the proper time, but contained a long article by him, implying careful study, upon the meat supply of our cities, with suggestions for its improvement. One of the most important among the numerous and manifold secretary's jobs, and a task that called for good feeling, tact, and judgment of a high order, as well as for firmness and intelligence, was that of going about as inspector, critic, and adviser among the

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to realize the quality of the new secretary. Friends of Dr. Bowditch will be reminded by it of the generous warmth which he threw alike into his friendships and his public work.

"Boston, June 25.

"My dear Dr., — I send by mail the Advertiser of to-day. I felt my heart almost jump as I read the fine compliment paid to you my dear Dr. in the editorial. I certainly echo the wish that you may long continue to occupy the position in which you are growing, not only in yourself, but in the estimation and love of the community. God be praised that you dropped a letter to me from Europe "just in the nick of time." . . .

"Faithfully yours,

"H. I. B."

then in the old familiar grounds at Somerville, and this he kept until the autumn of 1873. He threw himself, indeed, at this period, with great energy into the study of diseases of the mind, and came near to selecting this branch of medicine for his life work. Even as late as 1877 he writes to Mr. Gannett: "The bill has passed the Legislature requiring the Governor to appoint trustees, etc., to Danvers, and the question has been asked me square, whether I w'd be Supt. Although I said no more in reply than that I would *not* say no, I have since decided not to take it, and very largely because ——, who knows me for generations back, has convinced me that I am in many respects unsuited for that kind of work."

In the autumn of 1873 he went abroad for the sake of "seeing what asylums are there, etc." He was away about a year, studying mainly in Vienna and Berlin, but visiting also the hospitals of England and of Scotland and making valuable acquaintances. The full letters from Europe during this period (1873–1874), both to the various members of his family and to Mr. Gannett, show sound observation and an active mind. He found the English asylums the best, though by no means above criticism. The brutal manners of the Viennese doctors towards the poorer patients disgusted him, but did not prevent him from appreciating the splendid opportunities of these physicians for study nor their quality as teachers. Man for man he liked his own countrymen the best.

While he was still away an event occurred which proved to be for him of great significance. This was his selection for the secretaryship of the Massachusetts State Board of Health, just then thrown open by the regretted death of Dr. George Derby, a position in which an able physician could do more for the health of his fellow-citizens than in any other way whatever. The State Board of Health had then been in existence just four years. It had owed its life to the imagination and splendid zeal of Dr. Bowditch, and its remarkable development and career of usefulness at once to his labors and those of his public-spirited and able colleagues, and to the energy and spirit of Dr. Derby, fresh from service as army surgeon in the war and full of interest in matters relating to the public health. The Board as a whole was one of the best that ever served the State. Dr. Bowditch had been chairman from the first, and when the question came up of the appointment of a successor to Dr. Derby it was natural that his thoughts should turn to Dr. Folsom, young, free, of approved character and ability, and possessed already of experience in administrative work.* Dr. Derby died in June, 1874, and Dr. Folsom was appointed period, and in this way Dr. Folsom made, in 1868, the highly valued acquaintance of Dr. H. I. Bowditch. In a letter to Mr. Gannett, written in October of that year, he says: "Dr. Bowditch is simply splendid. He is one of the purest-minded men I ever knew, and the opportunities for study are very great." I had the privilege of following Dr. Folsom at this task and can warmly testify to its value. The duty of the assistant was to receive the patients in an anteroom of the delightful study at the house on Boylston Street, make full notes of their histories, which were to be submitted afterwards to close scrutiny, and a preliminary diagnosis. Then came the physical examination by Dr. Bowditch, at which the student was often invited to assist, and the frank comments of one of the best men and best physicians of his day. It was "section teaching" in its best form. Dr. Folsom's admiration for Dr. Bowditch was so great and the understanding between them became so fine, that the friendship then established proved one of the great forces in Dr. Folsom's life. There was some question in the next year (1869) whether he should become assistant at the City Hospital or at the Massachusetts General, for which he first applied. It was to the former that he went, and he found reason to congratulate himself for so doing, largely because it brought him again under Dr. Bowditch. It was not alone admiration for Dr. Bowditch's qualities as a man that drew his younger friend so strongly, but similarity in sentiment and opinion, likewise. Both of them had grown up in the atmosphere of abolitionism, and Dr. Bowditch's ardent advocacy, both of that cause and of the natural right of women to do what nature fitted them to do and especially to practice medicine if they wished, was met with quick and active sympathy on Dr. Folsom's part. In later years his cautious and conservative traits came more prominently forward, but the sentiments by which he was mainly moved were always those of unconventionality and freedom.

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* Dr. Bowditch's personal friendship for Dr. Folsom is testified to by the following note, evidently written at a period when observers had had a chance

He strongly advocated the plan of putting a woman physician on the medical board of Danvers Hospital and took an active part in furthering the admission of women to Johns Hopkins Medical School. In the bibliography which follows this paper a reference will be found to an address of his upon this latter subject.

The service at the City Hospital came to an end in the spring of 1870. As soon as it was over Dr. Folsom opened an office on Leverett Street and engaged in private practice, while at the same time he became physician to the Massachusetts Infant Asylum, then recently established. He was for a short time connected also with the Carney Hospital. At these tasks he remained until the spring of 1872, when he obtained a much desired position as assistant at the McLean Asylum, every morning, broom in one hand, water-bucket in the other, in my bare feet, shirt sleeves rolled up to my elbows, pants rolled up to my knees; or could you but see my dignified roll as I cross the main deck, slinging a tar bucket over one shoulder and the grease pot over the other; or the sad amble as I pace the deck in the lonely midnight watch, chanting the 'Gideonite's Lament' or 'Katie's gone to Roxbury.' I am exceedingly glad that I took the trip, and especially that I returned a tarry sailor as I did. It gave me insight into a new phase of life, and I am sure the benefit has been greater than if I had come back a passenger." Mr. Gannett recalls the following incident, important for our purpose: "A sailor fell from aloft, and broke himself all to pieces so hopelessly that they left him in a huddle to die. Folsom * could not stand that, went to work with what knowledge he had, patched him together as well as he could, nursed him, and brought him through alive to New York." This was, as Mr. Gannett says, "his first case," and a worthy one.

In 1866 Charles Folsom decided, after some hesitation, to study medicine. A small and favored portion of the would-be medical students of that period used to spend a few months in taking a preliminary course of Comparative Anatomy under Professor Jeffries Wyman. Dr. Folsom and I took this course together, and vividly do I remember our first meeting. I can see myself lingering about, on a summer morning, in the cool hall-way of Boylston Hall, where Professor Wyman's laboratory lay, watching the door swing open and observing the tall figure of Charles Folsom enter. I well recall his boyish yet thoughtful and intelligent expression, his pleasant smile, his light hair and sunburnt face, and his plain suit of homespun gray. We were entire strangers to each other then, but on the moment a bond of mutual sympathy was established and we became good friends. Professor Wyman, that rare man and teacher whom every one admired, loved, and trusted, soon recognized Dr. Folsom's ability and worth, and secured for him, a few years later, the Curatorship of the Natural History Museum, a position which he occupied for several years and abandoned with regret.

Between 1866 and 1869 came medical studies, diversified by half a year's tutoring in Charlestown, New Hampshire, which secured him some pleasant acquaintances and a gain in health, though it was felt as a somewhat rasping interruption to his work.

The old custom of supplementing one's class-room studies by serving as assistant in the private office of an established practitioner (even during the medical course) was still followed, to some extent, at that

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* Not yet a medical student.

It was a difficult task that was set before this company of willing but untried philanthropists, and it was well done. "Keenly as they felt the past suffering and the present helplessness of the freedmen, they had the supreme common-sense to see that these wrongs could not be righted by any method so simple as that of giving. They saw that what was needed was, not special favor, but even-handed justice. Education, indeed, they would give outright; otherwise they would make the negro as rapidly as possible a part of the economic world, a laborer among other laborers. All that has happened since has only gone to prove how right they were."

It was natural that friendships formed among fellow-workers under conditions such as these should be warm and lasting, and the small group of men and women of which Charles Folsom formed a member during the two years of their common labors in field and cabin on St. Helena Island remained firmly bound through life. Dr. Folsom's nearest friends were William C. Gannett and Miss Mary E. Rice, with whom he afterwards freely corresponded, Edward W. Hooper, and Charles P. Ware. Mr. Gannett in a recent letter writes as follows: "While we were together in Freedmen's work on St. Helena Island, in 1862–1864, he lived for a long time in our home, — Miss Rice's and mine; I remember well, when the malaria caught me, how he used to sit on my sick bed and tell stories until the room rang with our laughter, and how he journeyed ten or twelve miles to Beaufort and back through the sand just to get me a little ice for the fever."

The Port Royal experience was in some respects a disastrous one for Dr. Folsom, since he there received an accidental gun-shot wound in his arm which caused him a great deal of pain, and in addition contracted malaria and a valvular disease of the heart, both of which troubles are believed to have contributed more or less directly to his death. He also began to suffer from severe neuralgic headaches at about this time, due partly to the shot-gun accident,* partly, perhaps, to the malaria, and on this account he was advised by his physician, on his return to Boston, in 1865, to make a long voyage by sea. Following this advice he went around the Horn to San Francisco as passenger on a sailing vessel, and came back before the mast, much improved in health though not quite relieved of his headaches, which continued to trouble him during his medical studies and even later. He writes to Miss Rice of his experiences on this voyage: "How amused you would have been to see the calm and stately way in which I wash down decks the newly organized enterprise in behalf of the freedmen at Port Royal, and was sent to the island of St. Helena, where he remained for the next two years. The Port Royal enterprise, so far as the volunteer element in it was concerned, was the outcome of the sense of responsibility for the negroes on the part of Northern sympathizers with the movement of abolition. Dr. Folsom's father was an ardent abolitionist and this move on his son's part had his warm encouragement; there is some reason, indeed, to think that he suggested it. The story of the movement is well told in a recent book entitled "Letters from Port Royal," edited by Elizabeth Ware Pearson. Early in the war * the Sea Islands region of South Carolina, in the neighborhood of Port Royal and Beaufort, became, all of a sudden, untenable for its Southern occupants in consequence of the capture of two forts by Commodore Dupont, and the great plantations there were at once abandoned by their owners, who fled precipitately, leaving behind them several hundred negroes, incapable of caring for themselves, and a vast amount of cotton nearly ready for exportation. Not only this, but refugee negroes soon came pouring in, so that the number finally reached several thousand. Cotton agents were sent down by the Government to look after the cotton, and Mr. Edward L. Pierce of Milton was placed in charge of the negro problem and of the work of planting next year's crop. Mr. Pierce sought at once the aid of private citizens, at first in Boston, then in New York and Philadelphia. A Freedmen's Aid Society was formed, and very quickly a band of the best people of the North was under way, sufficiently well equipped in money, ability, and ardent devotion to the cause, but destitute of training or experience, to face the problems of "the housekeeper, the teacher, the superintendent of labor, and the landowner," under conditions strange and new. Especially prominent among them was Mr. Edward S. Philbrick of Boston, but the group comprised many other persons of intelligence and devotion, college graduates and women of the best sort. "For the first time in our history educated Northern men had taken charge of the Southern negro, had learned to know his nature, his status, his history, first-hand, in the cabin and the field. And though subsequently other Southern territory was put into the hands of Northern men and women to manage in much the same fashion, it was not in the nature of things that these conditions should ever be exactly reproduced. The question whether or not the freedman would work without the incentive of the lash was settled once for all by the Port Royal Experiment."

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* Some of the shot lodged in the scalp, and many, though perhaps not all of them, were extracted some years later.

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* L. c. Preface.

versally beloved. "The best boy in school and the foremost in scholarship" was the judgment of his teachers and school-fellows. It is a good test of a boy to be tried as the playmate of his younger sisters, and Charles was held by his an older brother without peer.

Both of his parents were natives of Portsmouth, New Hampshire. The major portion of his ancestors on both sides were of the English race, but the progenitors of the American branches came early to New England, the Folsoms * settling in Exeter, New Hampshire, and the Penhallows, whose name his mother bore, in Portsmouth. They were all active, respected people, many of them prominent in public life.

Nathaniel Smith Folsom, Dr. Folsom's father, was graduated one of the foremost in a somewhat notable class at Dartmouth College in 1828. He studied for the ministry at the Andover Theological Seminary, but was soon in the ranks of the Unitarians, and after some years of pastoral work in New England was appointed (in 1849) to a professorship in the Theological School at Meadville. He was a fine classical scholar, high-minded and conscientious. From him, as well as from his mother, Charles inherited the instinct for service to his fellow-men that was so prominent in his nature.

Mrs. Folsom was a woman of rare sweetness and evenness of temper, of fine and strong character, with the fidelity to duty and the steadiness of purpose that had been dominant traits in her family for generations.

In 1861 Mr. Folsom resigned the professorship in Meadville, and in 1862 moved to Concord, Massachusetts, where he engaged in teaching. Here the family remained for many years. I recall with pleasure a short visit to them at that place, a cross-country walk with Dr. Folsom, then a medical student, and the impression made upon me by his gentle, quiet manner, his simplicity and his love of nature. But during most of the Concord period he was away from home, at Port Royal, or studying his profession, and before this he was at Exeter Academy and Harvard College, where he was graduated with his class in 1862, the second year of the war.

Dr. Folsom would have enlisted in the army but for the solicitation of his parents. An elder brother was then living in the South and had been drafted into the Confederate ranks, and they could not bear the thought of their two sons meeting upon opposite sides. This brother was heard from once during the war, through a weather-beaten letter which he managed to get smuggled through the lines, and it was afterwards positively ascertained that he had fallen in 1862. Instead of entering the army, Dr. Folsom offered his services to aid in carrying out

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* The name of the first settler (1638) was written Foulsham.

instead of letting its smoke and brimstone out in the world,* whereas in knowing people well you get more than you give."

"Yes, I am going to Munich to study with Pettenkofer and Voit and Wolfhügel. I have the work to do and I want to do it as well and as much of it as I can.

"But I do not care when I stop, whether next year or next week or next century. So long as the machine runs, I want to keep some useful spindles going.

"I suppose I shall say Good-bye, next month, to many I may not see again, but I can't think of the 'gradual forgetting'; that seems hardly possible, and life is too short and too full of disagreeable things to ever forget one pleasant friend."

In another letter in which he discusses with deep feeling the sacrifice he made in relinquishing the practical work of a physician for the secretaryship of the Board of Health, he writes: "I have always been strongly drawn to a life which will be one to bring me in close relations with individuals needing help." And again, in the same letter, "If people will only place their ideals high enough, they may easily or with a fight make them real. . . . You know that I am conscientious from sense of duty, if at all, and not, like you, by instinct, and that duty does not come naturally to me, but only after toil and a fight."

The sentiments indicated by these citations point to Dr. Folsom's general characteristics and his plan of life; and the remarkable depth of feeling on the occasion of his death, shared in by the many persons whom he had befriended with his wise counsel and his generous purse, or who had worked side by side with him and knew his efficiency, his intelligence, his fidelity, and his power of accomplishment, is a sufficient warrant that the plan was carried out.

The feeling expressed by the word "loyalty," which underlies the best instincts of the moral life, was a fundamental feature of his character.

Charles Folsom was born in Haverhill, Massachusetts, April 3, 1842, the fifth of eight children. His father moved to Meadville, Pennsylvania, when Charles was but seven years old, and it was there that his boyhood was mainly spent. The life was simple and uneventful, but his was a case where in the boy could be read in great measure the character of the man. He gained new traits as he grew older, but lost none that were of value. Sweetness and evenness of temper, affectionateness, a strong instinct of helpfulness, untiring industry, skill in the use of brains and hands, — qualities such as these made him uni-ally I feel a great debt of gratitude for his aid and encouragement when I began the study of ophthalmology, and he was ever an interesting and interested and stimulating friend. He was one of the most valued members of the American Ophthalmological Society, was vice-president from 1873 to 1878, and would have been president had not his extreme modesty led him to decline the office; yet in spite of his retiring disposition he more than once took a stand in opposition to a popular judgment when he believed it to be an unjust one."

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* The order of the clauses in this sentence have been slightly changed, for greater clearness.

DR. CHARLES FOLLEN FOLSOM.

WHEN the news of the death of Dr. Charles Follen Folsom was telegraphed from New York to Boston, on August 20, 1907, a large circle of persons — social acquaintances, patients, and professional colleagues — felt that they had lost the support of a faithful adviser, the companionship of a dear friend.

It is a fortunate asset of the physician's life that he enters into intimate personal relationships with many of the individuals who turn to him for advice, and has an unusual chance to cultivate his powers of sympathy. But there have been few physicians of this neighborhood and generation in whom these fires of personal sympathy have burned so warmly as they did in Dr. Folsom, or who have been able to inspire with reciprocal emotions so many of their patients and their friends. The growth of these attachments was genuine and unforced, for they were based on well-grounded affection and respect.

Dr. Folsom had settled in Boston, with a record of two years' faithful service for the freedmen, but without influential connections and with no instinct for advertisement of himself. He showed, however, marked ability as a practitioner, marked willingness to labor for results worth having, a high standard of thoroughness and obligation, and the highest possible standard of friendship, and it was not long before these qualities made him a real figure among real men and women in our community. Some extracts from a letter to his intimate friend, Rev. William C. Gannett, written about 1881, will recall some of his characteristic traits. He says: ". . . I do not agree with you as to not making friends, even if it does hurt to tear up the roots. Go as deep, say I, into as many human hearts as you can. Never lose a single chance for knowing one person, even, well. In fact, it is the only thing in the world that pays. You do other things because you must, or it is your duty to do so, but that does not pay. You do not get back anything, and the volcano inside of one only rumbles and growls to itself president, of the American Ophthalmological Society, and one of the founders of the New England Ophthalmological Society.

After nearly fifty years of active and successful practice as an oculist at his office in Charles Street, and later in Marlboro Street, he retired in 1904, and died at his home in Jamaica Plain on the twenty-sixth of April, 1908, at the ripe age of seventy-eight.

Of the teachers under whom he studied during his residence in Cambridge as a Harvard undergraduate and as a member of the Lawrence Scientific School the one who made by far the deepest impression on his mind and character was Professor Benjamin Peirce, for whom and for whose favorite science his feeling was ever akin to reverence. Indeed to the end of his life, in spite of his mastery of his profession and his success in its practice, the love of mathematics held first place in his heart; and with him, as with many of the pupils of Benjamin Peirce, it was a romantic love, something that partook almost of the nature of religion. To it he always turned in his leisure moments as a solace and a joy.

His mathematical library, which was as well selected and almost as large as his medical library, was nearly as much used.

He was especially interested in the modern investigations into the foundations of geometry, and his one contribution to the Proceedings of the Academy, "On a Postulate respecting a Certain Form of Deviation from the Straight Line in a Plane," was on that subject.

Naturally his published contributions to science are mainly in the line of his profession: cases reported in the Boston Medical and Surgical Journal, contributions to the Archives of Ophthalmology, and numerous papers in the Transactions of the American Ophthalmological Society.

Of these papers a very considerable proportion are really mathematical investigations into optical problems, and one of the most important of them, "On the Position of the Eyeball during the Listing Rotation," — which showed that apparently contradictory results, reached and published by Helmholtz and Donders, which had caused much confusion and controversy among oculists, were really consistent, — might have been written by Poinst.

Dr. Hay was one of the most kindly and helpful, as well as most modest, of men. A fellow oculist says of him: "I need hardly write to you of Dr. Hay's many sterling qualities or of the esteem and affection with which he was regarded by his colleagues, especially by those who came into close contact with him; and yet I would say a word. He was always ready to give liberally of his time and thought to aid the younger members of the profession who sought his advice. Person-

REPORT OF THE COUNCIL.

Since the last report of the Council the deaths of ten members have been noted: three Resident Fellows, — Charles Eliot Norton, John H. Wright, Frederick I. Knight; four Associate Fellows, — James D. Hague, Wolcott Gibbs, W. K. Brooks, D. C. Gilman; four Foreign Honorary Members, — Sir John Evans, E. de Amicis, Gaston Boissier, Julius Thomsen.

DR. GUSTAVUS HAY.

Dr. Gustavus Hay was born in Boston on the eleventh of March, 1830. After going through the Boston Latin School he entered Harvard College at fifteen, and on completing successfully his four years' course he took the unprecedented step of petitioning the Faculty to be allowed to remain for a second Senior year, and thus received his degree of Bachelor of Arts with the class of 1850. He then entered the recently founded Lawrence Scientific School, where the most advanced educational theories were being put to the test, and took the degree of Bachelor of Science with honors in 1853.

By this time he had formed the "Harvard habit"; he was young, scholarly, and with no special professional bent. Neither theology nor law attracted him. There was only one other department of the University untested, so he entered the Harvard Medical School in 1854, and took the degree of Doctor of Medicine in 1857. Then accident turned his attention toward ophthalmology, and he went abroad to study that subject in Vienna, and on his return he began his long and successful practice as an oculist.

He was married in 1863 to Maria Crehore, who died a dozen years later, and in 1881 to Miriam Parsons, who survives him.

In 1861 he was appointed Surgeon at the Massachusetts Eye and Ear Infirmary, and held that position till 1873, and thereafter that of Consulting Surgeon till 1900.

He was a member of the American Academy and the American Mathematical Society; a member, and from 1873 to 1878 vice-

AMERICAN ACADEMY OF ARTS AND SCIENCES.

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REPORT OF THE COUNCIL. — PRESENTED MAY 12, 1909.

BIOGRAPHICAL NOTICE.

GUSTAVUS HAY	BY WILLIAM E. BYERLY.
CHARLES FOLLEN FOLSOM	BY JAMES JACKSON PUTNAM.

Hermann Georg Jacobi, of Bonn, as Foreign Honorary Member in Class III., Section 2 (Philology and Archæology).

Frederick James Furnivall, of London, as Foreign Honorary Member in Class III., Section 4 (Literature and the Fine Arts).

Dr. Theodore Lyman gave a communication entitled " A Vacation Trip to East Africa."

The following paper was presented by title: —

"The Burmese and Cingalese Tradition of Pāli Texts." By C. R. Lanman.

The following standing committees were chosen: —

Publication Committee.

Charles R. Sanger, of Class I,
Walter B. Cannon, of Class II,
Morris H. Morgan, of Class III.

Library Committee.

Harry M. Goodwin, of Class I, Samuel Henshaw, of Class II,
Henry W. Haynes, of Class III.

Auditing Committee.

A. Lawrence Lowell, Frederic J. Stimson.

House Committee.

William R. Ware, A. Lawrence Rotch,
Louis Derr.

On motion of H. C. Ernst the following Standing Vote was adopted: —

10. No report of any paper presented at a meeting of the Academy shall be published by any member without the consent of the author, and no report shall in any case be published by any member in a newspaper as an account of the proceedings of the Academy.

The following gentlemen were elected members of the Academy: —

Arthur Woolsey Ewell, of Worcester, as Resident Fellow in Class I., Section 2 (Physics).

Francis Gano Benedict, of Boston, as Resident Fellow in Class II., Section 3 (Zoölogy and Physiology).

William Wallace Fenn, of Cambridge, as Resident Fellow in Class III., Section 4 (Literature and the Fine Arts).

Gardiner Martin Lane, of Boston, as Resident Fellow in Class III., Section 4 (Literature and the Fine Arts).

James Hardy Ropes, of Cambridge, as Resident Fellow in Class III., Section 4 (Literature and the Fine Arts).

Vesto M. Slipher, of Flagstaff, Arizona, as Associate Fellow in Class I., Section 1 (Mathematics and Astronomy).

On the recommendation of the Rumford Committee, it was

Voted, To award the Rumford Premium to Professor Robert W. Wood for his discoveries in light, and particularly for his researches on the optical properties of sodium and other metallic vapors.

The annual election resulted in the choice of the following officers and committees: —

John Trowbridge, *President.*
Elihu Thomson, *Vice-President for Class I.*
Henry P. Walcott, *Vice-President for Class II.*
John C. Gray, *Vice-President for Class III.*
Edwin H. Hall, *Corresponding Secretary.*
William Watson, *Recording Secretary.*
Charles P. Bowditch, *Treasurer.*
A. Lawrence Rotch, *Librarian.*

Councillors for Three Years.

William R. Livermore, of Class I.
Theobald Smith, of Class II.
Charles R. Lanman, of Class III.

Finance Committee.

John Trowbridge,
Eliot C. Clarke,
Francis Bartlett.

Rumford Committee.

Charles R. Cross,
Edward C. Pickering,
Erasmus D. Leavitt,
Arthur G. Webster,
Elihu Thomson,
Theodore W. Richards,
Louis Bell.

C. M. Warren Committee.

Leonard P. Kinnicutt,
Henry P. Talbot,
Charles R. Sanger,
Theodore W. Richards,
Arthur A. Noyes,
George D. Moore,
James F. Norris.

The above estimates, less 5 per cent to be added to the capital, leave an income available for appropriation as follows: —

General Fund	$4263.29
Publication Fund	2791.80
Rumford Fund	2708.22
Warren Fund	263.78

The following appropriations are recommended: —

GENERAL FUND.

House expenses	$1450	
Library expenses	1400	
Books, periodicals, and binding	1050	
Expenses of meetings	50	
Treasurer's office	150	$4100

PUBLICATION FUND.

Publication	$2500

RUMFORD FUND.

Research	$1000	
Periodicals and binding	150	
Books and binding	50	
Publication	700	
To be used at discretion of Committee	808	$2708

WARREN FUND.

Research	$ 250

In accordance with the recommendation in the foregoing report it was

Voted, To appropriate for the purposes named the following sums: —

From the income of the General Fund, $4100.
From the income of the Publication Fund, $2500.
From the income of the Rumford Fund, $2708.
From the income of the C. M. Warren Fund, $250.

On the motion of the Treasurer, it was

Voted, That the assessment for the ensuing year be ten dollars ($10).

On May 1, 1908, there was an unexpended balance of $153.45 to the credit of the Publication Committee. The Academy appropriated $2400 for publications, and the income from sales, including $318.76 received from the author of the Memoir, has amounted to $713.91. The total amount available was therefore $3267.36. Bills have been approved by the chairman of the Committee to the amount of $3156.40, leaving an unexpended balance of $110.96.

Bills amounting to $279.12 incurred in publishing papers approved by the Rumford Committee have been forwarded to the chairman of that Committee for approval.

Edward L. Mark, *Chairman.*

May 12, 1909.

Report of the House Committee.

During the year 1908–09 the Academy's House has been occupied just as heretofore.

At the beginning of the year we had to our credit, as a balance in hand from the previous year, thirty-eight cents (.38). For the expenses of the year just elapsed, twelve hundred dollars ($1200) was appropriated in May 1908, and three hundred dollars ($300) in March 1909, making fifteen hundred dollars and thirty-eight cents ($1500.38).

During the year bills for current expenses have been approved to the amount of thirteen hundred and ninety dollars and ninety-three cents ($1390.93), leaving in the Treasurer's hands a balance to our credit of one hundred and nineteen dollars and forty-five cents ($119.45).

William R. Ware, *Chairman.*

May 12, 1909.

Financial Report of the Council.

The income for the year 1909–10, as estimated by the Treasurer, is as follows: —

General Fund	Investments	$1487.67	
	Assessments	1800.00	
	Rent of offices	1200.00	$4487.67
Publication Fund	Appleton Fund	$ 639.63	
	Centennial Fund	2299.11	$2938.74
Rumford Fund	Investments		$2850.76
Warren Fund	Investments		$ 277.66

The Committee has authorized the purchase of various missing volumes and numbers needed to complete the sets of certain periodicals belonging to the library of the Academy.

At two successive meetings held on February 10 and March 10, 1909, the Committee unanimously voted to recommend to the Academy that the Rumford Premium be awarded to Professor Robert W. Wood, of Johns Hopkins University, for his Discoveries in Light, and particularly for his Researches on the Optical Properties of Sodium and other Metallic Vapors.

CHARLES R. CROSS, *Chairman.*

May 12, 1909.

REPORT OF THE C. M. WARREN COMMITTEE.

The C. M. Warren Committee beg leave to report that grants have been made during the past year to the following persons, in aid of the researches specified: —

Professor A. W. Foote, Yale University, for his research on the nature of precipitated colloids	$300
R. C. Tolman, Research Laboratory, Massachusetts Institute of Technology, to aid in the construction of a centrifuge for the measurement of the electro-motive forces produced by the action of centrifugal forces	150

Reports have been received from Dr. Frederic Bonnet, Jr., from Professor Walter L. Jennings, and Professor James F. Norris, in regard to researches for which money has been contributed from the Warren Fund. None of these researches are yet ready for publication, but it is hoped all will be completed during the coming year.

LEONARD P. KINNICUTT, *Chairman.*

May 12, 1909.

REPORT OF THE PUBLICATION COMMITTEE.

Between May 1, 1908, and May 1, 1909, there were published six numbers of Volume XLIII (Nos.17–22), and seventeen numbers of Volume XLIV of the Proceedings, likewise two biographical notices, making in all 616 + v pages and nine plates. Two numbers of Volume XLIII (Nos. 18 and 21), and one number of Volume XLIV (No. 12) were paid for from the income of the Rumford Fund. Seven numbers of the Proceedings, Volume XLIV (Nos. 18–24) are in press.

One Memoir (Volume XIII, No. 6, pp. 217–469, plates xxxviii–lxxi) has been published as the final number of Volume XIII.

and $164.68 Rumford, Fund; making a total of $1057.47 for the General, and $249.23 for the Rumford, Funds, as the cost of subscriptions and binding. Of the appropriation of $50 from the Rumford Fund for books, only one book has been purchased, at a cost of $7.50, although more have been ordered, and will probably be received soon.

A. Lawrence Rotch, *Librarian.*

May 12, 1909.

Report of the Rumford Committee.

During the year 1908–09 the Committee has made grants in aid of researches in light and heat as follows: —

June 10, 1908. Professor Norton A. Kent, of Boston University, for the purchase of a set of echelon plates or other similar apparatus for his research on conditions influencing electric spark lines	$400
Professor Joel Stebbins, of the University of Illinois, an addition to a former appropriation for his research on the use of selenium in stellar photometry	100
Jan. 13, 1909. Professor W. W. Campbell, of the Lick Observatory, for the purchase of a Hartmann photometer to be used in the measurement of polarigraphic images of the solar corona	250
Feb. 10, 1909. Professor R. W. Wood, of the Johns Hopkins University, for his research on the optical properties of mercury vapor	150
May 12, 1909. Professor M. A. Rosanoff, of Clark University, for his research on the fractional distillation of binary mixtures.	300
Professor C. E. Mendenhall, of Wisconsin University, for his research on the free expansion of gases	300

Reports regarding the progress of their respective investigations have been received from Messrs. P. W. Bridgman, E. B. Frost, L. J. Henderson, L. R. Ingersoll, N. A. Kent, F. E. Kester, A. B. Lamb, H. W. Morse, E. F. Nichols, A. A. Noyes, J. A. Parkhurst, T. W. Richards, F. A. Saunders, J. Stebbins, J. Trowbridge, and R. W. Wood.

Since the last annual meeting the following papers have been published in the Proceedings, at the expense of the Rumford Fund: —

"A New Method for the Determination of the Specific Heat of Liquids." T. W. Richards and A. W. Rowe. June, 1908.

"Concerning the Use of Electrical Heating in Fractional Distillation." T. W. Richards and J. H. Mathews. June, 1908.

"Crystal Rectifiers for Electric Currents and Electric Oscillations." G. W. Pierce. March, 1909.

PUBLICATION FUND.

Receipts.

Balance, April 30, 1908	$ 344.30	
Appleton Fund investments	639.63	
Centennial Fund investments	2,303.86	
Sale of publications	713.91	$4,001.70

Expenditures.

Publication	$3,156.40	
Vault rent (part)	12.50	
Income transferred to principal	139.81	$3,308.71
Balance, April 30, 1908		692.99
		$4,001.70

The following reports were also presented: —

REPORT OF THE LIBRARIAN.

The work of cataloguing the library has been continued throughout the past year during such time as Miss Wyman has been able to devote to it. The books on the four upper floors of the stack-building, including the cases of folio plates, are completely catalogued. The cataloguing of the books on the first and second floors is now going on.

The work of completing the sets of society publications now in the library, because of lack of assistance, has not progressed beyond making the list of parts wanting in the various sets. The routine work of the business of the society and library takes all of the Assistant Librarian's time, although as Mrs. Holden lives in the building through the winter months, she gives much extra time to the library work.

The number of bound volumes in the library at the time of the last report was 29,089. 822 volumes have been added during the past year, making the number now on the shelves 29,911. The number added includes 130 old books which were in the fourth story of the house, and not before counted.

89 books have been borrowed from the library by 25 persons, including 20 Fellows, and by 5 libraries.

All the books borrowed during the year except eight have been returned.

The expenses charged to the library are as follows: Miscellaneous, $476.25 (which includes $141.00 for cataloguing); Binding, $555.60 General, and $84.55 Rumford, Funds; Subscription, $501.87 General, *Expenditures.*

Expenses of House	$1,390.93	
Expenses of Library	2,533.72	
Expenses of Meetings	149.91	
Treasurer	138.60	
Interest on bonds	68.75	
Charged to reduce premium on bonds	187.50	
Income transferred to principal	224.35	$4,693.76
Balance, April 30, 1909		507.57
		$5,201.33

Rumford Fund.

Receipts.

Balance, April 30, 1908	$ 751.18	
Investments	2,969.76	
Sale of publications	5.00	$3,725.94

Expenditures.

Research	$900.00	
Periodicals and binding	249.23	
Publication	279.12	
Books	7.50	
Income transferred to principal	134.90	$1,570.75
Balance April 30, 1909		2,155.19
		$3,725.94

C. M. Warren Fund.

Receipts.

Balance, April 30, 1908	$977.93	
Investments	352.66	$1,330.59

Expenditures.

Research	$700.00	
Vault rent (part)	4.00	
Charged to reduce premium on bonds	50.00	
Income transferred to principal	31.64	$ 785.64
Balance, April 30, 1909		544.95
		$1,330.59

The following communications were given: —

"The Present Status of Color Photography." By Louis Derr.

"The Algal Hypothesis of the Origin of Coal." By E. C. Jeffrey.

The following paper was presented by title: —

"Regeneration in the Brittle Star." By Sergeus Morgulis. Presented by E. L. Mark.

Nine hundred ninetieth Meeting.

MAY 12, 1909. — ANNUAL MEETING.

The PRESIDENT in the chair.

There were thirty-eight Fellows present.

The Corresponding Secretary read letters from the Società Ligure di Storia Patria, Genova, announcing its fiftieth anniversary, and enclosing a medal struck in commemoration of the event; from the International Committee in honor of Amedeo Avogadro, asking subscriptions for publishing the works of Avogadro and for a monument to be erected at Turin; from the Botanischer Verein der Provinz Brandenburg, announcing its fiftieth anniversary; from the Société de Géographie Commerciale de Bordeaux, announcing the death of its Secretary, M. Julien Manès; from the American Oriental Society, announcing its officers elected April 17, 1909.

The Chair announced the death of Daniel Coit Gilman, Associate Fellow in Class III, Section 2.

The annual report of the Council was read.*

The annual report of the Treasurer was read, of which the following is an abstract: —

GENERAL FUND.

Receipts.

Balance, April 30, 1908	$ 381.00	
Investments	1,660.33	
Assessments	1,870.00	
Admission fees	90.00	
Rent of offices	1,200.00	$5,201.33

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* See page 747.

The following papers were presented by title: —

"The Relations of the Norwegian with the English Church, 1066–1399, and their Importance to Comparative Literature." By Henry G. Leach. Presented by G. L. Kittredge.

"Some European Sandforms." By D. W. Johnson.

Contribution from the Gray Herbarium of Harvard University. New Series. No. XXXVII. 1. "Synopsis and Key to the Mexican and Central American Species of Castilleja." By A. Eastwood. 2. "A Revision of the Genus Rumfordia." By B. L. Robinson. 3. "A Synopsis of the American Species of Litsea." By H. H. Bartlett. 4. "Some Undescribed Species of Mexican Phanerogams." By A. Eastwood. 5. "Notes on Mexican and Central American Alders." By H. H. Bartlett. 6. "Diagnoses and Transfers of Tropical American Phanerogams." By B. L. Robinson. 7. "The Purple-flowered Androcerae of Mexico and the Southern United States." By H. H. Bartlett. 8. "Descriptions of Mexican Phanerogams." By H. H. Bartlett. Presented by B. L. Robinson.

"Crystallographic Notes on Minerals from Chester, Massachusetts." By Charles Palache and H. O. Wood.

Nine hundred eighty-ninth Meeting.

APRIL 14, 1909.

The Academy met at its house.

The PRESIDENT in the chair.

There were twenty-six Fellows and one guest present.

The Corresponding Secretary read letters from Herbert W. Rand and from W. M. Wheeler, accepting Resident Fellowship; from C. H. Toy and W. T. Porter, resigning Resident Fellowship; from the Academy of Natural Sciences of Philadelphia, Mineralogical and Geological Section, announcing a second annual meeting of geologists, to be held at Philadelphia, April 23 and 24, 1909; from the Holland Society of Sciences, announcing the resignation of its Permanent Secretary, J. Bosscha, and the appointment of J. P. Lotsy in his place from the Senckenbergische Naturforschende Gesellschaft, announcing the death of Professor Dr. Fritz Römer, the director of its Museum.

The following communication was given by Professor W. B. Cannon: —

"The Correlation of Gastric and Intestinal Digestive Processes and the Influence of Emotions upon Them."

The following paper was read by title: —

"A Photographic Study of Mayer's Floating Magnets." By Louis Derr.

Nine hundred eighty-eighth Meeting.

MARCH 10, 1909. — STATED MEETING.

The PRESIDENT in the chair.

There were twenty-nine Fellows present.

The Corresponding Secretary read a letter from Professor Maxime Bôcher, resigning Fellowship in the Academy.

The following deaths were announced by the Chair: —

Frederick I. Knight, Resident Fellow in Class II, Section 4; Julius Thomsen, Foreign Honorary Member in Class I, Section 3.

The following gentlemen were elected members of the Academy: —

Gilbert Newton Lewis, of Boston, as Resident Fellow in Class I, Section 3 (Chemistry).

Herbert Wilbur Rand, of Cambridge, as Resident Fellow in Class II, Section 3 (Zoölogy and Physiology).

William Morton Wheeler, of Boston, as Resident Fellow in Class II, Section 3 (Zoölogy and Physiology).

The Chair appointed the following Councillors to serve as Nominating Committee: —

James C. White, of Class II.

William R. Ware, of Class III.

Ira N. Hollis, of Class I.

On motion of the Librarian, it was

Voted, To appropriate from the income of the General Fund the sum of three hundred dollars ($300) for House expenses, and the sum of two hundred dollars ($200) for the binding of books.

The following communications were given: —

"Roman Calorifers." By Morris H. Morgan.

"The Titles of Pāli Texts and the Brief Designations of the Same." By Charles R. Lanman.

Dr. G. H. Parker read a paper entitled, "The Ears of Fishes in Relation to the Noise of Motor-boats, etc."

This was followed by a communication on the "Location of a Supposed Planet beyond Neptune." By Professor Percival Lowell.

The following papers were presented by title: —

"A Revision of the Atomic Weight of Chromium. First Paper: The Analysis of Silver Chromate." By. G. P. Baxter, Edward Mueller, and M. A. Hines.

"A Revision of the Atomic Weight of Chromium. Second Paper: The Analysis of Silver Dichromate." By G. P. Baxter and R. H. Jesse, Jr.

Nine hundred eighthy-seventh Meeting.

FEBRUARY 10, 1909.

The PRESIDENT in the chair.

There were twenty-three Fellows present.

The Corresponding Secretary announced that the following letters had been received: —

From Henry H. Edes, Henry Fay, Reginald A. Daly, and Harris H. Wilder, accepting Resident Fellowship; from the New York Academy of Sciences, inviting the Academy to attend its Darwin celebration on February 12; from the University of Geneva, inviting the Academy to send delegates to the celebration of its three hundred and fiftieth anniversary, July 7–10, 1909; from the American Antiquarian Society, announcing the retirement of its Librarian, Mr. E. M. Barton, and the appointment of Mr. C. S. Brigham to the position; from the Royal Society of Sciences, Gottingen, announcing a prize of $25,000 to be awarded to the first person proving the theorem that the equation $x^\lambda + y^\lambda = z^\lambda$ cannot be solved in integers if λ is an uneven prime number; from the Royal Academy of Sciences, Turin, announcing the seventeenth Bressa Prize.

On motion of the Corresponding Secretary, it was

Voted, That the invitation of the University of Geneva be accepted, and the selection of the delegates be made by the President.

Sciences de Finlande, announcing the death of its permanent Secretary, Lorenz L. Lindelöf, and the appointment of Anders Donner to the position; from the Philological Society of Rome, announcing the progress of the Graziadio Ascoli Fund and soliciting subscriptions; from William Z. Ripley, Resident Fellow, resigning Fellowship.

The Chair announced the death of Wolcott Gibbs, Associate Fellow in Class I, Section 3; and of W. K. Brooks, Associate Fellow in Class II, Section 3.

The following gentlemen were elected members of the Academy: —

Henry Fay, of Boston, to be a Resident Fellow in Class I, Section 3 (Chemistry).

Reginald Aldworth Daly, of Cambridge, to be a Resident Fellow in Class II, Section I (Geology, Mineralogy, and Physics of the Globe).

Harris Hawthorne Wilder, of Northampton, to be a Resident Fellow in Class II, Section 3 (Zoölogy and Physiology).

Henry Herbert Edes, of Cambridge, to be a Resident Fellow in Class III, Section 4 (Literature and the Fine Arts).

Upon the recommendation of the Committee on Amending the Statutes it was

Voted, To amend Chapter V, Section 7, to read as follows: —

"The House Committee to consist of three Fellows. This Committee shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to other Committees. This Committee shall report to the Council in March in each year on the appropriations needed for their expenses for the coming year. All bills incurred by this Committee within the limits of the appropriations made by the Academy shall be approved by the Chairman of the House Committee."

To amend Chapter X, Section 2, by adding to it the following: —

"In the case of officers of the Army or Navy, who are out of the state on duty, payment of the annual assessment may be waived during such absence if continued during the whole official year and if notification of such absence be sent to the Treasurer."

On adjournment to the Council Room, Professor J. E. Wolff gave an illustrated talk on "A Geological Tour in the Mountains of Montana and British Columbia."

Professor Percival Lowell spoke on his recent discovery, made through photographs, of the watery vapor surrounding Mars.

The following papers were presented by title: —

"A Revision of the Atomic Weight of Arsenic. Preliminary Paper: The Analysis of Silver Arsenate." By Gregory Paul Baxter and Fletcher Barker Coffin.

"Properties of Aluminium Anodes." By H. W. Morse. Presented by John Trowbridge.

Contributions from the Harvard Mineralogical Museum XIII: "Notes on the Crystallography of Leadhillite." By Charles Palache.

"Crystal Rectifiers for Electric Currents and Electric Oscillations. Part II. Carborundum, Anatase, Brookite, Molybdenite." By George W. Pierce.

"On the Joule-Thomson Effect in Air." By S. B. Serviss. Presented by John Trowbridge.

"The Measurement of High Hydrostatic Pressure: I. A Simple Primary Gauge. II. A Secondary Mercury Resistance Gauge." By P. W. Bridgman. Presented by W. C. Sabine.

"An Experimental Determination of Certain Compressibilities." By P. W. Bridgman. Presented by W. C. Sabine.

Nine hundred eighty-sixth Meeting.

JANUARY 13, 1909. — STATED MEETING.

The PRESIDENT in the chair.

There were twenty-four Fellows present.

The Corresponding Secretary announced that letters had been received from Professor William Trelease stating that he attended the inauguration of President Hill at the University of Missouri, as the representative of the Academy; from the Museo National of Mexico, offering the felicitations of the New Year; from Charles I. Kiralfy, announcing the Imperial International Exhibition in London in 1909; from the Société des

Treasurer, and referred to a committee consisting of W. H. Pickering, J. E. Wolff, and the Recording Secretary.

The President announced the appointment of Professor W. G. Farlow as representative of the Academy at the Darwin celebration of the University of Cambridge.

The following communications were given: —

Biographical notice of Dr. Charles Follen Folsom. By Dr. James J. Putnam.

"Location of a Hypothetical Planet beyond Neptune." By Professor W. H. Pickering.

The following papers were read by title: —

"The Preface of Vitruvius." By M. H. Morgan.

"The Theory of Ballistic Galvanometers of Long Period." By B. O. Peirce.

"The Magnetic Behavior of Hardened Cast Iron and Tool Steel at very High Excitations." By B. O. Peirce.

"The Use of the Magnetic Yoke in Measurements of the Permeabilities of Iron and Steel Rods in Intense Fields." By B. O. Peirce.

"A Study of Residual Charge in Dielectries." By C. L. B. Shuddemagen. Presented by E. H. Hall.

Nine hundred eighty-fifth Meeting.

DECEMBER 9, 1908.

The PRESIDENT in the chair.

There were thirty-one Fellows and one guest present.

The Corresponding Secretary read a notice of a prize to be given in 1910, by the Académie des Sciences et Lettres of Montpellier, to the author of the best work on the subject of General Pathology and Therapeutics. He also read the resignation of C. H. Toy, to take effect in May, 1909.

The following deaths were announced by the Chair: —

John H. Wright, Resident Fellow in Class III, Section 2; Gaston Boissier, Foreign Honorary Member in Class III, Section 4.

Professor George F. Moore read a paper entitled: —

"The Jewish Colony at Elephantine: Recently discovered Papyri."

Museo de la Plata, announcing the death of Enrique A. S. Delachaux; from the Belgian government, enclosing a prospectus of the First International Congress of Administrative Sciences at Brussels in 1910.

The Chair announced the following deaths: —

James D. Hague, Associate Fellow in Class I, Section 4; Henry C. Sorby, Class II, Section 1, and Sir John Evans, Class III, Section 2, Foreign Honorary Members.

It was *Voted*, To authorize the President to appoint one or more delegates to represent the Academy at the celebration of the University of Cambridge in commemoration of the centenary of the birth of Charles Darwin.

It was *Voted*, That the Corresponding Secretary explain to the Secretary of the American Association for the Advancement of Science the inability of the Academy to accept the invitation of the Association to participate in its meeting at Baltimore.

On the motion of the Recording Secretary, it was *Voted*, To meet on adjournment, on the 11th of November.

The President delivered his inaugural address, "Physical Science of To-day."

Professor Story gave an informal talk on Mathematical Puzzles.

The following paper was presented by title: —

"Binary Mixtures, a Contribution to Physical Chemistry," by William E. Story.

Nine hundred eighty-fourth Meeting.

NOVEMBER 11, 1908. — ADJOURNED STATED MEETING.

The PRESIDENT in the chair.

There were twenty Fellows present.

The Corresponding Secretary presented an invitation from the University of Missouri requesting delegates to attend the Inauguration of Albert Ross Hill as President of the University.

The Chair announced the death of Charles Eliot Norton, Resident Fellow in Class III, Section 4.

Certain amendments to the Statutes were proposed by the

RECORDS OF MEETINGS.

Nine hundred eighty-third Meeting.

OCTOBER 14, 1908. — STATED MEETING.

The PRESIDENT in the chair.

There were thirty-four Fellows and one guest present.

The Corresponding Secretary announced that letters had been received from Lady Evans, notifying the Academy of the death of Sir John Evans; from C. H. Warren, accepting fellowship; from Emil Fischer, accepting Foreign Honorary Membership; from William W. Goodwin, thanking the Academy for the resolution expressing its appreciation of his services as President; from Charles Gross, resigning Fellowship; from the Physikalisch-medizinische Sozietät, of Erlangen, inviting the Academy to attend its centennial celebration, June 27, 1908; from the American Association for the Advancement of Science, inviting the Academy to meet with them at Baltimore, Dec. 28, 1908, to Jan. 2, 1909; from the University of Cambridge, inviting the Academy to participate in the commemoration of the centenary of the birth of Charles Darwin; from the Comité Technique contre l'Incendie, enclosing the program of the Second International Congress; from the Nobel Prize Committee for Physics, and for Chemistry, inviting competition; from the Reale Università di Catania, inviting the Academy to attend the inauguration of a monument to the naturalist, Giuseppe Gioeni, July 19, 1908; from Dr. H. Morize, of the Rio de Janeiro Observatory, notifying the Academy of his appointment as Director; from the Königliche böhmische Gesellschaft der Wissenschaften, announcing the death of Johann Kvíčala, and Karl Pelz; from the Service Géologique du Portugal, announcing the death of its president, J. F. Nery Delgado; from the

Proceedings of the American Academy of Arts and Sciences.

Vol. XLIV. No. 26. — September, 1909.

RECORDS OF MEETINGS, 1908–1909.

REPORT OF THE COUNCIL: BIOGRAPHICAL NOTICES.

Gustavus Hay. By William E. Byerly.

Charles Follen Folsom. By James Jackson Putnam.

OFFICERS AND COMMITTEES FOR 1909–1910.

LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

STATUTES AND STANDING VOTES.

RUMFORD PREMIUM.

INDEX.

(Title Page and Table of Contents).

laws, and to reconcile them with the Principle of Relativity, which rests on the experiments of Michelson and Morley, and of Bucherer, that we have adopted the principles of non-Newtonian Mechanics.

These principles, bizarre as they may appear, offer the only method of preserving the science of mechanics substantially in its present form. If later, when more complex systems are considered, and especially when we deal with acceleration, these views prove untenable, it will then be necessary to revolutionize the whole of mechanics.

Research Laboratory of Physical Chemistry,
Mass. Inst. of Technology,
Boston, May 11, 1909.

Combining this equation with III gives

$$\frac{E}{E_0} = \frac{1}{\sqrt{1-\beta^2}}.$$

We thus see that energy changes with the velocity in the same way that mass does, and that the so-called kinetic energy is a "second order effect" of the same character as the change of length and the change of mass. The only reason that this effect is easily measured and has become a familiar conception in mechanics, while the others are obtainable only by the most precise measurements, is that we are in the habit of measuring quantities of energy which are extremely minute in comparison with the total energy of the systems investigated.

CONCLUSION.

We have shown how observers stationed on systems in motion relative to one another have been able to preserve their fundamental principles of mechanics only by adopting certain novel conclusions. These conclusions are self-consistent; in the one case where they have been tested they are in accord with experiment; and they enable us to save all the fundamental physical concepts which have been found useful in the past. We have, however, considered primarily only systems which are initially in uniform relative motion. Whether our conclusions can be retained when we consider processes in which the relative motion is being established, in other words, processes in which acceleration takes place, it is not our present purpose to determine.

The ideas here presented appear somewhat artificial in character, and we cannot but suspect that this is due to the arbitrary way in which we have assumed this point or that point to be at rest, while at the same time we have asserted that a condition of rest in the absolute sense possesses no significance.

If our ideas possess a certain degree of artificiality, this is also true of others which have long since been adopted into mechanics. The apparent change in rate of a moving clock, and the apparent change in length and mass of a moving body, are completely analogous to that apparent change in energy of a body in motion, which we have long been accustomed to call its kinetic energy. We may with equal reason speak of the kinetic mass found by Kaufmann and Bucherer, or the kinetic length assumed by Lorentz. We say that the heat evolved when a moving body is brought to rest comes from the kinetic energy which it possessed. We thus preserve the law of conservation of energy. It is in order to maintain such fundamental conservation

From equations I and II we may derive one of the most interesting consequences of the principle of relativity. If E is the total energy (including internal energy) of a body in motion, and E_0 is its energy at rest, the kinetic energy E'' is equal to $E - E_0$, and equation II may be written,

$$E - E_0 = m_0 c^2 \left(\frac{1}{\sqrt{1 - \beta^2}} - 1 \right) \qquad \text{III}$$

Moreover, we may write equation I in the form,

$$m - m_0 = m_0 \left(\frac{1}{\sqrt{1 - \beta^2}} - 1 \right) \qquad \text{IV}$$

and dividing III by IV

$$\frac{E - E_0}{m - m_0} = c^2 \qquad \text{V}$$

In other words, when a body is in motion its energy and mass are both increased, and the increase in energy is equal to the increase in mass multiplied by the square of the velocity of light. From the fundamental conservation laws we know that when a body is set in motion and thus gains mass and energy, these must come from the environment. So also when a moving body is brought to rest, it must give up mass as well as energy to the environment. The mass thus acquired by the environment is independent of the particular form which the energy may assume, and we are thus forced to the important conclusion that *when a system acquires energy in any form it acquires mass in proportion*, the ratio of the energy to the mass being equal to the square of the velocity of light. We might go further and assume that if a system should lose all its energy it would lose all its mass. If we admit this plausible although unproved assumption, then we may regard the mass of every body as a measure of its total energy according to the equation,

$$m = \frac{E}{c^2}. \qquad \text{VI}$$

For a body at rest,

$$m_0 = \frac{E_0}{c^2}.$$

Now let us assume that the whole system is in motion with velocity v in the direction bc. Obviously, merely by making such an assumption we cannot cause the lever to turn, nevertheless we must now regard the length bc as shortened in the ratio $\frac{\sqrt{1-\beta^2}}{1}$ while ab has the same length as at rest. We must therefore conclude that to maintain equilibrium the force at a must be less than the force at c in the same ratio. We thus see that in a moving system unit force in the longitudinal direction is smaller than unit transverse force in the ratio $\frac{\sqrt{1-\beta^2}}{1}$, and therefore, by the preceding paragraph, smaller than unit force at rest in the ratio $\frac{1-\beta^2}{1}$. It is interesting to point out, as Bumstead[13] has already done, that the repulsion between two like electrons, as calculated from the electromagnetic theory, is diminished in the ratio $\frac{\sqrt{1-\beta^2}}{1}$ if they are moving perpendicular to the line joining them, and in the ratio $\frac{1-\beta^2}{1}$ if moving parallel to the line joining them.

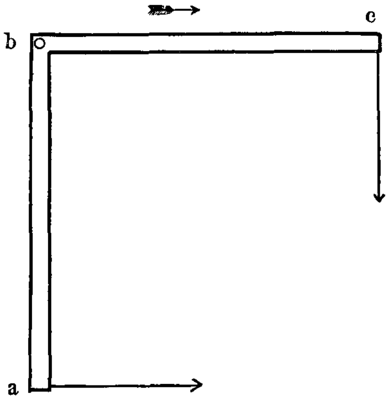

FIGURE 3.

From the standpoint of the principle of relativity, one of the most interesting quantities in mechanics is the so-called kinetic energy, which is the increase in energy attributed to a body when it is set in motion with respect to an arbitrarily chosen point of rest. Knowing the change of the mass with velocity as given by equation I, the general equation for kinetic energy,[14] E', may readily be shown to be

$$E' = m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) \qquad \text{II}$$

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[13] Bumstead, loc. cit.

[14] Consider a body moving with the velocity v subjected to a force f in the line of its motion. Its momentum M and its kinetic energy E' will be changed by the amounts $dM = fdt$, $dE' = fdl = fvdt$. Hence $dE' = vdM$, or substituting mv for M, $dE' = mvdv + v^2dm$. Eliminating m between this equation and equation I, and integrating, gives at once the above equation II.

agreement with equation I. Recently, however, Bucherer,[12] by a method of exceptional ingenuity, has made further determinations of the mass of electrons moving with varying velocities, and his results are in remarkable accord with this equation obtained from the principle of relativity.

This very satisfactory corroboration of the fundamental equation of non-Newtonian mechanics must in future be regarded as a very important part of the experimental material which justifies the principle of relativity. By a slight extrapolation we may find with accuracy from the results of Bucherer that limiting velocity at which the mass becomes infinite, in other words, a numerical value of c which in no way depends upon the properties of light. Indeed, merely from the first postulate of relativity and these experiments of Bucherer we may deduce the second postulate and all the further conclusions obtained in this paper. This fact can hardly be emphasized too strongly.

Leaving now the subject of mass, let us consider whether the unit of force depends upon our choice of a point of rest. An observer in a given system allows such a force to act upon unit mass as to give it an acceleration of one $\frac{\text{cm}}{\text{sec}^2}$, and calls this force the dyne. If now we assume that the system is in motion, with a velocity v, in a direction perpendicular to the line of application of the force, we conclude that the acceleration is really less than unity, since in a moving system the second is longer in the ratio $\frac{1}{\sqrt{1-\beta^2}}$ and the centimeter in this transverse direction is the same as at rest. On the other hand, the mass is increased owing to the motion of the system by the factor $\frac{1}{\sqrt{1-\beta^2}}$. Since the time enters to the second power, the product of mass and acceleration is smaller by the ratio $\frac{\sqrt{1-\beta^2}}{1}$ than it would be if the system were at rest. And we conclude, therefore, that the unit of force, or the dyne, in a direction transverse to the line of motion is smaller in a moving system than in one at rest by this same ratio.

In order now to obtain a value for the force in a longitudinal direction in the moving system, let us consider (Figure 3) a rigid lever *abc* whose arms are equal and perpendicular, and equal forces applied at *a* and *c*, in directions parallel to *bc* and *ba*. The system is thus in equilibrium.

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[12] Bucherer, loc. cit.

which we are considering is the same in both systems, the observer A, always using the units of his own system, concludes that the change in velocity of the ball b is smaller in the ratio $\frac{\sqrt{1-\beta^2}}{1}$ than the change in velocity of the ball a. The change in velocity of each ball multiplied by its mass gives its change in momentum. Now, from the law of conservation of momentum, A assumes that each ball experiences the same change in momentum, and therefore since he has already decided that the ball b has experienced a smaller change of velocity in the ratio $\frac{\sqrt{1-\beta^2}}{1}$, he must conclude that the mass of the ball in system b is greater than that of his own in the ratio $\frac{1}{\sqrt{1-\beta^2}}$.

In general, therefore, we must assume that the mass of a body increases with its velocity. We must bear in mind, however, as in all other cases, that the motion is determined with respect to some point *arbitrarily* chosen as a point of rest.

If m is the mass of a body in motion, and m_0 its mass at rest, we have[10]

$$\frac{m}{m_0} = \frac{1}{\sqrt{1-\beta^2}}.$$

The only opportunity of testing experimentally the change of a body's mass with its velocity has been afforded by the experiments on the mass of a moving electron, or β particle. The actual measurements were indeed not of the mass of the electron, but of the ratio of charge to mass $\left(\frac{e}{m}\right)$. It has, however, been universally considered that the charge e is constant. In other words, that the force acting upon the electron in a uniform electrostatic field is independent of its velocity relative to the field. Hence the observed change in $\frac{e}{m}$ is attributed solely to the change in mass. It might be well to subject this view to a more careful analysis than has hitherto been done. At present, however, we will adopt it without further scrutiny.

The original experiments of Kaufmann[11] showed only a qualitative distortion of the body in consequence of a real motion through a stationary ether, and his theory has aroused considerable discussion as to the nature of the forces which would be necessary to produce such a deformation. The point of view first advanced by Einstein, which we have here adopted, is radically different. Absolute motion has no significance. Imagine an electron and a number of observers moving in different directions with respect to it. To each observer, naïvely considering himself to be at rest, the electron will appear shortened in a different direction and by a different amount; but the physical condition of the electron obviously does not depend upon the state of mind of the observers.

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[10] This equation and others developed in this section are identical with those obtained through an entirely different course of reasoning by Lewis (Phil. Mag., **16**, 705 (1908)). The equations were there obtained for systems in motion with respect to a point at absolute rest. We shall show here, however, that they are true, whatever arbitrary point is selected as a point of rest.

[11] See Lewis, loc. cit.

Although these changes in the units of space and time appear in a certain sense psychological, we adopt them rather than abandon completely the fundamental conceptions of space, time, and velocity, upon which the science of physics now rests. At present there appears no other alternative.

NON-NEWTONIAN MECHANICS.

Having obtained these relations for the units of space and time, we may turn to some of the other important quantities used in mechanics.

Let us again consider two systems, *a* and *b*, in relative motion with the velocity *v*. An experimenter A on the first system constructs a ball of some rigid elastic material, with a volume of one cubic centimeter, and sets it in motion, with a velocity of one centimeter per second, towards the system *b* (in a direction perpendicular to the line of relative motion of the two systems). On the other system, an experimenter B constructs of the same material a similar ball with a volume of one cubic centimeter in his units, and imparts to it, also in his units, a velocity of one centimeter per second towards *a*. The experiment is so planned that the balls will collide and rebound over their original paths. Since the two systems are entirely symmetrical, it is evident by the principle of relativity, that the (algebraic) change in velocity of the first ball, as measured by A, is the same as the change in velocity of the other ball, as measured by B. This being the case, the observer A, considering himself at rest, concludes that the real change in velocity of the ball *b* is different from that of his own, for he remembers that while the unit of length is the same in this transverse direction in both systems, the unit of time is longer in the moving system.

Velocity is measured in centimeters per second, and since the second is longer in the moving system, while the centimeter in the direction

We thus see that a meter-stick, which, when held perpendicular to its line of motion, has the same length as a meter-stick at rest, will be shortened when turned parallel to the line of motion in the ratio $\frac{\sqrt{1-\beta^2}}{1}$, and indeed any moving body must be shortened in the direction of its motion in the same ratio.[9]

Let us emphasize once more, that these changes in the units of time and length, as well as the changes in the units of mass, force, and energy which we are about to discuss, possess in a certain sense a purely factitious significance; although, as we shall show, this is equally true of other universally accepted physical conceptions. We are only justified in speaking of a body in motion when we have in mind some definite though arbitrarily chosen point as a point of rest. The distortion of a moving body is not a physical change in the body itself, but is a scientific fiction.

When Lorentz first advanced the idea that an electron, or in fact any moving body, is shortened in the line of its motion, he pictured a real passes over the path mnm. If, however, the entire system is considered to be in absolute motion with a velocity v, the light must pass over a different path $mn'm'$ where nn' is the distance through which the

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[9] Certain of Einstein's other deductions from the principle of relativity will not be needed in the development of this paper, but may be directly obtained by the methods here employed. For example, the principle of relativity leads to certain curious conclusions as to the comparative readings of clocks in a system assumed to be in motion.

Consider two systems in relative motion. An observer on system a places two carefully compared clocks, unit distance apart, in the line of motion, and has the time on each clock read when a given point on the other system passes it. An observer on system b performs a similar experiment. The difference between the readings of the two clocks in one system must be the same as the difference in the other system, for by the principle of relativity the relative velocity v of the systems must appear the same to an observer in either. However, the observer A, considering himself at rest, and familiar with the change in the units of length and time in the moving system which we have already deduced, expects that the velocity determined by B will be greater than that which he himself observes in the ratio $\frac{1}{1-\beta^2}$, since he has concluded that B's unit of time is longer, and his unit of length in this direction is shorter, each by a factor involving $\sqrt{1-\beta^2}$. The only possible way in which A can explain this discrepancy is to assume that the clocks which B claims to have set together are not so in reality. In other words he has to conclude that clocks, which in a moving system appear to be set together, really read differently at any instant (in stationary time), and that a given clock is "slower" than one immediately to the rear of it by an amount proportional to the distance. From what has preceded it can be readily shown that if in a moving system two clocks are situated, one in front of the other by a distance l, in units of this system, the difference in setting will be $\frac{lv}{c^2}$. From this point Einstein's equations concerning the addition of velocities also follow directly.

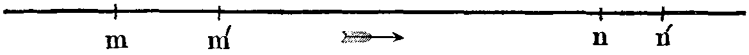

FIGURE 2.

mirror moves before the light reaches it, and mm' is the distance traversed by the source before the light returns to it.

Obviously then,

$$\frac{nn'}{mn'} = \frac{v}{c},$$

and

$$\frac{mm'}{mn'm'} = \frac{v}{c},$$

Also from the figure, $$mn' = mn + nn',$$

$$mn'm' = mnm + 2\,nn' - mm'.$$

Combining, we have

$$\frac{mn'm'}{mnm} = \frac{1}{1 - \frac{v^2}{c^2}} = \frac{1}{1 - \beta^2}.$$

Hence if we call the system in motion, instead of at rest, the calculated path of the light is greater in the ratio $\frac{1}{1 - \beta^2}$.

Now the velocity of light must seem the same to the observer, whether he is at rest or in motion. His measurements of velocity depend upon his units of length and time. We have already seen that a second on a moving clock is lengthened in the ratio $\frac{1}{\sqrt{1 - \beta^2}}$, and therefore if the path of the beam of light were also greater in this same ratio, we should expect that the moving observer would find no discrepancy in his determination of the velocity of light. From the point of view of a person considered at rest, however, we have just seen that the path is increased by the larger ratio $\frac{1}{1 - \beta^2}$. In order to account for this larger difference, we must assume that the unit of length in the moving system has been *shortened* in the ratio $\frac{\sqrt{1 - \beta^2}}{1}$.

and *mnm'* respectively, and that B's clock runs more slowly than A's. In general, whatever point may be arbitrarily chosen as a point of rest, it will be concluded that any clock in motion relative to this point runs too slowly.

Consider Figure 1 again, assuming system a at rest. We have shown that it is necessary to assume that B's clock runs more slowly than A's in the ratio of the lengths of the path *opo* to the path *mnm'*; in other words, the second of B's clock is longer than the second of A's in the ratio *mnm'* to *opo*. This ratio between the two paths will evidently depend on the relative velocity of the two systems v, and on the velocity of light c.

Obviously from the figure,

$$(op)^2 = (ln)^2 = (mn)^2 - (ml)^2.$$

Dividing by $(mn)^2$,

$$\frac{(op)^2}{(mn)^2} = 1 - \frac{(ml)^2}{(mn)^2}.$$

But the distance ml is to the distance mn as v is to c.

Hence

$$\frac{mn}{op} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Denoting the important ratio $\frac{v}{c}$ by the letter β, we see that in general a second measured by a moving clock bears to a second measured by a stationary clock the ratio $\frac{1}{\sqrt{1-\beta^2}}$.

Whatever assumption the observers A and B may make as to their motion, it is obvious that their measurements of length, at least in a direction perpendicular to their line of relative motion, will lead to no disagreement. For evidently, if each observer with a measuring rod determines the distance from his system to the other, the two determinations must agree. Otherwise the condition of symmetry required by the principle of relativity would not be fulfilled.

But let us now consider distances *parallel* to the line of relative motion.

A system (Figure 2) has a source of light at m and a reflecting mirror at n. If we consider the whole system to be at absolute rest, it is evident that a light signal sent from m to the mirror, and reflected back, which is reflected back to the starting point. He measures the time taken by the light in transit.

A, assuming that his system is at rest (and the other in motion), considers that the light passes over the path *opo*, but he believes that if a similar experiment is conducted by an observer, B, in the moving system, the light must pass over the longer path *mnm′* in order to return to the starting point; for the point *m* moves to the position *m′* while the light is passing. He therefore predicts that the time required for the return of the reflected beam will be longer than in his own experiment. A, however, having established communication with B, learns that the time measured is the same as in his own experiment.[8]

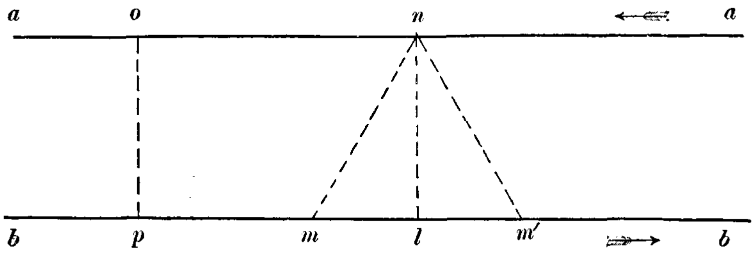

Figure 1.

The only explanation which A can offer for this surprising state of affairs is that the clock used by B for his measurement does not keep time with his own, but runs at a rate which is to the rate of his own clock as the lengths of the paths *opo* to *mnm′*.

B, however, is equally justified in considering his system at rest and A's in motion, and by identical reasoning has come to the conclusion that A's clock is not keeping time. Thus to each observer it seems that the other's clock is running too slowly.

This divergence of opinion evidently depends not so much on the fact that the two systems are in relative motion, but on the fact that each observer arbitrarily assumes that his own system is at rest. If, however, they both decide to call A's system at rest, then both will agree that in the two experiments the light passes over the paths *opo* quences to which it leads, however extraordinary they may be, provided that they are not inconsistent with one another nor with known experimental facts.

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[8] This is evidently required by the principle of relativity, for contrary to A's supposition the two systems are in fact entirely symmetrical. Any difference in the observations of A and B would be due to a difference in the absolute velocity of the two systems, and would thus offer a means of determining absolute velocity.

The consequences which one of us has obtained from a simple assumption as to the *mass of a beam of light*, and the fundamental *conservation laws* of mass, energy, and momentum, Einstein has derived from the *principle of relativity* and the *electromagnetic theory*. We propose in this paper to show that these consequences may also be obtained merely from the *conservation laws* and *the principle of relativity*, without any reference to electromagnetics.

In dealing with such fundamental questions as we meet here it seems especially desirable to avoid as far as possible all technicalities. We have endeavored to find for each of the following theorems the simplest and most obvious proof, and have used no mathematics beyond the elements of algebra and geometry.

THE UNITS OF SPACE AND TIME.

The following development will be based solely upon the conservation laws and the two postulates of the principle of relativity.

The first of these postulates is, that there can be no method of detecting absolute translatory motion through space, or through any kind of ether which may be assumed to pervade space. The only motion which has physical significance is the motion of one system relative to another. Hence two similar bodies having relative motion in parallel paths form a perfectly symmetrical arrangement. If we are justified in considering the first at rest and the second in motion, we are equally justified in considering the second at rest and the first in motion.

The second postulate is that the velocity of light as measured by any observer is independent of relative motion between the observer and the source of light.[7] This idea, that the velocity of light will seem the same to two different observers, even though one may be moving towards and the other away from the source of light, constitutes the really remarkable feature of the principle of relativity, and forces us to the strange conclusions which we are about to deduce.

Let us consider two systems moving past one another with a constant relative velocity, provided with plane mirrors *aa* and *bb* parallel to one another and to the line of motion (Figure 1). An observer, A, on the first system sends a beam of light across to the opposite mirror,

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[7] We will imagine that the observer measures the velocity of light by means of two clocks placed at the ends of a meter stick which is situated lengthwise in the path of the light.

This possibility being excluded, the only satisfactory explanation of the Michelson-Morley experiment which has been offered is due to Lorentz,[3] who assumed that all bodies in motion are shortened in the line of their motion by an amount which is a simple function of the velocity. This shortening would produce a compensation just sufficient to offset the predicted positive effect in the Michelson-Morley experiment, and would also account for the result obtained by Trouton and Noble. It would not, however, prevent the determination of absolute motion by other analogous experiments which have not yet been tried.

Einstein[4] has gone one step farther. Because of the experiments that we have cited, and because of the failure of every other attempt that has ever been made to determine absolute velocity through space, he concludes that further similar attempts will also fail. In fact he states as a law of nature that absolute uniform translatory motion can be neither measured nor detected.

The second fundamental generalization made by Einstein he calls "the law of the constancy of light velocity." It states that the velocity of light in free space appears the same to all observers, regardless of the motion of the source of light or of the observer.

These two laws taken together constitute *the principle of relativity*. They generalize a number of experimental facts and are inconsistent with none. In so far as these generalizations go beyond existing facts they require further verification. To such verification, however, we may look forward with reasonable confidence, for Einstein has deduced from the principle of relativity, together with the electromagnetic theory, a number of striking consequences which are remarkably self-consistent. Moreover the system of mechanics which he obtains is identical with the non-Newtonian Mechanics developed from entirely different premises by one of the present authors.[5] Finally, one of the most important equations of this non-Newtonian mechanics has within the past year been quantitatively verified by the experiments of Bucherer[6] on the mass of a β particle, to which we shall refer later.

Therefore, in as far as present knowledge goes, we may consider the principle of relativity established on a pretty firm basis of experimental fact. Therefore, accepting this principle, we shall accept the conse-

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[3] Abhandlungen über Theoretische Physik, Leipzig, 1907, 443.

[4] An excellent summary of the conclusions drawn from the principle of relativity, by Einstein, Planck, and others, is given by Einstein in the Jahrbuch der Radioaktivität, **4**, 411 (1907). An interesting treatment of certain phases of this problem is given by Bumstead, Amer. Jour. Sci., **26**, 493 (1908).

[5] Lewis, Phil. Mag., **16**, 705 (1908).

[6] Ber. Phys. Ges., **6**, 688 (1908); Ann. Physik, **28**, 513 (1909).

CONTRIBUTIONS FROM THE RESEARCH LABORATORY OF PHYSICAL CHEMISTRY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.—No. 42.

THE PRINCIPLE OF RELATIVITY, AND NON-NEWTONIAN MECHANICS.

By Gilbert N. Lewis and Richard C. Tolman.

Presented May 18, 1909. Received May 18, 1909.

Until a few years ago every known fact about light, electricity, and magnetism was in agreement with the theory of a stationary medium or ether, pervading all space, but offering no resistance to the motion of ponderable matter. This theory of a stagnant ether led to the belief that the absolute velocity of the earth through this medium could be determined by optical and electrical measurements. Thus it was predicted that the time required for a beam of light to pass over a given distance, from a fixed point to a mirror and back, should be different in a path lying in the direction of the earth's motion, and in a path lying at right angles to this line of motion. This prediction was tested in the crucial experiment of Michelson and Morley,[1] who found, in spite of the extreme precision of their method, not the slightest difference in the different paths.

It was also predicted from the ether theory that a charged condenser suspended by a wire would be subject to a torsional effect due to the earth's motion. But the absence of this effect was proved experimentally by Trouton and Noble.[2]

The skill with which these experiments were designed and executed permits no serious doubt as to the accuracy of their results, and we are therefore forced to adopt certain new views of far-reaching importance.

It is true that the results of Michelson and Morley might be simply explained by assuming that the velocity of light depends upon the velocity of its source. Perhaps this assumption has formerly been dismissed without sufficient reason, but recent experimental evidence to which we shall revert seems to prove it untenable.

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[1] Amer. Jour. Sci., **34**, 333 (1887).

[2] Phil. Trans. Roy. Soc. (A), **202**, 165 (1904).

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By Gilbert N. Lewis and Richard C. Tolman.

sigla is already so great (p. 703) that it is well-nigh impossible to remember their meanings. To superimpose the difficulty of remembering a new set of collective designations is a most regrettable procedure, and all the more so because they are so indirect and so needless.

Postscript. — May 21, 1909. Letters received this morning from H. R. H., Prince Vajira-ñāṇa, and dated Pavara-nivesa Vihāra, Bangkok, Siam, April 11, 1909, report that the publication of the second edition of the Siamese Tipiṭaka (referred to above, at page 667) and of the first edition of the commentaries is at a standstill, apparently on account of the difficulties with the introduction of the Kambodian types. His Royal Highness adds that he is editing Buddhaghosa's Dhammapada-commentary, and expects to complete the first volume, containing one half of it, in May, 1909. — C. R. L.

Mahā-vaŋsa. — In the edition of 1908 (see p. LVI), the Burmese mss. are designated by B, the Cingalese by S, and the Kambodian by C.

Group-letter, without exponent, for a group of mss. — It is a very considerable advantage of the system proposed by me, that a group-letter may be used, without the exponents, to designate collectively all the manuscripts of that group. Thus, in the forthcoming Visuddhi-magga, B1 and B2 represent two Burmese mss., and B9 a Burmese printed text; while B, without exponents, is the simple and natural designation of all three Burmese authorities collectively. Similarly C1, C2, C3, C4 represent four Cingalese mss., and C9 the Colombo printed text; while C alone means all these five authorities. In like manner, when occasion arises, K may be used alone for all the Kambodian authorities, and S for all the Siamese.

At first I thought of this advantage merely as one incidental to the use of the system of group-letters; but I now deem this simple and natural way of designating all the mss. of a group collectively to be an essential and very valuable part of the system. The presence or absence of exponents is therefore also an essential matter. The question then arises, What shall we do when a single ms. forms a "group"? When an editor has only one ms. of a given group (Burmese, for instance), so that that ms. alone constitutes the entire group, it seems at first blush immaterial whether he calls it B1 or B; but for this case I propose the following rule: If he cites the ms. *as an individual ms.*, let him cite it with an exponent, thus, as B1 or Ba; if he cites it with other groups (for example, with CKS) *as a group*, let him cite it without an exponent. Thus BCKS would mean each and every authority of all four groups.

Feer, in the Saŋyutta, I. (1884), p. xii., uses SS. as a designation of S^1, S^2, S^3, taken collectively. Morris, in the Añguttara, I. (1885), p. 102 and later, uses SS. and later S.S., apparently to designate his Cingalese authorities collectively. He gives no explanation that I can find, but seems to be following Feer. Since, in designating an individual ms., an exponent should always be used with the group-letter, it follows that the use of the group-letter without an exponent is amply sufficient and characteristic as a designation for all the mss. of that group collectively. Feer's duplication of the group-letter is therefore needless.

In the Mahā-vaŋsa of 1908 (see pages V, VI, VII, LVI), the editor comprehends his Burmese mss. B1 and B2 under the designation X; his Cingalese mss. S1, S2, S3, S4, S5, and S6 under the designation Y; and his Kambodian mss. C1 and C2 under the designation Z. In practice, this is extremely confusing. The confusion in the use of

Udāna. — Steinthal designates his Burmese ms. by A; his Cingalese mss. by B and D; and his ms. of the commentary by C.

Iti-vuttaka. — Windisch enumerates his mss. very properly in two distinct series, and his first Burmese ms. is called B and his first Cingalese ms., C; but he has not carried out this good beginning.

Sutta-nipāta. — Fausböll's edition is not mentioned here as an instance of confusion, but rather by way of calling attention to his admirable procedure described above, p. 702.

Peta-vatthu. — Minayeff uses B for his Burmese ms., and C, D, C^1, D^1 for his Cingalese.

Thera-gāthā, Therī gāthā. — In the prior text, the Burmese mss. are A and B, and the Cingalese are C and D. In the latter, the Burmese mss. are B, L (London), P (Paris), and C (commentary); and the Cingalese ms. is S (Subhūti).

Jātaka. — As early as 1877 Fausböll used the excellent method described above, p. 702. In his preliminary remarks to vol. 4 (1887), he gives B, C, and S as the proper abbreviations for Burmese, Cingalese, and Siamese; and in vol. 5, a Siamese ms. is cited in the notes as S^{dr}.

Paṭisambhidā-magga. — Fausböll's good example is wholly disregarded. Burmese is M (Mandalay); Cingalese is S; and (as in Dīgha II.) Siamese is K.

Vinaya. — The designations of the London ed. (1879–1883) vary by volumes, and so perplexingly as to baffle even a good memory. If, in designating the editions of the Mahā-bhārata, we called the Bombay edition C and the Calcutta edition B, we might remember it as a case of contraries; but not even that unhappy makeshift will serve us here, as the table shows.

Volume.	Burmese mss.					Cingalese mss.	
I.	A		C		E	B	D
II.	A		C	D		B	
III.	A		C			B	D
IV.	A	B	C	D			
V.	A		C	D		B	

Vibhaṅga. — Here, as in Dīgha II., Burmese is B, Cingalese is S, and Siamese is K (Kambodian).

Kathā-vatthu. — Burmese is M (Mandalay); Cingalese is S for palm-leaf mss., and P for the paper ms. (cp. p. 699, note, above), and (as in Dīgha II.) Siamese is K.

Paṭṭhāna. — The Burmese authorities are B and R (the Rangoon print); Cingalese is S; and Siamese (again: Behold how great a matter a little fire kindleth!) is K.

and unsystematic that it is neither possible to memorize them, nor worth the while, if possible; and one set of them has, as a rule, nothing to do with another set. — 2. In other cases, the mss. are designated with reference to the groups to which they belong, but the group-letters are in part ill-chosen and the choices of different editors disagree. C is used for Cingalese and Kambodian, and S is used for Cingalese and Siamese. Or, to put it the other way, Cingalese is designated by C and S; Kambodian, by C and K; and Siamese, by S and K and Si. The details follow.

Dīgha-nikāya. — In vol. I. (1890) the readings of the Burmese mss. were designated by B with exponents, and those of the Cingalese mss. by S with exponents. This was in accord with the principle stated in 1886 in the preface to D.cm. (reprinted above, p. 703). When vol. II. (1903) appeared, the Royal Siamese ed. had meantime become available, and it was necessary to cite its readings. Instead of changing from S to C for Cingalese (so as to have S free to use for Siamese), the editor stuck to his short-sighted error, and, quite forgetting his promise (above, p. 703) to use K for Kambodian, he designated the Siamese readings by K, because, forsooth, they are (preface to D., II., p. viii) "the readings of mss. written in the Kambojian character"! Since a new edition in Kambodian characters is now expected from Bangkok, it remains to see how confusion will be still further confounded.

Majjhima-nikāya. — In vol. I., Trenckner designated his Burmese ms. by M, and his Cingalese ms. by A. In vols. II.–III., his successor, Chalmers, adopting the correct principle (as in D.cm.), but with the faulty application, changed the sigla and designated his Burmese ms. by B^m, his Cingalese by S^k S^t, and the Siamese ed. by Si.

Saŋyutta-nikāya. — Feer designated his Burmese mss. by B^1 B^2, his Cingalese mss. by S^1 S^2 S^3, and his ms. of S.cm., "in Siamese-Cambodgian characters," by C.

Añguttara-nikāya. — In vol. I. (1885) Morris designates his Burmese ms. by Ph (= Phayre); his Cingalese mss. by T (Turnour), Ba and Bb (British Museum), D (Davids), Tr (Trenckner); and his Cingalese mss. of the A.cm. by Com. In vol. II. (1888) he changes his system of designations, probably in deference to the views of the ed's of D.cm. (given above, p. 703): here his Burmese ms. is B.K. and his Cingalese mss. are S.T., S.D., S.Tr., S.M. — typographically most awkward. In vol. III. the lamented Hardy designates his Burmese authorities as M., Ph., M_8; and his Cingalese as T., M_6, M_7, M_9, M_{10}, T_1; and adds new confusion by introducing S with the meaning, not of Cingalese, but of Siamese. In short, the whole system (or rather hotch-potch) of sigla is so desperately muddled as almost wholly to defeat the purpose of an apparatus criticus.

should be Roman and not Italic (Cursivschrift).[61] The exponents should be separated, each from its neighbors (but not from the group-letter), by a comma (thus : B1, e, 9).[62]

Confusion of the designations in texts already issued. — In what follows, the editions of the Pāli Text Society [63] are intended, except in the case of the Jātaka and Vinaya. Some of the texts (Vv., Bu., Cr.; Dhs., Pug.) have no apparatus criticus and hence no sigla codicum.

The principle underlying Fausböll's procedure in the Sutta-nipāta (1885) was expressly enunciated in 1886 by the editors of the D.cm. (preface, p., xii), who say: We "give the Sinhalese tradition as our text, and . . . add the Burmese readings in our notes. And it is to make this perfectly clear and easy to the reader that we have adopted the plan of naming the Sinhalese mss. not D., T., etc., but S^d, S^t, etc. When we are able to quote mss. in Kambojan characters, we shall designate them on the same principle as K^d, K^t, etc."

The principle is absolutely correct; but its enunciators or authors, in using S instead of C for Cingalese, have applied it with such lack of prevision and circumspection as largely to defeat their purpose. For the results, see below, under Dīgha, etc. It is most amazing and unfortunate that Fausböll's good example was not duly and generally heeded, and that the principle just rehearsed was put into practice so badly. The editors of Pāli texts assuredly *possess* discernment enough to recognize the excellence of Fausböll's procedure, and wisdom enough to follow it; but in this matter they have been simply heedless and have failed to *use* those qualities. If scholars would uniformly adopt the sigla here proposed, the economy and convenience and utility of them would be very great, and would be surely recognized by all who tried them.

To make this clear, it is worth while to show up the existing confusion. This may be summarized as follows: 1. In some cases, the mss., without any reference to the groups to which they belong, are designated by haphazard sigla, which convey no idea as to the origin of the ms. or the alphabet in which it is written. These sigla are so arbitrary choice left us, as between S and C, for the mss. of Ceylon, and must perforce use C. And although either K or C would serve for "Kambodian" or "Cambodian," it is most fortunate that we have a choice [57] and can avoid using the preëmpted and ambiguous C by employing the unambiguous K.

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[61] Fausböll's Sutta-nipāta shows small italics used as exponents.

[62] I may say in this place (for lack of a better): If the apparatus criticus is given in the foot-notes, with reference-numbers corresponding to numbers in the text above, then the reference-numbers at the foot (not in the text) may well be set in a black-faced type, and they should certainly be set with columnar alignment.

[63] I beg the reader not to think that I wish to detract in the smallest degree from the very great merit of the services rendered to science by the Managing Chairman of the Pāli Text Society. Nothing could be farther from my wish. My sole purpose is to show how hurtful the present lack of agreement and system is, and to put an end to it.

The most important one of these four designations, C (and not S) for Cingalese, was employed in 1877 by Fausböll in the first volume of the Jātaka. Again, in 1885, in his ed. of the Sutta-nipāta, he goes still farther on the right course, and designates his Burmese mss. by B and his Cingalese mss. by C, distinguishing the individuals of each class by suggestive exponents. Thus B^a is the Burmese ms. of the Asiatic Society of London, and B^i is that of the India Office. C^k is the Cingalese ms. in Kopenhagen, and C^b is that of the British Museum. Two years later, in 1887, no less than three, and those the most important, of the four designations (B = Burmese, C = Cingalese, S = Siamese) were all settled, and settled wisely, by Fausböll in his preface to the Jātaka, vol. 4, p. vi.

The exponents. — The exponents may very well be either Arabic numerals or small letters, or both numerals and letters may be used together. I think the numerals (but only from 1 to 9) are better than the letters, unless it is desired to suggest by a small letter the name of some especially famous library or scholar. Numbers with two digits should be avoided; if there are more than nine authorities in a given group, numbers and letters may be used together as exponents.

Typography of the designations of the mss. — The group-letter should always be a capital letter, and no period or other mark of punctuation should be used after it as a part of the designation.[58] The use of a digraph as siglum is not to be tolerated: thus Ph for Phayre should be avoided. The exponents may be set either as "superiors" (thus: B^1) or else so as to be on a line with the group-letter (thus: B1);[59] but the best and easiest way of all is to set the exponents with a hybrid type, of which the face is two points smaller than the body (thus: B1). If letters (not numbers) are used as exponents, they should certainly be small letters, never capitals;[60] and I think it is better that they

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[57] In the Mhvṇ. of 1908, the editor chooses C for Kambodian, although he had already chosen K for it in List 13.

[58] After the group-letter as a part of a sentence in which it may occur, any appropriate mark of punctuation may of course be put.

[59] Never below the line; the bad effect of this method is exemplified in vol. III. of the Aṅguttara.

[60] Volume II. of the Aṅguttara shows the clumsy effect of capitals used as exponents.

Both the Burmese and the Cingalese alphabets abound in groups of confusingly similar letters. Thus in Burmese we have the groups: bh and h and ṡ; te and vo; dh and m; ṭ and ḍ; ñ and u (initial). In Cingalese we have: bh and h and g; t and n; s and y; v and c; ch and j; ph and th and e (initial); m and o (initial).[50] It is because the points of confusion are differently located in the several alphabets that a ms. of one class often proves to be an effectual check (Kontrolle) upon a ms. of another.[51]

Group-letter with exponent, for an individual ms. — The logical conclusion from all this is clear. The sigla must show, each on its face, to which one of the four groups or classes the ms. belongs. Nor is there the slightest difficulty in devising such sigla, as the next paragraph shows. The letter which indicates the group I call the group-letter. This in the first place. — In the second place, each siglum must of course indicate the individual ms. of the group to which the ms. belongs. This also is very simply and easily done, namely, by placing after the group-letter (which must be a capital) a small letter or an Arabic numeral. This letter or numeral I call an exponent.

Determination of the group-letters. — B = Burmese; C = Cingalese; K = Kambodian; S = Siamese. — The word "Burmese" is never written[52] with any other initial than B. Nor can there be any doubt that S is the only available abbreviation[53] for "Siamese." It is quite true that "Singhalese" or "Sinhalese," like the older forms of the name of the island, Sanskrit Siṅhala-dvīpa, Pāli Sīhala-dīpa,[54] is very commonly spelled with an S, in English as in German; and true also that "Cingalese" and "Ceylonese" are in irreproachably good use[55] and are spelled with a C; but for the name of the island, "Ceylon," although it was formerly written[56] with S and Z, the spelling with C is now the fully established one in English and French and German. And since the necessity of employing S for "Siamese" is inexorable, we have no follow blindly the procedure of Hellenists or Latinists, good or bad as that may be. And in fact, in looking over the prefaces of the various editions of Pāli texts, I have been so struck by the abominable and needless confusion of the sigla codicum, that I take this opportunity to urge a rational course of procedure.

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[50] On the other hand, both in Burmese and in Cingalese, t is clearly distinguishable from ṭ, and n from ṇ.

[51] Windisch has made most useful observations on this subject in the preface to his Iti-vuttaka (1889), p. iv.; and so has Hardy in the preface to his ed. of the Pv.cm. (1894), p. vii. Cp. also Hardy's remarks on p. v. of his preface to Aṅguttara, vol. V., and among them this: "There is no ms. nor any set of mss. which can be relied upon indiscriminately."

[52] Since we are not likely to be so pedantic as to adopt the form Mranmā.

[53] It would indeed be far-fetched pedantry to use a Th (for Thaĭ)!

[54] For the origin of the name, see Mhvṇ., vii. 42, ed. Geiger.

[55] Linschoten, in 1598, writes Cingalas: see Yule-Burnell, Hobson-Jobson, s.v. Singalese.

[56] See Hobson-Jobson, s.v. Ceylon.

Four classes of Pāli mss. to be clearly distinguished. — The material for editions of Pāli texts consists of mss. in the Pāli language, and written, some in Burmese letters, some in Cingalese, some in Kambodian, and some in Siamese letters. It is, in the first place, to any one who has even a slight knowledge of these four alphabets, as plain as a pikestaff that the really important thing for us to know concerning a given reading as reported in an apparatus criticus is not whether the ms. in which it appears belonged twenty or thirty years ago to Richard Morris or to Sir Arthur Phayre, nor whether it was kept in Copenhagen or Chicago.[46] What we do greatly need to know about a given reading is this, In what country did the ms. containing it originate, and in what alphabet is it written?

Country of origin. Alphabet used. — Why these two matters should be indicated by the siglum may be shown by an example or two. There are certain peculiarities of orthography proper to mss. coming from Burma, and others proper to mss. coming from Ceylon. If, in a given passage, we know from the sigla that, for instance, the ms. which reads veḷu is from Burma, while the ms. reading veṇu is from Ceylon, we may very well discount that fact[47] and let it pass without special comment. The provenience of the ms. is here the essential question. In other cases the essential question may be, In what alphabet is the reading given? In the Cingalese alphabet, for example, y and s are confusingly similar, while t and n are almost desperately indistinguishable. In Burmese, on the other hand, there is not the slightest danger of confusing t and n. Now, taking for example[48] the passage Pv. iv. 6^5, if we know that the distinction between santo and yan no in Cingalese letters is not worth a fig, and that one Burmese ms. reading yaŋ no is worth twenty Cingalese mss. with the unintelligible santo, the fact that the unintelligible santo is in Cingalese letters is the fact of prime importance.[49]

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[46] The sigla used in the Añguttara (see below) tell us just the things that we do not need to know, and most effectually conceal from us all that we do. They are models of badness.

[47] See Davids and Carpenter, preface to D.cm., I., p. xv.

[48] Cp. Minayeff's ed. of Pv., p. 63, verse 5, with Hardy's ed. of Pv.cm., page 261.

[49] And yet this one little fact is not to be known from Minayeff's ed. except at a cost of precious minutes! See his preface, p. iii., top, p. v., bottom, p. vi., top.

mentario = L. commentarium = G. Commentar). Hence, not Cy. nor Comm. (which last is long). Lists 2 and 11 have C, which is a capital and is too short and suggests Culla, etc. Either com. or cm. would serve very well; but since cm. is as readily suggestive as com., and shorter, and does not suggest anything else, I think that cm. is on the whole the best.

Supercommentaries. — The same objections to fanciful titles are cogent here as before. Moreover, the Hindus often employ a special word for a supercommentary, namely, ṭīkā. Thus they apply this name to the very important supercommentary of Ānandagiri upon Çaŋkara's commentary (bhāshya) upon the Upanishads. This word ṭīkā is a short and convenient one; and since it begins with a characteristic and very rare initial, ṭ, and one which is very suggestive, and since supercommentary is a long word and difficult to abbreviate satisfactorily, I favor designating these works by ṭ. For Dhammapāla's supercommentary upon Buddhaghosa's commentary entitled Destroyer of Error or Papañca-sūdanī, we write, not Līnattha-ppakāsinī (which may be any one of six different things : cp. p. 696), but simply M.cm.ṭ.; and read it as Supercommentary on the Majjhima-commentary.

Methods of designating the manuscripts. — In classical philology, the codices are named after persons who once owned them (thus the Vossianus of Ovid), or after the places where they are kept (thus, Parisinus, Guelferbytanus; Bodleianus, Vaticanus).[45] In a discipline which has so long been cultivated, it would be a questionable proceeding to depart from long-accepted usage, especially in the case of mss. celebrated the world over. But Pāli philology is very young, and definitive designations are in large measure yet to be made. Considering broadly the ways of literary tradition in the Orient, the multiplicity of the mss., and the inevitable modernity of many of them, the complete insignificance of temporary ownership, and the comparative insignificance of the place of keeping, — it is evidently a headless thing to enumerating Buddhaghosa's works, says : The commentary, Sumañgala-vilāsinī by name, upon the Dīgha-nikāya, Dīgha-nikāyassa Sumañgala-vilāsinī nāma aṭṭhakathā. The colophon to the Kvu.cm., p. 199, JPTS. 1889 (cp. p. 231, ed. Rangoon), says : Kathāvatthu-ppakaraṇaŋ . . . tassa niṭṭhitā atthavaṇṇanā. . . . Kathāvatthu-ppakaraṇa-aṭṭha-kathā niṭṭhitā. Why should we be more Hindu than the Hindus ?

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[45] Sometimes even the material employed gives the name to a ms. Thus the world-famed ms. of Ulfilas at Upsala is called the Codex argenteus, because it is in letters of silver on purple parchment. The first Cingalese ms of the Kathāvatthu is designated as P, either because it belonged to a Professor, or, more probably, because it is written on Paper leaves as distinguished from Palm-leaves. This reminds me of the old woman who always marked the upper crust of her pies, not only her mince-pies but also her apple-pies, with "TM" meaning in the one case "'T is mince," and in the other case "'T aint mince." — For the benefit of the dwellers in partibus, I observe that mince-pies are made of pastry filled with minced meat, that 'T is = It is, and that 'T aint = It is not.

The fanciful titles should be ignored by us also. — Long ago I heard a jocose account of the method of weighing hogs in Arkansas. They make fast the hog to one end of a rail, balance the rail on a fence with stones fastened to the other end, and then guess how much the stones weigh. Those stones correspond to our fanciful titles. Why tell a student that a citation is from the Par. Jot. ? He has first to find out that Par. Jot. means Paramattha Jotikā. Secondly, he must find out what the texts are which have a commentary bearing that name. Thirdly, he must find out which of those texts (in this case Khuddaka-pāṭha or Sutta-nipāta) is intended. Having got so far, he is just as far as he would have been, if, in the first place, we had told him that the citation was, for example, from the commentary on the Sutta-nipāta or, briefly, from the Suttanipāta-commentary or Sn.cm.

The abbreviation "cm." for "commentary." — Since then the use of the fanciful titles is a blameworthy indirectness, the commentary on a given text should be spoken of by us uniformly as "the commentary on" that text, or, briefly, "the . . . -commentary." Thus we ought not to speak of "the Sumañgala-vilāsinī," but rather "the commentary on the Dīgha," or, briefly, "the Dīgha-commentary." For this phrase, "the . . . -commentary," it remains to devise a uniform and direct and suggestive and simple abbreviation.

In the "Contractions" given on p. xvii of Davids and Carpenter's ed. of Sumañgala-vilāsinī, we find three commentaries designated in three different ways : namely, Dhammapada-commentary as Dhp. Com. ; Jātaka-commentary as J. ; and Vinaya-commentary as S.P. Such lack of uniformity, if carried far, would be exceedingly embarrassing. — Lists 1 and 4 and 10 use A., the initial letter of aṭṭha-kathā, the Pāli word for "commentary," and List 13 uses Ak. This again is a useless indirection. — Aufrecht, in his Catalogus catalogorum, uses a turned C (ↄ) for commentary, and two turned C's with a stroke (ↄ̶ↄ̶) for super-commentary. Personally, I like this ; but as it is too arbitrary for general use, and suggests withal the "scruples" of Apothecaries' Weight, I scruple to use it. — The designation may best be something that suggests, not only the English word "commentary," but also its various equivalents (Fr. commentaire = It. commentario = Sp. counqualifiedly condemned. They have been common, however, not only with writers of Pāli and Sanskrit in Ceylon and India, but also with those of other lands and ages. In Sanskrit, for instance, we have a work entitled The Poet's Secret, Kavi-rahasyam. This is not a vision of Calliope in the grove upon Helicon, but (God save the mark!) a treatise of Sanskrit roots. A work upon Hebrew synonyms by Salomon Urbinas (Venice, 1548) is entitled Tabernacle of the Covenant (Tentorium Conventus or Ohel Mo'ed). A supercommentary to the biblical commentary Rashi, as being the offspring begotten from the spiritual loins of Rabbi Leo of Prague (about 1590), is called The Lion's Whelp (Catulus Leonis, Gur aryeh, with reference to Genesis 49.9). A treatise of the Divine clemency by William Sibb is entitled Bowels Opened, and is cited as Sibb's Bowels Opened. Among the fanciful titles of Cotton Mather's works is found one, "Edulcorator. A brief essay on the waters of Marah sweetened."

Unserviceableness of the fanciful titles of these commentaries. — In giving English equivalents of these titles (in Table III), I have used the utmost pains to reproduce the essential peculiarities of the originals. If a Pa-jjotikā is an Il-luminator, then a Jotikā should be a Luminator. As serving the second purpose of a title (cp. page 696), nothing could belie itself worse by emptiness than "The Fulfiller of Wishes." My equivalents make clear how utterly unserviceable the fanciful titles are. What difference in meaning is there between a Destroyer of Error and a Dispeller of Folly (no's 2 and 22) such as might help us to associate the one with the Majjhima and the other with the Vibhañga-ppakaraṇa? And when it comes to holding surely in memory the fact that the Illuminator (Pajjotikā) of the Good Religion is the comm. on the Niddesa, while the Illustrator (Pakāsinī) of the same is the comm. on the Paṭisambhidā, — for me, I confess, it 's like trying to keep my grip on a pendent icicle. The differences between no's 3 and 5 and 7 (see Table III) are just as elusive. Even if this were not so, the fact that the same fanciful name is applied to more than one text quite defeats the usefulness of the name (see p. 694, end).

The Hindus often ignore these fanciful titles. — Buddhaghosa does indeed refer (in his Attha-sālinī, p. 97) to his Complete Clarifier by its fanciful title, but explains that it is the comm. on the Vinaya: Atthikehi pana Samanta-pāsādikaŋ Vinay-aṭṭhakatham oloketvā gahetabbo (cp. p. 98). Later writers, like the author of the Gnvŋ. (passim), speak of a given commentary, just as we should do, simply as a commentary, that is, as an aṭṭhakathā or vaṇṇanā or atthavaṇṇanā or saŋvaṇṇanā of such and such a text, and add the fanciful title or not, as the case may be. And so do the writers of the colophons. Thus the Gnvŋ., p. 59, which the volume forms a comment; and the cover of no. 13 (which I fortunately preserved) adds the information (not given on the title-page!) that that is "Part V."

How the numbers "V., III., IV.," as designations of these "Parts" of Paramattha-dīpanī, were arrived at, — this passes my comprehension. I do not find the individual commentaries designated as "Parts" in the colophons.[43] The matter is so confusing that even the confusion cannot be shown without a little table. In this the Arabic numbers at the left refer to Table III, and the Roman numerals at the left give the Parts according to their order in the canon.

No. 7	Part I.	Ud.cm.			
No. 8	Part II.	It.cm.			
No. 10	Part III.	Vv.cm.	Issued in	1901 as	"Part IV."
No. 11	Part IV.	Pv.cm.	Issued in	1894 as	"Part III."
No. 12	Part V.	Th.1.cm.			
No. 13	Part VI.	Th.2.cm.	Issued in	1893 as	"Part V."

If numbered according to the order in the canon, "Part IV." should have been called Part III., "Part III." should have been called Part IV., and "Part V." should have been called Part VI. If numbered according to the order of publication, "Part IV." should have been called Part III., "Part III." should have been called Part II., and "Part V." should have been called Part I. Evidently to cite any one of these six commentaries as a "Part" of Paramattha-dīpanī is sheer folly; and to cite it simply as "Par. Dīp." is wholly futile.[44]

Līnattha-ppakāsinī, Illustrator of the Hidden Meaning, is the title of at least six supercommentaries, namely, Dhammapāla's ṭīkās to nos. 1, 2, 3, 4, and 14 of Table III, and also a ṭīkā on the Kaṅkhāvitaraṇī. — But enough! a glance at Mrs. Bode's most convenient Index will give a bird's-eye view of the thickets of this endless jungle and convincing proof of the folly of citing the fanciful titles.

Fanciful titles of books. — The main purposes of a title are two: (1) like the name of a man, it is to serve as a designation; and (2) it is to indicate the general subject of the book. Except for works of fiction and the like, titles which do not serve the second purpose are to be

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[43] The ṭīkās on The Meaningful (no. 21), The Dispeller of Folly (no. 22), and The Five-Treatise-Commentary (no's 23–27) are indeed called respectively, by the Saddhamma-saṅgaha, "The First, Second, and Third Illustrators of the Supreme Meaning," Paṭhama-, Dutiya-, and Tatiya-Paramattha-ppakāsinī: see JPTS.1890, p. 60. Likewise at p. 59 we find "First, Second, Third, and Fourth Chest of Essential Meanings" (Sāratthamañjūsā) as names of ṭīkās on the four Nikāyas.

[44] The author of List 4 seems to have had glimpses of trouble ahead, when, after "Par. Dīp. = Paramattha Dīpanī," he added "Parts 3 and 5 quoted as Thig. A. and P. V. A."

Khuddaka-nikāya have a comm. bearing this title, to wit, nos. 7–8 and 10–13. Curiously, the title Paramattha-dīpanī is not even mentioned by the Gnvŋ. (see Bode, p. 67), except as *title* of a Ṭīkā on Bu.cm. (see below); but it is vouched for as a true title of the comm. on Therī-gāthā, on Peta-vatthu, and on Vimāna-vatthu (nos. 13, 11, 10: that is, the three published parts) by a line found in the colophon of each of them, to wit:

pakāsanā Paramattha-dīpanī nāma nāmato.

Cp. the Sdhs., p. 63, verses 32 and 27. The comm. on Udāna is spoken of by Steinthal, p. vii. of his ed. of the text, as "entitled the Paramattha-dīpanī;" and the comm. on Thera-gāthā is "called Paramattha-dīpanī," according to Oldenberg, p. xii. of his ed. of the text. Only in the case of the comm. on Iti-vuttaka was I unable to cite authority for entitling it Paramattha-dīpanī. Accordingly I wrote to Professor A. Cabaton of the Bibliothèque Nationale to inquire, and he very kindly informed me that in the colophon of the ms. in that library the comm. is indeed called "Paramattha-dīpanī, comm. on Iti-vuttaka."

Paramattha-dīpanī is a title applied, by the Sdhs. at least, to five other commentaries also, namely those on the last five texts of the Abhidhamma, nos. 23–27: for at p. 60, the supercommentary called "The Third Illustrator of the Supreme Meaning" (p. 696, note 43, below) is described as "a statement of the meaning of the Five-Treatise-commentary styled The Elucidator of the Supreme Meaning" (Pañca-ppakaraṇ-aṭṭhakathāya Paramattha-dīpaniyā attha-vaṇṇanā). I suppose this Paramattha-dīpanī must be Buddhaghosa's. And finally Dhammapāla's supercommentary on the comm. to the Buddha-vaŋsa is styled Paramattha-dīpanī.[41]

The "Parts" of the Paramattha-dīpanī. — As to the three "Parts" published by the PTS., namely, no. 13, in 1893, on Therī-gāthā; no. 11, in 1894, on Peta-vatthu; and no. 10, in 1901, on Vimāna-vatthu. — No. 13 is lettered on the back (from the bottom upwards) "Paramattha Dīpanī." No. 11 is lettered on the back (from the bottom upwards) "Dhammapāla's Paramattha-Dīpanī. Part III." No. 10 is lettered on the back (from the top downwards) "Paramattha-Dīpanī. Part IV." No further indication of the contents of any of these volumes is given on the back;[42] but the title-page of each does name the text to

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[41] Gnvŋ., p. 60: cp. Bode's Index, pp. 67, 70.

[42] These negligences are doubtless petty ones. It is only a petty annoyance to take the book from the shelf upsidedown, and only a petty annoyance to have to take down two or three volumes before you get the right one; but such annoyances are gratuitous and have a cumulative tendency to impede rapid work.

Aṭṭha-sālinī, p. 408, lines 18–19 and 26, we read Aṭṭhasālinī nāma Dhammasaŋgah-aṭṭhakathā,[40] but in line 27, Dhammasaŋgaṇī-aṭṭhakathā.

The titles of such texts are justly the despair of Occidental librarians and bibliographers, who are inevitably at their wit's end in trying to perform the well-nigh impossible task of making these Oriental books available to Orientalists. Perhaps we ought not to blame the Hindus. With their erudition, profound in many ways, but narrow, they had no more conception of the many-sided knowledge indispensable for a modern librarian than they had of aerial automobiles or wireless telegraphy.

Different names for the same commentary. — Comm's on books of the Khuddaka-nikāya. — Comm. on Iti-vuttaka. The title Paramatthadīpanī belongs of right to this text (see below, p. 695); but Childers, as noted above, calls it Abhidhammattha-dīpanī. Where he got this title I do not know. It is not given in the Gnvŋ. (p. 60), which simply calls it Itivuttaka-commentary. — Comm. on Jātaka. Buddhaghosa himself, at the beginning (pages 1^{21}, 2^{1}), calls the work Jātakass' Atthavaṇṇanā. — Comm's on Vimāna- and Peta-vatthu. Although the Gnvŋ., at p. 69, calls them simply Vimānavatthu-Petavatthu-ṭṭhakathāgandha, it gives to each of them somewhat earlier, at p. 60, the fanciful style of The Spotless Charmer, Vimala-vilāsinī. This title does not appear in the mss. of these two texts, according to Hardy, p. 107. Cp. again below, p. 695. — Comm. on Niddesa. I do not find the colophon of this anecdoton in any of the ms. catalogs. The Gnvŋ., at p. 70, says Saddhamma-ppajjotikā nāma Mahāniddesass' aṭṭhakathāgandho; but at p. 61, it is called (if I may coin the word) The Maintenancer of the Good Religion, Saddhamma-ṭṭhitikā nāma.

Comm's on Abhidhamma treatises. — The first and second have each a fanciful name, while the last five (see Table III) have one comprehensive title, The Five-Treatise-commentary; but all seven also are comprehended under the broader title, Account of the Supreme Meaning or Paramattha-kathā, by the Gnvŋ., which says, at p. 59, satta-abhidhamma-gandhānaŋ Paramattha-kathā nāma aṭṭhakathā. At p. 68 it is called simply the "commentary-book of the seven Abhidhamma books;" cp. also sattābhidhammagandha-aṭṭhakathā, at p. 60, line 3, and Abhidhamm-aṭṭhakathā, at p. 60, l. 15, and p. 69, l. 18, and Sdhs., p. 60, l. 18.

The same title for different texts. — **Paramattha-dīpanī.** — This means a dozen commentaries, if not more. Not less than six texts of the

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[40] And so in Westergaard's Catal., p. 44, b, and in E. Müller's ed., p. 430.

For Paramattha-dīpanī, the comprehensive fanciful name of the commentary on these six texts, we have in Lists 2, 3, 4, 5 the abbreviation Par. Dīp. (compare Par. Jot., above); but since this is an indication which does not indicate, List 4 adds: "Parts 3 and 5 quoted as Thig.A. and P.V.A."

For no. 9, see under no. 5; for no's 10–13, see under no. 7.

No. 14, Jā.cm. List 4 gives Jāt. for the commentary, and J. for the verses; but see p. 686, Canon 8.

No's 15 and 16, Nd.cm. and Ps.cm. Here again (as in the case of Par. Jot. and Par. Dīp.), cumbrous double designations are needed, Sad. Paj. and Sad. Pak. (so List 5).

No's 17–19 are unpublished, but List 5 gives Madh. Vil. for no. 18.

No. 20, Vin.cm. This is usually designated as Smp., but as Sam. in Lists 5 and 7.

No. 21, Dhs.cm. This is oftenest Asl.; but it is Aṭṭh. in List 5 and As. in List 8.

No. 22, Vbh.cm. This work, published (like no. 21) in a volume by itself in the Rangoon ed. of the P. G. Mundyne Piṭaka Press, has hardly received any designation among Occidental scholars.

No's 23–27. In the ed. just named, these last five form one volume and are printed in the order given by Buddhaghosa (D.cm. I. 17) or as in Table III, Kvu.cm. being put in the third place among these five (thus: Dhk.cm., Pug.cm., Kvu.cm.), instead of being put in the first. It would be useless to invent a comprehensive designation for the five. No. 25 has received the designation K. V. A. in Lists 4 and 10, and Kathāv. P. A. in List 13. No. 23 appears as Dhk. A. in List 4.

Different names for the same thing. — Polyonymy. We have heard of the student who, undergoing examination on the Homeric question, answered that "The Iliad was not written by Homer, but by another man of the same name." In India the trouble is often the other way, — it is the same man with another name. "The Hindus, even in historical documents and works, had the bad habit of designating one and the same person by different names of the same significance. Thus Vikrama-arka = Vikrama-āditya; Sūrya-matī = Sūrya-vatī."[39] So one of the three Elders at whose request Buddhaghosa wrote the Jā.cm., is called by him (I. 1) Buddha-deva, but by the Gnvŋ., p. 68, Buddha-piya. — Unfortunately, this is true not only of men, but also of texts. The Dhamma-saŋgaṇi is called Dhamma-saŋgaha by great Buddhaghosa himself at D.cm., I. 17; while in the Rangoon (Mundyne) ed. of

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[39] So Bühler, Zeitalter des Somadeva, Stzbr. der Wiener Ak., 1885, p. 554.

Excursus: Books about Pāli books. — It is well to notice here a few books which treat of the titles and authors of Pāli books. — First, the Book-history or History of the books or Gandha-vaŋsa. The text was edited by Ivan P. Minayeff in the JPTS. for 1886, pages 54–80, and reprinted in his Recherches sur le Bouddhisme (Annales du Musée Guimet), 1894, pages 235–263. In this connection Mrs. Bode's extremely useful Index to the Gnvŋ., JPTS.1896, pages 53–86, should not be overlooked. — The text of the Saddhamma-saŋgaha was edited by a Cingalese in JPTS. for 1890, pages 21–90. — In 1892 Professor James Gray of Rangoon College published his Buddhaghos-uppatti or the historical romance of the rise and career of Buddhaghosa (London, Luzac & Co.). — The text of the Sāsana-vaŋsa, a modern work by Paññā-sāmī, A. D. 1861, was edited for the Pāli Text Society by Mrs. Bode, 1897. — In the Journal of the German Oriental Society for 1897, li. 105–127, Edmund Hardy published a paper on Dhammapāla. — All these works are of use in this connection and are cited by the following designations: Gnvŋ. (thus, when the original ed. is meant, JPTS.1886); Bode's Index; Minayeff, Recherches; Sdhs.; Gray; Sāvŋ.; Hardy.

The fanciful titles: confusions and uncertainties. — It is necessary to show the results that have come from the use of these titles, and that are to be expected from the continuance of this most reprehensible practice. We will take the numbers in their order.

No. 1, D.cm. This is designated oftenest as Sum., but in List 8 as Sv., which means Sutta-vibhañga in List 12. Parts of it are designated in List 1 as Br. J. S. A., Par. S. A., and Sám. S. A.: as to this, cp. page 678.

No. 2, M.cm. This has the euphonious designation "Pap." in Lists 2 and 5, and the biblical designation Ps. (suggesting Psalm) in List 8. But Ps. means Paṭisambhidā-magga in List 2, and so does PS. in List 5.

No. 3, S.cm. This is Sār. Pak. in List 5, Sār. being needed to distinguish it from Sad. Pak., no. 16.

No. 4, A.cm. This is Man. in Lists 5 and 10; but in List 8 it is Mp., which means Milinda-pañha in List 12, and Mahāparinibbāna-sutta in List 8.

No's 5 and 9, Kh.cm. and Sn.cm. In Lists 2 and 5 these are designated as Par. Jot., the addition of Jot. being needed because we have Par. Dīp. (no. 7); and in List 8 they are designated as Pj. But even if we use the cumbrous Par. Jot., it is impossible to know whether no. 5 or no. 9 is intended.

No. 6, Dh.cm. This is Dhp.C. in List 2; Dhp.A., in Lists 4 and 10; and Dhp.Com. in the PTS. ed. of D.cm.

No's 7–8, 10–13, Ud.cm., It.cm., Vv.cm., Pv.cm., Th.1.cm., Th.2.cm.

TABLE III. — COMMENTARIES OF BUDDHAGHOSA, ETC.

1. On the First Four Nikāyas.

1	D.cm.	Buddhaghosa's	Sumañgala-vilāsinī	Auspicious Charmer
2	M.cm.	“	Papañca-sūdanī	Destroyer of Error
3	S.cm.	“	Sārattha-ppakāsinī	Illustrator of the Essential Meaning
4	A.cm.	“	Manoratha-pūraṇī	Fulfiller of Wishes

2. On the Khuddaka-nikāya.

5	Kh.cm.	Buddhaghosa's	Paramattha-jotikā	Luminator of the Supreme Meaning
6	Dh.cm.	“	Dhammapad-aṭṭhakathā	Dhammapada-commentary
7	Ud.cm.	Dhammapāla's	Paramattha-dīpanī	Elucidator of the Supreme Meaning
8	It.cm.	“	“ “	Elucidator of the Supreme Meaning
9	Sn.cm.	Buddhaghosa's	Paramattha-jotikā	Luminator of the Supreme Meaning
10	Vv.cm.	Dhammapāla's	Paramattha-dīpanī	Elucidator of the Supreme Meaning
11	Pv.cm.	“	“ “	Elucidator of the Supreme Meaning
12	Th.1.cm.	“	“ “	Elucidator of the Supreme Meaning
13	Th.2.cm.	“	“ “	Elucidator of the Supreme Meaning
14	Jā.cm.	Buddhaghosa's	Jātak-aṭṭhakathā	Jātaka-commentary
15	Nd.cm.	Upasena's	Saddhamma-ppajjotikā	Illuminator of the Good Religion
16	Ps.cm.	Mahānāma's	Saddhamma-ppakāsinī	Illustrator of the Good Religion
17	Ap.cm.	Buddhaghosa's	Visuddhajana-vilāsinī	Charmer of the Purified
18	Bu.cm.	Buddhadatta's	Madhurattha-vilāsinī	Charmer by Sweet Meanings
19	Cr.cm.	Dhammapāla's	Cariyāpiṭak-aṭṭhakathā	Cariyāpiṭaka-commentary

3. On Vinaya. — On Abhidhamma.

20	Vin.cm.	Buddhaghosa's	Samanta-pāsādikā	Complete Clarifier
21	Dhs.cm.	Buddhaghosa's	Attha-sālinī	The Meaningful
22	Vbh.cm.	“	Sammoha-vinodanī	Dispeller of Folly
23	Dhk.cm.	“	Pañca-ppakaraṇ-aṭṭhakathā	Five-Treatise-commentary
24	Pug.cm.	“		
25	Kvu.cm.	“		
26	Yam.cm.	“		
27	Ptn.cm.	“		

Pāli commentaries upon the 27 canonical works: namely, the commentaries of Buddhaghosa (17),[34] Dhammapāla (7), Upasena (1), Mahānāma (1), and Buddhadatta (1). All of these commentaries are constantly and very naturally spoken of by Buddhist writers as "commentaries upon" or "explanations of" this or that work; but they nearly all have also each a fanciful name, by which it has become usual to designate them in the Occident. Some may raise the objection that it is premature to settle upon the best short designations of these commentaries now, while only so few are accessible to Pāli scholars in European editions. In reply I say (as I have already said, p. 679), that we must look into the future. At present only numbers 1 (part), 6 (part), 10, 11, 13, 14, 20 (extracts), 21, 23, and 25 have been published in Europe. But the Burmese editions either include, or will doubtless soon include, so many of these commentaries, and it will be so easy to make reprints of them in Roman letters, that we may well hope soon to have a large part of them available for easy use in good Roman type. And what more useful preliminary for a lexicon can there be than a systematic and careful exploitation of Buddhaghosa's glosses, as given in his commentaries? It is highly important, therefore, to settle, promptly and rightly and once for all, upon a system of brief designations of these valuable sources of lexicography.

To do this, we must see these fanciful Pāli titles set in a list, with their nearest English equivalents. They may best be put in tabular form, with the designation proposed by me at the left of the author-names, and with a number for convenient reference. All are ascribed to Buddhaghosa,[35] excepting ten. Of these ten commentaries, six (to wit, numbers 7, 8, 10, 11, 12, 13), all bearing the title Paramattha-dīpanī, and no. 19, are ascribed to Dhammapāla,[36] and the remaining three are ascribed, one (no. 15) to Upasena,[37] one (16) to Mahānāma,[37] and one (18) to Buddhadatta.[38]

The following list accords with that of Childers, Dictionary, p. 67, except at numbers 8 and 13. To the Itivuttaka-commentary (no. 8) he gives the name Abhidhammattha-dīpanī, and he omits the Therīgāthā-commentary (no. 13), perhaps by oversight.

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[34] Or 13, if we count the comm's on the last five of the seven books of the Abhidhamma as one; or 11, if we count all seven as one.

[35] Gnvŋ., pp. 59, 68.

[36] Gnvŋ., pp. 69, 60.

[37] Gnvŋ., pp. 70, 61.

[38] Gnvŋ., pp. 59–60.

Comment 3. The triliterals. — For Vinaya, the use of Vin. is discussed at pages 679, 680. Next, Dhamma-saŋgaṇi. The biliteral Dh. stands for the oft-cited Dhamma-pada: and the s which converts that biliteral into the triliteral Dhs. is the only natural differential for Dhamma-saŋgaṇi. — For Vibhañga, Canon 6 forbids the use of Vibh. as a triliteral and of Vib. with split digraph, and so we are forced to take Vbh. — Coming to the difficult Kathā-vatthu: Kthv. counts as 4 letters and Ktv. is barred (Canon 6); Kav. and Kāv. are not suggestive; and since uncombined K does not stand for anything else than Kathā, it would seem that Kvu. is the best available designation (in spite of the vatthu and the v: see page 681). — Next, Dhātu-kathā. That Dh. should stand for two things, Dhamma- and Dhātu-, is a pity, but it does not stand for anything else; and, of the alternatives Dhā. and Dht. for Dhātu-kathā, neither seems to me better (Canon 2) than the only other feasible one, Dhk. — For Puggala-paññatti, Pug. is more suggestive than Pup., although both are very doggy. — For Yamaka, Yam. is satisfactory. — For Paṭṭhāna, Paṭ. is much too vague; despite Canon 10, we must needs take the initial of each syllable and combine them to Pṭn.

Comment 4. The quadriliterals. — The Vaŋsas are so numerous that their designations should unquestionably be uniform (Canon 5), and nothing could possibly be more suggestive than vŋ. Far the most important are Dīpa-vaŋsa and Mahā-vaŋsa. It has been made amply clear that D. and M. may be put to much better use as designating Dīgha and Majjhima. With due regard to Canon 5, nothing could be more natural and suggestive than Dpvŋ. and Mhvŋ. The designations Tpvŋ. and Dṭvŋ. involve split digraphs: hence Thvŋ. and Dāvŋ.; and, by analogy with Dāvŋ., rather Sāvŋ. than Ssvŋ. (cp. Canon 10).

Of the "other books," Visuddhi-magga, Milinda-pañha, and Abhidhāna-ppadīpikā are by far the most important. Although I have for years myself written Vm. for Visuddhi-magga, I think, since a quadriliteral is required, that Visu. is more suggestive than Vism. (which makes us think of vismaya) or Vsdm. or Vsmg. For Milinda, I choose Miln., rather than Mlnd. or Mlnp., as being more suggestive and because we are so familiar with Mil. For Abhp. and Abhs., see p. 681. On more than one account, Khus. and Mūls. are better than the Khus. and Mūl. of List 4. The commentaries are much better designated in the manner explained below. The very familiar and important Asln. may perhaps be tolerated, and perhaps also Samp. and Sumv.; but, on the whole, Dhs.cm. and Vin.cm. and D.cm. are vastly better. For the rest, comment is dispensable.

Counting these two couples (Th.1 and Th.2; Nd.1 and Nd.2) each as one, and rightly so, it then appears that of all the texts of the Khuddaka-nikāya only two, namely Peta-vatthu and Paṭisambhidā-magga, collide with their initials. Were these initials digraphs, my system would be wrecked. Happily they are not. It remains to difference them. — First, the differential for Peta-vatthu. The objections to vatthu and vaŋsa and their initial v (p. 681) are cogent, and I think Pe. is more suggestive than Pv. (Peṭakopadesa cuts no figure) and Bu. than Bv. (for Buddha-vaŋsa). But since Peta-vatthu follows Vimāna-vatthu in the usual lists and the two titles thus form a couple, I waive my objections and tolerate Vv. and Pv. This I do the less reluctantly, because Vv. and Pv. already appear in several of the older lists, because (despite the biliterality) Vi. may suggest Vinaya or Vibhaṅga and Vm. may suggest Visuddhi-magga, and because PV. (for Vin. 5) is now, I hope, quite out of court. — Secondly, the differential for Paṭisambhidā-magga. Neither Pa. nor Pt. will serve, since both are too vague. Possible are Ps. and Pm. Since this work is very often spoken of (so by Buddhaghosa) simply as the Paṭisambhidā (without magga), I deem Ps., despite its biblical suggestion (Psalm), preferable to Pm. — But this paragraph shows well how intricately the pros and cons interlace, and how full of compromises a system of this kind must needs be.

Khuddaka-pāṭha and Dhamma-pada: for these, the designations Kh. and Dh. go back to Childers, and Dh. appears in Lists 5 and 7; we need not regret that p does not figure in them (p. 681). For Udāna, Ud. is on several accounts better than U. For Iti-vuttaka, the designation "It." is better than "I.," which suggests the Roman numeral for 1 and looks too much like J. Nor will any one prefer Iv. to It. For Sutta-nipāta, Sn. is surely better than Su. (p. 680) or St. The next four in Table I have just been discussed. For Jātaka, the designation Jā. is better (all things considered, p. 678) than J. and more suggestive than Jt. For the next two, Nd. and Ps., see pages 687, 688. There is no objection to Ap. for Apadāna. On Bu. we have already touched. The use of p as non-initial part of a combination (compare page 681) in Cp., for Cariyā-piṭaka, might pass, if Cp. were not used also for the English word "Compare." The combination "Cp. Cp." in the sense of "Compare Cariyā-piṭaka" would be an inexcusable stone of stumbling. Juxtaposed *c* and *r* will not be mistaken for a Pāli phonetic combination (as in English *cross*), but will naturally be pronounced *char;* and since Ca. is vague, and identical with the enclitic conjunction ca (τε), I think Cr. is the best available biliteral for this text.

typographically awkward.[31] Thus the **V. V.** of the Lists will at once be condemned by any one who has the "typographic sense;" and similarly **A. A.** (cf. List 10) for Aṅguttara-aṭṭhakathā.

The capitalization of the second of the initials representing the two members of a compound is unnecessary, and, as increasing sensibly the number of obtrusive characters on the printed page, gives to it an unrestful effect (macht das Satzbild unruhig: Baensch-Drugulin). This effect is aggravated by the interposition of a period between the two. Examples: **P. V.**, **K. V.**, **P. P.**; so **P. V. A.**, **K. V. A.**, List 4.

Comments on the abbreviations in their order. — Much of what is to be said in justification of this or that abbreviation has already been said by way of illustration of this or that canon. What remains I give in the order of the abbreviations concerned (as they appear in Table I) and with references to previous discussion.

Comment 1. The uniliterals. — D. and M. and S. and A. for the first four Nikāyas. Other things being equal, the fewer the letters, the less suggestive is the abbreviation. Hence the class of uniliterals should be kept within the very narrowest limits. They are in fact so few and have to be used so often, that they will be easily remembered. To maintain their efficiency, abbreviations, like domestic plumbing, should be used constantly.

Comment 2. The biliterals. — Texts of the Khuddaka-nikāya. Although the diaskeuasts have grouped the Stanzas of the Male Elders and the Stanzas of the Female Elders separately as Thera-gāthā and Therī-gāthā, the two texts are so truly one[32] that they should certainly be designated by the same letters, Th. The difference is most clearly and unobtrusively indicated by an appended Arabic 1 and 2; and is so indicated in fact by Davids and Carpenter in the edition of Sumaṅgala-vilāsinī, p. xvii. The like applies to the texts of the Exposition, Major and Minor; it is far more practical to have the differentiated term, Niddesa, come first, and the differentials last[33] (especially since those differentials are Mahā and Culla: see p. 681). Even Nd.$^{\text{m.}}$ and Nd.$^{\text{c.}}$ or Nd.$^{\text{maj.}}$ and Nd.$^{\text{min.}}$ are better than MN. and CN.; but Nd.1 and Nd.2 are better still.

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[31] Or can be read as an ill-sounding or unpleasantly suggestive combination. To me, at least, the abbreviations Thag. and Thīg. (sic) have always suggested cruel Thugs and Dacoits rather than gentle Theras and Therīs.

[32] Observe that Buddhaghosa (D.cm., I. 15) says Vimāna-peta-vatthu Thera-therī-gāthā, treating these four texts of the second pentad as two groups.

[33] This principle is duly recognized by the administrations of the great metropolitan post-offices. Thus we have "London EC.," "Berlin SW.," not "EC. London," "SW. Berlin."

Canon 7. — With due regard to Canon 1, the designations should be as brief as possible.

"Brevity is the soul of wit," and, no less truly, the soul of an abbreviation. So no comment is needed upon this canon, but rather only upon its limitations. "What then," asks the etymologist, "what do you look for in an ab-*breviation*, if not for *brevity?*" Much, I answer, and above all things, ready suggestiveness (Canons 2 and 5). Brevity gained at a sacrifice of easy intelligibility is to be condemned absolutely. — To illustrate: the use of P., V., K., and Y., for Paṭṭhāna, Vibhaṅga, Kathā-vatthu, and Yamaka (as in Lists 2, 5), is most objectionable. Two counts lie against P.: it stands for Paṭisambhidā-magga and Parivāra (and seven other pertinent elements: p. 681); and it is not triliteral (p. 683). That the brevity of M. and D. for Mahā-vaŋsa and Dīpa-vaŋsa is too dearly bought appears from pages 681, 677. Although we are very familiar with Mil. for Milinda-pañha and there is nothing else that it can be mistaken for, we willingly add an n, simply to make it a quadriliteral, and for no other reason. As to J. for Jātaka, see pp. 677–678. Indeed, the comments on the previous canons abundantly illustrate the limitations of this one.

The foregoing canons state the more important principles which should govern the determination of a workable set of abbreviations. Clearly, they are well worthy of the consideration of a scholar. Several minor prescriptions, however, touching lesser but yet essential matters, ought not to go unheeded.

Canon 8. — Arbitrary distinctions. These should be carefully avoided. Thus the use of J. for the stanzas of the Jātaka alongside of Jāt. for the commentary (as in List 4) is too arbitrary, and needlessly so (use Jā. and Jā.cm.). This prescription condemns also the use, side by side, of SN. and Sn. in List 8.

Canon 9. — Alternative designations for the same text should be strictly excluded. Thus the "V. or Vin." of List 4, the C. and Vin. [2] for Culla-vagga of List 6, and the "Sās. or Sās. V." of List 13, are objectionable. So the "CAR. oder AR." of List 12.

Canon 10. — Typographical form should be duly regarded. The chosen designations should avoid, as far as possible, the use of letters requiring diacritical marks (macrons, dots, etc.). All the abbreviations of one or of two or of three letters here proposed by me do in fact dispense with diacritics, excepting Jā. for Jātaka and Pṭn. for Paṭṭhāna. The quadriliterals show five macrons and one dotted ṭ, namely, in Dāvŋ., Sāvŋ., Jinā., Mūls., and Yogā., and Peṭa., all unimportant texts.

They should also avoid the juxtaposition of elements which are oldest strata of redactional precipitates, and no question at all that the Abhidhamma represents the latest.[28]

Buddhaghosa, in explaining at D.cm., I.22, how the Tipiṭaka, as an aggregation of collections (nikāyas), may be regarded as five-fold, says that it consists of the Dīgha, Majjhima, Saŋyutta, Aṅguttara, and Khuddaka, and proceeds: "Apart from the four Nikāyas, all the rest, namely, the entire Vinaya and Abhidhamma, and the fifteen aforesaid works, Khuddaka-pāṭha etc., are the word of Buddha." Then, continuing with a verse of "the ancients," he says: "And apart from these four Nikāyas, Dīgha and so forth, the words of Buddha other than those, are held to be the Khuddaka-nikāya."

Ṭhapetvā caturo p' ete nikāye Dīgha-ādike
Tad-aññaŋ Buddha-vacanaŋ Nikāyo Khuddako mato ti.[29]

The Gandha-vaŋsa expressly says [30] that the Khuddaka-nikāya consists of the usual 15 texts plus the Vinaya and the Abhidhamma. Accordingly, if we take Sutta and Vinaya and Abhidhamma as the sequence of the texts in our scheme, doubtless no one will make serious objection.

Quadriliterals for uncanonical texts. — If the scheme thus far has been rightly settled, we need have no hesitation in designating the titles of the post-canonical books by quadriliterals. Herewith are not included the commentaries (especially those of Buddhaghosa and Dhammapāla), which are discussed below.

Canon 6. — A digraph must be counted as two letters, never as one. This rule, as applied to Canon 5, is so absolutely essential and has been so wholly ignored, that it demands special and separate mention. If, on looking at an abbreviation, we must stop and go through the mental process of considering whether two separately printed characters are to be counted as one or as two, it is obvious that the advantage of a scheme of abbreviations in which the number of letters employed is highly significant, is wholly lost. This will be clear to any one upon examining List 4 as it appears in the original typography, JPTS.1896, pages 103–106. Here D. and Dh. alike are to be understood as uniliteral; Vbh. as biliteral; and Thig. as triliteral, — all being Piṭaka texts and intended to be designated with one letter. — Digraphs must on no account be split, as in List 1, where Abhidhāna-ppadīpikā is designated by Ab.

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[28] Cp. Pischel's Buddha, p. 6.

[29] D.cm., I. 23; repeated for substance, Dhs.cm., p. 26; also Sdhs., p. 30, in JPTS.1890. Cp. also Childers, Dic'y, p. 282.

[30] Ed. JPTS.1886, p. 57, top; or Minayeff, Recherches, p. 237.

however (at pages 680, 682), that Vin. is by all odds the best designation for Vinaya; and (at p. 682) that each one of the first four Nikāyas demands one comprehensive designation, and that the texts of the fifth Nikāya stubbornly resist any such treatment. Taken by and large, the first four Nikāyas are surely the longest and most important texts of the second and third Piṭakas. Convenience and economy therefore dictate for the first four Nikāyas the briefest possible designations, that is, uniliterals; and (by extraordinarily good luck) the names of these four begin each, not only with a different letter, but also with an Oriental character for the transliteration of which only one Roman letter is needed and not a digraph. We shall surely make no mistake in settling upon D., M., S., and A. as designations for the Dīgha-nikāya, Majjhima-nikāya, Saŋyutta-nikāya, and Aṅguttara-nikāya respectively. So far, so good.

Biliterals for the Khuddaka-nikāya. — Coming now to the Khuddaka, the case is not so simple. The general title of the Nikāya cannot possibly be abbreviated by a single Roman letter, since it begins with kh. And even if it could, each of the 15 titles of the constituent texts demands (as we saw at p. 683) an independent abbreviation. Moreover, of those titles not less than four begin with a sound requiring a digraph (kh, dh, th) for its transliteration. It is evident that for the books of this Nikāya naught less than a biliteral will suffice. But (once more) this necessity is a very lucky one as fitting admirably into a scheme which modulates smoothly from uniliterals up (or down) to quadriliterals.

Triliterals for Vinaya and Abhidhamma. — For Vinaya texts we have already (p. 680) settled on Vin. Next, the seven texts of the Abhidhamma-piṭaka. The titles of two begin with Dh, while Vibhaṅga, like Vimāna-vatthu, begins with Vi. It is obviously impossible to give to all seven a distinctive designation of less than three letters without abandoning the whole system.

Excursus: Sequence of the Piṭaka-texts. — The sequence in which the Piṭakas are usually named is Vinaya, Sutta, Abhidhamma. Thoroughly cogent reasons, however, compel us to put the Sutta-piṭaka, with its uniliteral and biliteral designations, at the beginning of our scheme. After it comes naturally the Vinaya, with its triliteral designation; and along after the Vinaya comes the Abhidhamma, also with its triliteral designations. But this order (Sutta, Vinaya, Abhidhamma) is one which we may well regard as according with that of the historical development of the several parts of the canon. For there can be little question that the Sutta-piṭaka[27] represents in general the constituent texts of which do not by any means form one larger coherent text. That collection is an omnium-gatherum. As a whole, it differs greatly from each of the units that make up the four Nikāyas; and so does each of its 15 constituent parts. These parts, moreover, differ so, each from the others, that the title of each requires to be taken account of separately.

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[27] Cp. Neumann, Majjhima, **1**, pp. x.-xi.

Canon 5. — The abbreviations should conform to some easily remembered general scheme of a set of classes. Unquestionably, for Pāli texts, the best scheme is one that shows at a glance the class to which a given text belongs by the number of letters employed in abbreviating its title: that is, a scheme of uniliteral, biliteral, triliteral, and quadriliteral abbreviations, — the abbreviations for each text of a given class consisting uniformly of one letter, of two, of three, or of four.

This canon is designed to increase the ready suggestiveness of the abbreviations, and so is close akin with Canon 2. The traditional classification[24] of the Pāli texts is such that they lend themselves with great ease to this scheme.

Davids's guiding principles are reprinted above, at p. 668, and his proposals appear in my Table II (p. 676), as List 4. For Vinaya he gives on p. 104, as alternative designations, "V. *or* Vin." Of the other 26 abbreviations of the names of Piṭaka texts, just 10 are uniliteral.[25] Of the remaining 16, 8 are biliteral, 6 are triliteral, and 2 are quadriliteral; or, if we count (as we most certainly should not: Canon 6) the digraphs kh, th, dh, bh, each as one letter, then 13 are biliteral and three are triliteral. — The underlying idea of the proposals of Davids is most valuable, and to his proposals I am indebted for the suggestion of my own. On the other hand, the actual working out of his own ideas is very unpractical and fragmentary. I am absolutely certain that his list of 1896 (List 4) would prove highly unsatisfactory for lexicon use.

Reverting to Canon 5. As that eminent and sagacious mariner, Cap'en Bunsby, justly observes,[26] "The bearings of this observation lays in the application on it." To the application of my observations, accordingly, let me address myself.

Uniliterals for the first four Nikāyas. — These first. If we followed the usual order of the books, we should designate Vinaya texts with uniliterals, Suttanta texts with biliterals, Abhidhamma texts with triliterals, and uncanonical texts with quadriliterals. We have seen,

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[24] See Minayeff, Recherches sur le Bouddhisme, pp. 257–259.

[25] The principle of 1896 was to designate the Piṭaka texts, "as far as possible," with one letter, and later texts with three.

[26] Dickens, Dombey and Son, **1**, chap. xxiii. Cp. **2**, chap. xxx.

Ambiguous combinations. — It is bad enough, albeit unavoidable, to use ambiguous single letters; but it is inexcusable to use ambiguous combinations.[23] Nevertheless, we find SN. for Saŋyutta-nikāya in List 8; and S.N. or SN. for Sutta-nipāta in Lists 2, 3, 4, 5, 6, 9, 10. Again, in List 5, MN. means Mahā-niddesa; while in Lists 2, 8, 9, 11, and 12 M.N. or MN. means Majjhima-nikāya. In List 2, Ps. means Paṭisambhidā-magga; while in List 8 it means Papañca-sūdanī. In List 8, Sv. = Sumañgala-vilāsinī; in List 12, SV. = Sutta-vibhañga. In List 7, Mp. = Mahāparinibbāna-sutta, and in List 8, Mp. = Manoratha-pūraṇī; but it might just as well mean Milinda-pañha, and MP. does so in List 12. In List 10, Mhv. means Mahā-vastu, although it suggests Mahā-vaŋsa quite as easily and is in fact used in that sense by Davids and Carpenter, in Sumañgala-vilāsinī, I., p. xvii. In List 5, PV. means Parivāra; but P.V. or Pv. means Peta-vatthu in various lists. Of the ambiguity of Abh. I have just spoken. If these things must needs be, then life is too short for us to spend it in trying to hold the eel of science by the tail.

Canon 4. — The individual titles of briefer texts which together form one larger coherent text with a comprehensive title, should be ignored, and the abbreviation should be based on the comprehensive title.

To illustrate: In List 3, as designations of parts of the Sutta-vibhañga of the Vinaya-piṭaka, we find Bhnīpār. for Bhikkhunī-pārājika, BhnīS. for Bhikkhunī-saŋghādisesa, and Bhnīpāc. for Bhikkhunī-pācittiya; but we are obliged to interpret, ex silentio, simple Pār. and S. and Pāc. as Bhikkhu-pārājika, etc. Although to these are added the very objectionable N. for Nissaggiya and P. for Parivāra, yet, even so, by no means all the parts of the Sutta-vibhañga are covered. Nor do the designations suggest the volume in which we are to look for the designated text. The texts themselves are lexicographically and otherwise so important that the constant recurrence of such illogical and blind and cumbrous abbreviations would be an annoyance as intolerable as it is gratuitous. The last volume of Oldenberg's Vinaya had appeared five years before List 3. Surely the logical and suggestive and simple Vin. 3, Vin. 4, Vin. 1, Vin. 2, Vin. 5 would have been vastly better, as we have already shown in another connection, pp. 679–680.

That this canon applies to the Vinaya-piṭaka and (see p. 678, ¶ 2) to the first four Nikāyas is as clear as sunshine. It is just as clear that it does not apply to the fifth, the Khuddaka-nikāya, the briefer general and therefore lacking in suggestiveness, and all this apart from the fact (see below) that the initials P and N and S stand for so many other important Pāli words.

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[23] Unless unavoidable, as in the digraph Dh for Dhamma- and Dhātu-, p. 689.

Secondly, and for like reasons, it is even more important that Mahā and Culla should not be included among the elements of text-titles to be abbreviated. The Bhikkhu-vibhaṅga, vol's 1 and 2 of the Siamese ed., is called Mahā-vibhaṅga, just as vol's 4 and 5 of the same are called Mahā-vagga. Part 2 of the Dīgha-nikāya (vol. 10) is the Mahā-vagga, and of the ten suttas of that vagga, the names of not less than seven begin with Mahā. Part 2 of the Khuddaka-nikāya (vol. 26) is the Mahā-niddesa; and the first of the three divisions of part 4, Paṭisambhidā-magga (vol. 28), is named Mahā-vagga. In short, there are so many Mahā-this's and Culla-that's, that, even if you are right in *taking* M *as* = Mahā and C *as* = Culla, the *suggestiveness* of the rightly guessed words is practically nil.[22] Hence the M. and C. of List 2 (for Vin. 1 and Vin. 2) are to be condemned without reservation, as are also the M. and Mah. of List 3 (for Vin. 1 and Mahā-vaŋsa).

Similarly, the terms vatthu and vagga and vaŋsa are objectionable. Hence I have preferred Bu. to Bv. for Buddha-vaŋsa. In like manner Abhi were better avoided. In List 2, Abh. means Abhidhammattha-saŋgaha; but in List 5 Abh. means Abhidhāna-ppadīpikā, for which List 1 has Ab. For these two words, both important, I do not see how the use of Abh can be avoided, and it is tolerable if we add for the one an s (Abhs.) and for the other a p (Abhp.).

Not only are certain words to be avoided; certain letters also are either to be avoided or else used with caution. This will be clear to any one on glancing over the table (II, p. 676) of what the American newspapers call "deadly parallels." The letter P has 10 meanings and stands for Paṭisambhidā-magga, Paṭṭhāna, and Parivāra; and (less objectionably, because in combination) for Peta-, Puggala-, and Pañca-, and also for -pāṭha, -pada, -piṭaka, and -paññatti. — Again, V has 8, and stands for Vibhaṅga and (List 4) Vinaya; for Vimāna-; for -vatthu, -vaŋsa, -vibhaṅga, -vagga, and (!)-vāra. — The letter S has 7 values: Saŋyutta-nikāya, Saŋghādisesa (! List 3); Sutta-; -sambhidā-magga, -saŋgaṇi, -saŋgaha, -sikkhā. — The letter N, or even the combination Ni, has 4: to wit, Niddesa and (List 3) Nissaggiya; -nikāya and -nipāta. And so has M, namely, Majjhima-nikāya, Mahā-vagga (= Vin. 1), and Mahā-vaŋsa (List 13); and -magga. In List 12, moreover, M. means (not Majjhima-nikāya, but) Münze; "aber natürlich auch Meile und Mitte." Finally, C means Cariyā-piṭaka and Culla-vagga; and I (besides suggesting the Roman numeral I) is too much like J.

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[22] List 7 employs Mp., Mv., M-vaŋsa, and M-vastu.

two groups Bhikkhu-vibhaṅga and Bhikkhunī-vibhaṅga, which two might be styled Mahā-vibhaṅga (it is in fact so styled) and Culla-vibhaṅga (with a propriety no less than that with which the two khandhaka-groups are styled Mahā-vagga or MV. and Culla-vagga or CV.) and accordingly designated also as MV. and CV., a futile duplication.

Now the Vinaya-piṭaka forms a clean-cut body of treatises on the perfectly definite subject of discipline (which is the natural and usual meaning of the word vinaya), and it forms a clean-cut group of volumes in all the three editions. Canon 1 bids me ask first whether anything is gained by making the abbreviations such that they will tell us whether a given passage is in the Rule-Division (Big or Little) or in the Little-Division (schlechthin); and since I must answer No, and since, to a Pāli scholar, Vin. readily and naturally suggests Vinaya, and since (Canon 3) it does not suggest anything else, and since the uniliteral V. meets neither of the last two requirements, and since the biliteral Vi. might easily be mistaken for the Vibhaṅga of the Abhidhamma, — therefore there is (considering Canons 4 and 5) no choice left us[20] but to take the admirably suggestive Vin. as comprehensive designation of the whole Vinaya-piṭaka, and to distinguish its different parts simply by the volume-numbers.[21]

About so small a matter, my colleagues will ask, why so much wordy talk? kiṁ anenātivistareṇa? And I answer, Nāivātra doshaḥ, the case is a typical one. It clearly shows how many-sided is the circumspection which may be used in the choice of fit designations. Let the scholar who has never been vexed and whose time has never been wasted by the lack of such circumspection in his predecessors, tell me that such circumspection is profitless!

Canon 3. — In the chosen designations, elements which are not readily suggestive or which are easily susceptible of several interpretations, should be studiously avoided.

This is indeed a corollary of Canon 2, or also, in some aspects, so to say, the converse of Canon 2; but the violations of Canon 3 have been so many and so gross as to call for special illustration and express condemnation.

To begin with, I need hardly say that the words piṭaka and nikāya and sutta should not be used as the basis of an abbreviation, for this practice has gone out of vogue, and rightly, since the words are far too

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[20] Assuredly, no one would prefer Vna. or Vny. to Vin.

[21] This plan admits of easy reference to each of the editions, ed. O., ed. B., ed. S.

The maker of a list must look a good bit into the future and scrupulously avoid methods that are sure to waste the time and patience of his colleagues for years to come. Each of these wastes is small, a fraction of a minute or more, but the wastes are innumerable, and in the aggregate large, and wholly needless.

Canon 2. — The abbreviations of text-titles should be so readily suggestive as to be easily understood, — if possible, without any explanation, or, at most, with a very little explanation once given.

The phrase "without explanation" means, of course, without explanation to those who know the names of the texts. This canon I deem the most important of all, next after Canon 1. Strictly, Canons 3 and 4 and 5 are ancillary to Canon 2; but there is so much to say in illustration and enforcement of Canons 3 and 4 and 5 that they may best be set up by themselves.

To illustrate Canon 2, take the parts of the Vinaya-piṭaka, namely: Sutta-vibhañga, "Rule-Division," Mahā-vagga, "Big (Group or)-Division," Culla-vagga, "Little (Group or)-Division," Parivāra, "Entourage, Following, Appendix." The designations of these parts in List 5 were SV., MV., CV., and PV. It is true that V is the initial of the second part of each of these titles, if we reckon, as we certainly should not, -vāra (as it were, -dix of Appen-dix) as such a part. The uniform second letter would serve to characterize all these four abbreviations as belonging to one group, and so tend (according to Canon 5) to make them acceptable ones, were it not for the fact that V stands for so many extremely common parts of Pāli text-titles or text-divisions (vibhañga, vagga, vatthu, vaŋsa, Visuddhi-magga) as to be readily suggestive of nothing at all in particular. Consider too the unsuggestive vagueness of the meanings of the designations themselves! how palpable it is, if we turn them into English, and use RD., BD., LD., and AD. respectively for Rule-Division, Big-Division, Little-Division, and Appen-dix! Moreover these four groupings do not wholly coincide with the five volumes of Oldenberg's edition and of the Burmese, nor with the eight of the Siamese.[19] Nor do they take account of the accessible text, and (unlike the Cariyā-piṭaka) an exceedingly important one, and likely to be cited oftener even than the Dīgha. The uniliteral designation J. is therefore recommended not only by its brevity, but also by the importance of the text and the extreme frequency of citation. So weighty are these considerations in themselves, that I hesitated for no little time and thought I might treat Jātaka as the sole exception among titles of the Khuddaka-nikāya, and make its designation uniliteral and not biliteral. Finally I became convinced that the practical value of Canon 5 is so great that the considerations just adduced should be allowed no weight at all.

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[19] It is a thousand pities (as we look back!) that Oldenberg inverted the native sequence (3, 4, 1, 2, 5) of the volumes in his admirable and timely edition. — The division of the Vinaya-text into volumes coincides as between Oldenberg's ed. and the Hanthawaddy ed. In the Siamese ed., the Mahā-vagga (Oldenberg's 1) forms vol's 4 and 5, and the Culla-vagga (O's 2) forms 6 and 7, and the Parivāra (O's 5) forms 8. It is otherwise with the Bhikkhu- and Bhikkhunī-vibhañgas: of the latter, the Siamese ed. makes a whole volume (3d); and of the former, it puts kaṇḍas 1–3 into volume 1, and 4–7 into volume 2; while Oldenberg's puts kaṇḍas 1–4 into volume 3, and kaṇḍas 5–7 with all of the Bhikkhunī-vibhañga into vol. 4. In like manner the Hanthawaddy ed.

Once more: Childers's designations of suttas 1 and 16 of the Dīgha-nikāya, to wit, of the Brahmajāla-sutta and the Mahāparinibbāna-sutta, are Br. J. S. and Par. S., and, for Buddhaghosa's commentary on those two suttas respectively, Br. J. S. A. and Par. S. A. Even as late as 1902, List 10 has M. P. S. for the latter sutta. Now undeniably these two are suttas of transcendent importance, and these designations were entirely excusable, or indeed hardly objectionable, in Childers's time, because printed suttas were then so few that no serious complications arose. But if to-day we were to invent analogous abbreviations for the titles of each one of the 34 suttas of the Dīgha and of the 152 of the Majjhima, to say nothing of the multitudinous suttas of the Saŋyutta (7762)[18] and Aṅguttara (9557),[18] the result would prove bewildering, intolerable, futile. We should simply be driven to writing each sutta-title out in full. And yet even this would not suffice: for although "Antelope-shin sutta" (S., I. 16), as a title, is distinctive enough, there are, for instance, more "Loka-suttas" in the Saŋyutta than there are volumes in the edition.

Again: It is inadvisable to lengthen the list of abbreviations by including designations of such small and insignificant texts as the Cha-kesa-dhātu-vaŋsa (11 pages in JPTS.1885).

To make an end: Not even because a given book is of modest compass and purpose may its author disregard this canon. Andersen's Glossary is intended merely for the text of his Reader and of the Dhamma-pada. His abbreviations (List 8) are so few that he might naturally ask, Do they not serve well enough, considering how few the volumes are to which I refer? The answer is Yes, if that is all there is to the question; and a most emphatic No, if you are to use his book (where, for example, SN. means Saŋyutta-nikāya) on the same day with various others (see Table II) in which S.N. means Sutta-nipāta.

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[18] These are Buddhaghosa's numbers (D.cm., I. 23). Mrs. Davids (S., VI. 204–233) indexes about 1150 sutta-titles for the Saŋyutta; and Hardy (A., V. p. vi.) gives 2344 as the number for the Aṅguttara.

List 15 1909 Lanman.	List 2 1886 T. Davids.	List 3 1888 E. Müller.	List 4 1896 T. Davids.	List 5 1898 H. Warren.	List 6 1900 C. Davids.	List 8 1901 Andersen.	List 10 1902 E. Hardy.
Asln.	Asl.		Asl.	Aṭṭh.	Asl.	As.	Asl.
Abhp.				Abh.			
Abhs.	Abh.		Abh.S.		Abh.S.		
Kacc.			Kacc.				
Khus.	Kh.S.		Khus.				
Jinā.			Jin.				Jin.
Nett.	Net.		Nett.			Nett.	Nett.
Peṭa.			Peṭ.				Peṭ.
Pgdp.	Pgd.		Pgd.				
Miln.		Mil.	Mil.	Mil.	Mil.	Mil.	Mil.
Mūls.	Mls.		Mūl.				
Yogā.			Yog.				
Visu.	Vsm.				Vis.M.		Vis.M.
Sdhs.			Sad.S.				Sad.S.
Sank.	San.		San.K.				
Samp.	Smp.	Smp.	Smp.	Sam.			
Sumv.	Sum.	Sum.	Sum.	Sum.	Sum.	Sv.	Sum.

Canons governing the determination of the new abbreviations. — Since the determination has been made with careful consideration of certain principles, it is needful to state them. For convenience of reference, I call them canons.

Canon 1. — Full weight should be given to general considerations of broad scope. —This canon should dominate all the rest. It should be regarded as a paribhāshāsūtram[17] for all that follow.

One such general consideration may be instanced : the designations of the most important texts should be settled first, and those of the rest afterwards, as well as may be with the resources then available (cp. Canon 5). — Of other such let me give examples. Thus brevity (Canon 7) in itself is just as desirable for the designation of the Cariyā-piṭaka as it is for the Dīgha-nikāya. But when we look at the matter from a larger point of view, and consider that the Cariyā-piṭaka is a text as insignificant in its contents as it is in extent, and that, as such, it needs very seldom to be cited, it is palpably injudicious to assign to it the great distinction of referring to it by a single letter (C). And the like holds for B and the Buddha-vaŋsa. This distinction must not be cheapened ; it must be reserved for the most important and most frequently cited texts of the four great Nikāyas, to wit, Dīgha, Majjhima, Saŋyutta, and Aṅguttara.

To take a different example. Thanks to Fausböll, the Jātaka is a very

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[17] "A general rule or definition applicable throughout a whole system, and more binding than any particular rule." Max Müller, SBE., xxx. 311.

TABLE II. — PREVIOUSLY PROPOSED ABBREVIATIONS COMPARED.

List 15 1909 Lanman.	List 2 1886 T. Davids.	List 3 1888 E. Müller.	List 4 1896 T. Davids.	List 5 1898 H. Warren.	List 6 1900 C. Davids.	List 8 1901 Andersen.	List 10 1902 E. Hardy.
D.	D.		D.	D.	D.	DN.	D.
M.	M.N.	Majjh.	M.	M.	M.	MN.	M.
S.	S.	Saṃy.	S.	S.	S.	SN.	S.
A.	A.	A.	A.	A.	A.	AN.	A.
Kh.	Kh.P.		Kh.P.	KhP.		Khp.	Kh.P.
Dh.	Dh.P.	Dhp.	Dhp.	Dh.	Dhp.	Dhpd.	Dhp.
Ud.	Ud.	Ud.	Ud.	U.			Ud.
It.	I.		It.	I.		It.	It.
Sn.	S.N.	S.N.	S.N.	SN.	S.N.	Sn.	S.N.
Vv.	V.V.		V.V.	VV.			V.V.
Pv.	P.V.		P.V.	PeV.		Pv.	P.V.
Th.1.	Th.I.	Th.I.	Thag.	Th.		Th.	Thag.
Th.2.	Th.II.	Th.II.	Thig.	Th.		Thī.	Thig.
Jā.	J.	Jāt.	J.	J.	Jāt.	Jāt.	Jāt.
Nd.1.	N.		N.	MN.			
Nd.2.	N.		N.	CN.			
Ps.	Ps.		P.	PS.			
Ap.	Ap.		Ap.				
Bu.	B.	Bv.	B.	BV.			B.
Cr.	C.P.	Cariyāp.	C.	CP.			C.
Vin.3.	S.V.	Pār.,S.,N.	V.3.,V. *or* Vin.	SV.	Vin.3.	Vin.3.	Vin.3.
Vin.4.	S.V.	Pāc.[16]	V.4.	SV.	Vin.4.	Vin.4.	Vin.4.
Vin.1.	M.	M.	V.1.	MV.	Vin.1.	Vin.1.	Vin.1.
Vin.2.	C.	C.	V.2.	CV.	Vin.2. *or* C.	Vin.2.	Vin.2.
Vin.5.	Pr.	P.	V.5.	PV.	Vin.5.	Vin.5.	Vin.5.
Dhs.	Dh.S.		Dh.S.	DhS.	Dh.S.		Dh.S.
Vbh.	V.		Vbh.	V.			
Dhk.	Dh.K.		Dh.K.	DhK.	Dh.K.		
Pug.	P.P.	P.P.	P.P.	PP.	P.P.		P.P.
Kvu.	K.		K.V.	K.	K.V.		K.V.
Yam.	Y.		Y.	Y.			
Pṭn.	P.		Paṭ.	P.			
Anvŋ.	An.V.		An.V.				
Gnvŋ.	G.V.	Gv.	G.V.	Gan.			G.V.
Thvŋ.			Thpv.				
Dāvŋ.	D.V.		Dāṭh.				
Dpvŋ.		Dīp.	Dīp.	Dīp.			Dīp.
Mhvŋ.		Mah.	Mah.	Mah.	Mah.		
Sāvŋ.	Sās.		Sās.				Sās.

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[16] Also Bhnīpār., BhnīS., Bhnīpāc. for parts of Bhikkhunī-vibhañga.

dhist Psychology (translation of the Dhs.), page xiii. This is an improvement in using Vin. for Vinaya-piṭaka.

List 7, year 1900, is that given by Jyunjiro Takakusu in his Pāli Chrestomathy, Tokio, 1900, page 129.

List 8, year 1901, is the one given by Dines Andersen in his Pāli Reader, part 1, page 131.

List 9, year 1901, is chiefly a list of translations referred to by Mrs. Bode in her Index to Pāli words discussed in translations, JPTS. 1901, p. 3.

List 10, year 1902, is the one given by Edmund Hardy in his Netti-pakaraṇa, page v. Both Lists, 9 and 10, adhere to the improvement (Vin.) of List 6.

List 11, year 1902, is the one given by R. O. Franke in his Geschichte und Kritik der einheimischen Pāli-grammatik und -Lexicographie, pages 97–99. The scope of this and the three next mentioned works is such that it is not fair to judge the system of abbreviations as if the works in question had the needs of a lexicographer primarily in view. But even so, it is desirable that, in technical works on Pāli, the most important parts of the Pāli literature should have uniform designations.

List 12, year 1902, is the one given by R. O. Franke in his Pāli und Sanskrit, pages 171–174. Here M. = Münze ; but see note to List 11.

List 13, year 1905, is a list given by W. Geiger in his critical essay entitled Dīpavaŋsa und Mahāvaŋsa, Leipzig, 1905. See note to List 11.

List 14, year 1908, refers to the abbreviations and sigla codicum given by Geiger in the course of his introduction (pages V–XI and LVI) to his PTS. ed. of the Mahāvaŋsa, 1908.

List 15, year 1909, is the one proposed to my colleagues by me in this article in the hope of receiving from them any adverse criticisms which they may be kind enough to send me. For convenience, this list is printed in the left-hand column of Table II.

Previously proposed abbreviations. — The critic who would pass judgment upon the abbreviations proposed by me, and upon the principles which guided me in determining them, ought first to consider the various sets of abbreviations previously put forward by Pāli scholars, and the principles (so far as there were any) by which those scholars were guided. With this in mind I studied half a score of such lists, or more, and found, in the first place, that there were almost no such guiding principles, and, in the second, that no argument for a thoroughgoing discussion of the subject could be more convincing than a simple typographical juxtaposition of some of the abbreviations of some of these lists with all their maddening perplexities.

Bibliography of 14 lists of abbreviations. — The necessary bibliographical notes for each list follow, with certain general comments. The abbreviations themselves (not all) will then be tabulated in Table II, p. 676. Specific comments on this and that one may best be given in the notes on the Canons, below.

List 1, year 1872. Given by Childers at the beginning of the first half of his Dictionary. At that time very few canonical texts had been printed in Europe, so that this list contains hardly more than two abbreviations (Dh., and the unimportant Kh.) which can now be used to advantage. The rest (like Das. for Jā.iv.124 or Ten J. for Ten Jātakas) are for the most part antiquated.

List 2, year 1886. Proposed by Rhys Davids, JPTS.1886, pages xiii–xv. For this list the author states a "guiding principle" (reprinted by me at p. 668, above); but the principle is too loose and too loosely followed.

List 3, year 1888. Given by Edward Müller in his "Pāli proper names," JPTS.1888, p. 106. This list is to be disapproved almost in toto.

List 4, year 1896. Proposed by Davids, JPTS.1896, pages 102–106, in an article entitled "Abbreviations of titles of Pāli books." So far as I know, this is the earliest article devoted expressly to this subject. In the preface he says: "The principle adopted is that all Piṭaka texts should be designated, as far as possible, with one letter; and later texts with three letters." (See p. 668, above.) The list in fact departs much too far from this principle (see p. 683, below). Apart from D., M., S., A. for the four Nikāyas, very many of its designations need to be revised typographically and otherwise.

List 5, year 1898 or thereabouts, is the unprinted list prepared by Henry Clarke Warren for use in his edition of the Visuddhi-magga. In its entirety this list also is far from acceptable.

List 6, year 1900, is the one given by Mrs. Rhys Davids in her Bud-

TABLE I. — NEWLY PROPOSED ABBREVIATIONS, UNILITERALS, ETC.

1. Uniliterals. — First four Nikāyas.

D.	Dīgha-nikāya.	[Add the number of the volume (1, 2, 3) and page.]
M.	Majjhima-nikāya.	[Add the number of the volume (1, 2, 3) and page.]
S.	Saŋyutta-nikāya.	[Add the number of the volume (1, 2, 3, 4, 5) and page.]
A.	Aṅguttara-nikāya.	[Add the number of the volume (1, 2, 3, 4, 5) and page.]

2. Biliterals. — Khuddaka-nikāya: 15 books, 3 pentads.

Kh.	Khuddaka-pāṭha.
Dh.	Dhamma-pada.
Ud.	Udāna.
It.	Iti-vuttaka.
Sn.	Sutta-nipāta.

Vv.	Vimāna-vatthu.
Pv.	Peta-vatthu.
Th.1.	Thera-gāthā.
Th.2.	Therī-gāthā.
Jā.	Jātaka.

Nd.	Niddesa.*
Ps.	Paṭisambhidā.
Ap.	Apadāna.
Bu.	Buddha-vaŋsa.
Cr.	Cariyā-piṭaka.

3. Triliterals. — Vinaya-piṭaka. Abhidhamma-piṭaka.

Vinaya. Oldenberg's ed.: 5 vols.

Vin.3.	Pārājika, etc.
Vin.4.	Pācittiya, etc. Bhikkhunī-vbh.
Vin.1.	Mahā-vagga.
Vin.2.	Culla-vagga.
Vin.5.	Parivāra.

Abhidhamma: 7 books.

Dhs.	Dhamma-saŋgaṇi.
Vbh.	Vibhaṅga.
Dhk.	Dhātu-kathā.
Pug.	Puggala-paññatti.
Kvu.	Kathā-vatthu.
Yam.	Yamaka.
Ptn.	Paṭṭhāna.

4. Quadriliterals. — Uncanonical books.

1. The Vaŋsas.

Anvŋ.	Anāgata-vaŋsa.
Gnvŋ.	Gandha-vaŋsa.
Cuvŋ.	Culla-vaŋsa.
Thvŋ.	Thūpa-vaŋsa.
Dāvŋ.	Dāṭhā-vaŋsa.
Dpvŋ.	Dīpa-vaŋsa.
Povŋ.	Porāṇa-vaŋsa.
Bovŋ.	Bodhi-vaŋsa.
Mhvŋ.	Mahā-vaŋsa.
Sāvŋ.	Sāsana-vaŋsa.

2. Other books.

Asln.	Attha-sālinī = Dhs.cm.
Abhp.	Abhidhāna-ppadīpikā.
Abhs.	Abhidhammattha-saŋgaha.

Kacc.	Kaccāyana's Grammar.
Khus.	Khudda-sikkhā.
Jinā.	Jinālaŋkāra.
Jinc.	Jina-carita.
Nett.	Netti-pakaraṇa.
Peṭa.	Peṭakopadesa.
Pgdp.	Pañca-gati-dīpana.
Miln.	Milinda-pañha.
Mūls.	Mūla-sikkhā.
Yogā.	Yogāvacara Manual.
Visu.	Visuddhi-magga.
Sdhs.	Sad-dhamma-saŋgaha.
Sank.	Sandesa-kathā.
Samp.	Samanta-pāsādikā = Vin.cm.
Sumv.	Sumaṅgala-vilāsinī = D.cm.

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* Nd.1. = Mahā-Nd. Nd.2. = Culla-Nd.

example of the harm done by ignoring the native divisions of a work is seen in Grassmann's Dictionary to the Rig-veda, where the hymns are numbered from 1 to 1017, with entire disregard of the historically most important division into maṇḍalas. It is a lamentable fact that usable minor divisions are indeed often lacking in the Pāli prose books, as in the Vinaya or the Visuddhi-magga. The minor divisions (chapters and paragraphs) of Oldenberg's Vinaya were made by the editor. In metrical texts or texts of mingled verse and prose which show minor divisions in native mss., the editors have often treated those divisions so unpractically and unclearly as to render it very inconvenient to make practical use of them. Thus it would be far better to cite Dhamma-pada by vagga and stanza (for with this method we need not mention the edition), and the Sutta-nipāta by vagga, sutta (pucchā), and stanza, had not Fausböll numbered the stanzas consecutively.[15] And the Siamese ed. gives no usable indication of the minor divisions.

It would seem, therefore, that, until the editiones principes are replaced by better ones made with more regard to the needs of Occidental students, we must content ourselves by indicating the place in the book by stating the volume (and edition) and page. The place on the page may be indicated by stating the number of the stanza (a poor makeshift) or (better) by designating the four quarters of the page as **a**, **b**, **c**, and **d**.

Indication of the title of the book. — This part of our problem is not affected by the shortcomings of the editors. The abbreviations are given in the following table (Table I), opposite the titles concerned. Apart from the Vinaya, the canonical books are given in the usual order, and then follow the post-canonical books, first the vaŋsas, and then the "other books," in alphabetic arrangement. I earnestly beg my colleagues to criticise my proposals most rigorously and to send me any suggestions of improvement, in order that I may be enabled, if necessary, to publish my list soon again in revised form for the use of lexicographers and of editors of texts.

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[15] The numbering in his own translation does not even coincide with that of his own text!

titles (Dīgha, etc.), but also very commonly by the specific title of the sutta in question: thus, at i. 982, Brahmajāle (=D. I. 1); at xxi. 654, Poṭṭhapāda-suttante (=D. I. 178). — Of the Khuddaka-nikāya, he cites Udāna and Niddesa and Paṭisambhidā-magga by name (the last, twenty-five times), but none of the rest except Sutta-nipāta and Jātaka. These he cites not by the general title, but by the special title of the sutta or Jātaka-tale in question: thus, in quoting a couplet from Sutta-nipāta i. 8^3, he says (ix. 36) simply yaŋ ca Metta-sutte . . . tiādi vuttaŋ. It is most instructive to note that in citing, for instance, the Ratana-sutta, he cites it (at xiii. 166) as a well-known and much-used text (a paritta), and does not care whether we think of it as constituting Sutta-nipāta ii. 1 or as Khuddaka-pāṭha vi (ed. Childers, p. 6 or 314). — Of the Abhidhamma books, he cites four, Vibhañga, Kathā-vatthu, Dhātu-kathā, and Paṭṭhāna. The Vibhañga he cites perhaps[14] oftenest of all, and mostly by that title, but sometimes by chapter-titles, thus, iii, iv, and v, as Dhātu-vibhañga, Sacca-vibhañga, Indriya-vibhañga. — Typical forms of Buddhaghosa's citations are the following: "Such indeed is the opinion of the Dīgha- and Saŋyutta-professors. But the Majjhima-professors will have it that . . . " Idaŋ tāva Dīghabhāṇaka-Saŋyutta-bhāṇakānaŋ mataŋ: Majjhimabhāṇakā pana . . . icchanti (viii. 952). Similarly, Evaŋ tāva Dīghabhāṇakā: Majjhimabhāṇakā pan' āhu . . . (viii. 1179). Evaŋ tāva Majjhimabbāṇakā: Saŋyuttabhāṇakā pana . . . ti vadanti (xiii. 541).

Requirements for a good system of citations. — The essential parts of a citation are two, — the title of the book and the indication of the place in the book. The requirements of a good system are ready intelligibility, brevity, convenience, and precision. The first three concern especially the abbreviations of the titles, and the last two concern the indication of the place. Moreover, to be readily intelligible, the abbreviations must be unambiguous and easily remembered. It is evident that the citations of the ancients fail to meet most or all of these requirements. And, as appears in the sequel, the like is true of the abbreviations that have hitherto been in use among Pāli scholars. I have good reason to hope that the designations here proposed will prove to be so suggestive and so easily remembered as to win general acceptance.

Indication of the place in the book. — This is a subject which I should like to discuss at length if it were not so hopeless. An extreme knowledge included even texts which have not come down to us. Thus at ii. 3^{43} he cites as part of an "Atharvan Brahma-sūkta" the verses *brahma dāçā brahma dāsā brahmāiveme kitavāḥ*, which do not appear even in Bloomfield's Concordance.

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[14] This Vibhañga is liable to confusion with the Vibhañga of the Vinaya and is in fact so confused in an Index of Proper Names in the Visuddhi-magga made, I presume, by an amanuensis of H. C. Warren.

Sāyaṇa, in his preface to his Rig-veda commentary (Müller's 1st ed., p. 2, l. 10), cites the famous passage, "I study, exalted one, the Rig-veda," etc. (Ch. up. vii.1^2); but he indicates its provenience in a merely incidental way, by introducing it with the words, "The Chandogas cite a speech of Nārada to Sanat-kumāra," etc. And so, in his comment on x.129.2, he indicates the locus of a verse which he quotes from the Kaṭha Upanishad, merely by the words "It is handed down by the Kaṭhas," Kaṭhāir āmnāyate.

The Jātakas, as they appear in the Bharhut sculptures of 250 B.C., are exceedingly instructive. The scenes of the stories are chiselled on the rails or medallions, and the titles are inscribed above or below them. Many of these scenes have been certainly identified[10] with tales of the Jātaka-book, and it is a most illuminating fact that the incised titles often fail to correspond with the titles as we know them from Fausböll's text. Thus the story which appears in Fausböll's text (1.295) as Aṇḍabhūta-jātakaŋ, appears on the medallion[11] as the Yaŋ bramano avayesi jatakaŋ, the title for the sculpture being made from the first pāda of the gāthā, Yaŋ brāhmaṇo avādesi (i. 293). The Nacca-jātakaŋ of the text (i. 208) appears in the inscription as Haŋsa-jātakaŋ, plate xxvii. 11. The well-known story of the Banyan deer is alluded to in the Milinda (p. 203 top), and the substance of it is given by Hiouen-Thsang,[12] and it is called in the text Nigrodha-miga-jātakaŋ, but on the sculpture (plate xxv. 1), simply Miga-jātakaŋ. This is all as natural as can be.

Buddhaghosa's citations. — The tradition is that Buddhaghosa's commentaries are a recast of the old Cingalese commentaries in vogue at the school of the "Great Minster" of Anurādhapura in Ceylon. In his Dhamma-saŋgaṇi commentary, Attha-sālinī, he cites "the ancients," "the commentary-teachers," "the commentaries," and so on, Nāgasena (of the Milinda) by name, and in particular also his own Visuddhi-magga frequently, and his own commentary on the Vinaya.[13] In his Visuddhi-magga he cites all the first four Nikāyas by their general texts — those which are most often quoted — marked with a single letter that is easy for scholars to identify."

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[10] See Oldenburg, JAOS. xviii. 183–201 (1897), and Rhys Davids, Buddhist India, p. 209.

[11] A. Cunningham's Bharhut, plate xxvi. 8; Hultzsch, ZDMG. xl. 76.

[12] Si-yu-ki, book vii., near beginning; Julien, **1**, 361; Beal, **2**, 50.

[13] The details are given by Mrs. Rhys Davids in her Buddhist Psychology, p. xxiii. Cp. Davids's Introd. to Milinda, SBE. 35, pp. xxvii.–xxxvi.

Citations in antiquity. The Greeks. — Herodotus, speaking (at ii. 116) of the wanderings of Paris, cites Homer's mention of them, and cites it, not as occurring "at Iliad vi. 289–292," but as occurring "in the exploits of Diomede," *ἐν Διομήδεος ἀριστείῃ.* Thucydides (at i. 9), to prove Agamemnon's power at sea, cites a line of Homer, not as occurring "at Iliad ii. 108," but as occurring "in the handing down of the sceptre." Lucian, in one of his very frequent references to Homer (at Charon, § 7), speaks of Κύκλωψ, meaning by that word the last part of what is now called book ix. of the Odyssey. Indeed, for citing Homer, the titles (*ἐπιγραφαί*) of what are now books or parts of books were often used in antiquity (see Aelian, Varia historia, xiii. 14). We might still cite Iliad xxi. as *μάχη παραποτάμιος*, and the last part of Odyssey xii. as *βόες Ἡλίου*; but all this is too cumbrous and lacking in precision for modern technical works.

Citations in antiquity. The Hindus. — One of the commonest ways of referring to the ancient texts is by the phrase "Because it says thus and so in the Sacred Word," iti çruteḥ. So iti mantravarṇāt. Certain hymns of the Rig-veda have traditional names (made from their opening words) by which they are cited. Thus, x. 9 is the Āpohiṣṭhīya; and the Āitareya-āraṇyaka (at p. 37, ed. Bibl. Ind.) speaks of i.165 as the Kayāçubhīya. The Kāuçika-sūtra (at 47.12) refers to Atharva-veda ii. 12 as "Bharadvāja's cleaver." In the commentary to the Vedānta-sūtras, Çaŋkara's citations from older texts are simply multitudinous.[6] They imply a stupendous knowledge of memorized texts which rises far above the necessity (under which we Occidentals labor) of "looking the passage up."[7] Hence his references are commonly vague,[8] and made with a simple çrūyate or smaryate. It suffices him to cite a certain text, now (as at iii. 2^{41}) by the familiar title Bhagavad-gītā, and now (at ii. 3^{45}) by the title Īçvara-gītā. But at iii.1^{17}, for instance, with more precision than usual, he cites the famous question "Knowest thou how it is that the other world does not become overfilled?" (Ch. up. v. 3^{3}) as occurring in the "Doctrine of the Five Fires."[9] This vast of good typography, and send out its lucid and forceful articles, all the more lucid and all the more forceful because of the admirable form in which they are presented? while some learned writer on some Oriental topic presents his lucubrations with an indifference to ready intelligibility and to the rules of logical and typographical clarity which we might call at once sublime and ridiculous, if our whole force of will were not required to resist the temptation to profanity?

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[6] Over 2500, I judge; see Deussen's System des Vedānta, Citaten-Index.

[7] Cp. my Notes on the Externals of Indian Books, in Hertel's Panchatantra, HOS. xi., pp. xix. end, xxi.

[8] See Deussen, ibid., p. 30.

[9] The old Bibliotheca Indica ed. does not give the precise references, but they are most conveniently given by the admirable new ed. of Dhūpakar and Bākre (Bombay, 1904), and I mention this fact as showing that European needs are coming to be considered in India, and as well illustrating the progressive attitude of the Nirṇaya Sāgara Press.

Need of agreement as to designation of book-titles. — Without adequate funds,[5] Professor Rhys Davids is now bravely trying to supply the new dictionary. Before that is printed, it is manifestly of the utmost importance that scholars should agree upon some uniform system of designating the Pāli texts and of abbreviating their titles, which shall be so well-considered and easily mastered as to command the general assent of Pāli scholars and come into use not only in the dictionary, but in general technical works on Buddhism and Pāli literature. A scholar has to handle a score or perhaps scores of different works in a single day, and ought not to be perplexed and hindered by the uncertainties entailed by lack of uniformity.

What I have just said is something that sorely needs to be said, even if it is not new. Long ago, in speaking of the preliminaries for the Dictionary, JPTS.1886, p. xiii., Rhys Davids observed: "For such work it is of importance that scholars should, when abbreviations of the titles are desirable, use the same or similar ones. I therefore venture to suggest that Piṭaka Texts might, in most cases, be referred to by one or two letters, and the subsequent texts by three." And again, JPTS.1896, p. 102, ten years later: "It is very desirable for dictionary work, and for notes to text, to have short abbreviations, on which all scholars shall agree, for the titles of Pāli books. The use of different abbreviations by different scholars causes confusion, and is a hindrance to memory. I therefore venture to submit to my co-workers the following scheme. And I should be glad to receive, for publication as soon as possible, any suggestions upon it."

"The principle adopted is that all Piṭaka texts should be designated, as far as possible, with one letter, and later texts with three letters. It is indeed impossible to adhere strictly to the one and the three. But it is possible to preserve a practical distinction of the kind, and to have all the most important and longest of the Piṭaka and the six brief texts Vimāna- and Peta-vatthu, Thera- and Therī-gāthā, Buddha-vaŋsa and Cariyā-piṭaka. Moreover, a new Bangkok edition in Kambodian letters is reported to be under way, although I have thus far failed to elicit answers to my inquiries about it.* But this is not all. There stand actually on my shelves not less than forty-seven volumes of the new Rangoon editions of Tipiṭaka books and commentaries in Burmese letters. Twenty of them are from the Hanthawaddy Press and cover all the Vinaya and Abhidhamma, while of the Sutta-piṭaka they contain most unfortunately only the Dīgha-nikāya. In short, they give largely the texts of which we already have good editions and leave out much of that of which we are most in need. Twenty-six are from the P. G. Mundyne Piṭaka Press and contain Buddhaghosa's commentaries and various Ṭīkās.[3] Childers's dictionary is hardly to be had for love or money; and, if it were, it is wholly inadequate for reading the vast amount of texts since published. A new one must be made.

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[5] With sufficient money to maintain a staff of young but properly trained readers for several years and (what is an inexorable necessity) an adequate organization of their labors, I believe it would be quite possible to produce a good Pāli dictionary within a reasonable time.

Result of lack of organization as concerns the dictionary. — Reverting to the matter with which we began, it is safe to say that within the last twenty-five years good and efficient labor has been expended by competent scholars upon the work of gathering materials for a Pāli dictionary, of an amount which would have been amply sufficient to produce a good dictionary if only it had been properly organized. As it is, B has unwittingly duplicated part of A's labor; C, part of B's; and so on; and we are about where we were when we started, and all for lack of some central organization. This is a pitiful result, and is due to a state of things of which we Indianists ought to be thoroughly ashamed.

Some forty odd years ago a beautiful melody from Weber's Freischütz came to be used in the church-choirs of some rather remote New England villages. The ultra-conservatives were scandalized and remonstrated: "Shall the sons of Belial possess themselves of our holy altars?" "Not so," answered the innovators; "say rather, 'Shall the devil have a monopoly of all the good tunes?'"[4] In like manner (without suggesting any likeness between business and deviltry), why should mercantile undertakings have the monopoly of good organization? or again, why should, for instance, that excellent periodical, Collier's Weekly, with its very wide circulation, avail itself of the advantages facturing company were conducted as unsystematically and wastefully as are many of the most laudable undertakings of philological science, such a railway or company would be speedily overwhelmed by bankruptcy. The Director of the Astronomical Observatory of Harvard College, Professor Edward C. Pickering, has recently called the attention of his colleagues far and wide[2] to the tremendous gains in the progress of that science which would be made possible by the organization of a central bureau through which the useless duplication of observations and of researches might be avoided and comprehensive plans be made and laid before the numerous eager workers whose labors are now more or less misdirected and wasted.

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[3] The other volume contains the Buddhist Acta Sanctorum, Buddhaghosa's commentary on the Dhammapada. Here is a splendid chance for a young man to win his spurs in exploiting this rich mine of Buddhist legend.

[4] Substantially the same remark is attributed to Whitfield in R. Southey's Life of Wesley, **2**, 374 (London, 1858).

* See Postscript, p. 707.

Organization as applied to Oriental studies. — No sane scholar will for a moment underrate the value of individualism and of individual initiative. But the question remains, How may those invaluable factors in the advancement of knowledge best be brought into well-directed and harmoniously organized activity and so most fully utilized? An Oriental Society, even the strongest, is not strong enough for this task; nor even an International Congress, of which the meetings, albeit frequent, are always too preoccupied and hurried. The most helpful agency would seem to be the Union of the great National Academies. But the undertakings (such as an edition of the Mahā-bhārata) with which that Union can as yet concern itself are limited in number and of large scope. Accordingly, it behooves us, meantime, to make as much use as possible of the Journals of the Societies in the task of urging scholars to unite in a common method touching this and that and the other matter of common interest.

Need of a new Pāli dictionary. — This is a need most keenly felt by all students of Southern Buddhism. The admirable work of Childers was completed in 1875. It is very hard to get, for the unsold remainder of the edition of the first half of the work was destroyed in a conflagration. And it is far behind the times, for, in the generation that has since lapsed, there have been published most of the books of the Tipiṭaka. Not only do we have European editions of the Vinaya-piṭaka from the hand of Oldenberg, and of most of the Sutta-piṭaka and Abhidhamma-piṭaka from Rhys Davids and his collaborators in the Pāli Text Society; even the East is awaking to the needs of the day, and we have the Bangkok edition, in Siamese letters, of the Vinaya and Abhidhamma entire, and of all of the Sutta-piṭaka except the Jātaka (already published by Fausböll), the Apadāna (a considerable text),

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[2] In A Plan for the Endowment of Astronomical Research, No. 2, published by the Observatory, 1904.

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Organization of science. — Whatever may be thought of the economic or political or moral results of the work of such "captains of industry" as Carnegie or Rockefeller, one thing is certain: the efficiency of their great business organizations, the United States Steel Corporation and the Standard Oil Company, is nothing less than marvellous. Professor Hermann Diels, in his admirable treatise[1] entitled The Organization of Science, has set forth much of what has already been done to further the progress of science by united human endeavor; but his exposition makes very clear how little has been done, in comparison with what should have been done. It is a moderate statement to say that, if the business of a great American railway or steel manuthey are written (Burmese, Cingalese, Kambodian, Siamese), by a group-letter (B, C, K, S) with an exponent (Arabic numeral or small Roman letter). A group of mss. is to be designated by the group-letter without the exponent: thus, B means all the Burmese authorities. — It will be convenient to have a Table of Contents.

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[1] In Die Kultur der Gegenwart, **1**, i. 591–649.

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PĀLI BOOK-TITLES AND THEIR BRIEF DESIGNATIONS.

By Charles Rockwell Lanman.

Presented March 10, 1909. Received April 19, 1909.

Purpose and scope and outcome of this article. — Its purpose is to devise a system of brief designations of the titles of Pāli books. By books are meant both printed and manuscript books. The abbreviations intended are such as may properly be used in a lexicon or in the apparatus criticus of a text-edition or in technical works on Pāli and Buddhism.

The scope or range of the article includes the canonical books of the Ti-piṭaka (Sutta-piṭaka, Vinaya-piṭaka, Abhidhamma-piṭaka), many of the miscellaneous uncanonical books (like the Visuddhi-magga), and especially the Pāli commentaries (by Buddhaghosa and Dhammapāla) on the books of the canon, and the supercommentaries. Just after Table II comes a discussion of the principles by which the new designations should be and have been determined. To these principles, for convenience of reference, I have given the name of Canons. Especially important is Canon 5, and under this are discussed the most essential features of the system. After the Canons comes a series of Comments on the individual abbreviations proposed. The paper ends with an attempt to devise a good system of designations for the manuscripts, for the use of the editors of texts.

Outcome. The proposed designations are: for the first 4 Nikāyas, the uniliterals, D., M., S., and A.; for the 15 books of the Khuddaka-nikāya, the biliterals, Kh., Dh., etc.; for the Vinaya and the 7 books of the Abhidhamma, the triliterals, Vin., Dhs., Vbh., etc.; and for the miscellaneous uncanonical books, the quadriliterals, Dpvŋ., Mhvŋ., Miln., Visu., etc. As to the commentaries (mostly by Buddhaghosa and Dhammapāla) on the 27 books of the canon: the use of all fanciful titles must be abandoned; the commentaries must be spoken of, for instance, as "commentary on the Dīgha" or "Dīgha-commentary," and be designated by adding to the abbreviation of the name of the text the abbreviation "cm." for the word "commentary" (thus, "D.cm." for "Buddhaghosa's commentary on the Dīgha-nikāya"). A supercommentary is to be designated by an added ṭ (thus, D.cm.ṭ.). The manuscripts are to be designated, according to the characters in which

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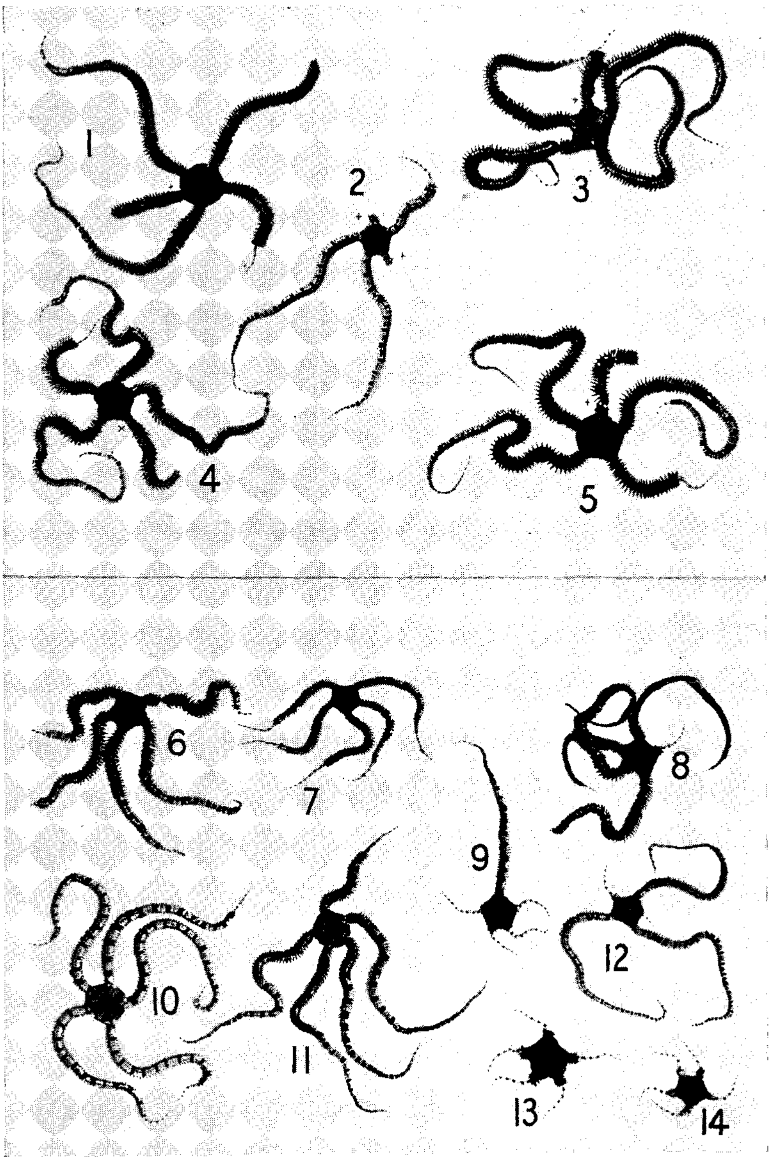
PĀLI BOOK-TITLES AND THEIR BRIEF DESIGNATIONS.

By Charles R. Lanman.

MORGULIS.—REGENERATION OPHIOCOMA. PLATE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on his study of regeneration in the brittle-star Ophioglypha lacertosa. He formulated his rule in these words: "The rate of regeneration of a removed arm increases as the number of uninjured arms still remaining decreases." According to my own observation specimens of Ophiocoma pumila with 1 to 3 arms removed regenerate in the course of thirty days new arms ranging in length from 10 to 11 mm., while those deprived of 4 or 5 arms regenerate arms from 10 to 13 mm. long. It will also be observed that there was an equally rapid regeneration in the two animals, one with all five arms cut off at the middle and the other with one arm only thus cut off (Figures 7 and 11).

It is evident from this that there is some correlation between the degree of injury and the rate of regeneration, but that this relation is not of the nature of a close parallelism, such as is suggested by Zeleny's rule. Furthermore, it is stated that in Ophioglypha lacertosa "the regenerated lengths are on the whole at least twice as great in Series IV, where four arms were removed, as in Series I, where only one arm was removed." This, again, differs from my results in Ophiocoma pumila, where the regenerated lengths never presented such wide variations.

(as will be seen from Figure 2), while other arms in the same specimen with the nerve intact have regenerated normally.[3]

There are three possible interpretations of this phenomenon : (1) It may be essential for the regeneration of an arm that the cut nerve should present a free end, as was the case with the earthworm. Or, (2) it may be possible that the undestroyed portion of the radial nerve between the point of injury and the cut end of the arm could furnish sufficient impetus to cause a slight regeneration. Finally, (3) the explanation may be that in those cases where the wound was made deep the continuity of all parts of the nervous tissue present in both the superficial and deeper portions of the arm was destroyed. Which of these explanations is the correct one must be decided by future experiments.

Experiments by injuring the ring-nerve have not yet been successful, owing to the great difficulty of such an operation.

Before concluding I wish to mention some of the observations made on the *rate* of regeneration of arms. This matter was examined from two standpoints : the relation of the rate of regeneration, first, to the level at which the arms were cut, and, secondly, to the number of arms removed. A few specimens represented by Figures 6 to 14 show the nature of the results. If we compare the rate of regeneration of arms cut at the base with that of those cut near the middle of their length, making proper allowance for individual variations, it will be almost impossible to say which regenerates most. On the other hand, comparing arms cut off at the base, or at the middle (Figures 7, 11, 13, and 14), with those cut off near the tip (Figures 6 and 10), the difference in the rates of regeneration becomes very striking. The total amount regenerated during the same period is much greater in the case of the shorter stubs than in the case of the very long one; indeed, it would not be an exaggeration to say that the greatest regeneration from the arm cut near its tip does not exceed the least regeneration from one cut at its middle. These results are in perfect agreement with Miss King's results on the regeneration of arms in Asterias.

As regards the second point — the relation of the rate of regeneration to the number of removed arms — my experiments with brittle-stars from which 1, 2, 3, 4, or even 5, arms had been removed by being cut off at the base, do not fully conform to Zeleny's rule, which was based imposed upon the other; they are usually designated as ectoneural and hyponeural systems. There are in addition many ganglia and an elaborate peripheral system of nerves, but we shall not be concerned with the latter.

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[3] Miss H. D. King, working on the regeneration of the star-fish, found that on cutting the arms horizontally just above the vertebral ridge the edges of the dorsal parts curled under, but did not regenerate, while the ventral parts, containing the radial nerve, reproduced a new dorsal surface.

The operation consisted in destroying a small portion of the radial nerve in order to break its connection with the ring-nerve, and was performed in the following manner: The calcareous plate on the oral surface of an arm was first punctured with a needle (see Figure *A*) a very short distance from the disc. In this way an opening was established leading into the canal in which the nerve lies. If released at this phase of the operation, the animal would crawl away, using all its arms, and behaving in an absolutely normal fashion, showing thus that the injury was not serious. Next, a red hot needle was introduced into the opening already made, burning the nerve at that particular spot, as will be seen from the diagram. To prevent the needle from injuring the deeper portions of the arm, its point was bent at an angle of 45°. After this operation the animal would crawl away, but would use only the uninjured arms, while the injured arm would be practically paralyzed and curled up about the point of injury, being dragged along passively. In nearly all cases when the wound was not made deep the arm was not cast off even at the end of thirty days, when the animals were preserved. Whenever the wound was made too deep, the arms were subsequently cast off.

When this preliminary operation had been accomplished, the arm was cut off at about the middle of its length. In a number of animals another arm with the nerve intact was also cut off at about the middle, this serving as a control for the arm with an injured nerve. In every case the arms with the radial nerve intact regenerated from the cut surface, and so far as I could ascertain, they regenerated quite normally. On the other hand, if the radial nerve was injured before cutting off the arm,[2] the latter in the course of thirty days regenerated only a small stump, which might easily be overlooked unless the specimen were examined closely. Figures 1 to 5 of the Plate represent several brittle-stars in all of which the same results appear. Where the radial nerve was left intact, a long new part was regenerated (Plate, Figures 1, 3, and 5), whereas if the nerve was destroyed near the disc, so little new tissue was formed that it is difficult to recognize it at all. The interesting thing in this connection, however, is that in cases where the animal threw off the arm at the place of injury to the nerve, there was absolutely no regeneration from the cut surface thus produced system does not exert any appreciable influence upon the process of regeneration.

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[2] The place of injury to the nerve is indicated by a cross in Figures 1 to 5.

While working at the Bermuda Biological Station for Research last summer, I undertook a study of some phases of this problem on the brittle-star Ophiocoma pumila with a view to determining certain points, especially whether or not there exists a relation between the nervous supply and the rate with which a part of an organism regenerates. The brittle-stars present certain advantages for such a study, (1) because the operation is not connected with a profuse bleeding, (2) because there are several similar parts which may be operated upon simultaneously, and (3) because the same animal with its five similar arms can be used both for the experiment and for the control, the variations incident to the use of different individuals being thus eliminated.

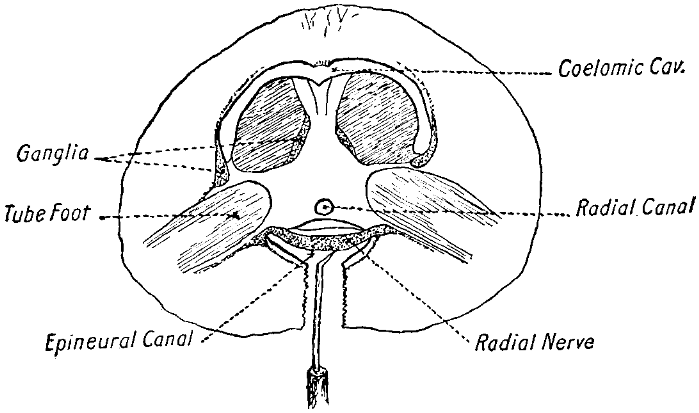

FIGURE *A*.

Unfortunately the want of an abundant material and the great mortality among the operated animals prevented me from obtaining a decisive answer to all the questions which interested me, and the facts to be presented here form merely the beginning of a more extensive investigation which I hope to pursue at the earliest opportunity.

Before discussing my experiments and their outcome I will say a word about the anatomy of the nervous system of the brittle-star and about the method of operation. The central nervous system of the Ophiuroids, unlike that of the star-fish, is a deeply seated organ, and consists of the ring-nerve around the oesophagus, and radial nerves extending out from the ring-nerve into each of the five arms. The ring-nerve and the radial nerves are really double structures, one system being super-

CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION FOR RESEARCH. — No. 15.

REGENERATION IN THE BRITTLE-STAR OPHIOCOMA PUMILA, WITH REFERENCE TO THE INFLUENCE OF THE NERVOUS SYSTEM.[1]

By Sergius Morgulis.

Presented by E. L. Mark, April 14, 1909. Received April 8, 1909.

In animals with a well differentiated nervous system all functions are so intimately associated with this system that the severing of the connection between an organ and its nervous supply leads to a loss of function, and at times also to an atrophy of the organ itself. Furthermore, the nervous system exercises an important rôle in regulating the interrelation of parts of the organic complex, so that interference with, or loss of, one function may — through the nervous system — lead to a more or less profound disturbance of another function. Indeed cases of abnormalities or monstrosities are not infrequently attributable to some disturbance in the nervous system.

Leaving aside entirely those instances which fall within the scope of embryology, Herbst, it may be recalled, found in the crustacean Porcellana that whether there was regenerated an eye or an antenna in place of an extirpated eye depended wholly upon whether or not the optic ganglion had been injured by the operation. It may also be recalled that the exposure of the cut end of the nerve cord is a condition *sine qua non* for the regeneration of the head in the earthworm, as was discovered by Morgan.

The evidence concerning this problem of the influence of the nervous system is, however, very conflicting in some important points, and so far as vertebrates are concerned there is apparently no agreement among writers, although the opinion is strong that the central nervous

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[1] I am under obligation to Dr. E. L. Mark, both for the opportunity of research which I enjoyed at Bermuda, and for the careful revision of the manuscript.

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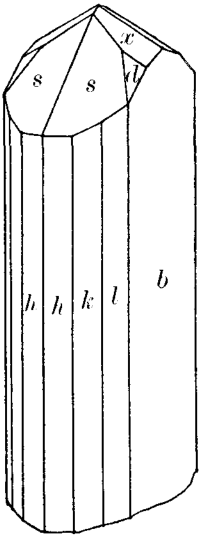
CONTRIBUTIONS FROM THE BERMUDA BIOLOGICAL STATION
FOR RESEARCH. — NO. 15.

REGENERATION IN THE BRITTLE-STAR OPHIOCOMA PUMILA, WITH REFERENCE TO THE INFLUENCE OF THE NERVOUS SYSTEM.

BY SERGIUS MORGULIS.

WITH A PLATE.



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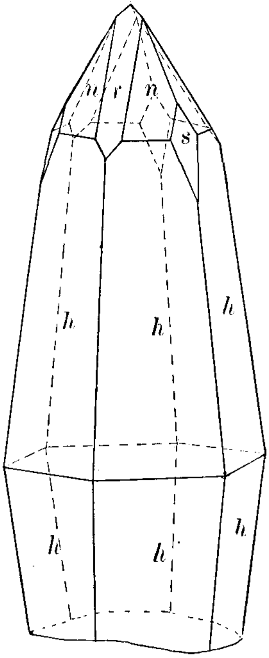
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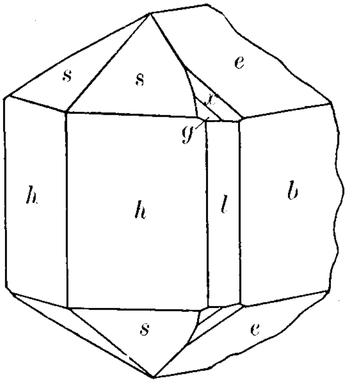
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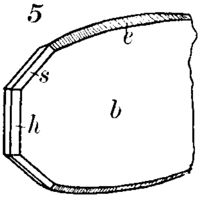

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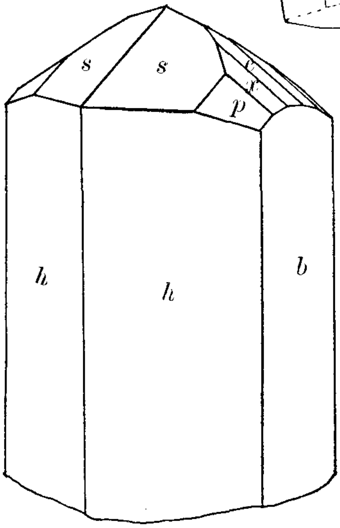

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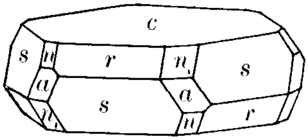
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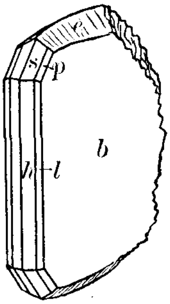

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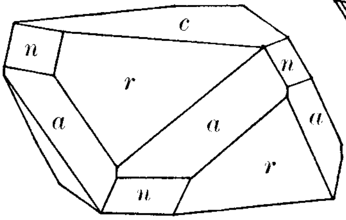

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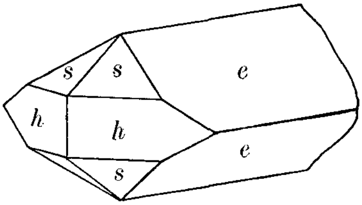

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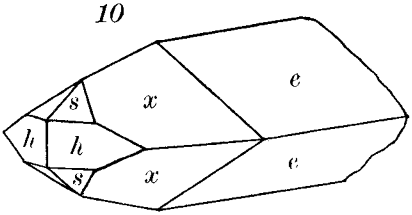

7

EXPLANATION OF PLATE.

FIGURES	1–7.	Diaspore.
FIGURE	8.	Corundum.
FIGURES	9, 10.	Ilmenite.

mineral species recorded in lists of Chester minerals are either found in the schists which have a widespread occurrence in the town or in the granites and quartz veins which intersect them ; they hence have no genetic relationship with those minerals contained in the emery bed and the associated amphibolite formation as a whole.

HARVARD UNIVERSITY,
March, 1909.

termination, closely resembling hornblende, are abundant in chlorite schist at Chester, the only common form of this mineral there. Two exceptional occurrences were noted in our collections. One specimen shows a sharp vein about 4 cm. thick, consisting of margarite plates set on edge on both walls, the central suture completely filled by radiating needles of black tourmaline. In a second specimen black prisms of tourmaline, intimately intermixed with epidote needles and plates of ilmenite, occupy a calcite-filled vein in amphibolite. The tourmaline is crystallized against the calcite and shows singly terminated crystals with the forms a (11$\bar{2}$0), m (10$\bar{1}$0), r (10$\bar{1}$1), and o (02$\bar{2}$1) in typical development. In both of these cases the tourmaline belongs to a later genetic stage than any recorded for the mineral by Emerson.

Albite. Veins of snow-white feldspar are frequently found in the amphibolite about Chester. In cavities the crystals are sometimes quite large and well formed. This feldspar was determined by its extinction angles as almost pure albite. The crystals are albite twins, tabular parallel to x ($\bar{1}$01), of pronounced pericline habit; the forms noted (by inspection only) were c (001), b (010), f (130), m (110), M (1$\bar{1}$0), and x ($\bar{1}$01).

Chlorite. Although beautifully sharp pseudo-hexagonal crystals of corundophyllite and amesite, respectively the dark and light green forms of chlorite common at Chester, are abundantly present in our collections, attempts to study them goniometrically were quite unsuccessful. The basal plane is alone of good quality; the pyramid planes are too deeply striated to yield any measurements. The appearance of these crystals is well described by Emerson,[9] and we can add nothing to his statements of the facts.

Other Minerals. A number of other minerals are represented in our collections from the Chester emery mine, but not in crystals permitting of even approximate measurement. A list of them is appended, to which are added four species, recorded by Emerson from the mine, which we did not see: margarite, chloritoid, hornblende, talc, oligoclase, titanite, calcite, aragonite, dolomite, malachite, azurite, hematite, pyrrhotite, and molybdenite, making in all some twenty-six species known from this locality.

In the large area of serpentine north of Chester Village, which, while it is not in physical connection with any part of the emery bed, is believed to have a genetic relation to it, are found the minerals chromite, magnetite, brucite, siderite, olivine, and picrosmine; brucite and olivine are new to the region and have been described elsewhere.[10] Other walls of corundophyllite and cut massive emery. The cleavage and general physical appearance of the mineral, together with distinct chemical tests obtained for cobalt, arsenic, and sulphur, permit no doubt that these specimens represent cobaltite. The material at hand is not sufficient in amount for a quantitative analysis.

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[9] Loc. cit., Lexicon, pp. 16, 61; Monograph, p. 143.
[10] Am. Journ. Sci., **24**, 491 (1907).

It is interesting to note in this connection that the analyses of serpentines of the Chester Formation recorded in Emerson's work[8] show in a number of cases the presence of minute amounts of cobalt and nickel; in view of this evident source of the material for the formation of the cobaltite it seems probable that analysis of the cobaltite would reveal a nickel content.

Pyrite. Pyrite is abundant in the chlorite schist containing tourmaline which traverses the emery deposit on North Mountain. The crystals are quite large, somewhat rounded, and deeply striated cubes. It is also disseminated rather commonly in the amphibolite that encloses the emery deposit on South Mountain.

A number of isolated crystals were obtained after removal of calcite from a veinlet in chlorite schist in which were present also magnetite octahedrons, epidote, titanite in rounded crystals, scales of chlorite, and feldspar anhedra. These crystals show dominant cube with subordinate faces of e (210), p (111), and n (211). The crystals are deeply pitted and contain magnetite octahedrons embedded in their mass.

Pyrite is very rare in the immediate vicinity of the emery. In but a single specimen of one of the corundophyllite veins containing crystallized magnetite, corundum, diaspore, etc., we found tiny pyrite crystals of cubical habit with narrow faces of e (210).

Chalcopyrite. The occurrence of this mineral in crystals has been mentioned above in describing cobaltite. These crystals were attacked by the acid used to remove calcite from the veins and were not measurable. They appeared to the eye to be steep, much striated sphenoids. The mineral is very sparingly present in the emery deposit.

Epidote. Epidote is abundant in the wall rocks of the emery bed and is found in many of the secondary veins. The best crystals obtained by us came from a calcite-filled vein in chlorite schist, together with chlorite and specular ilmenite. The slender epidote needles are pale yellow and quite transparent; the one crystal measured showed the forms c (001), a (100), u (210), m (110), k (012), o (011), n (111), and ρ (113); most of the needles, however, show no terminal planes and are deeply striated parallel to their length.

Tourmaline. Hexagonal prisms of black tourmaline without distinct

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[8] Loc. cit., Monograph, **29**, 116.

Magnetite. Magnetite in crystals more or less perfect is frequently found in veins in the emery. In our specimens we find it most often with dark green corundophyllite crystals and with the sapphire corundum, diaspore and rutile being present occasionally.

The crystals are of two habits: 1, simple octahedrons, often quite large, showing excellent octahedral parting; 2, dodecahedrons with slight modification by octahedral planes, faces of the former always striated parallel to the longer diagonal of its faces, the latter bright. Tiny crystals of the second habit, embedded in amethystine diaspore, have the symmetrical perfection of a model.

Rutile. As stated by Emerson rutile was abundantly formed, following corundophyllite and diaspore, chiefly in the form of acicular and sagenitic growths. These are generally imbedded in calcite. Our material presents abundant illustrations of such growths, the dull to bright red needles showing every variety of sagenitic network and of cyclic and repeated twinning, the groups minute for the most part and very beautiful as examined under high magnification, as with the Zeiss stereoscopic microscope. Much of the sagenitic rutile is apparently in the form of ilmenite plates, which have been altered to rutile and magnetite.

Occasionally crystals of rutile of stouter proportions are revealed in cavities from which calcite has been removed by acid. One such crystal which was measured showed a prism zone deeply striated by oscillatory combination of the forms, a (100) and m (110); the terminal forms comprised e (101), s (111), and g (212), the latter and other uncertain ditetragonal pyramids forming a striated zone between e and s.

Cobaltite. This uncommon mineral was found on a number of specimens collected by us at the emery mine in 1903. It has not been hitherto described from the locality, and this seems indeed to be the first established occurrence of the mineral in the United States.

It occurs in well-formed cubical crystals up to 2 mm. on an edge and in irregular masses surrounded by chalcopyrite. The crystals are brilliant, silver white in color, and show the cube, a (100), octahedron p (111), and pyritohedron e (210), the cube generally dominant. A few crystals, however, show pyritohedral outline, the faces deeply striated, and on this type the octahedron is lacking. Crystals with octahedron dominant were not seen. The free crystals, revealed by removing with acid the enclosing calcite, are implanted on acicular diaspore or on the pale-green amesite variety of chlorite; associated with them are magnetite, ilmenite, rutile, and chalcopyrite, all in distinct crystals. The massive cobaltite surrounded by rims of chalcopyrite occurs in the same veins with the crystals, in parts where it was not so free to develop. The veins in which it occurs are always bordered by comparatively thick

Ilmenite. Ilmenite in the form of thin bent plates is one of the familiar minerals in secondary veins at the emery bed at Chester. A phase of the alteration of such a plate to rutile and magnetite was observed in several specimens. The mass of the plate is changed to dull massive rutile, and tiny brilliant octahedral crystals of magnetite are grouped in parallel strings on its surface. Sagenitic rutile in orientated groups on ilmenite plates was also observed.

A second type of ilmenite was discovered in the form of exceedingly brilliant tiny crystals implanted on acicular diaspore in open or calcite-filled cavities. These crystals do not exceed 0.5 mm. in diameter, but attracted attention by their adamantine lustre, which caused them to be mistaken for brookite at first. There is little doubt in the authors' minds that the brookite long since reported from Chester by Shepard and not afterwards observed there was of the same nature as these tiny ilmenite crystals. They are thin, tabular parallel to the base, and are attached by an edge of the table. The base is marked by triangular striations, but, like all the faces, reflects the signal well considering its minute size. Measurement of a number of them revealed the same forms on all: c (0001), a $(11\bar{2}0)$, r $(10\bar{1}1)$, s $(02\bar{2}1)$, n $(22\bar{4}3)$, n_1 $(2\bar{4}23)$. These forms are shown in Figure 9 in average development; there is considerable variation in the relative size of the different forms on different crystals.

Shepard in his report on the emery mine[6] refers to the occurrence of large crystals of ilmenite (called by him Washingtonite) in white quartz veins within a mile of the northern end of the vein.

Concerning this occurrence Emerson[7] makes the following statement: "There were in the Shepard collection at Amherst, destroyed by fire, great tabular crystals 6 to 8 inches across and 1 inch thick of model-like perfection from the locality mentioned above. They were tabular by the predominence of OP. I cannot find that they were ever described by Professor Shepard."

While in Chester in 1904 the senior author secured from an old local collection a specimen which clearly represents this "lost locality" and which seems worthy of description. It consists of two attached crystals, three and two inches across and half an inch thick, partially embedded in glassy white quartz. The crystals are dull black and more or less covered with scales of rusty mica. They show the forms c, a, r, and n, very sharply developed in about the proportions of the accompanying figure 10. No further information as to the exact location of the vein which yielded the ilmenite crystals was secured.

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[6] Reprinted in Monograph, **29**, pp. 122–135.

[7] A Mineralogical Lexicon, p. 107.

line faces in a striated zone; measurement of more crystals would probably increase their number, and they are variable in position. They occur on crystals of type a.

Corundum. Veins traversing the emery and containing massive corundum have long been known and are well described by Emerson.[4] One such vein, about 2 cm. thick, was collected in 1904 which showed the following minerals symmetrically developed on the two walls:

1. Ilmenite in thin plates with thin films of chlorite along parting surfaces.
2. Alternating thin layers of margarite and corundophyllite, the latter also projecting into the vein in larger crystals.
3. Rich blue corundum forming the vein centre without open spaces.

Where the corundum-bearing veins are filled at the centre by calcite, crystals are not seldom developed, generally with rounded or rough faces and not measurable. One tiny veinlet in chlorite, however, yielded us exquisite crystals of pure sapphire blue color, transparent and with symmetrical faces, brilliantly lustrous. Although minute, the crystals gave good measurements for the following forms: r $(10\bar{1}1)$, s $(02\bar{2}1)$, n $(22\bar{4}3)$, and h $(33\bar{6}1)$. The last named is new and is established by the following angles measured on two crystals with the two-circle goniometer:

Angle between 0001 and $33\bar{6}1$, crystal	1	83° 09′
		83 02
		83 00
		83 00
		83 03
		83 09
"	2	82 59
		82 52
Average		83 02
Calculated		83 02

This form is recorded by Melczer[5] who observed it on ruby crystals from Burmah as one of a series of weak images given by rounded portions of the crystals. He did not regard the form as established, but only as indicated.

Figure 8 shows the observed combination with little alteration of the actual proportions. The crystal figured was about 1.5 mm. in length.

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[4] Loc. cit., Monograph, **29**, p. 144.

[5] Melczer, G. Zeitschr. für Kryst., 1901, **35**, 570.

and c only moderately well placed would hardly establish this form if it were not already known.

u (344). One face each on three crystals, all of poor quality and only one really definite, are all that could be referred to this form. These fell near the computed position.

*d (455). This form is *new*. All four faces of good quality occur on one crystal of prismatic habit in close agreement with the computed position. The form must be regarded as established. The data follow :

φ	ρ	
41° 50′	38° 36′	image of *good* quality
40 52	38 34	" " " "
40 29	38 29	" " " "
40 18	38 15	" " " "

whence,

40 52   38 28· is the *mean* observed position of this form.
40 29   38 27 is the calculated position of this form.

Another crystal of prismatic habit shows one face in approximate agreement with this position, but it is less definite. Because it is so poorly placed that it might equally well be referred to the pyramid *g, another new form described below, it is not included in the tables nor allowed to disturb the mean of the observed values for either d or g.

*g (788). All four faces of this form are found on the same crystal of prismatic habit that showed the form d, the best developed crystal of the suite. It does not agree as well as could be desired with its computed place, but it occurs more definitely on this crystal than some of the established forms occur on any of the crystals examined ; therefore it is accepted and presented, but it needs confirmation. The data follow :

φ	ρ	
41° 23′	38° 42′	image of *poor* quality
41 25	38 48	" " *good* "
42 42	39 18	" " *good* "
43 03	39 35	" " *fair* "

whence,

42 08   39 05· is the *mean* observed position for this form.
43 02   39 34 is the calculated position of this form.

A face is described under d which might better be referred to this form, except that this is in greater doubt.

Uncertain forms : Except the first two listed, these forms all fall in the same zone with the two last described, between p and e. They are faces on crystals of different habits. Were the form not already known, it would not be recognized on the evidence furnished by this suite.

h (210). This form was present on all twelve crystals. It is uniformly good in quality with bright faces very little striated. On the prismatic habit its faces are necessarily very narrow, but they are definite and of excellent quality. It is the dominant prism on Chester diaspore.

m (110). Only five faces of this form were observed, all on crystals of the prismatic habit. Therefore all the faces were very narrow. They varied in quality from very good to very poor, but they were definite and placed close to the calculated position.

k (230). Seven faces of this form were observed, all on crystals of the prismatic habit. The faces are good, unstriated, and well placed.

l (120). Eleven faces of this form were found. It was not confined to any habit, but occurred on all types. It was better developed on the slender prismatic crystals. Sometimes it is badly striated, but again it is found with bright clear faces.

e (011). Ten faces of this dome were seen. It occurs on all types of the Chester crystals. It is seldom quite good, being usually the centre point of a zone of striations. For this reason the readings in azimuth were often slightly displaced. While sometimes dull, it is usually sharp and bright, but sometimes very small.

w (101). One disturbed, doubtful face lay approximately in the position of this form. The form is established or no mention of the observation would be made.

p (111). Nine faces of this form were observed distributed among all three habits. But it finds its best development on the disc-like type where occasionally it is comparatively large and usually sharp and good. The prismatic habit furnished only one of these readings, but in that case the face was quite definite.

*s (212). This form was well developed on ten crystals. It is the dominant pyramid on the Chester diaspore. It is always sharp and sometimes of comparatively good size, but in some of the disc-like crystals its faces are not so large as those of the pyramid p.

q (232). Two faces, both doubtful, one each on two different crystals (one prismatic, the other disc-like), are all the evidence the Chester suite presents of the development of this form.

x (133). Eight faces of good average quality, fairly well placed and confined to the prismatic habit, establish this form on the Chester species. Most of the faces are well defined but small.

v (122). Seven faces of poor quality on three crystals of habits a crystals measured. Only three of the observed faces were cleavage planes, one each on three different crystals. Some of these faces were smooth with a few hair-like striations on them, but for the most part the form is striated considerably from oscillatory combination with the prisms. On crystals of the slender prismatic habit its faces are necessarily narrow, broad on the disc-like crystals and medium on the stout habit. It is the dominant form on the Chester diaspore, and perfect cleavage parallel to it is characteristic of the species.

TABLE OF OBSERVED MEASUREMENTS ON DIASPORE.

Letter.	Symbol.		Calculated.		Measured.		Limits.		No. of Readings.	Quality of Images.
	G'dt.	Miller.	ϕ	ρ	ϕ	ρ	ϕ	ρ		
			° ′	° ′	° ′	° ′	° ′ ° ′	° ′ ° ′		
b	0 ∞	010	0 00·	90 00	0 00	90 00	... ...	... ...	24	excellent
a	∞ 0	100	90 00	"	90 00	"	... ...	... ...	2	...
h	2 ∞	210	64 53·	"	64 58·	"	65 25–63 43	... ...	38	very good
m	∞	110	46 51·	"	46 46	"	47 00–46 26	... ...	5	fair
k	∞ $\frac{3}{2}$	230	35 25·	"	35 06	"	35 48–34 01	... ...	7	good
l	∞ 2	120	28 05	"	27 59	"	29 15–27 39	... ...	11	fair
e	0 1	011	0 00	31 07·	...	31 06	... ...	31 13–30 50	10	poor
w	1 0	101	90 00	32 48	...	...	... ...	... ...	1?	...
p	1	111	46 51	41 27	47 18	41 04	47 52–46 06	41 29–41 00	9	good
*s	1 $\frac{1}{2}$	212	64 53·	35 26	65 01	35 31	66 00–64 15	35 46–35 24	25	very good
q	1 $\frac{3}{2}$	232	35 25·	48 01·	...	...	... ...	... ...	2?	...
x	$\frac{1}{3}$ 1	133	19 35	32 39·	19 45·	32 40	20 21–18 51	32 49–32 32	8	good
v	$\frac{1}{2}$ 1	122	28 05	34 23·	28 22·	34 22·	29 41–26 48	34 49–33 32	7	poor
u	$\frac{3}{4}$ 1	344	38 40	37 43	38 43	38 09	38 59–37 10	38 49–37 28	3	poor
*d	$\frac{4}{5}$ 1	455	40 29	38 27	40 52	38 28·	41 50–40 18	38 36–38 15	4	good
*g	$\frac{7}{8}$ 1	788	43 02	39 34	42 08	39 05·	43 03–41 23	39 35–38 42	4	fair
UNCERTAIN FORMS.										
	0 $\frac{4}{3}$	043	00 00	38 50	2 01	39 02	... ...	... ...	1	fair
	1 $_5$	545	53 08	38 51	54 09	38 59	54 41–53 37	39 14–38 45	2	good
	$\frac{1}{12}$ 1	1·12·12·	5 05	31 13	4 51	31 10	5 29– 4 13	... ...	2	poor
	$\frac{1}{8}$ 1	1·8·8	7 36	31 20·	7 21	31 26	7 45– 7 01	31 29–31 12	3	fair
	$\frac{1}{6}$ 1	166	10 05	31 31·	9 28	31 24	9 49– 9 06	31 33–31 22	5	fair
	$\frac{1}{4}$ 1	144	14 56	32 01	15 42	32 29	16 59–13 54	33 20–32 06	5	fair
	$\frac{4}{9}$ 1	499	25 22	33 46	25 24	33 53	... ...	... ...	1	good
	$\frac{6}{11}$ 1	6·11·11	30 12	34 57	30 32	34 53	... ...	... ...	1	fair

a (100). Only two faces of this form were observed as line-like dance in the mine and were much prized by collectors, the broad surfaces, covered with richly colored amethystine crystals, making showy specimens. Such a specimen now in the Harvard Mineral Cabinet, presented by the Ashland Emery and Corundum Company, measures about 20 cm. square and is covered over most of its area with platy crystals, backed by pale green chloritic emery.

Type c. Crystals of stout prismatic habit characterize the most recent discoveries of diaspore at Chester. The combinations are simple, generally showing only b, h, e, and s, with l, x, and v less frequently developed. The crystals are always implanted upon a prism plane, and the two developed faces of the prism h are prone to show deep vertical striations without, however, losing their brilliant lustre. Occasionally the brachypinacoid is reduced in size or lacking, and the faces of e more or less curved, giving the crystal a curious lense-shaped form. The crystals are glassy and transparent, with rich colors, ranging from rich brown through wine yellow and green to pure amethystine, often mingled in the same crystal.

The largest crystal seen was a square prism 1 cm. long with diameter of 5 mm.; smaller crystals are, however, the rule. They are implanted, singly or in small groups, in cavities in well-crystallized corundophyllite; a second generation of microscopic crystals of the same type is often present in the cavities, dusting the larger diaspores and chlorite crystals with sparkling gem-like points of light. The habit was also found on very brilliant crystals of about 1 mm. size coating cracks of but a few mm. width in solid emery. All veins containing diaspore of this habit seem to have had a final filling of all open spaces by dolomitic calcite, the removal of which with acid revealed these very beautiful and unusual crystallizations of diaspore. The habit is illustrated by Figures 5, 6, and 7.

Crystallography: Fifteen crystals were measured, the results of the observations being presented in the annexed table. Besides the two new forms there given a number of measurements were obtained from pyramid forms which, either because of poor quality of the faces or complex indices indicated, did not seem established with certainty. These are recorded at the end of the table. It is to be noted that in all forms the agreement between calculated values and mean observed angles is less close than could be desired, or, from the appearance of the measured crystals, expected. The variation is, however, quite irregular, and because of this no attempt was made to calculate a new axial ratio for diaspore from the measurements.

Observations on the forms:

b (010). Natural faces of this form occurred on all but two of the

There is of course more or less gradation between these types.

Type a. Diaspore of this habit occurs as the filling or inner lining of drusy lenses or veins of corundophyllite in emery. Usually the space is completely filled with bladed diaspore, and when broken open presents an attractive network of long narrow cleavage surfaces of brilliant lustre. Occasionally irregular angular openings are left in which grow the delicate acicular crystals, sometimes quite spanning the cavity, sometimes with one end free and showing terminal planes. They vary in color from amethystine to gray or water-white with brilliant vitreous lustre. Isolated needles were noted with a length of 15 mm. or more, and a diameter of not more than 1 mm., but most of them are shorter and stouter. With them in these cavities are beautiful bipyramidal crystals of pale green amesite, sagenitic rutile, and magnetite crystals, giving a most attractive appearance under a powerful lense.

The following forms are found on crystals of this habit: b (010), a (100), h (210), m (110), k (230), l (120), e (011), p (111), s (212), u (344), x (133), d (455), and g (788). Two of these, d and g, are new forms; all are discussed below. The prism zone is striated in the direction of its length, as is the zone of pyramids between p and e. Figures 1 and 2 illustrate this habit of crystal. To this type belongs also the crystal described by Dana,[3] on which were the forms b, a, h, l, e, p, s, u, and v (122).

Type b. The disc-like diaspores occur in lenticular druses which have remained partly open, and on the walls of open cracks in emery. The backing of these druses is usually the emery itself with admixed chlorite and without the distinct layer of corundophyllite, as described for the first type. In color the crystals are usually light green, yellowish, or amethystine, and are less brilliant in lustre than those of type a. They are tabular parallel to the brachypinacoid, with maximum dimensions across the plate of 8 to 10 mm. and thickness of 1 to 2 mm.; usually, however, they are much smaller and paper thin. They are ordinarily attached by prism faces to the vein wall and stand out at right angles, exposing both upper and lower terminations; the disc-like form of the plates is due to the rounded surface, resulting from the oscillation of pyramids and brachydome as shown in Figures 3 and 4. While the crystals are usually implanted separately, they sometimes are in contact to form a drusy surface not unlike that which prehnite ordinarily presents.

The forms observed on this type are but few: b, h, e, p, and s.

Crystals of this type were at one time found in considerable abun-

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[3] Loc. cit.

CONTRIBUTIONS FROM THE HARVARD MINERALOGICAL MUSEUM — XIV.

CRYSTALLOGRAPHIC NOTES ON MINERALS FROM CHESTER, MASS.

By Charles Palache and H. O. Wood.

Presented March 9, 1909. Received March 16, 1909.

The minerals of Chester, Mass., have long been the subject of investigations by many mineralogists, especially from the chemical and genetic standpoints. All such studies are cited, and their substance, together with very much more that is original, is fully presented in Emerson's well-known works.[1] The following notes, chiefly crystallographic, are presented because this aspect of the Chester minerals has been almost wholly overlooked in what has been hitherto published. The material studied was collected by the authors during the years 1902, '03, and '04, at the end of the last working period of the emery mine. The observations on diaspore were made by Mr. Wood; the remainder of those presented in the paper, by the senior author.

Diaspore. Diaspore crystals from Chester were first described by Dana,[2] whose brief paper remains the sole crystallographic study of any Chester mineral. Since his description appeared the mineral has been found in several new phases which seem to deserve added record.

Diaspore occurs in three fairly distinct habits:

Type a, long and slender, acicular or bladed crystals.

Type b, flat, disc-like crystals, tabular parallel to the brachypinacoid, with narrow prism and pyramid faces and larger, curved brachydomes.

Type c, short, stout crystals having prisms and pyramids about equally developed, sometimes quite without the brachypinacoid, and then prismatic parallel to the ă axis.

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[1] B. K. Emerson, A Mineralogical Lexicon of Franklin, Hampshire, and Hampden Counties, Mass., Bull. U. S. G. S., **126**, 1895. The Geology of Old Hampshire County, Mass., Monograph U. S. G. S., **29**, 1898.

[2] Dana, E. S., Mineralogical Notes: Diaspore from Chester, Mass., Am. J. Sci., **32**, 388 (1886).

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CONTRIBUTIONS FROM THE HARVARD MINERALOGICAL MUSEUM. — XIV.

CRYSTALLOGRAPHIC NOTES ON MINERALS FROM CHESTER, MASS.

By Charles Palache and H. O. Wood.

With a Plate.

extus papilloso-pubescenti; foliis maximis 5 cm. longis. — Specimens examined: "Mexico," Alaman; "Valle de Toluca pr. Tenancingo," State of Mexico, September, 1874, and 1 October, 1876, *Schaffner;* Guanajuato, State of Guanajuato, *Dugès;* rocky hills, Cuyamaloya Station, alt. 2300 m., Hidalgo, *Pringle*, no. 12,070.

PEREZIA ADNATA Gray involucri bracteis ca. 28 viscido-pubescentibus coriaceis anguste lanceolatis ochraceis, basi insigniter callosis; pappi setulis ca. 84; corolla glabra; floribus ca. 14; foliis maximis 8–9 cm. longis. Morelia, Michoacan, *Ghiesbreght*, no. 378 (type).

Perezia adnata var. **oolepis**, n. var., involucri bracteis ca. 21 viscido-pubescentibus coriaceis ochraceis apice viridiusculis vel purpureo-tinctis, basi insigniter callosis, exterioribus ovatis, interioribus lanceolatis; pappi setulis ca. 63; corolla glabra; floribus ca. 11; foliis maximis 10–12 cm. longis. — Rocky hills at an altitude of 2500 m., Tultenango, State of Mexico, *Pringle*, nos. 3244 & 9945.

ceeded by the leaves, and in the lack of leaf-like bracts subtending the larger branches of the inflorescence. From one another they differ most markedly in the size and dentation of the leaves, but also in the character of the pubescence on the upper leaf-surface. In *P. Stramonium* it is velvety, in *P. Arctium* arachnoid-tomentose. In *P. Stramonium* the panicle is nodding, in *P. Arctium* it is upright. Both species occupy the same floral region and are the northwestern congeners of the southeastern *P. tomentosum* and its allies.

Parthenium Lozanianum, n. sp., fruticosum ramosum usque ad 2.5 m. altum, ramis ochraceis subsulcatis juventate exigue albo-tomentosis, aetate glabris; internodiis quam foliis saepe duplo brevioribus. Folia plerumque lyrato-partita 2–4.5 cm. lata 4–9 cm. longa, supra viridia exigue crispo-pubescentia, subtus molliter albido-tomentosa, parte terminali circumscriptione triangula vel cuneato-lanceolata ipsa fere generis *Aceris* modo obtuse dentata lobataque, partibus inferioribus parvis vel nullis basi in petiolum 3–6 mm. longum decurrentibus. Inflorescentia terminalis ex corymbis 5–6 sublaxis constans. Bracteae deorsum foliis superioribus similes sursum gradatim minores et lanceolatae vel lineares. Inflorescentiae ramuli pedicellique puberulo-tomentosi graciles. Capitula diametro et altitudine ca. 5 mm. Involucri squamae 10 biseriatae exteriores late ovatae acutiusculae interiores suborbiculâres basi truncatae apice obtusissimae. Radii flores 5, tubo brevi, limbo suborbiculari apice emarginato aut raro tridentato. Achenia nigra hirtella compressa cuneata 2.5 mm. longa ad margines singula palearum aristis florum duorum sterilium adnata. Pappi aristae 2 nigrae arcuato-ascendentes tubum superantes albo-pubescentes. Disci flores ca. 26 in axillis palearum cuneatarum pubescentium positi. — Nuevo Leon, State of Nuevo Leon, alt. 300 m., *Lozano*, no. 10,247 (type, in hb. Gray). A member of De Candolle's section *Parthenichaeta* and very closely allied to *P. incanum* HBK., from which it may be distinguished by its incurved, ascending pappus-awns and green upper leaf surface. In *P. incanum* the pappus-awns are divergent or often recurved, and the leaves are whitened above.

PEREZIA ADNATA Gray. This species has long been considered identical with *Perezia Alamani* Hemsl. Specimens which have accumulated in recent years afford evidence that not only may *Perezia adnata* and *P. Alamani* be distinguished, but also a third plant which is here described as a variety of the former. The following brief descriptions contrast the diagnostic characters of the three plants.

PEREZIA ALAMANI (DC.) Hemsl. involucri bracteis ca. 14 paene glabris submembranaceis anguste lanceolatis viridibus apice purpureo-tinctis basi vix callosis; pappi setulis ca. 49; labro corollae interiore

Viburnum cuneifolium, n. sp. Frutex 3–5 m. altus novellis ferrugineis lepidotis. Lepides glandulos 8 brunneos radiantis gerentes. Ramuli modice crassi obscurissime angulati grisei glabrati; lenticellis brunneis; gemmis nudis; internodiis 2–6 cm. longis. Foliorum laminae juventate secus nervos perexigue lepidotae, aetate utrinque glabratae virides late cuneatae leviter denticulatae, in specimine florenti maximae 3.5 cm. longae 3.5 cm. latae, basi acutae, apice truncatae emarginatae; petioli 2–4 mm. longi anguste membranaceo-marginati, subtus persistenter ferrugineo-lepidoti, supra glabri atropunicei. Inflorescentiae umbelliformes diametro ca. 6 cm., floribus exceptis lepidotae, in ramulis lateralibus terminales, radiis 4 primariis 1–1.5 cm. longis. Bracteae bracteolaeque minutae glabrae obtusae scariosae saepe puniceo-tinctae. Pedicelli usque ad 3 mm. longi. Flores omnes conformes. Calycis tubus glaber subcylindricus 2 mm. longus; limbus expansus lobis brevibus obtusis. Corolla alba rotata 4 mm. longa lobis suborbicularibus. Stamina tubo inserta, corollae lobis aequilonga. Stylus perbrevis fere nullus. Stigma capitatum obscure trilobum. — Collected in the Sierra Madre above Monterey, Nuevo Leon, alt. 760 m., 27 March, 1906, *Pringle*, no. 10,234 (type, in hb. Gray). *Viburnum cuneifolium* is very readily distinguished from all the other Mexican species of the genus by its broadly cuneate emarginate leaves. It is allied to *Viburnum prunifolium* L.

Parthenium Arctium, n. sp., fruticosum, ramis juventate niveo-tomentosis aetate glabris ochraceis; internodiis quam foliis ca. 10-plo brevioribus. Folia deltoidea crenato-dentata usque ad 10 cm. lata 30 cm. longa, apice angustata acuta vel obtusa, basi cordata abrupte in petiolum usque ad 5 cm. longum decurrentia, supra viridia tenuiter arachnoideo-tomentosa, subtus niveo-tomentosa. Inflorescentia terminalis corymbosa a foliis longe superata omnibus partibus dense albo-tomentosa. Bracteae minutae nec deorsum foliis similes. Capitula densius aggregata diametro et altitudine ca. 3.5 mm. Involucri squamae 10 biseriatae exteriores oblongae apice obtusae interiores suborbiculares basi truncatae apice obtusissimae. Radii flores 5, tubo brevi; limbo oblongo apice dilatato truncato emarginato. Achenia (immatura) nigra compressa ovoidea 1.5 mm. longa epapposa ad margines singula palearum aristis florum duorum sterilium adnata. Disci flores ca. 18 in axillis palearum pubescentium cuneatarum positi. — Southwestern Chihuahua, August to November, 1885, *Palmer*, no. 123 (type, in hb. Gray). *P. Arctium*, so named because its leaves so closely resemble those of the common burdock, and *P. Stramonium* Greene constitute a well defined group in De Candolle's section *Partheniastrum*. From the other species of the section they differ in having the inflorescence much ex-apice curvatus, sub lente leviter bifidus. — Santa Eulalia Mountains, Chihuahua, April, 1885, *Pringle*, no. 133 (type, in hb. Gray), distributed as *H. costata* Gray. Its nearest affinity is with *H. plicata* Torr. From this species it is at once distinguished by the color of the foliage and shape of the leaf-base. *Hedeoma costata* Gray, based upon Ghiesbreght's no. 815, was obscurely published in the Synoptical Flora in 1878 (Vol. II, Part II, p. 363), and thus has priority over Hemsley's *H. costata*, published in the Biologia Centrali-Americana. This is indeed fortunate, for although Hemsley's description was drawn up from *Ghiesbreght*, no. 815, the first specimen which he cited, *Palmer*, no. 1095, from Chihuahua, is clearly the more recently published *H. Pringlei* Briq. (including *H. permixta* Briq.). True *H. costata* is represented in the Gray Herbarium by only the type specimen from Chiapas, and is doubtless a species of strictly southern range. Specimens which have been distributed under the name are for the most part *H. plicata* Torr., a species which, to judge from the material at hand, is confined to the arid region of northern Mexico and the southwestern United States.

Hedeoma quinquenervata, n. sp. Herba perennis ca. 2 dm. alta, ubique cano-pubescens, caulibus e basi lignosa numerosis, sparsim ramosis vel simplicibus. Internodia media 3–4 cm. longa. Foliorum laminae usque ad 12 mm. latae, 18 mm. longae, basi obtusae, apice obtusiusculae vel acutae, margine leniter revolutae, subargute 10–12-denticulatae, exigue punctatae, utrinque permanenter pubescentes, supra virides, subtus pallidiores, nervis alterutrinque 5(–6), ad denticulorum apices terminantibus, solum subtus prominulis. Petioli usque ad 5–6 mm. longi. Verticillastri plerumque 7-flori axillares in caule summo aggregati, folia bractiforma occultantes, pedunculis usque ad 2 mm. longis. Pedicelli 4–6 mm. longi. Bracteolae omnes uniformes pedicellis multo breviores lineares. Calyx maturus 9 mm. longus anguste cylindricus antice levissime gibbosus, intus a pilorum annulo obseptus, valde nervosus; labri dentibus aristiformibus leviter incurvatis quam eis labioli divergentibus vix longioribus. Corolla 18 mm. longa extus minute puberula, e basi tenui sursum gradatim ampliata, labro oblongo apice truncato emarginato; labiolo trilobo, lobis lateralibus semiovatis, medio oblongo apice truncato. Stamina antica fertilia in tubo summo inserta vix labioli lobos superantia, duo postica 1 mm. longa, longe infra alia inserta, antheras capitatas nec polliniferas gerentia. Stylus nudus integer. — Sierra Madre, Monterey, State of Nuevo Leon, *Pringle*, no. 10,241 (type, in hb. Gray). A species most closely allied to *Hedeoma tenella* Hemsl., but differing in the nervation of the leaves, the more profuse and persistent pubescence, and the larger flowers.

paulo elevatis quam gemmis axillaribus bis terve latioribus. Folia laminis 6.5–8.5 cm. latis 15–18 cm. longis, apice basique acutis, supra glabris, subtus in nervis axillisque nervorum hispidulis; petiolis 2–2.5 cm. longis appresse hispidulis, supra canaliculatis. Inflorescentia paucibracteata, ramis 4–5 primariis subumbellatim insertis, perlongis, floris terminalis rhachin multo superantibus; ramulis ultimis atris dense glutinoso-puberulis; bracteis foliaceis lineari-lanceolatis. Calyx cylindricus 10-sulcatus minute puberulus 6.5 mm. longus leviter 5-dentatus seu potius 5-apiculatus. Corolla alba 2.5 cm. longa, tubo quam calyce vix longiore; faucibus 11 mm. longis; segmentis limbi 5 obtrapezoideis, 6 mm. longis, inter sinus 10 mm. latis, sub angulis rotundatis 11 mm. latis. Stamina 5 ad loborum baseis vix attingentia, tubo in summo adnata; filamentis deorsum ligulatis sursum teretibus; antheris 4 mm. longis. Pistillum 14 mm. longum staminibus multo brevius. — Iguala Cañon, State of Guerrero, alt. 760 m., 28 December, 1906, *Pringle*, no. 13,912 (type, in hb. Gray). The Mexican allies of *Cordia igualensis* are *Cordia tinifolia* Willd. and *Cordia gerascanthoides* HBK. From the former it differs in its shorter, less pubescent, shallowly dentate calyx, and from the latter in its relatively short stamens, short broad corolla lobes and shallowly dentate calyx.

Hedeoma albescentifolia, n. sp. Herba perennis 1.5 dm. alta undique cano-hirta, caulibus e basi lignosa ramosa pernumerosis gracilibus purpureo-tinctis saepissime ramosis. Internodia media 1.5–3 cm. longa. Foliorum laminae circumscriptione fere orbiculares basi obtusae vel rotundatae, apice cuspidato-acuminatae margine leniter revolutae, utrinque perpallide virides, saepe generis *Chenopodii* modo purpurascentes, pubescentes, supra demum glabratae, exigue punctatae, dentibus 8–10 solito acutioribus altioribusque. Petioli ca. 2 mm. longi. Verticillastri 1–3-flori, axillares, post anthesin foliis aequilongi vel longiores, breviter pedunculati, supremi fere sessiles. Pedicelli 4–5 mm. longi. Floris terminalis bracteolae calycis basin paulo superantes, anguste cuneatae, triaristatae; aliae quam pedicelli dimidio breviores, lineari-subulatae. Calyx maturus 7 mm. longus prominule nervosus, antice leviter gibbosus, intus a pilorum annulo in faucibus posito obseptus; labri dentibus setaceis leviter arcuatis quam eis labioli divergentibus paulo longioribus. Corolla gracillima 15–18 mm. longa, extus minute puberula, intus nuda; tubo anguste cylindrico, sursum vix ampliato; labro ovato apice leviter-bilobato; labiolo trilobato, lobis lateralibus ovatis apice rotundis, medio obovato apice levissime obcordato et apiculato, quam lateralibus longiore. Stamina antica fertilia in tubo summo inserta, vix lobos labioli lateralis superantia; duo postica ad staminodia 0.5 mm. longa reducta, longe infra alia inserta. Stylus nudus quoque loculo solitaria ovoidea pendula. Stigma in rostrum cylindricum integrum quam antheras longius productum. Folliculi teretes acuminati tenues laeves. — Herbae perennes volubiles tenues, foliis oppositis parvis linearibus petiolatis; floribus parvis solitariis vel in cymata pauciflora aggregatis. Genus habitu et squamis coronae simplicibus *Metastelmati* accedit, sed corollae lobis aestivatione obtegentibus facile distinguendum est. *Basistelma* squamis coronae simplicibus corollae adnatis et rostro integro nec bifido *Meliniae Pattaliadique*[2] dissimile est: a *Pattaliade* differt etiam lobis corollae reflexis nec rectis patentibusve, appendicibus antherarum magnis rectis vel interdum reflexis nec perparvis nec rostro adpressis. Species duae, **Basistelma angustifolium** (Torr.) n. comb. (*Metastelma angustifolia* Torr.) et **Basistelma mexicanum** (Brandegee) n. comb. (*Melinia mexicana* Brandegee), Sonorae Sinaloaeque incolae.

Marsdenia trivirgulata, n. sp., lignosa volubilis, ramis gracilibus juventate griseis aetate griseo-brunneis, in lineis longitudinalibus puberulis; lenticellis magnis conspicuis; internodiis foliis fere aequilongis. Folia opposita ovato-lanceolata, maxima 5 cm. longa 2 cm. lata, apice basique acuminata, supra viridia sparsim puberula, subtus, praecipue secus nervos, densius puberula, petiolis longitudine plerumque infra 10 mm. Cymata fere sessilia ca. 8-flora, pedicellis 2–3 mm. longis, basi bracteas ovatas minutas gerentibus. Calyx 2 mm. longus infra mediam 5-fidus, segmentis late ovatis obtusis, extus puberulus intus sub sinubus glandulis 5 papilliformibus praeditus. Corolla 6 mm. longa usque ad calycis apicem 5-fida sub sinubus callosa et appendicibus perbrevibus truncatis emarginatis praedita, segmentis angustis oblongis plus minusve patentibus, lineis tribus rectis longitudinalibus purpureis maculisque concoloribus ornatis; coronae squamis 5 carnosis late ovatis basi connatis, margine liberis, supra sinus in auriculas callosas productis, infra antherarum loculos columnae brevi adnatis. Antherarum membranae terminales latae apice truncatae erosae mucronatae rostro adpressae. Pollinia erecta oblonga 0.4 mm. longa corpusculo virguliformi paululo breviora. Stigmatis rostrum conicum 1.8 mm. longum, apice leviter bidentatum. Folliculi ignoti. — Iguala Cañon, State of Guerrero, *Pringle*, no. 10,333 (type, in hb. Gray). In flower 13 October 1906. A species well marked by its small, thin leaves, attenuate at the base.

Cordia igualensis, n. sp., sectionis *Gerascanthi* arbor. Ramuli grisei ca. 4 mm. crassi, aetate glabri, juventate puberuli, cicatricibus foliorum

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[2] Examination of the type material has shown that in *Pattalias Palmeri* the rostrum is distinctly bifid, and not entire, as stated in the original characterization of the genus.

centia terminalis 5–11-flora, floribus in rhachi quam ramo crassiore subumbellatim dispositis. Pedicelli ca. 6 mm. longi. Sepala marginibus atrotincta integra. Fructus subglobosus 1.5–1.8 cm. longus, 1.4–1.6 mm. latus, apice abrupte mucronatus, seminibus 8 aut abortu paucioribus. Flores ignoti. — Type (in hb. Gray) collected at Iguala Cañon, State of Guerrero, alt. 750 m., 3 October, 1906, *Pringle*, no. 10,337.

Melinia angustifolia (Torr.) Gray and M. mexicana Brandegee. In the Botany of the Mexican Boundary Survey Torrey published *Metastelma (?) angustifolia*, based upon Wright's no. 1677 from Santa Cruz, Sonora, commenting upon it as follows: "We refer this plant to *Metastelma* with much doubt, but there is no other genus to which it seems to be more allied." Gray transferred Torrey's species to *Melinia*, but with some misgivings as to its true affinity, as is evidenced by the following quotation from the Synoptical Flora: "Melinia, Decaisne. . . . Two or three extra-tropical S. American species, which have cordate leaves and slender peduncles; to which is appended the following, doubtfully, for its habit is that of Metastelma." When, in 1889, Watson described the genus *Pattalias*, the type species of which was *Pattalias Palmeri* Wats., he wrote: "A second species of this genus is *P. angustifolius*, a Sonora plant doubtfully referred by Dr. Torrey in the Mexican Boundary Report to *Metastelma*, and more recently by Dr. Gray to the extra-tropical South American genus *Melinia*. It is of similar habit [to *P. Palmeri*], but has petiolate leaves, a longer calyx, the crown at the base of the column, the anther-tips much more conspicuous, and the beak of the stigma narrow and columnar."

Another plant of the same dubious affinity was published in Zoe for August, 1905 (Vol. V, p. 216), as *Melinia mexicana* Brandegee. Although habitally similar to *Metastelma angustifolia* Torr., it is clearly distinguished from that species by its shorter rostrum, longer and more fleshy corona-scales, and its recurved anther-membranes, which are much less constricted at the base than are those of *Metastelma angustifolia* Torr. The two species are congeneric, and since they cannot be placed with *Metastelma* nor with *Melinia* nor with *Pattalias*, a new genus is here characterized for their reception.

BASISTELMA, gen. nov. Calyx alte 5-lobus, lobis saepius angustis acutis. Corolla campanulata, lobis intus infra mediam saepius retrorsum pilosis, aestivatione leviter sed manifesto dextrorsum (externe visis) obtegentibus. Coronae squamae 5 carnulosae triangulo-subulatae vel lanceolatae, ad columnae basin corollae adnatis. Stamina prope corollae basin affixa, filamentis in columnam brevem connatis. Antherarum membranae rectae vel reflexae, haud inflexae. Pollinia in

VIII. DESCRIPTIONS OF MEXICAN PHANEROGAMS.

By Harley Harris Bartlett.

Struthanthus Alni, n. sp., lignosus 20–40 cm. altus omnibus partibus glaber; novellis viridibus glaucescentibus; ramis teretibus nodosis a cortice argyraceo-brunneo tectis. Folia subcoriacea lanceolata vel obovata 2–3.5 cm. longa 8–15 mm. lata, ad basin acutam in petiolum perbrevem decurrentia, apice acuta vel obtusa saepe mucronulata. Inflorescentiae fere glomerulatae 3- vel 6-florae quam folia triplo breviores, plerumque in ramulis lateralibus terminales sed rarius axillariae; ramuli idem aut solitarii aut binis trinisve fasciculati. Pedunculi crassiusculi saepissime perbreves nunc fere obsoleti nunc usque ad 5 mm. longi. Pedicelli nulli. Bracteae bracteolaeque carnosae delapsu apicium truncatae, partem calycis inferiorem obtegentes et pedicellos brevis simulantes. *Flores* ♂. Calyx ut in floribus ♀, sed brevior. Petala linearia 6 inaequalia 7–8 mm. longa. Stamina sex dimorpha, alterna brevia atque longiora. Staminum filamenta petalis ex toto adnata sed propter colorem formamque carinatam faciliter videnda, longiorum antherae oblongae quam stylus longiores quam filamenta sua subduplo breviores, breviorum antherae usque ad aliarum baseis attingentes filamentis suis aequilongae. Ovarium quam in floribus ♀ multo brevius, stylo paululo tenuiore, stigmate rudimentario disciformi nec capitato. *Flores* ♀. Calyx urceolatus 2.2 mm. longus leviter 5-denticulatus. Petala 5 linearia, tria 5 mm. longa usque ad basin libera, dua aliis paulo breviora fere usque ad styli apicem connata. Staminodia omnia subaequilonga quam petala paulo breviora et eisdem connata, antheris rudimentariis liberis exceptis. Ovarii subcylindrici discum annuliforme; stylus 4 mm. longus; stigma capitatum. Fructus ignotus.—Parasitic on *Alnus jorullensis* var. *exigua* Fern., collected on the summit ridge of the Sierra de San Felipe, above the City of Oaxaca, State of Oaxaca, alt. 3000 m., *Pringle*, no. 10,244 (type, in hb. Gray). A peculiar species on account of the difference between the corollas of the staminate and pistillate flowers.

Jacquinia Pringlei, n. sp. Arbor parva ramulis junioribus novellisque exigue pubescentibus. Folia lanceolata 3.5–5.5 cm. longa 7–11 mm. lata perbreviter petiolata, utrinque lepidoto-punctata, basi acuta, apice saepissime acuta et in mucronem rigidum producta. Inflores-

8 mm. diametro. — Plains near Chihuahua, State of Chihuahua, 30 September, 1885, *Pringle*, no. 604 (type, in hb. Gray).

Solanum tenuipes, n. sp. Caulis glanduloso-hirsutus aculeatus. Folia bipinnatifida utrinque subscabra, subtus exigue stellato-pilosa, segmentis ultimis obtuse angulatis, petiolis nervisque aculeatis glandulosis. Racemus elongatus ca. 8-florus, pedicellis gracilibus aetate quam internodiis longioribus. Calycis pars inflata 10 mm. longa nervosa inter aculeos minute glanduloso-pilosa, aculeis ca. 10 magnis, paucis minoribus: laciniae inermes lineares persistentes 5 mm. longae. Corolla purpurea 21–23 mm. longa, tubo 2.1 mm. longo, lobis quam in *S. citrullifolio* angustioribus. Staminum 5 filamenta 2.7 mm. longa; antherae 4 aequilongae rectae 9 mm. longae, quinta arcuata 18 mm. longa. Bacca globosa calyce obtecta; seminibus lateraliter compressis 2.5 mm. latis 3 mm. longis atrobrunneis foveatis. — Coahuila: mountains 39 km. northeast of Monclova, September, 1880, *Palmer*, no. 939 (type, in hb. Gray); 180 km. west of Saltillo, June, 1880, *Palmer*, no. 940.

Solanum Lumholtzianum, n. sp., omnibus partibus aculeatum, caule subherbaceo, basi glabriusculo, superne viscoso-hirto. Folia quam in speciebus sectionis *Androcerae* reliquis parviora, sub-bipinnatifida utrinque minute viscoso-hirta, juventate subtus perexigue stellato-pilosa; segmentis ultimis angustis, eis *Botrichii lanceolati* similibus. Inflorescentia 1–3-flora, pedunculo 7–11 mm. longo; pedicellis quam pedunculo crassioribus, longitudine e 3.5 mm. in inflorescentiis trifloris usque ad 11 mm. in inflorescentiis unifloris variantibus. Calyx maturus 17 mm. longus, 11 mm. latus, nervosus glabriusculus, aculeis longioribus (ca. 10) 12–15 mm. longis, brevioribus pernumerosis. Corolla purpurea (?) profunde 5-lobata, tubo 1.7 mm. longo, 1.5 mm. diametro, faucibus ca. 2–2.5 mm. longis, segmentis 2 inferioribus 8 mm. longis, 3 superioribus 5 mm. longis. Staminum filamenta 1.7 mm. longa; antherae duae summae rectae 5 mm. longae, duae intermediae arcuatae 6.5 mm. longae, quinta (infima) arcuata 8 mm. longa. Stylus curvatus stamina superans. Bacca ovoidea, seminibus 2.5 mm. latis 3 mm. longis, configuratione formaque cornui Ammonis similibus. — Collected at La Tinaja, Sonora, alt. 1100 m., 19 November, 1890, *C. V. Hartman*, no. 246, in Plants of the Lumholtz Expedition (type, in hb. Gray).

ca. 5 in racemi apice aggregati. Calyx pilosus aculeatus, sub fructus maturitatem 12–14 mm. longus, aculeis minoribus pilisque atro-olivaceis: segmenta gradatim acuta in apices exaculeatos persistentis 2 mm. longos desinentia. Corolla purpurea ca. 7 mm. longa profunde subaequaliter 5-partita, extus puberula, tubo 1.4 mm, longo. Stamina 4 aequalia, filamentis 1.35 mm. longis, antheris rectis 2.5 mm. longis; quintum filamento 1.4 mm. longo, anthera arcuata 3 mm. longa. Stylus 5 mm. longus curvatus. Bacca globosa calyce obtecta, diametro ca. 9 mm.; seminibus nigris lateraliter compressis rugoso-foveatis, 2.5 mm. latis 3 mm. longis. — Mexico: Zacoalco, Valley of Mexico, *Bourgeau*, no. 542. San Luis Potosi: *Parry & Palmer*, no. 634½; *Schaffner*, no. 696. Vera Cruz: Mt. Orizaba, *Seaton*, no. 468. — *Thurber*, no. 750, from Chihuahua is perhaps a variety of this species.

S. heterodoxum var. **novomexicanum**, n. var., a varietate typica differt partibus omnibus densius glanduloso-pubescentibus aculeatisque; calycis segmentis aetate ad apicem versus abrupte obtusatus, in lacinias angustas exaculeatas terminantibus, aculeis pilisque aureo-brunneis, nec, ut in varietate typica, olivaceis. Corolla 10 mm. longa, tubo 1.3 mm. longo. Staminum filamenta 2 mm. longa; antherae 4 rectae 3 mm. longae, quinta arcuata 5 mm. longa. — New Mexico, *Fendler*, no. 673 (type, in hb. Gray).

Solanum citrullifolium A. Br. This species does not appear to reach Mexico in its typical form. It is clearly distinct from the Mexican *S. heterodoxum*, with which it has long been considered identical. The original description (Ann. Sci. Nat. ser. 3, xii. 356) is entirely adequate. Specimens examined: Fayette, Iowa, 1894, *Fink* (introduced?); Texas, August, 1848, *Lindheimer;* Hort. Freiburg, 1849, *A. Braun* (cotype, grown from Lindheimer's Texan seed); Hort. Cantab., 1849, *Gray* (from Texan seed), and 1852 (from Texan or New Mexican seed).

S. citrullifolium var. **setigerum,** n. var. Caulis persetiger aculeis reflexis violaceo-tinctis. Folia sub-bipinnatifida longe petiolata aculeata (aculeis in petiolis venisque quam his in caule inter se distantioribus) utrinque scabriuscula, subtus exigue stellato-pilosa. Inflorescentia unilateralis elongata ca. 12-flora, pedunculo 4–6 cm. longo; pedicellis aetate 1 cm. longis glanduloso-pilosis. Calyx (apicibus segmentorum persistentibus angustis 5 mm. longis exceptis) aculeatus, aetate inter spiculos fere glaber, spiculis majoribus 13 mm. longis. Corolla purpurea irregularis 18 mm. longa, tubo 1.5 mm. longo; segmentis aliquanto incurvatis. Staminum filamenta 2.1 mm. longa; antherae 4 rectae 9 mm. longae, quinta arcuata 15 mm. longa. Stylus 17 mm. longus curvatus. Bacca globosa calyce obtecta ca.

VII. THE PURPLE-FLOWERED ANDROCERAE OF MEXICO AND THE SOUTHERN UNITED STATES.

By Harley Harris Bartlett.

The Mexican Solanums of the sub-genus *Androcera* divide naturally into two sections, one of which is characterized by purple or white flowers and the lack of stellate pubescence except on the leaves, the other by yellow flowers and extreme development of stellate pubescence on all parts of the plant. In the only apparent exception to this grouping, *Solanum macroscolum* Fernald, the flowers are tinged with purple, but the basal color, over which the purple is suffused, is yellow. The pubescence is that of the second section, to which the plant evidently belongs. All of the species of the first section, with the single exception of *S. Grayi* Rose, which has white flowers, are purple-flowered. Those in the Gray Herbarium may be determined by the following key:

- Anthers of two kinds; four subequal and straight, the fifth longer and curved.
  - Corolla 1 cm. long or less.
    - Pubescence and spines of young fruiting calyx olive-green. *S. heterodoxum.*
    - Pubescence and spines of young fruiting calyx golden-brown. *S. heterodoxum* var. *novomexicanum.*
  - Corolla about 2 cm. long.
    - Pedicels stout, about as long as the fruiting calyx.
      - Spines on stem scattered, separated from one another by their own length. . . . . *S. citrullifolium.*
      - Stem densely bristly with slightly reflexed spines. *S. citrullifolium* var. *setigerum.*
    - Pedicels slender, longer than fruiting calyx. . . . . *S. tenuipes.*
- Anthers of three kinds, two short and straight, two longer and curved, forming a transition to the still longer and more curved fifth. *S. Lumholtzianum.*

Solanum heterodoxum Duval. Caulis sparsim vel dense aculeatus, pilosus vel in parte inferiore subglaber, pilis apice glanduliferis. Folia petiolata sub-bipinnatifida, partibus 5–7 oppositis, utrinque aculeata, supra glabra vel pilis paucis simplicibus conspersa, subtus et pilis stellatis et simplicibus tecta. Pedunculus 3–5 cm. longus. Pedicelli 8–12 mm. longi crassiusculi aculeati glanduloso-pilosi. Flores

achaeniis brunneis graciliter cylindricis puberulis apice a cupula albida pappifera coronatis; pappi setis numerosis albis tenuissimis obscure barbellatis. —Tepic, Mexico, 5 January to 6 February, 1892, *Dr. Edward Palmer*, no. 2018 (type, in hb. Gray and hb. U. S. Nat. Mus.). A species readily recognized by its slender at first sight apparently naked stems and loose corymbose inflorescence. It is probably related to *P. Seemannii* Gray, which, however, has smaller heads and narrower green and granular involucral scales, larger leaves, etc.

Perezia platyptera, n. sp., herbacea robusta 1.5 m. alta; caule glabro striato basibus foliorum valde decurrentium conspicue lateque alato; alis cuneiformibus ad insertionem folii ca. 1 cm. latis herbaceis reticulato-venosis deorsum gradatim decrescentibus saepissime denticulatis; foliis lanceolati-oblongis firmiusculis acute acuminatis ca. 12 cm. longis 3–4 cm. latis argute denticulatis utrinque reticulato-venosis; inflorescentia corymboso-paniculata, ramis folioso-bracteatis; bracteis lanceolatis ca. 3 cm. longis subintegris conspicue decurrentibus; capitulis ca. 15-floris 1.5 cm. longis; involucri campanulati squamis multiseriatim imbricatis linearibus attenuatis valde inaequalibus glanduloso-puberulis; corolla rosea ca. 1 cm. longa alte bilabiata; achaeniis subteretibus fusco-brunneis glandulosis; pappi setis numerosis laete albis ca. 7 mm. longis. — In clayey soil, Sierra Madre Mountains, Michoacan or Guerrero, Mexico, 22 January, 1899, alt. 1700 m., *E. Langlassé*, no. 773 (type, in hb. Gray). A species readily distinguishable by its broadly winged stems.

achaeniis graciliter obconicis tomentellis 2 mm. longis; pappi squamulis ca. 23 anguste linearibus attenuatis ca. 5 mm. longis scariosis maturitate patentibus. — In thickets, British Honduras, *Prof. Morton E. Peck*, no. 64 (type, in hb. Gray). A species somewhat resembling *C. prunifolia* HBK., but differing in having smaller leaves, sessile fascicles from the axils of leaf-like bracts, etc.

Calea scabra (Lag.), n. comb. *Calydermos scaber* Lag. Gen. et Spec. Nov. 25 (1816); DC. Prod. v. 669 (1836). *Calea peduncularis*, var. *epapposa* "HBK. Nov. Gen. et Spec. iv. 296, t. 408, f. 5" ex DC. Prod. v. 669 (1836); Robinson & Greenman, Proc. Am. Acad. xxxii. 23 (1896). — Foliis ovatis vel ovato-lanceolatis; achaeniis calvis.

Var. **longifolia** (Lag.), n. comb. *Calydermos longifolius* Lag. Gen. et Spec. Nov. 25 (1816); DC. Prod. v. 669 (1836). *Calea peduncularis*, var. *longifolia* Gray, Proc. Am. Acad. xxii. 430 (1887), as to synon.; Robinson & Greenman, Proc. Am. Acad. xxxii. 23 (1896). — Foliis anguste lanceolati-oblongis elongatis; achaeniis calvis.

Var. **peduncularis** (HBK.), n. comb. *Calea peduncularis* HBK. Nov. Gen. et Spec. iv. 295, t. 408, f. 1–4 (1820). *Calebrachys peduncularis* Cass. Dict. lv. 277 (1828), acc. to Hook. f. & Jack. Ind. Kew. i. 383 (1895), but the combination merely implied not actually made by Cassini. — Foliis ovatis vel ovati-lanceolatis; involucri squamis luteis; achaeniis papposis.

Var. **livida** (Robinson & Greenman), n. comb. *Calea peduncularis*, var. *livida* Robinson & Greenman, Proc. Am. Acad. xxxii. 24 (1896).— Foliis lanceolatis vel lanceolati-oblongis; involucri squamis atropurpureis; achaeniis papposis.

PEREZIA HEBECLADA (DC.) Gray, var. **urolepis**, n. var., capitibus quam ea formae typicae majoribus 2.5 cm. longis; involucri squamis exterioribus longis conspicue caudato-attenuatis interiores longitudine subaequantibus; ceteris formae typicae simillima. — Sierra de Pachuca, Hidalgo, Mexico, alt. 2900 m., 10 December, 1907, *Pringle*, no. 13,975 (type, in hb. Gray).

Perezia nudiuscula, n. sp., gracilis erecta verisimiliter perennis; caule gracili tereti purpurascenti glabro sparse foliato; foliis linearibus vel lineari-oblanceolatis erectis firmiusculis acutis 2–4 cm. longis 2–5 mm. latis glabris patente denticulatis sessilibus basi subamplexicaulibus; capitibus ca. 12-floris laxe corymboso-paniculatis 1.5–2.2 cm. diametro graciliter pedicellatis; pedicellis ascendentibus 1–3.5 cm. longis saepe bracteolas 1–2 subulatas gerentibus; involucri squamis valde inaequalibus apice acuminatis et purpurascentibus glabris, interioribus lanceolato-oblongis ca. 1 cm. longis, intermediis ovati-lanceolatis brevioribus, extimis brevissimis parvis lanceolatis; corollis purpureis;

Guadalupe y Calvo, alt. 2150 to 2300 m., *Nelson*, no. 4853; near Colonia Garcia, 25 August, 1899, *Nelson*, no. 6097; near Casas Grandas, 15 August, 1899, *Townsend & Barber*, no. 438. This species has been variously referred to *C. scabiosoides* HBK. and *C. diversifolius* Otto. From the former it is readily distinguished by its larger and much paler rays, yellow disk-flowers, and puberulent stem; from the latter in having the stem puberulent instead of sparingly to copiously beset with longer hairs, also in having a firmer leaf-texture, a more leafy stem, etc.

COSMOS SCABIOSOIDES HBK. Nov. Gen. et Spec. iv. 242 (1820). This species presents leaf-forms so diverse that without the numerous transitions now known it would be difficult to believe them conspecific. The extremes are certainly so marked as to merit at least formal recognition. The typical form, described as having "folia pinnatipartita, foliolis aut laciniis quinque, sessilibus, lanceolato-oblongis, acutis, basi cuneatis, apicem versus subserratis," was collected near Patzcuaro in Michoacan, and appears to be exactly represented by Pringle's no. 4263 from that locality. Differing markedly from this typical form are the following:

Forma **indivisus**, n. f., foliis indivisis integriusculis vel irregulariter serratis lanceolatis vel lanceolato-ovatis. — Hills of Patzcuaro, Michoacan, 11 October, *Pringle*, nos. 4263 in part, and 3589 in part; in shady places near San Miguelito, San Luis Potosi, *Schaffner*, no. 200; on the Sierra Madre, Zacatecas, 18 August, 1897, *Rose;* near Santa Teresa, Tepic, *Rose*, no. 3433; in the Sierra Madre, west of Balaños, Jalisco, *Rose*, no. 2957. Transitions to the typical form are frequent and are well illustrated by Purpus's no. 1551 (Salto de Agua, Mexico) in which the lower leaves are undivided and the upper pinnatifid with lanceolate segments.

Calea Peckii, n. sp., fruticosa scandens; caule volubile tereti lignoso lenticellis minutis prominulis scabro atrobrunneo oppositirameo; foliis oppositis ovatis subintegris breviter petiolatis acutis 2–4 cm. longis 1–2.2 cm. latis basi subrotundatis 3-nerviis utrinque scabriusculis quamquam aspectu glabris subtus paulo pallidioribus aureo-atomiferis, petiolo gracili puberulo ca. 3 mm. longo; pedicellis in axillis superioribus binis vel trinis; inflorescentia fasciculiformi vel corymbiformi rotundata multicapitulata; capitulis ca. 8 mm. diametro homogamis; involucri subcylindrici squamis valde inaequalibus exterioribus brevibus late ovatis puberulis ciliolatisque subherbaceis plus minusve squarrosis, intermediis longioribus ovato-oblongis flavescentibus rubro-striatis, intimis anguste-lanceolatis laete flavis rubro-striatis acutis; corollis flavis aureisve involucrum modice superantibus;

was secured southwest of the City of Oaxaca, alt. 2300–2900 m., 10–20 September, 1894, *Nelson*, no. 1363 (hb. U. S. Nat. Mus.); and in the Valley of Oaxaca, alt. 1700–2300 m., 20 September, 1894, *Nelson*, no. 1449 (hb. U. S. Nat. Mus.). This species is nearly related to *C. scabiosoides* HBK., *C. Uhdeanus* Kunth, and *C. caudatus* HBK. From *C. scabiosoides* it differs in its pale rays, yellow disk-flowers, and bipinnatifid leaves; from *C. Uhdeanus* (which seems to be represented by Pringle's no. 8238) it differs in having larger heads, lighter rays, and yellow disk-flowers; and from *C. caudatus* it is distinguished by having the involucral scales of subequal length and achenes usually 4-aristate and much less caudate-attenuate.

Cosmos Palmeri, n. sp., herbacea 3–5 dm. alta; radice e fibris 2–5 tuberiformibus graciliter fusiformibus elongatis 5–8 mm. crassis; caule tereti folioso puberulo; foliis oppositis vel alternis bipinnatifidis 3–5 cm. longis, lobis linearibus 1-nerviis acutiusculis in margine et in nervo breviter hispidulis, 4–17 mm. longis 1–2 mm. latis; pedunculis ca. 2 dm. longis nudis 1-capitatis; capitibus (ligulis inclusis) 6–8 cm. diametro; involucri squamis exterioribus ca. 8 lanceolati-oblongis ascendentibus vel saepe reflexis 8 mm. longis 2 mm. latis viridibus striatis gradatim ad apicem obtusiusculum angustatis apice paulo incrassatis interioribus ovati-oblongis acutiusculis viridi-stramineis glabris striatis margine tenuibus ca. 1.5 cm. longis ca. 5 mm. latis apice ciliolatis; ligulis ca. 8 lilacinis ellipticis 2.5–3.5 cm. longis 1.2 cm. latis; corollis disci flavis; achaeniis (valde immaturis) fusiformibus in costis hispidulis apice aristas 2 rigidiusculas erectas gerentibus; aristis levissimis apice solum aculeolis binis patenti-deflexis munitis. — Moist spots on hills and plains at Otinapa, Durango, Mexico, 25 July–5 August, 1906, *Dr. Edward Palmer*, no. 388 (type, in hb. Gray).

Cosmos Pringlei Robinson & Fernald, n. sp., e radicibus 1–2 tuberiformibus crassiusculis 5–7 cm. longis erectus 6–9 dm. altus; caule tereti flexuoso griseo-puberulo vel -pulverulo praecipue in media parte folioso; foliis petiolatis firmiusculis ab ovato-oblongis indivisis apice dentatis ad formas profunde partitas vel pinnatifides cum segmentis linearibus integris obtusis variantibus; capitibus magnis (ligulis inclusis) 6 cm. diametro, pedunculis 1–3 saepe 3 dm. longis; involucri campanulati squamis ovato-oblongis exterioribus 8–11 mm. longis interioribus ca. 13 mm. longis; flosculis disci flavi; achaeniis graciliter rostratis 16 mm. longis sursum sparse hispidulis apice aristas binas arcuato-ascendentes retrorsum barbatas gerentibus; ligulis late ellipticis laete purpureis nec atro-violaceis. — Chihuahua, Mexico: pine plains at the base of the Sierra Madre, 20 September, 1887, *Pringle*, no. 1299 (type, in hb. Gray); at base of Mt. Mohinora, 12 km. from

its discoid heads and in the character of its corolla and achene, this species resembles *O. curviflorus* (R. Br.) Hemsl., but it is readily distinguished by its different inflorescence, the heads being very short-pedicelled; the leaves are longer and relatively narrower, and carefully examined the throat of the corolla is found to be campanulate and the deltoid segments of the limb straightish, while in *O. curviflorus* the throat is very short and funnel-formed, the limb being of lanceolate spreading-recurved segments.

Var. **glabratus** (Coulter), n. comb. *O. curviflorus*, var. *glabratus* Coulter, Bot. Gaz. xx. 50 (1895). — Foliis tenuioribus utrinque viridibus supra scabridis subtus solum in nerviis venisque obscure puberulis; inflorescentia floribusque ut formae typicae. — Volcano of Jumaytepeque, Department Santa Rosa, Guatemala, alt. 1850 m., November, 1892, *Heyde & Lux*, no. 4235 (of Mr. J. Donnell Smith's distribution).

Otopappus tequilanus (Gray), n. comb. *Zexmenia tequilana* Gray, Proc. Am. Acad. xxii. 425 (1887), pro parte, i. e. quoad pl. Palmeri no. 359. — Foliis tenuioribus levioribus vix rugosis vix reticulatis.

Var. **acuminatus** (Wats.), n. comb. *Zexmenia tequilana* Gray, Proc. Am. Acad. xxii. 425 (1887), pro parte, i. e. quoad pl. Palmeri no. 394. *O. acuminatus* Wats. Proc. Am. Acad. xxvi. 140 (1891). — Foliis quam ea formae typicae multo rugosioribus subtus tomentosis reticulato-venosis.

Cosmos Nelsonii Robinson & Fernald, n. sp., herbaceus perennis 6–8 dm. altus; caule tereti erecto subsimplici glabro; foliis oppositis petiolatis bipinnatifidis 5–8 cm. longis, 4–9 cm. latis, segmentis lanceolatis acutis plerisque 1–2 cm. longis, 4–6 mm. latis integris vel 2–3-lobatis supra puberulis subtus paulo pallidioribus margine scabriusculo-ciliolatis basi cuneato-decurrentibus, rhachi glabro gracili vix alato; capitibus saepe 3 nutantibus 4–4.5 cm. (radiis inclusis) diametro; pedunculis 9–12 cm. longis; involucri campanulati squamis exterioribus ca. 8 lineari-oblongis acutatis ca. 1 cm. longis 1.7 mm. latis saepe 5-nerviis, squamis interioribus ovato-oblongis quam exteriores haud longioribus crebre striatis margine tenuibus pallidisque; flosculis disci flavis; antheris linearibus brunneo-violaceis; achaeniis graciliter fusiformibus glabris; aristis pappi saepissime 4 retrorsum barbatis quarum duae longae, aliae multo breviores; ligulis 8–10 ellipticis vel oblongis pallide purpureis ca. 2 cm. longis 1 cm. latis. — Vicinity of Cerro San Felipe, Oaxaca, Mexico, alt. 2900–3300 m., 1 September, 1894, *Nelson*, no. 1176, in part (type, in hb. Gray). — Unfortunately specimens of *Bidens pilosa* L. were by some oversight or transposition of labels distributed under the same number. — Further material of *C. Nelsonii*

Gymnolomia scaberrima (Benth.), n. comb. *Tithonia scaberrima* Benth. in Oerst. Vidensk. Meddel. 1852, p. 91. *Tithonia platylepis* Sch. Bip. ex Benth. & Hook. f. Gen. ii. 368 (1873). *Mirasolia scaberrima* Benth. & Hook. f. ex Hemsl. Biol. Cent.-Am. Bot. ii. 168 (1881). *Gymnolomia platylepis* Gray, Proc. Am. Acad. xix. 5 (1883); Robinson & Greenman, Proc. Bost. Soc. Nat. Hist. xxix. 102 (1899). *G. decurrens* Klatt, Leopoldina, xxiii. 90 (1887). *Perimeniopsis perfoliata* Sch. Bip. ex Klatt, Leopoldina, xxiii. 90 (1887).

Verbesina (§ **Saubenetia**) **Langlassei**, n. sp., fruticosa 2 m. alta; ramis 4-angulatis angustissime 4-alatis striatis scabro-tomentellis; foliis lanceolatis oppositis sessilibus utroque acuminatis serratis vel serratulis utrinque viridibus 10–12 cm. longis 2–3 cm. latis supra scaberrimis subtus vix pallidioribus flavescenti-viridibus tomentellis; capitulis radiatis 9 mm. altis ca. 20-floris in corymbo plano densiusculo ca. 6 cm. diametro basi foliaceo-bracteato dispositis; pedicellis tomentosis 4–9 mm. longis; involucri ovoideo-subcylindrici squamis subtriseriatim imbricatis extimis brevissimis suborbicularibus glabriusculis vix herbaceis, intermediis late ovati-oblongis stramineis intimis paulo longioribus angustioribusque laete flavis; flosculis ♀ 4–5 fertilibus liguliferis, ligulis flavis ellipticis 5–7 mm. longis, tubo gracillimo glabro; flosculis ☿ ca. 15, corollis flavis, tubo proprio brevi, faucibus multo longioribus, dentibus limbi deltoideis brevibus erectis; achaeniis nigris saepe sursum albido-tuberculosis 3 mm. longis bialatis biaristatis. — Granitic soil, Sierra Madre Mountains, Michoacan or Guerrero, Mexico, 1300 m. alt., 7 November, 1898, *E. Langlassé*, no. 595 (type, in hb. Gray). This species appears to belong near *V. acapulcensis* Robinson & Greenman, but is readily distinguished by its considerably smaller leaves, smaller fewer-flowered heads, and non-herbaceous involucre.

Otopappus brevipes, n. sp., fruticosus; caulibus teretibus griseis striatulis vix puberulis, internodiis 4–5 cm. longis; foliis ovati-lanceolatis acuminatis ca. 1 dm. longis 3–4 cm. latis basi attenuatis margine mucronulato-denticulatis supra scabris rugulosis subtus griseo-tomentosis reticulato-venosis; panicula 1.5–1.8 dm. longa 1–1.2 dm. diametro folioso-bracteata puberula, ramis capituliferis late patentibus racemiformibus vel spiciformibus; capitulis discoideis brevissime pedicellatis ca. 1 cm. diametro; flosculis numerosis; corollis albidis, tubo proprio gracili valde curvato sursum in fauces campanulatos abrupte dilatatis, dentibus limbi deltoideis subrectis; achaeniis 2-aristatis in latere interiore a media parte ad apicem aristae interioris late alatis. — Temperate region, Chiapas, Mexico, 1864–1870 (flowering in November and December), *Dr. Ghiesbreght*, no. 541 (type, in hb. Gray). In deflexis puberulis; capitulis minimis primo erectis 3–3.5 mm. diametro, involucri squamis exterioribus 3–4 ovatis herbaceis acuminatis maturitate late patentibus 2.5 mm. longis; disco valde convexo, receptaculo columnari; flosculis ♀ 3–5, ligulis minimis flavis ca. 1 mm. longis; fructu (achaenio in squama involucri interioris involuto) obovato compresso apice dentibus 2 parvis rectis conicis instructo quorum uno antico altero postico, faciebus lateralibus fructus utrinque cum cavulis 2 parvis profundis insignibus, facie postica rotundata vix carinata inconspicue tuberculato-scabrido. — Clayey soil, on prairies, El Calabazal, Michoacan or Guerrero, Mexico, alt. 300 m., 20 October, 1898, *E. Langlassé*, no. 482 (type, in hb. Gray). A species related perhaps most nearly to *M. microcephalum* Less., which, however, is described as having leaves sessile by a much narrowed base. There is nothing furthermore in Lessing's description of the achene to suggest that he had before him the peculiar fruit of the present species.

Melampodium tepicense, n. sp., gracile parvum annuum basi decumbens plus minusve repens deinde erectum 5–9 cm. altum dichotomo-ramosum; caule tenui bifariam puberulo folioso; foliis ovatis vel rhomboideis obtusis vel obtusiusculis paucidentatis basi cuneatis 3-nerviis supra viridibus sparse pilosis subtus paulo pallidioribus praecipue marginem versus hirsutulis 10–14 mm. longis 4–7 mm. latis, petiolo 3 mm. longo gracili angustissime alato; capitulis parvis 3 mm. diametro inconspicuis in dichotomis breviter pedicellatis, pedicellis ca. 1 mm. longis 1-capituliferis; involucro exteriori 5-partito, lobis obovatis obtusis 2.5–3 mm. longis 3–5-nerviis ciliatis viridibus; receptaculo parvo conico; flosculis liguliferis 5, ligulis ovato-oblongis cucullatis viridescentibus 3-nerviis 1.8 mm. longis apice 2-dentatis, fructu (i. e. achaenio in bractea involuto) compresso semiobovato dorso tuberculato apice ecupulato exappendiculato; flosculis disci ca. 5. — Tepic, Mexico, 5 January to 6 February, 1892, *Dr. Edward Palmer*, no. 1814 (type, in hb. Gray). This species should stand nearest to *M. arvense* Robinson, but it is readily distinguished from that species by its leaf-form, the shape of the rhombic-ovate bracts, the more numerous ray-flowers, etc.

Jaegeria glabra (Wats.), n. comb. *Sabazia glabra* Wats. Proc. Am. Acad. xxiii. 277 (1888). *Jaegeria petiolaris* Robinson, Proc. Am. Acad. xxxv. 316 (1900). When this species was transferred some years ago to *Jaegeria* its specific name was changed owing to the existence of *J. hirta*, var. *glabra* Bak. in Mart. Fl. Bras. vi. pt. 3, 167 (1884). According to the Vienna rules, however, the existence of a varietal name in a genus is no obstacle to the use of the same name in the specific category and, therefore, the combination *J. glabra* is required by priority.

graciliter tubulosis supra mediam partem paulo in fauces ampliatis granulosis 4 mm. longis limbum versus purpureo-violaceis, dentibus limbi brevissimis obtusis; pappi setis albis corolla distincte brevioribus; achaeniis 5-gonis glabris 1.1 mm. longis basim versus paulo decrescentibus. — Alturas de Oaxaca, Mexico, 1800 m. alt., 20 February, 1907, *Prof. C. Conzatti*, no. 1738 (type, in hb. U. S. Nat. Mus., fragments in hb. Gray). A species somewhat approaching *E. oresbium* Robinson in many of its more technical characters, but readily distinguished by its hastate-angled leaves, more globular inflorescence, purple pubescence, etc.

Eupatorium ramonense, n. sp., herbaceum vel basi paulo lignescens a basi valde decumbens multirameum, ramis oppositis teretibus flexuosis foliosis viridibus pubescentibus ascendentibus 1.5–2 dm. vel ultra altis; foliis oppositis petiolatis ovato-lanceolatis argute serratis vel biserratis acuminatis basi obtusis vel saepe plus minusve cuneatis trinerviis 3–4.5 cm. longis 1.6–1.8 cm. latis supra atroviridibus minute pubescentibus subtus laete viridibus in nerviis breviter sparseque pilosis, petiolis 0.8–3 cm. longis hispidulis; capitulis 75-floris longipedicellatis 1 cm. diametro in cymis multicapitulatis quasi fastigiatis; pedicellis filiformibus 2–3.5 cm. longis erectis breviter pubescentibus; bracteis lineari-lanceolatis acutis 3–5 mm. longis; involucri campanulati squamis subaequalibus (exterioribus 2–3 brevioribus) lanceolati-linearibus attenuatis viridibus 2–3-nerviis breviter hispidulis 4–5 mm. longis; corollis laete albis 3.8 mm. longis glabris vel sparse pilosis, tubo proprio gracili 1.7 mm. longo, faucibus subcylindrici-campanulatis distincte ampliatis, dentibus limbi deltoideis brevissimis; achaeniis nigris 2 mm. longis 5-angulatis in costis sursum hispidulis apicem basimque versus paulo decrescentibus. — In shady moist places, forming compact masses, San Ramon, Durango, Mexico, 21 April–18 May, 1906, *Dr. Edward Palmer*, no. 74 (type, in hb. Gray). This species is nearly related to *E. petiolare* Moc., but is readily distinguished by its smaller ovate-lanceolate (never cordate) and much smoother leaves, as well as by the somewhat harsher non-glandular pubescence of the pedicels and involucral scales.

Melampodium dicoelocarpum, n. sp., gracile 4 dm. altum; caule dichotomo flexuoso striato-costato viridi sparse pubescenti vel puberulo nodos versus atropurpureo, internodiis ca. 1 dm. longis; foliis oppositis graciliter petiolatis ovato-rhomboideis tenuibus acuminatis paucidentatis basi abrupte acutatis vel etiam acuminatis 3-nervatis supra laete viridibus sparse pilosis subtus paulo pallidioribus subglabris 3.5–6 cm. longis 1.2–3.3 cm. latis; petiolo 5–10 mm. longo; pedunculis filiformibus 3–5 cm. longis in dichotomis caulis solitariis nutantibus vel etiam scariosa coronatis, aristis nullis. — Rocky slopes, Cerro de Gentile, Puebla, Mexico, August, 1907, *C. A. Purpus*, no. 2539 (type, in hb. Gray). This species most nearly approaches *S. arachnoidea* Robinson, but differs in its much narrower, entire, and revolute-margined leaves, grayish-puberulent involucre, etc.

Eupatorium malacolepis, n. sp., perenne 3–12 dm. altum herbaceum vel basi lignescens fere a basi oppositirameum; ramis teretibus brunnescentibus pubescentibus vel puberulis; foliis oppositis petiolatis ovatis vel rhomboideis tenuibus crenato-dentatis 4–6 cm. longis 2.4–5 cm. latis a basi cuneato 3-nervatis ad apicem obtusiusculum angustatis; inflorescentia trichotomo-corymbosa; capitulis parvis numerosissimis ca. 5 mm. diametro 40-floris; involucri campanulati squamis subaequalibus oblanceolati-oblongis 2.5 mm. longis pallide viridibus tenuibus 2–3-nerviis dorso tomentellis margine tenuissimis saepissime ciliolatis; corollis albis 2.4 mm. longis, tubo proprio gracili faucibus distincte ampliatis campanulatis paulo longiore; achaeniis nigris lucidis glaberrimis 5-angulatis 1 mm. longis; pappi setis paucis corollam subaequantibus laete albis tenuissimis barbulatis. — In dense woods along water courses, San Ramon, Durango, Mexico, 21 April–18 May, 1906, *Dr. Edward Palmer*, no. 90 (type, in hb. Gray); oak woods on hills near Huachinango, alt. 1375–1675 m., 4 March, 1897, *E. W. Nelson*, no. 4011 (hb. Gray, distributed as *E. pazcuarense* HBK.). *E. malacolepis* differs from *E. pazcuarense* HBK. and *E. isolepis* Robinson, to which it bears a considerable resemblance, in having much smaller flowers and shorter glabrous achenes.

Eupatorium oresbioides, n. sp., perenne lignescens; ramis teretibus plus minusve flexuosis foliatis fulvo-tomentellis; ramulis et pedunculis et petiolis purpureo-lanatis, pilis creberrimis tenuissimis moniliformibus; foliis oppositis graciliter petiolatis late ovatis hastatis 8–11 cm. longis 6–10 cm. latis tenuibus duplice mucronato-serratis caudato-acuminatis basi rotundatis vel subtruncatis cum angulis vel lobis lateralibus 1(–3) acuminatis divaricatis utroque munitis supra viridibus glabriusculis subtus praecipue in nerviis venisque tomentellis, nerviis ca. 7 paulo supra basin pinnatim oriuntibus, petiolis 1.5–5 cm. longis; panicula corymbiformi subglobosa multicapitulata 8–10 cm. diametro; bracteis petiolatis inferioribus foliaceis superioribus minimis; bracteolis filiformibus 2 mm. longis; pedicellis gracillimis patentibus 1–3 mm. longis; capitulis ca. 17-floris 8 mm. altis; receptaculo parvo convexo brevissime setulifero; involucri anguste campanulati squamis valde inaequalibus 3–4-seriatim imbricatis, extimis minimis linearibus, intermediis lanceolatis nunc appressis nunc laxe patentibus vel reflexis, interioribus oblongis obtusis erosis puberulis violaceo-tinctis; corollis part, not Gray. — San Luis Potosi, Mexico, 22° N. Lat., alt. 1830–2400 m., *Parry & Palmer*, no. 322 (type, in hb. Gray); in mountains, San Miguelito, San Luis Potosi, August, 1876, *Schaffner*, no. 247.

Var. **anadenotricha,** n. var., dense crispeque puberula; foliis quam ea formae typicae paulo majoribus 4–5 cm. longis 3.5–4 cm. latis; involucri squamis brevioribus 3–4 mm. longis crispe puberulis, pilis omnino eglandulosis. — Southwestern Chihuahua, August to November, 1885, *Dr. Edward Palmer*, no. 257 (type, in hb. Gray).

Stevia dictyophylla, n. sp., fruticosa ramosa; caulibus teretibus foliosis brunneis crispe tomentellis; foliis oppositis ovatis vel ovati-ellipticis acutiusculis integerrimis vel obsolete crenato-dentatis 3.5–6 cm. longis 12–25 mm. latis basi cuneatis punctatis supra scabriusculis subtus paulo pallidioribus crispe puberulis prominenter reticulato-venosis supra basin subtrinerviis deinde pinnatim venosis, petiolo 3–7 mm. longo cuneato-alato; corymbis densis multicapitulatis valde convexis 12–14 cm. diametro; bracteis ovatis vel ellipticis foliaceis; capitulis subsessilibus 5-floris; involucri squamis lineari-oblongis acutiusculis dorso rotundatis vel plus minusve carinatis griseo-tomentellis 4 mm. longis; corollis 3.8 mm. longis albidis valde exsertis, tubo proprio 1.3 mm. longo extus granuloso, dentibus limbi ovatis patentibus minute hispidulis; achaeniis gracillimis nigrescentibus acute 5-gonis glabriusculis basi callosis apice cupulo brevissimo coronatis. — *S. subpubescens* Benth. Pl. Hartw. 19 (1839); Hemsl. Biol. Cent.-Am. Bot. ii. 90 (1881), in part; not Lag. — Guanajuato, Mexico, *Hartweg*, no. 37, (type, in hb. Gray); hills near Guadalajara, Jalisco, Mexico, 11 December, 1889, *C. G. Pringle*, no. 2832 (hb. Gray). *S. subpubescens* Lag., as ordinarily and with probable correctness interpreted, differs in its more oblong leaves, which are decidedly more pubescent and much less venose-reticulate, also in its smoother involucre, etc.

Stevia revoluta, n. sp., fruticosa dichotomo-ramosa griseo-puberula; ramis teretibus nodosis a cortice griseo tectis; ramulis teretibus rectiusculis foliosis griseo-puberulis; foliis oppositis lanceolato-linearibus integerrimis 5–7 cm. longis 4–6 mm. latis 1-nerviis pinnatim obscure venosis supra viridibus puberulis subtus canescenti-tomentosis margine valde revolutis; corymbis multicapitulatis densiusculis griseo-puberulis leviter convexis terminalibus; bracteis linearibus ramos ramulosque inflorescentiae subaequantibus; involucri squamis 5 oblongis acutis purpurascentibus, exterioribus dorso rotundatis nec carinatis crispe puberulis, interioribus plus minusve carinatis; flosculis 5; corollis 5 mm. longis, tubo externe sparse granulifero saepius purpureo, dentibus limbi 5 albis ovatis dorso hispidulis; achaeniis nigrescentibus gracilibus subglabris lucidulis acute 5-gonis 4.3 mm. longis apice cupula brevi

alt. 1900 m., 14 November, 1897, *Conzatti & González*, no. 563 (hb. Gray). This species appears to be most nearly related to *V. Karvinskiana* DC. and *V. jaliscana* Gleason. It is distinguished from both by its somewhat larger and considerably more numerously flowered heads, as well as by the tomentose pubescence on the stem and lower surface of the leaves.

Elephantopus micropappus Klatt, Jahrb. Hamburg. wissensch. Anstalt. ix. pt. 2, p. 124 (1892). Dr. Klatt's memorandum regarding this plant was grounded upon Ule's no. 1184, collected "in campo bei Laguna [Brazil] März 1889." The specimen examined and labelled by Dr. Klatt and now preserved in the Gray Herbarium has nothing whatever to do with *E. micropappus* Less. but is GOMPHRENA PERENNIS L.

Phania Curtissii, n. sp., suffruticosa oppositiramea tomentella; caulibus teretibus obscure striatulis; foliis oppositis graciliter petiolatis late ovatis supra puberulis subtus paulo pallidioribus tomentellis puncticulatis, caulinis late cordatis 1.5–2.2 cm. longis et latis grosse crenato-lobatis vel subtripartitis, petiolo ca. 1 cm. longo; foliis ramealibus multo minoribus basi obtusis vel raro acutiusculis nec cordatis 7–15 mm. longis 5–12 mm. latis, petiolo 3–4 mm. longo; capitulis parvis graciliter pedicellatis numerosis cymosis ca. 25-floris; involucri squamis oblanceolati-linearibus acutis viridibus ca. 3 mm. longis subaequalibus; corollis albis; achaeniis nigris glabris deorsum decrescentibus 5-angulatis lucidis; pappi squamellis 5 saepissime 3–5-fidis ciliolatis dorso granuliferis. — Near Nueva Gerona, Isle of Pines, West Indies, 17 December, 1903, *A. H. Curtiss*, no. 239 (type, in hb. Gray). This species most nearly approaches *P. matricarioides* (Spreng.) Griseb., but may be readily distinguished by the very different form of its leaves, which in most cases are fully as wide as long and on the main stems are cordate.

STEVIA BERLANDIERI Gray. In this species, now known from several states of northern Mexico, it is easy to remark certain rather striking differences of pubescence and glandularity, though these do not seem to be correlated with other distinctions of importance. In the typical form, occurring in Tamaulipas and Nuevo Leon, the branchlets, leaves, and petioles are minutely and often sparingly glandular-pulverulent rather than pubescent, and the involucral scales are rather conspicuously covered with sessile globular aureous atoms. From this very constant typical form the following varieties are easily distinguished.

Var. **podadenia**, n. var., ramulis et foliis et petiolis laxe crispeque griseo-pubescentibus; involucri squamis cum glandulis stipitatis hispidulis. — *S. Berlandieri* Hemsl. Biol. Cent.-Am. Bot. ii. 84 (1881), in

Amoen. Monac. (Auswahl merkwürdiger Pflanzen — Choix des plantes remarquables) 11, t. 8 (1830). *Caconapea gratioloides* Cham. & Schlecht. Linnaea, viii. 29 (1833). *Herpestis gratioloides* Benth. in Hook. Comp. Bot. Mag. ii. 57 (1836). *Monniera semiserrata* Ktze. Rev. Gen. ii. 463 (1891). *Bacopa gratioloides* Chod. & Hassl. Bull. Herb. Boiss. ser. 2, iv. 288 (1904).

Bacopa stricta (Schrad.), n. comb. *Herpestis stricta* Schrad. in Link, Enum. ii. 142 (1822). *H. domingensis* Spreng. Syst. ii. 801 (1825). *H. polyantha* Benth. in Hook. Comp. Bot. Mag. ii. 57 (1836). *Monniera stricta* Ktze. Rev. Gen. ii. 463 (1891).

Heterotoma Pringlei, n. sp., annua pusilla erecta 5–11 cm. alta glaberrima glaucescens; foliis radicalibus parvis ovato-rhomboideis dentato-angulatis obtusis 3–5 mm. longis 1–4 mm. latis saepius purpurascentibus basi cuneatis, petiolo glaberrimo 3–7 mm. longo; foliis caulinis 1–2 minimis bracteiformibus linearibus vel anguste lanceolatis; racemo ca. 4 cm. longo 3–5-flora; bracteis linearibus 2–4 mm. longis; pedicellis gracilibus flexuosis patentibus 6–8 mm. longis 1-floris; calyce 3–4 mm. longo valde gibboso vel breviter calcarato, dentibus limbi subaequalibus brevibus lineari-oblongis; corolla azurea 7 mm. longa, dentibus superioribus 2 angustis erectis 1.5 mm. longis, labio inferiore 3-lobato, lobis obovatis rotundatis patentibus. — Chalky mountains, Nuevo Leon, Mexico, 7 November, 1904, *C. G. Pringle*, no. 13,274 (type, in hb. Gray).

Vernonia Conzattii, n. sp., herbacea, erecta; caulibus striato-angulatis sordide tomentosis foliosis apice corymboso-ramosis; foliis ovato-oblongis vel ovato-lanceolatis firmiusculis obscure serrulatis vel integriusculis acuminatis breviter petiolatis basi acutiusculis vel obtusis penninerviis supra rugosis scabris subtus paulo pallidioribus reticulato-venosis saltim juventate tomentosis; inflorescentia umbelliformi terminali valde convexa; pedicellis rectiusculis 1.2–3 cm. longis glanduloso-tomentellis cum bracteolis 1–2 parvis lanceolati-linearibus saepissime munitis; capitulis ca. 35-floris 1.2 cm. diametro; involucri campanulati squamis pluriseriatis valde inaequalibus purpurascentibus acutis mucronatisque ciliolatis, interioribus oblongis, exterioribus lanceolatis vel lanceolati-linearibus multo brevioribus; corollis purpureis glabris ca. 1 cm. longis, dentibus limbi 5 lineari-oblongis obtusis: achaeniis (immaturis) 1.8 mm. longis costatis subglabris plus minusve granuliferis; pappi setis numerosis albidis, interioribus 6–7 mm. longis, exterioribus paucis ca. 2 mm. longis. — Sta. Ines del Monte, Zimatlan, Oaxaca, Mexico, alt. 2700 m., 8–9 December, 1905, *Prof. C. Conzatti*, no. 1327 (type, in hb. Gray); also previously collected in somewhat less mature condition on the Cerro de San Felipe, Oaxaca,

Gratiola oresbia, n. sp., perennis (Sect. GRATIOLARIA § SUBDIDYNAMAE PEDUNCULATAE) erecta 7–18 cm. alta; radicibus fibrosis numerosis; caulibus flexuosis viridibus mollibus foliosis fere a basi floriferis obscure praesertim apicem versus glanduloso-puberulis; foliis lanceolati-oblongis sessilibus auriculato-amplexicaulibus 1.5–2.4 cm. longis 4–6 mm. latis plerumque acutatis rarius obtusis 3(vel obscure multi)-nerviis saepissime crenulatis vel rarius subintegris utrinque viridibus glabris; pedicellis axillaribus 1.5–2 cm. longis nutantibus glanduloso-puberulis; bracteolis sepaloideis lineari-oblongis obtusis 4–5 mm. longis; calycis segmentis anguste oblongis obtusis 3-nerviis ca. 5 mm. longis glanduloso-puberulis; corolla intense aurea 1.3 cm. longa extus glanduloso-puberula intus villosa, lobis latis brevibus retusis; staminibus fertilibus 2, connectivo membranaceo-expanso, loculis transversis; rudimentis 2 parvis filiformibus in tubo quam stamina fertilia altius affixis; capsula compressa ovata acuta maturitate segmenta calycis aequanti. — Sierra Madre Mountains, near Colonia Garcia, Chihuahua, Mexico, 25 August, 1899, *E. W. Nelson*, no. 6099 (type, in hb. Gray); also earlier at the same station, alt. 2285 m., *Townsend & Barber*, no. 31. This species appears to be most nearly related to *G. Drummondi* Benth., which, however, has narrower more attenuate leaves and a suborbicular obtuse capsule scarcely half the length of the lance-linear calyx-segments.

In a recent attempt to revise and label in accordance with the Vienna Rules of Nomenclature the material of the genus *Bacopa* in the Gray Herbarium the writer has found it necessary to employ several apparently new combinations, which it may be well to record here, as follows:

Bacopa Beccabunga (Griseb.), n. comb. *Herpestis Beccabunga* Griseb. Cat. Pl. Cub. 182 (1866). *Monniera Beccabunga* Ktze. Rev. Gen. ii. 463 (1891).

Bacopa humifusa (Griseb.), n. comb. *Herpestis humifusa* Griseb. Cat. Pl. Cub. 183 (1866). *Monniera humifusa* Ktze. Rev. Gen. ii. 463 (1891).

Bacopa micromonnieria (Griseb.), n. comb. *Herpestis micromonnieria* Griseb. Cat. Pl. Cub. 183 (1866). *Monniera micromonniera* Ktze. Rev. Gen. ii. 463 (1891).

Bacopa monnierioides (Cham.), n. comb. *Ranaria monnierioides* Cham. Linnaea, viii. 31 (1833). *Herpestis Ranaria* Benth. in Hook. Comp. Bot. Mag. ii. 57 (1836). *Monniera monnierodes* Ktze. Rev. Gen. ii. 463 (1891). *Bacopa Ranaria* Chod. & Hassl. Bull. Herb. Boiss. ser. 2, iv. 288 (1904).

Bacopa semiserrata (Mart.), n. comb. *Bramia semiserrata* Mart.

VI. DIAGNOSES AND TRANSFERS OF TROPICAL AMERICAN PHANEROGAMS.

By B. L. Robinson

Antigonon grandiflorum (Bertol.), n. comb. *Polygonum grandiflorum* Bertol. Bologn. Nov. Comm. iv. 412 et Florula Guatimalensis, 12 (1840). *Antigonon guatimalense* Meisn. in DC. Prod. xiv. 184 (1856). *A. guatemalense* Hemsl. Biol. Cent.-Am. Bot. iii. 37 (1882).

Tamonea euphrasiifolia, n. sp., fruticosa ramosissima; ramis flexuosis a cortice flavido-griseo tectis; ramulis elongatis foliatis 4-gonis striatis griseo-puberulis; foliis subdeltoideo-ovatis flabelliformi-nervatis brevibus 4–6 mm. solum longis aequilatis quam internodia plerumque brevioribus argute dentatis breviter petiolatis supra glabris rugosis viridibus subtus praecipue in nerviis puberulis; racemis spiciformibus pedunculatis 5–10 cm. longis; bracteis parvis subulatis ca. 2 mm. longis; pedicellis inferioribus ca. 4 mm. longis; calyce cylindrico demum turbinato maturitate 6 mm. longo 5-costato costis excurrentibus extus puberulo; corolla 1.7 cm. longa glabra; fructu obovoideo spinis solis e calyce exsertis. — Alta Mira, Tamaulipas, Mexico, 14–22 May, 1898, *E. W. Nelson*, no. 4415 (type, in hb. Gray and hb. U. S. Nat. Mus.).

Russelia cuneata, n. sp., modice robusta 1 m. alta verisimiliter frutescens; caulibus acute 4-gonis 4-costatis glabris laevisque prope nodos solum sparse pubescentibus, internodiis 5–10 cm. longis folia saepissime superantibus; foliis oppositis firmiusculis 5–8 cm. vel ultra longis rhomboidei-oblongis supra mediam partem crenato-dentatis basi longe cuneatis integriusculis utrinque sparse puberulis vel subglabris; cymis multifloris ca. 3 cm. longis saepissime binis in axillis superioribus oppositis oriuntibus, pedunculis crassiusculis sordide pubescentibus ca. 6 mm. longis, pedicellis puberulis 4–6 mm. longis flexuosis ascendentibus; calycis lobis ovatis acuminatis brevissimis praecipue in costa media hispidulis margine subscariosis ca. 2 mm. longis; corolla tubulosa sanguinea in sicco nigrescenti 1 cm. longa glabra, lobis limbi brevissimis suberectis. — On granitic soil, El Ocote, Michoacan, Mexico, December, 1898, alt. 300 m., *E. Langlassé*, no. 723 (type, in hb. Gray). From its square stem and numerously flowered cymes near *R. floribunda* HBK. and *R. syringaefolia* Schlecht. & Cham., but clearly distinct by its entirely different leaf-contour, smaller flowers, etc.

recte divergentia, duo summa propinqua. Nuculae 4 mm. longae 2.5–3 mm. latae basin versus angustatae, quam in speciebus affinibus latius coriaceo-alatae, apice saepius auriculatae. — GUATEMALA: San Lucas, Department of Zacatepequez, alt. 1700 m., *J. Donnell Smith*, no. 2188 (type, in hb. Gray); Antigua, Department of Zacatepequez, *Kellerman*, no. 4966; San Miguel Uspantán, Department of Quiché, alt. 1800 m., *Heyde et Lux*, no. 2923. It was from the type of this species, in the main, that Professor Fernald drew up the description of *Alnus acuminata* in his Synopsis of the Mexican and Central American Species of Alnus. There the peculiar ashy-brown color of the bark and strobiles is mentioned, a character afterward emphasized as of diagnostic worth in his characterization of *Alnus Pringlei* Fern. The color is peculiar to the type specimen and seems to be due to a thin deposit of clay, perhaps wind-borne dust. Professor Thaxter has kindly examined the specimen for fungi, with negative results.

ALNUS PRINGLEI Fern. The range of this species probably extends northward to Durango. At least the following specimens in the National Herbarium are nearer to *A. Pringlei* than to any other species: Terreria, Jalisco, *M. E. Jones*, no. 439 a; San Ramon, Durango, 21 April–18 May, 1906, *Palmer*, no. 207.

ALNUS RHOMBIFOLIA Nutt. The accrediting of this species to Mexico in the Pflanzenreich is based upon an error in determination. The number cited as *A. rhombifolia* is *A. glabrata* Fern.

can element, in the synonymy of *A. arguta* (Schlecht.) Spach var. *cuprea* Bartlett.

Alnus glabrata var. durangensis, n. var. Arbor trunco a cortice griseo sublevi tecto. Ramuli glabri ochracei modice graciles. Folia lanceolata 14 cm. longa infra mediam 6 cm. lata argute dentata, dentibus subremote serratis, supra glabra olivaceo-viridia, subtus glauca glabra vel secus nervos minute pubescentia, exigue resinoso-punctata, apice longe acuminata, basi acuta in petiolum 1.5 cm. longum decurrentia; nervis utrinque 9–10; petiolis anguste canaliculatis exigue albido-pubescentibus. Amenta ♀ ca. 4 maturitate cylindrica 2.5 cm. longa 8 mm. crassa, pedunculis saepe 6 mm. longis. Nuculae alis coriaceis anguste cinctae. — Collected in the vicinity of the City of Durango, State of Durango, April to November, 1896, *E. Palmer*, no. 965 (type, in hb. Gray). Readily distinguished from the typical form of the species by the glaucous lower leaf-surface.

ALNUS JORULLENSIS HBK. This species has been seen from the States of Jalisco, Michoacan, Mexico, Hidalgo, and Oaxaca, the var. EXIGUA Fern. from the States of Guanajuato and Oaxaca. The material from Oaxaca, both of the species (*Pringle*, no. 10,248) and of the variety (*Pringle*, no. 10,249), is in young foliage, and future collections may show that it belongs elsewhere.

ALNUS JORULLENSIS var. *E. W. Nelson*, no. 3661, collected near the Hacienda of Chaucol, Guatemala, has small sessile buds and cuneate leaves very much like those of *A. jorullensis*, but since the pistillate strobiles are unknown it seems better to leave the form undescribed rather than to risk adding another name to the involved synonymy of this species.

A. MIRBELII Spach. The only material in American herbaria which answers to the description and plate of this species is *Bang*, no. 1893, from Bolivia. Perhaps a sheet in the Gray Herbarium collected by *Seemann* and labelled by Dr. Gray "And. Quitensis — Panama" should be placed here also.

Alnus ovalifolia, n. sp., *A. acuminata* Fern. *pro parte, non* HBK. Arbor ramulis junioribus brunneis glabris subangulatis. Gemmae glutinosae brevipedicellatae sparsim pubescentes vel glabratae. Folia ovalia subregulariter denticulata, apice basique rotundata obtusa vel raro acutiuscula, supra solum in nervis perexigue pilosa, subtus secus nervorum latera plus minusve pilosa, alias glabra, laminis 1.5–5.5 cm. latis 2.5–8.0 cm. longis; petiolis 2–10 mm. longis, supra canaliculatis pilosis, subtus glabratis. Amenta ♂ 4–6 usque ad 13 cm. longa fere sessilia vel longipedunculata. Amenta ♀ in uno ramulo 3–4 ovoidea ca. 2.5 cm. longa 1.4 cm. crassa maturitate plerumque ficte sessilia

Alnus arguta var. **cuprea,** n. var. Arbor aspectu inter formam speciei typicam *Alnumque glabratam* Fern. media. Ramuli glabri juventate rubescentes aetate griseo-brunnei. Folia magnitudine valde variabilia, usque ad 8.5 cm. lata 14 cm. (petiolo excluso) longa, basi rotundata vel leviter cordata, apice acuta vel acuminata, argute dupliciter vel irregulariter dentata, utrinque paene glabra, subtus vel nihil vel minus quam ea formae typicae glauca, colore saepe cuprea, supra atriora. Amenta ♀ pedunculata 9–11 mm. crassa ca. 2 cm. longa. — Oaxaca: wet cañon near base of the summit ridge of the Sierra de San Felipe above the City of Oaxaca, alt. 2135 m., *Pringle*, no. 10,251; west slope of Mt. Zempoaltepec, alt. 2350–2440 m., *Nelson*, no. 599; road from Juquila to Nopala, alt. 1220–2135 m., *Nelson*, no. 2415; vicinity of Cerro San Felipe, alt. 2900–3350 m., *Nelson*, no. 1154. Vera Cruz: Orizaba, *Bilimek*, no. 404; Mt. Orizaba, alt. 1830–2440 m., *Nelson*, no. 296; Orizaba, *Botteri*, no. 191.

Alnus arguta var. **subsericea,** n. var. *A. ferruginea* Fern. Proc. Am. Acad. xl. 27 (1904) *pro parte, non* HBK. ? *A. rufescens* Liebm. ex Hemsl. Biol. Cent.-Am. Bot. iii. 165. Arbor ramulis griseo-brunneis, junioribus ferrugineo-puberulis. Folia laminis late ovatis maximis 14 cm. longis 9 cm. latis, basi leviter cordatis vel rotundatis, apice acutis vel breviter acuminatis, supra atroviridibus tenuiter sericeo-pilosis demum glabratis, pilis longis rectis valde appressis, subtus molliter glauco-pubescentibus, nervis rufescentibus in foliis maximis utrinque 16; petiolis subferrugineo-pubescentibus saepissime quam 2 cm. brevioribus. Gemmae parvae glutinosae puberulae pedicellis suis valde longiores nec raro sessiles. Amenta ♀ maturitate ca. 1 cm. crassa 3 cm. longa vel multo breviora. Nuculae alis percoriaceis angustissime cinctae. — Oaxaca: wet cañon near the base of the summit ridge of the Sierra de San Felipe, above the City of Oaxaca, *Pringle*, no. 10,252. This is also the locality cited by Hemsley for *A. rufescens* Liebm. From the name which Liebmann chose there can be little doubt that he had this plant before him, for the only other reddish-leaved *Alnus* from the same locality is so glabrous that Hemsley would certainly not have placed it with *A. acuminata* var. *ferruginea*. Since Liebmann's name is a *nomen nudum* it does not seem at all desirable to take it up in a changed category without having seen his type. To var. *subsericea* may be referred *Ghiesbreght*, no. 160, from Chiapas, the plant upon which Mr. Fernald's description of *A. ferruginea* is largely based.

ALNUS CASTANEAEFOLIA Mirb. It is clear from the original plate and characterization that this species can have no close affinity to the Mexican plant cited by Hemsley under the name *A. jorullensis* HBK. *β castaneaefolia*. The latter name should be placed, as to the Mexi-

V. NOTES ON MEXICAN AND CENTRAL AMERICAN ALDERS.

By Harley Harris Bartlett.

Alnus acuminata HBK. *A. acuminata α genuina* Regel, Monog. 89 (1861), *A. jorullensis* var. *acuminata* (HBK.) Ktze. Rev. Gen. ii. 638 (1891), not *A. acuminata* Mirb. Mém. Mus. Par. xiv. 464, t. 22 (1827), not *A. acuminata α genuina* Hemsl. Biol. Cent.-Am. Bot. iii. 165 (1883), not *A. acuminata* Sarg. Silva ix. 79, t. 457 (1896), not *A. jorullensis* var. *η acuminata* Winkl. Pflanzenreich, iv. **61**, 127 (1904), not *A. acuminata* Fern. Proc. Am. Acad. xl. 25 (1904). Here are placed *Seemann*, no. 942, Loja, Ecuador, and, with considerable doubt, *Tonduz*, no. 11,680, " Bords des rivières au Copey," Costa Rica. The latter specimen is much more ferrugineous than the former and forms a transition to what has been called

Alnus acuminata var. ferruginea (HBK.) Regel. ?? *Alnus ferruginea* HBK. *A. ferruginea* Fern. Proc. Am. Acad. xl. 27 (1904) *pro parte*. This name may be provisionally accepted for *Tuerckheim*, no. 351, Coban, Department of Alta Verapaz, Guatemala, which seems to be a very ferrugineous extreme of the plant here called *A. acuminata*. The pubescence is very dense, and is persistent on all but the oldest leaves.

Alnus arguta (Schlecht.) Spach. *Betula arguta* Schlecht. *Alnus arguta* Spach *α genuina* Regel, Monog. 93 (1861). In its typical form this species is accepted as interpreted by Professor Fernald. It presents, however, two variations which seem worthy of recognition. Neither of them appears to fall into any of Regel's four varieties. His var. *genuina* is here taken up as the type form of the species. Var. *Benthami* is so inadequately characterized as to be unrecognizable without access to the type. Moreover it came from Zacualtipan, to the north of the known range of either of the two varieties here proposed. Var. *ovata* was based upon material from three Mexican localities, and one Peruvian locality, but since Regel cited as a synonym *A. Mirbelii* var. Grisb. in Lechl. Pl. Peruv. the type is definitely fixed as the Peruvian element, which it is almost inconceivable should be the same as the Mexican. Var. *punctata* was purely South American.

erecto, inferiore declinato; stylo glabro latitudine filamenta aequanti; stigmate obscure bidentato; ovario ovato-acuminato, inserto in receptaculo cupulato; capsula elliptica basi ad stipam latam contracta, apice minute glandulifera; seminibus suborbiculatis minutissime muriculatis et palmate nervatis. — DURANGO: San Ramon, April–May, 1906, *Dr. Edward Palmer*, no. 73 (type, in hb. Gray). Dr. Palmer notes that this is a loosely branching plant 12–15 dm. high, with many scarlet flowers, growing at the edge of shady woods. It belongs to Sect. *Sphenostegia* Nees in DC. Prodr. xi. 479, and is near *D. sexangularis* Juss. and *D. brachiata* Spreng. The corolla in this is larger, with the lips less spreading.

Tetramerium flavum, n. sp., caule erecto divaricate ramoso 6–12 dm. alto quadrangulato, inter angulos striato, scabridulo investo pilis tenuibus adpressis et pilis articulatis longioribus; ramis oppositis, junioribus glandulosis et dense albo-pubescentibus; foliis penninervatis ovato-acuminatis basi ad petiolum brevem inaequaliter attenuatis longissimis, in specimine viso 12 cm. longis, 5 cm. latis; petiolis 2 cm. longis; spicis axillaribus et terminalibus simplicibus vel compositis (ultima spica longissima); floribus imbricatis, bracteis distichis oblanceolatis aristatis trinerviis 5 mm. longis 1.5 mm. latis, apice recurvatis; involucri bracteolis connatis carinatis obovatis apice aristatis quinquenervatis 12 mm. longis, floram solitariam includentibus; calycis laciniis lineari-setaceis glandulifero-pilosis membranaceis 3 mm. longis; corollae flavae tubo anguste cylindrico 4 mm. longo, laciniis patenti-divaricatis 1 cm. longis, labio superiore erecto spatulato 1 cm. longo, inferiore ternato, segmentis patenti-divaricatis obovatis 1 cm. longis; filamentis faucibus insertis, glabris; antherarum loculis parallelis paulo inaequalibus muticis conjunctis; ovario crasso-stipitato apice hispido breve acuminato calycis lacinias superanti; receptaculo crasso clavato; stylo bifido antheras superanti. — DURANGO: San Ramon, April–May, 1906, *Dr. Edward Palmer*, no. 75 (type, in hb. Gray). This is most closely related to *T. aureum* Rose, which, however, has bracts and bracteoles obovate obtuse, leaves truncate or subcordate at base. From all other species it differs in having the cauline bracts narrower and much shorter than the involucral. It is a showy plant, rather woody, growing at base of mountains near the edge of woods. It is a free bloomer with "canary yellow flowers that close at night."

insertis ad mediam tubi; antheris exsertis obscure mucronulatis; filamentis glandulosis et pubescentibus; stylo crasso, apice bifido, stigmatibus peltatis. — OAXACA: on hills, altitude 1300 m. at Jayacatlan, *Lucius C. Smith*, July 27, 1895, no. 549 (type, in hb. Gray), also Cuicatlan hills, June 17, 1895, no. 399.

Seymeria deflexa, n. sp., scabrida et glandulosa; foliis deflexis, majoribus oblongo-ovatis obtuse dentato-laciniatis, segmentis inferioribus crenatis apice obtusis, basi decurrentibus ad petiolum; minoribus oblongis integris basi cuneatis, superiore parte dentatis; petiolis brevibus; floribus divaricate-paniculatis, pedicellis capillaribus saepe decurvatis 5 mm. longis; calycis laciniis tubum campanulatum aequantibus, oblongo-ovatis obtusis recurvatis 2 mm. longis, in fructu patentibus; corolla campanulata 8 mm. longa, laciniis inaequalibus suborbiculatis ciliatis reflexis basi auriculatis; filamentis subulatis brevibus crassis pilosis; antheris exsertis flavis nervatis 3 mm. longis 1.3 mm. latis papillosis, apice dehiscentibus; stylo antheras superante, in fructu declinato apice tenuiter clavato; ovario punctato-scabrido sub-cydoniformi. — NUEVO LEON: limestone ledges of the Sierra Madre above Monterey, 19 September, 1907, *C. G. Pringle*, no. 10,398 (type, in hb. Gray). This differs from other described species in having deflexed less dissected leaves, and pyramidal paniculate inflorescence. The color of the flowers is not known, but the exserted stamens, as well as the recurved divisions of the calyx and corolla, give the flowers a slight resemblance to some Californian species of *Dodecatheon* belonging to the *D. patula* group. The type specimen consists of the upper part of the stem, therefore the lowest leaves are unknown.

Dicliptera floribunda, n. sp., perennis, erecte et diffuse ramosa, 12–15 dm. alta; ramis sexangulatis sparse pubescentibus, nodis remotis foliatis et floribundis supra axillas geniculatis; foliis integerrimis ovato-lanceolatis acuminatis 1 dm. longis, 5 cm. latis apice mucronatis, basi ad brevem petiolum decurrentibus, scabridule pubescentibus subter penninerviis et investis cum pilis brevibus furcatis; junioribus partibus albo-tomentosis; capitulis glomeratis, pedunculis brevibus vel abeuntibus; bracteis involucri 2, obovatis basi cuneatis chartaceis apice foliaceis saepe purpureo-tinctoreis scabridulis; umbellis inclusis 3-floris; bracteolis lineari-acuminatis calycem superantibus costatis et carinatis apice aristatis basi connatis: calycis segmentis trinerviis chartaceis attenuatis obscure glandulifero-pilosis 6 cm. longis; corolla verisimiliter flammea (coccinea fide Palmeri) leviter investa cum pilis furcatis 3 cm. longa, tubo gradatim ampliato, faucibus 5 mm. diametro, labiis paulo divergentibus, postice integris, antice 3-crenulatis; filamentis paulo pilosis; antheris exsertis, loculis discretis, superiore loculo brevibus stigma superantibus; stylo brevi recto; stigmate bilamellato, partibus obovatis 1 mm. longis 0.5 mm. latis; capsulis calycem superantibus oblongo-ellipticis ad basim dehiscentibus cum duabus valvulis divergentibus; placentis paulo intrusis muricatis; seminibus numerosis brunneis suborbiculatis minute papillosis vel irregulariter et interrupte corrugatis. — MICHOACAN: Morelia, on a bare damp mesa, 29 November, 1907, *C. G. Pringle*, no. 10,408 (type, in hb. Gray). This tiny plant seems nearest to **Centaurium tetramerum** (Schiede), n. comb. (*Erythraea tetramera* Schiede ex Schl. in Bot. Zeit. xiii 920), and resembles that species in its 4-merous flowers and dehiscent fruit. It differs, however, in the short corolla-tube not exceeding the calyx, the leaves with scarcely perceptible nerves, the fasciculate flowering stems, the persistent basal leaves, the striate peduncles, and the bilamellate stigma. The color of the flowers is not readily discernible in the dried specimens, but the lower part of the limb of the corolla appears to be yellow and the tips of the lobes tinged with pink.

Spigelia quaternata, n. sp., radicibus fasciculatis; caulibus multis ex caudice breve, 3 dm. altis purpureis minute scabridis, parte superiore angulatis; foliis saepissime quaternatis supremis oppositis ovato-oblongis apice acuminatis 4–8 cm. longis 1–3 cm. latis integris superne glabris vel scabridulis subter pallidioribus et glabris, nerviis primariis et secondariis hispido-scabridis; stipulis brevibus triangularibus basi semi-amplexicaulibus folia conjungentibus; tot ramulis quot foliis ad nodos, terminantibus in spicis gracilibus; floribus flavis sessilibus secundis, in alabastro confertis, in fructu 3–6 mm. distantibus; sepalis lineari-lanceolatis acuminatis 4 mm. longis 1 mm. latis cum duabus glandibus interioribus; corollae tubo 8 mm. longo, laciniis oblongis acutis 3 mm. longis, superiore paulo longiore; capsulae basi persistente; seminibus globosis punctatis. — SAN LUIS POTOSI: Rascon, *Dr. Edward Palmer*, 19–22 June, 1905, no. 671 (type, in hb. Gray). This species is most closely related to *S. Humboldtiana* Cham. & Schlecht. and is easily distinguished by its much smaller flowers and its scabrid and more or less hispid pubescence.

Bourreria obovata, n. sp., ramulis senioribus minute albo-punctatis, junioribus canescentibus cum pilis brevibus adpressis; foliis obovatis superne scabridulis cum pilis brevibus adpressis basi minute pustulatis subter pallidioribus non scabridis, apice truncatis obtusis vel retusis, basi ad petiolum brevem attenuatis; pedunculis terminalibus cymosis cum pedicellis brevibus; bracteis foliaceis; calyce fere ad mediam partem 5-dentato, dentibus triangularibus acutis 4 mm. longis, utrinque adpresso-pilosellis; corolla rotata, tubo calycem aequanti, lobis 5–6 rotundatis, 5 mm. longis, basi auriculatis; staminibus 5–6, subter reticulatis; calyce fructifero quinquepartito, segmentis inflexis obovatis vel oblongis parallele nerviis praeter basim fusco-puberulentem glabris; fructibus globosis depressis glabris nitidis 2.5 cm. diametro; pedunculis solitariis 5 mm. longis fusco-pubescentibus; seminibus oblongis 12 mm. longis 7 mm. latis, una facie convexa, altera plana. — San Luis Potosi: San Dieguito, 7–10 June, 1905, *Dr. Edward Palmer*, no. 631 (type, in hb. Gray). Dr. Palmer notes this as a large shrub or small tree 2–4 m. high with considerable top and a profusion of dark green leaves, the fruit thinly scattered, having the appearance of persimmons, light green but with a patch of red and brown at the exposed or lower end. Without the flowers its affinities are doubtful. Compared with the species listed by Hemsley (Biol. Cent.-Am. Bot. ii. 300) it differs as follows: from *D. ciliata* A. DC. in having obovate instead of ovate leaves; from *D. cuneifolia* Hiern, in being glabrous instead of hispid or pubescent, as well as in having leaves larger, and fruit three times the size; from *D. Ebenaster* Retz. it differs in the shape and size of leaves, much smaller fruit, and quite entire calyx-lobes; from *D. velutina* Hiern, it differs in the shape of leaves and absence of fulvous velutinous pubescence, and from *D. texana* Scheele it also differs in leaves and pubescence.

Forestiera puberula, n. sp., divaricate ramosa; ramulis griseis et atro-puberulis, verrucosis cum squamulis marcescentibus alabastrorum; foliis fasciculatis lineari-spatulatis apice obtusis basi breve petiolatis 5–10 mm. longis 1-nervatis, margine revolutis, superne puberulis, subter glabris porulosis; pedunculis cum foliis fasciculatis, 3–5 mm. longis; fructibus (immaturis) cylindraceis falcatis obtusis 8 mm. longis, 3 mm. diametro. — Zacatecas: in arroyas, Cedros, June, 1908, *J. E. Kirkwood*, no. 12 (type, in hb. Gray).

Related to *F. angustifolia* Torr., differing chiefly in the puberulent stems and leaves, the latter smaller and strongly revolute. The cylindrical falcate fruit also distinguishes it. The flowers are unknown.

Centaurium pusillum, n. sp., nanum 4–8 cm. altum ramosissimum glabrum; ramis tenuissimis quadrangulatis; foliis imis rosulatis, primis spatulatis, ceteris oblanceolatis acutis 1 cm. longis 2 mm. latis, nerviis obscuris; foliis caulinis lanceolatis acuminatis vel apice acutis basi amplexicaulibus; floribus longe pedunculatis non-numquam sessilibus tetrameris 7 mm. longis; pedunculis inter angulos striatis; calycis laciniis fere liberis lanceolatis acutis carinatis, margine membranaceis 3–4 mm. longis, tubo brevi multo longioribus; corollae laciniis oblongis vel ellipticis obtusis 4 mm. longis 2 mm. latis contortis et supra capsulam marcescentibus, tubo calycem aequanti, faucibus constrictis; filamentis in faucibus insertis capillaribus 2 mm. longis; antheris ovato-cordatis

Aristolochia Nelsonii, n. sp., suffruticosa prostrata; caule prope basim ramoso velutino, ramis diffusis angulatis; foliis ovato-cordatis vel saepius auriculato-trilobatis, apice acuminatis, basi ad petiolum cuneate excurrentibus (auriculis rotundatis), palmate trinerviis et reticulatis, supra investis regulariter adpressis pilis basi minute pustulatis, subter subvelutinis; petiolis canaliculatis 1–2 cm. longis; floribus solitariis in axillis, pedunculis gracilibus 2 cm. longis, apice cum bractea sessili ovato-cordata acuminata 1 cm. longa 5–7 mm. lata; calycis limbo patulo peripherico longe caudato, basi purpureo-marginato, flavo circa os, cauda flava 4–5 cm. longa 2 mm. lata; tubo geniculato 4 mm. diametro; columna superne quinquelobata basi stipitata; antheris 5; ovario clavato albo-pilosissimo pedicellato. — OAXACA: San Geronimo, 61 m. altitude, July 1–5, 1895, *E. W. Nelson*, no. 2769 (type, in hb. Gray). This approaches *A. longicaudata* Watson, but differs in much broader limb, and in the form of the leaves. It belongs to Sect. *Gymnolobus* Dctre. l. c. and to the pentandrous group.

Passiflora platyneura, n. sp., caulibus angulatis et striatis hispidis pilis albis uncinatis; cirrhis nullis; foliis infra mediam partem trilobatis 2–4 cm. longis 3–5.5 cm. latis, lobo medio oblongo-ovato lateralibus inaequaliter bilobatis, basi late reniformibus, lobis margine integris vel saepissime irregulariter dentatis, dentibus apice aristatis, nerviis supra filiformibus subter planis; petiolis ca. 1 cm. longis apice biglandulosis glandulis crasse stipitatis; stipulis viridibus oblique ovatis subfalcatis apice aristate attenuatis 3 mm. longis; floribus axillaribus pedunculis 15.–2 cm. longis; bracteis 2–3 proximis angustissime linearibus attenuatis 2 mm. longis; calycis tubo rotato-campanulato 1.5 cm. lato, lobis lineari-oblongis apice obtusis 1.5 cm. longis 6 mm. latis uninerviis, interiore glabris exteriore hispidis; petalis tenuibus oblongo-lanceolatis ca. 1 cm. longis 2.5 mm. latis; corona exteriori filamentosa, filamentibus 1.3 cm. longis ad basim liberis; corona interiori membranacea, apice fimbriata, duos annulos inferiores occultanti; gynandrophora 9 mm. longa glabra; fructibus globosis basi cuneatis. — OAXACA: Cuilopan Mountains, altitude 2135 m., 27 July, 1894, *Rev. Lucius C. Smith*, no. 44 (type, in hb. Gray); Sierra de San Felipe, altitude 2287 m., 31 May, 1894, *C. G. Pringle*, no. 5750. This species is probably nearest *P. Pringlei* Robinson & Greenman, differing most noticeably in the shape of the leaf, the position of the stipular glands, and the white instead of dark pubescence. The differences in the flowers seem to be rather of degree than of kind.

Diospyros Palmeri, n. sp., arborescens; ramulis divaricatis griseis glabris; foliis alternis obovatis 3–5 cm. longis 2 cm. latis, basi cuneatis breviter petiolatis, apice rotundatis vel truncatis, coriaceis superne nitida

IV. SOME UNDESCRIBED SPECIES OF MEXICAN PHANEROGAMS.

By Alice Eastwood.

Aristolochia oaxacana, n. sp., caulibus 1–paucis ex radice tuberosa prostratis tenuiter pilosissimis ramosis 1–2 dm. longis; foliis ovato-cordatis 2–4 cm. longis 2 cm. latis, apice acutis basi cordato-auriculatis ad petiolum brevem inter auriculas cuneate decurrentibus; floribus in axillis solitariis, bracteis obscuris ovatis ad basim pedunculi brevis insertis; calyce albo-purpureo unilabiato 3.5 cm. longo recto, tubo 12 mm. longo paulo constricto ad squamam interiorem infundibuliformem, limbo lineari antice ad tubum decurrenti; columna crasso-stipitata; antheris 5; stigmate peltato quinquelobato; ovario clavato pilosissimo; capsula turbinata quinquevalvata pilosa, apice dehiscenti. — Oaxaca: Clajiaco, *Galeotti*, no. 214. This belongs to Sect. *Gymnolobus* Dctre. in Ann. Sci. Nat. ser. 4, ii. 30, and is related to *C. cordata*, which, however, has a bilabiate calyx.

Aristolochia cordata, n. sp., caulibus plurimis ex radice longa flava, simplicibus vel ramosis prostratis gracilibus striatis tenuiter albo-pilosissimis; foliis subsecundis ovato-cordatis 2–4 cm. longis et latis, apice obtusis, basi cordatis, palmate quinquenerviis reticulatis, investis sparse, supra, densiore subter cum pilis tenuissimis obscure articulatis adpressis simplicibus vel basi bifurcatis; petiolis 5–12 mm. longis pilosissimis; floribus solitariis in axillis, pedunculis laminatis striatis pilosissimis cum bractea ovata apice inserta; calyce bilabiato albo-purpureo exteriore piloso, interiore glabro; labio superiore cucullato 6–10 mm. longo, inferiore deflexo et conduplicato obcordato 1 cm. lato; tubo flavo-lineato 11–12 mm. longo paulo constricto ad squamam interiorem infundibuliformam et sub os; columna sessili; antheris 5; stigmate peltato quinquelobato; ovario clavato pilosissimo basi ad pedicellum attenuata; capsula oblongo-turbinata quinquevalvata rugulosa, valvulis dorso crenati-alata, apice dehiscentibus. — Durango: Otinapa, July–August, 1906, *E. Palmer*, no. 431 (type, in hb. Gray). This belongs to Sect. *Gymnolobus* Dctre. l. c. 30 and is distinguished from the other pentandrous species of the section by the remarkable two-lipped flower.

furcatis ; internodiis quam foliis 3–4-plo brevioribus. Folia glabra vel glabrata, laminis ovato-lanceolatis 1.2–3 cm. latis 3–7 cm. longis, supra viridibus, subtus albido-viridibus, apice acuta saepe mucronulata, basi rotundata vel aetate subcordata; petiolis 4–7 mm. longis. Inflorescentiae in foliorum axillis solitariae vel fasciculatae. Pedunculi 5–7 mm. longi glabri, prope apicem glauci. Involucrum 3(–5)-florum, squamis saepissime 4 suborbicularibus, duabus exterioribus extus mox glabratis, apice mucronatis, interioribus utrinque pubescentibus, apice obtusis. Pedicelli ante florum anthesin tomentosi, maturitate fructus glabri valde incrassati pedunculis aequilongi, apice in discum diametro 5 mm. expansi. *Flores* ♂ (a gemmis nondum florescentibus descripti). Perianthii tubus brevis ; segmenta 6 ovata apice obtusa. Stamina 9, interiorum 3 filamentis biglanduliferis. Loculi inferiores seriei antherarum interioris simulate lateraliter, ceteri introrsum, dehiscentes. Ovarium abortivum sine stylo. *Flores* ♀ non visi. Fructus (siccatus) niger usque ad 11 mm. diametro. Nucula 7 mm. diametro, cotyledonibus apice emarginatis corculum minutum includentibus. — Nuevo Leon: Sierra Madre near Monterey, *Pringle*, nos. 2837 (type, in hb. Gray) & 2078. San Luis Potosi: Alvarez, Sept. 1902, *Palmer*, no. 62 ; mountains, San Jose Pass, *Pringle*, no. 3146.

interiora tria (an staminodia?) acuta. Staminodia vera 9, ea seriei interioris biglandulifera, glandulis longe stipitatis. Stylus apice stigma disciforme lateraliter gerens. Fructus (siccatus) globosus, diametro usque ad 9 mm., niger (?). — This species constitutes a part of *Litsea parvifolia* (Hemsl.) Mez, as defined by Mez. The following specimens may be referred to it. San Luis Potosi: "*in montibus* San Miguelito," *Schaffner*, nos. 23 (type, in hb. Gray) & 710; *Schaffner*, nos. 431 & 463; *Parry & Palmer*, no. 798. State of Guanajuato: near Guanajuato, 1880, *A. Dugès;* Palmilla, Dept. Victoria, *Berlandier*, no. 2185. The last specimen is cited by Meissner in the original description of *Litsea glaucescens* var. *subsolitaria*, and is the same as the unnumbered plant cited by Hemsley as follows: "Vittoria to Tula *(Berlandier)*." The original label reads: "No. 2185 = 765. Arbuste 8–10 pds., d'les gorges ombragées — avant d'arriver à Palmilla. De Victoria à Tula, Nov. 1830." Probably the citation by Mez, under *Litsea glaucescens*, of "*Berlandier* n. 2158 *(non vidi)*" is an error for no. 2185, since there is no record of a "no. 2158" in the manuscript catalog of Berlandier's collections at the Gray Herbarium. According to the Parry & Palmer label, *Litsea Schaffneri* is the "Sacred Laurel" of the Mexicans. The Schaffner labels give "Laurel" as the vernacular name.

10. Litsea parvifolia (Hemsl.) Mez, fruticosa, ramulis gracilibus juventate griseis puberulis, aetate ochraceo-brunneis glabris. Internodia foliis 2–3-plo breviora. Folia utrinque glabra, laminis orbiculari-ovatis vel maximis non raro ovatis 1.2–3 cm. latis 1.3–4 cm. longis, supra pallide viridibus, subtus albidis, basi cordatis vel subcordatis, apice plerumque rotundatis vel obtusis sed in foliis maximis saepe acutiusculis; petiolis 2–5 mm. longis ochraceo-olivaceis. Inflorescentiae axillares saepissime solitariae, raro fasciculatae. Pedunculi 5–9 mm. longi nutantes tenuissime puberuli. Involucrum 3(–5)-florum, squamis 3(–5) deciduis puberulis. Pedicelli subaequales floribus multo breviores albo-tomentosi. *Flores* ♂. Perianthii tubus perbrevis; segmenta 6 ovata obtusa. Stamina 9. Antherae late rectangulares filamentis longiores apice truncatae minute apiculatae, loculis omnibus introrsum dehiscentibus. Filamenta seriei staminum interioris glandulos brevistipitatos convolutos gerentia. Ovarium abortivum stylo apice indistincte trilobo instructum. *Flores* ♀ non vidi. Fructus diametro 8–10 mm. globosus. — The original characterization of this species was probably drawn up from insufficient material. Mez's description includes at least two and perhaps even three species. Specimens examined: Mexico, 1848–'49, *Gregg*, no. 314: Saltillo, State of Coahuila, 15–30 April, 1898, *Palmer*, no. 68.

11. **Litsea novoleontis**, n. sp. Frutex 3–5 m. altus, ramulis

deciduis 5 (vel 7) suborbicularibus, extus mox glabratis, exterioribus apice acutiusculis, interioribus obtusis. *Flores* ♂. Pedicelli quam perianthium breviores vel idem aequantes, juventate tomentosi. Perianthii tubus fere nullus; segmenta 6 oblonga 3 mm. longa, basi paulo angustata. Stamina 9. Filamenta antheris paululo breviora, tria interiora biglandulifera, glandulis mediocribus varie lobatis sed non convolutis. Antherae oblongae ad apicem versus sensim angustatae, apice obtusae. Loculi antherarum omnes introrsum dehiscentes, sed ei inferiores seriei interioris aspectu sublaterales. Ovarium abortivum sine stylo. *Flores* ♀. Pedicelli quam perianthium longiores (is floris medii duplo longior), juventate tomentelli, aetate glabri incrassati. Perianthii tubus fere nullus; segmenta 6 anguste oblonga 2.4 mm. longa. Staminodia 9 graciliter spatuliforma, tria interiora biglandulifera, glandulis reniformibus ad hilum stipitatis. Stylus curvatus, stigmate disciformi irregulariter bilobo. Fructus immaturus ovoideus. — "Petit arbre, à port élancé. Collines au dessus de Belmira près Santa Maria de Dota," Prov. San José, Costa Rica, alt. 1600 m., January, 1898, *Tonduz*, no. 11,638 (= no. 7352 of Mr. John Donnell Smith's distribution, type, in hb. Gray); Cuesta de Tarrazú, April, 1893, *Tonduz*, no. 7796. Vernacular name, "Lentisco." In all probability the Costa Rican specimens cited by Mez under *Litsea glaucescens* var. *subsolitaria* belong to this species, but unfortunately none of them are available for examination. *Litsea flavescens* may be distinguished from *L. glaucescens* not only by the characters given in the key, but also by its smaller flowers, tomentose pedicels, and obtuse, not emarginate, anthers.

9. **Litsea Schaffneri,** n. sp. Frutex 2–3 m. altus, ramulis gracilibus ochraceis; internodiis quam foliis 3–4-plo brevioribus. Folia glabra subtus glaucescentia, laminis anguste lanceolatis 6–14 mm. latis 2–5 cm. longis, basi acutis, apice acutis saepe mucronulatis; petiolis 5–10 mm. longis. Inflorescentiae solitariae vel fasciculatae. Pedunculi nutantes 5–10 mm. longi glabri. Involucrum triflorum, squamis 5 suborbicularibus deciduis, duabus exterioribus mucronulatis extus glabratis intus pubescentibus, interioribus obtusis utrinque pubescentibus. Pedicelli aut glabri aut tomentosi, inaequales, is floris medii aliis multo longior sed ipse perianthio brevior. *Flores* ♂. Tubus perianthii brevis; segmenta 6 ovata obtusa 3 mm. longa. Stamina 9. Filamenta antheris paulo breviora, tria interiora biglandulifera, glandulis valde stipitatis convolutis. Antherae subquadrangulares ad apicem versus paulo angustatae apice emarginatae, loculis omnibus introrsum dehiscentibus. Ovarium abortivum stylo brevi integro instructum. *Flores* ♀. Perianthii segmenta 9 ovata 2 mm. longa, exteriora 6 obtusa,

florum lateralium aliquanto breviores. Perianthium tubo brevi, segmentis ovatis obtusis 3.2 mm. longis. Stamina 9, filamentis glabris antheris aequilongis vel eis paulo brevioribus, tribus interioribus biglanduliferis, glandulis majusculis convolutis. Antherae subrectangulares supra mediam paulo angustatae, apice emarginatae, loculis omnibus introrsum dehiscentibus. Ovarium abortivum. *Flores* ♀ quam masculi multo parviores. Perianthii tubus brevis; segmenta ovata obtusa 2.2 mm. longa. Staminodia 9, interiora 3 glandulifera, glandulis eis florum ♂ similibus. Stylus 1.2 mm. longus. Stigma discoideum subreniforme. Fructus ignotus. — On limestone ledges in the Sierra Madre above Monterey, State of Nuevo Leon, Mexico, alt. 850 m., 8 March, 1906, *Pringle*, no. 10,238 (type, in hb. Gray).

4. LITSEA GLAUCESCENS HBK. The following specimens, all from the State of Vera Cruz, are in the Gray Herbarium: Orizaba, *Botteri*, nos. 7 & 549 (error for 945 ?); Orizaba, 10 April, 1867, *Bilimek*, no. 359; hills near Jalapa, 16 April, 1899, *Pringle*, no. 8156. Since the Pringle specimen shows a strong habital resemblance to *Litsea guatemalensis* Mez, it may represent the *Litsea glaucescens* var. *major* (Meissn.) Hemsl., from which Mez segregated his species.

Var. SUBSOLITARIA (Meissn.) Hemsl. — Mexico, 1848–'49, *Gregg*, no. 639. Leaves much more glaucous beneath than in the typical form. None of the inflorescences arranged in axillary corymbs.

5. LITSEA GUATEMALENSIS Mez. — MEXICO. Chiapas: "Bergwald zwischen Huitztan und Oxchuc," 11 March, 1896, *Caec. & Ed. Seler*. GUATEMALA. Department of Quiché: San Miguel Uspantan, alt. 2440 m., *Heyde & Lux*. Department of Zacatepequez: San Rafael, alt. 1980 m., *John Donnell Smith*, no. 1276.

6. LITSEA NEESIANA (Schauer) Hemsl. Nothing has been seen which answers to the description of this species. The plant cited by Mr. John Donnell Smith as *Litsea Neesiana* in his *Enumeratio Plantarum Guatemalensium* is *Litsea guatemalensis*.

7. LITSEA ORIZABAE (Mart. & Gal.) Mez. — State of Vera Cruz: Orizaba, alt. 2440 m., *Liebmann*, *Lauraceae* no. 65. This shrub has larger leaves than any other American member of the genus.

8. **Litsea flavescens**, n. sp. Arbuscula (fide cl. Tonduz), ramis numerosis ochraceo-brunneis, gemmis quam in speciebus *Litseae* ceteris majoribus. Folia coriacea glabra quam internodia 6–7-plo longiora, laminis griseo-viridibus, supra subnitidis, subtus pallidis, lanceolatis ca. 2 cm. latis 6.5 cm. longis, basi acutis, apice caudato-acutis mucronatis; petiolis 1–1.5 cm. longis. Petioli et costae laminarum mediae marginesque latiusculi flavescentes. Inflorescentiae solitariae vel fasciculatae. Pedunculi glabri 7–9 mm. longi. Involucrum 3–7-florum, squamis

1. LITSEA GENICULATA (Walt.) Benth. & Hook. Mez says of this species : " Hab. in paludosis a Virginia ad Floridam." There seem to be no specimens in American herbaria from further north than North Carolina. Perhaps the reference to Virginia is merely traditional, coming from the name of the work (Gronovius's *Flora Virginica*) in which this shrub was first described, as "*LAURUS foliis lanceolatis enerviis annuis.*"

2. **Litsea pedicellata**, n. sp. Frutex 1–2 m. altus, ramulis furcatis glabris atro-bruneis. Folia glabra coriacea quam internodia duplo longiora, laminis orbiculari-ovatis utrinque albicantius viridibus 2–3 cm. longis 1.5–2 cm. latis, basi subcordatis, apice obtusis saepe mucronulatis, petiolis brunnescentibus 2–3 mm. longis. Inflorescentiae solum in axillis superioribus positae, plerumque in ramulis brevibus quorum terminalis paniculiformis est et foliis multo longior. Ramuli floriferi in gemmam parvam paucisquamosam terminantes. Pedunculi 6–9 mm. longi glabri prope apicem incrassatum glauci. Involucrum triflorum, squamis tribus late suborbicularibus deciduis, extus mox glabratis intus pubescentibus. *Flores* ♂. Pedicelli quam in speciebus aliis mexicanis multo longiores, saepissime pedunculis fere aequilongi, superne glabrati, prope basin aeque quam in pedunculi apice, intra involucrum, albo-tomentelli. Perianthii tubus brevis ; segmenta ovata apice obtusa. Stamina 10, filamentis glabris quam antheris brevioribus vel eis aequilongis, tribus interioribus biglanduliferis, glandulis majusculis convolutis. Antherae subrectangulares ad apicem versus valde angustatae. Loculi superiores inferioribus parviores, semper introrsum dehiscentes. Loculi inferiores staminum glanduliferorum sublateraliter, reliqui omnes introrsum, dehiscentes. Ovarium abortivum stylo apice breviter bilobato. *Flores* ♀ fructusque desunt in specimine authentico. — Mountains near Saltillo, State of Coahuila, Mexico, alt. 2135 m., 12 April, 1906, *Pringle*, no. 10,239 (type, in hb. Gray).

3. **Litsea Pringlei**, n. sp. Frutex 1–2 m. altus, ramulis gracilibus glabris olivaceis. Internodia plerumque quam folia duplo breviora. Folia glabra coriacea, laminis ovato-lanceolatis 4.5 cm. longis, supra basin 2 cm. latis, apice acutis saepe mucronulatis, basi subcordatis vel rotundatis ; petiolis subolivaceis 5–7 mm. longis. Axillae foliorum omnes ramulos breves floriferos gerentes quorum terminalis haud paniculiformis est, sed aliis similis et foliis brevior. Ramuli floriferi, ut in *L. pedicellata*, apice gemmiferi. Pedunculi 6–9 mm. longi apice excepto glabri, ad apicem, intus in involucro, albido-hirtelli. Involucri squamae 3 late suborbiculares deciduae, extus mox glabratae, intus pubescentes. Involucrum 3- vel 5-florum. *Flores* ♂. Pedicelli glabri inaequilongi, is floris medii longitudine perianthium saepe aequans, ei

III. A SYNOPSIS OF THE AMERICAN SPECIES OF LITSEA.

By Harley Harris Bartlett.

The following synopsis of the American species of *Litsea* includes the six species recognized by Mez[1] in 1889, together with five heretofore undescribed species from Mexico and Central America. No attempt has been made to cite full synonymy, nor, with one exception, to redescribe species recognized by Mez, hence this paper may be considered as supplementary to his treatment of the genus.

For the loan of valuable Central American material, without which the new species from Costa Rica must have remained undescribed, I am indebted to Captain John Donnell Smith. Except for the specimens from his herbarium, the *exsiccatae* cited are all at the Gray Herbarium.

Folia decidua. . . . . 1. *L. geniculata.*
Folia persistentia.
Inflorescentiae plerumque corymbosae, rarius paniculatae.
Folia subtus glabra.
Folia basi rotundata vel subcordata.
Pedicelli quam flores multo longiores; inflorescentiae fere omnes in paniculam terminalem dispositae . . . . . . 2. *L. pedicellata.*
Pedicelli quam flores breviores; inflorescentiae non modo terminales sed etiam in axillis foliorum mediis corymbosae. 3. *L. Pringlei.*
Folia basi acuta. . . . . 4. *L. glaucescens.*
Folia subtus pubescentia.
Folia subtus plus minusve strigosa. . . . . 5. *L. guatemalensis.*
Folia subtus ochraceo-tomentosa. . . . . 6. *L. Neesiana.*
Inflorescentiae solitariae vel fasciculatae.
Folia basi acuta.
Folia subtus albescentia, molliter tomentosa. . . . 7. *L. Orizabae.*
Folia glabra.
Folia plus quam 2 cm. lata.
Folia subtus glauca. . . . . (4) *L. glaucescens var. subsolitaria.*
Folia haud glauca. . . . . 8. *L. flavescens.*
Folia maxima 1.5 cm. lata. . . . . 9. *L. Schaffneri.*
Folia basi subcordata vel rotundata.
Folia orbiculari-ovata, apice obtusa. . . . . 10. *L. parvifolia.*
Folia ovato-lanceolata, apice acuta. . . . . . . 11. *L. novoleontis.*

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[1] Carl Mez, Lauraceae Americanae monographice descriptae. Jahrbuch des königlichen botanischen Gartens und des botanischen Museums zu Berlin. Band V, 1889.

montium Sierra Madre inter Michoacan et Guerrero, alt. 2250 m., 16 Feb. 1899, *E. Langlassé*, n. 878 (specimine typico in hb. Grayano conservato).

6. R. POLYMNIOIDES Greenman, verisimiliter herbacea vel subherbacea; caule purpurascenti striatulo-angulato crispe albido-pubescenti fistuloso; foliis oppositis late ovatis acute acuminatis 1–1.2 dm. longis ca. 8 cm. latis 3-nerviis reticulato-venosis mucronato-denticulatis supra viridibus breviter pubescentibus subtus pallidioribus griseo-tomentellis et resinoso-atomiferis basi primo abrupte deinde cuneate ad petiolum 2–3 cm. longum per totam longitudinem alatum angustatis; capitulis in paniculam laxam planiusculam 2–3 dm. diametro dispositis, pedicellis griseo-hirsutis gracilibus 1–3 cm. longis saepe nutantibus; involucri squamis exterioribus 5 late ovatis acutis herbaceis 3-nerviis et reticulato-venosis extus laxe griseo-hirsutis intus paulo pallidioribus glaberrimis margine albido-puberulis vel -pulverulis, squamis interioribus linearibus conduplicatis attenuatis hispidulis; flosculis ♀ ca. 15, ligulis minimis, tubo gracili hispido ca. 3 mm. longo, lamina oblonga ca. 4 mm. longa 1.8 mm. lata apice 3-lobata flava; flosculis disci ca. 80, corolla flava, tubo proprio hispidulo 3 mm. longo basin versus non ampliato fauces cylindricos aequanti; achaeniis laevibus pallide brunneis oblique obovatis modice compressis plus minusve 4-gonis. — Proc. Am. Acad. xxxix. 99 (1903). — In agris ubi colitur *Zea Mais*, Copey, Costa Rica, alt. 1800 m., Apr. 1898, *Tonduz*, n. 11,947.

pubescentibus; involucri squamis exterioribus 5 ovati-ellipticis acutis 8–10 mm. longis 3–4 mm. latis herbaceis glabriusculis margine albide granuloso-puberulis; squamis interioribus ovatis acuminatis cucullatis dorso breviter hispidulis; flosculis ♀ ca. 6–8, ligulis lineari-oblongis 10–12 mm. longis flavis conspicue exsertis et patentibus; corollis disci hispidulis 6 mm. longis, tubo gracili fauces cylindricos subaequanti; achaeniis glabris. — In terra humo pingui montium Sierra Madre inter Michoacan et Guerrero, alt. 1750 m., 26 Apr. 1899, *E. Langlassé*, n. 800 (specimine typico in hb. Grayano conservato).

4. R. ARAGONENSIS Greenman, verisimiliter fruticosa; caulibus teretibus fistulosis; foliis rhomboideo-ovatis mucronulato-denticulatis membranaceis supra glabriusculis subtus sparse pubescentibus ca. 1.2 dm. longis 9–10 cm. latis latere utrioque unilobatis vel uniangulatis basi ad petiolum per totam longitudinem alatum 3–4 cm. longum angustatis; foliis supremis ovati-lanceolatis caudato-acuminatis non angulatis; panicula planiuscula laxa; involucri squamis exterioribus 5–6 ovatis acuminatis venosis 1.6 cm. longis 7–8 mm. latis tenuibus inconspicue puberulis, squamis interioribus ovatis acuminatis dorso breviter glanduloso-hispidulis 5 mm. longis; ligulis linearibus tenuibus 5 mm. solum longis 0.8 mm. latis flavis, tubo 2 mm. longo pubescenti; flosculi disci 20–30, corollis 5–6 mm. longis, tubo proprio gracili pubescenti fauces subcylindricos subaequanti basin versus bulboso-ampliato; achaeniis obovatis nigrescentibus nitidis 2 mm. longis. — Proc. Am. Acad. xl. 38 (1904). — Arbusculetis prope Aragon, Turrialba, Costa Rica, alt. 630 m., *Pittier*, n. 13,246.

5. **R. oreopola** Robinson, n. sp., verisimiliter fruticosa 3 m. alta; ramis trichotomis subteretibus fistulosis glabriusculis purpurascentibus; foliis oppositis ovatis caudato-acuminatis serrulatis ca. 1 dm. longis ca. 7 cm. latis a loco paulo supra basin 3-nervatis cum dente unico arcuato acuminato in latere utrioque instructis utrinque viridibus inconspicue sparseque puberulis basi rotundatis deinde cuneatis, petiolo proprio brevissimo obcompresso margine lanoso-ciliato; capitibus modice numerosis in paniculam laxam folioso-bracteatam dispositis 3–3.5 cm. diametro (ligulis inclusis); ramulis paniculae glanduloso-tomentosis; involucri squamis exterioribus viridibus plerumque 5 lanceolatis attenuatis 1.3–1.9 cm. longis 6 mm. latis tenuibus subtrinerviis laxe glanduloso-pilosis, pilis albidis longiusculis moniliformibus; flosculis ♀ ca. 10, ligulis anguste oblongis 1.6 cm. longis 4 mm. latis flavis late patentibus; flosculis disci numerosis flavis, corollae tubo proprio gracili fauces cylindricos vix aequanti pilosiusculo basin versus bulboso ampliato, dentibus limbi brevibus deltoideis; achaeniis obovoideis atrobrunneis glaberrimis lucidulis. — In terra argillacea summorum

flavis, tubo proprio gracili 1.3 mm. longo pubescenti, faucibus cylindricis 3 mm. longis glabriusculis; achaeniis nigrescentibus compressiusculis obovatis striatulo-sulcatis 2.5 mm. longis. — Prod. v. 550 (1836); Deless. Ic. Sel. iv. t. 30 (1839); Hemsl. Biol. Cent.-Am. Bot. ii. 157 (1881). — Locis montanis mexicanis praecipue in terra argillacea prope rivulis altitudine 1500–2500 m. haud rara. JALISCO: *Nelson*, nn. 4024, 4172. MICHOACAN: *Pringle*, n. 3940; *Nelson*, nn. 6570, 6889. MORELOS: *Pringle*, nn. 9955, 13,902, 13,086 (infeliciter sub nomine *Trigonospermum floribundum* errore distributa). OAXACA: *Ghiesbreght*, anno 1842. Sierra Madre inter Michoacan et Guerrero, *Langlassé*, nn. 83, 801.

Forma PUBESCENS Greenman, foliis subtus saltim nervos versin permanenter laxeque floccoso-lanosis; ligulis quam eae formae typicae paulo longioribus etiam ad 2 cm. attingentibus. — Proc. Am. Acad. xli. 261 (1905). — Cerro de San Felipe, alt. 2500 m., *Conzatti*, n. 30.

2. R. CONNATA Brandegee, herbacea perennis multicaulis 1–2 m. alta; caulibus teretibus striatulis pubescentibus apicem versus trichotomoramosis; foliis ovati-lanceolatis regulariter serratis gradatim acutatis basi paulo angustatis late perfoliato-connatis 5–9 cm. longis 2–4 cm. latis utrinque pubescentibus; capitulis laxe paniculatis; pedicellis 3–6 cm. longis saepissime nutantibus glanduloso-pubescentibus; squamis involucri exterioribus 5 inaequalibus ovati-oblongis vel ellipticis integris vel apice 2–3-dentatis ca. 13–16 mm. longis ca. 8 mm. latis utrinque laxe glanduloso-pubescentibus, squamis interioribus tenuibus pallide viridibus ovato-lanceolatis conduplicatis acutis 5–6 mm. longis dorso glanduloso-pubescentibus; flosculis ♀ ca. 19, ligulis saepissime bilabiatis, labio inferiore 1 cm. longo ca. 7-nervio 3–4 mm. lato apice 3-dentato, labio superiore e lobulis 1–2 lineari-oblongis saepe obscuris 1.7–2 mm. longis composito; flosculis disci numerosissimis (ca. 100), corollis 8 mm. longis, tubo proprio 2.5 mm. longo pubescenti, faucibus graciliter cylindricis 5.5 mm. longis; achaeniis valde immaturis glabris. — Zoe, iii. 241, t. 23 (1892). — In montibus prope capnum Sancti Lucae Californiae inferioris australis, *Brandegee.*

3. **R. attenuata** Robinson, n. sp., verisimiliter fruticosa 2.5 m. alta glabriuscula; ramis trichotomis subteretibus fistulosis striato-angulatis, internodiis 1–1.5 dm. longis; foliis oppositis lanceolatis vel rhomboideolanceolatis tenuissimis breviter petiolatis 1.4–1.8 dm. longis 2–7 cm. latis longissime attenuatis in latere utrioque 8-angulatis mucronulatodenticulatis vel subintegris utrinque viridibus subglabris, petiolo 3–5 mm. longo vix alato; capitibus 1.5–2 cm. diametro laxe cymosopaniculatis; pedicellis gracilibus saepe nutantibus glanduloso-

saepe fistulosis laxe ramosis. Folia opposita saepissime ovata vel rhomboideo-lanceolata nunc petiolata nunc connata et perfoliata, petiolo plerumque cuneato-alato, lamina serrata vel denticulata nunc margine rotundata nunc utrioque latere unilobata vel uniangulata. Capitula in paniculam laxiusculam ovoideam vel planiusculam disposita.

Species hujusque cognitae 6, quarum tres mexicanae sunt, una in montibus Californiae inferioris inventa est, ceterae reipublicae Costae Ricae incolae sunt.

Clavis specierum.

a. Folia utrioque latere regulariter rotundata nec lobata nec angulata, *b.*
  b. Flosculi disci ca. 12. Involucri squamae exteriores obovati-spatulatae integerrimae ca. 6 mm. longae. Folia omnino disjuncta vel obscure angustissimeque connata. . . . . 1. *R. floribunda.*
  b. Flosculi disci ca. 100. Involucri squamae ovati-oblongis vel ellipticis ca. 15 mm. longae, aliae integrae aliae 2–3-dentatae. Folia late conspicueque connati-perfoliata. . . . . 2. *R. connata.*
a. Folia utrioque latere unilobata vel uniangulata subhastatiformi-rhomboidea, *c.*
  c. Involucrum exterius puberulum solum vel quasi pulverulentum, *d.*
    d. Pedicelli glanduloso-puberuli. Ligulae 10–12 mm. longae conspicue exsertae. Petioli veri breves 3–5 mm. solum longi vix alati. 3. *R. attenuata.*
    d. Pedicelli puberuli sed eglandulosi. Ligulae 5 mm. longae ex involucro vix exsertae. Petioli per totam longitudinem conspicue alati 3–4 cm. longi. . . . . 4. *R. aragonensis.*
  c. Involucrum exterius laxe pubescens, pilis albidis moniliformibus modice longis, *e.*
    e. Ligulae conspicuae 16 mm. longae valde exsertae. Petioli basin versus graciles exalati. . . . . 5. *R. oreopola.*
    e. Ligulae parvae inconspicuae involucrum non superantes. Petioli per totam longitudinem alati. . . . . 6. *R. polymnioides.*

1. R. FLORIBUNDA DC. (*Palo gogo* mexicanorum) fruticosa elata speciosa; foliis ovatis serratis breviter acuminatis firmiusculis utrinque glabriusculis 7–16 cm. longis 5–12 cm. latis supra basin conspicue 3-nerviis basi in petiolum abrupte contractis deinde cuneatis; panicula ovoidea 1–2 dm. diametro multicapitulata oppositiramea, bracteis primariis foliaceis, secundariis multo minoribus quam ramuli pedicellique saepius brevioribus; involucri squamis exterioribus 5 patentibus obovato-spatulatis striato-venosis integerrimis obtusis 6 mm. longis utrinque granuloso-puberulis, squamis interioribus cucullato-cymbiformibus 4–5 mm. longis acutiusculis dorso glanduloso-scaberrimis; flosculis ♀ 7–11, tubo proprio gracili 2 mm. longo pubescenti, ligula elliptica striato-nervia ca. 12 mm. longa 8 mm. lata apice breviter obtuseque 2–3-dentata maturitate durescenti et persistenti; flosculis disci 10–14, corollis

II. A REVISION OF THE GENUS RUMFORDIA.

By B. L. Robinson.

The genus *Rumfordia*, originally described by the eldest De Candolle and dedicated to Count Rumford, was founded upon a single species, *R. floribunda*, a showy-flowered shrub from the uplands of central and southern Mexico. The genus was for more than fifty years believed to be monotypic, but in 1892 Mr. T. S. Brandegee published the description of a second and very distinct species, which he had discovered in the mountains of southern Lower California. From 1903 to 1905 Dr. Greenman amplified the records of the genus by characterizing two species from Costa Rica and a pubescent form of the original *R. floribunda*. As two more new species of *Rumfordia* have now been found in a very interesting collection of plants secured by the late E. Langlassé, it seems worth while to present here a résumé of the genus as far as it is known to date. The group is notable for its entire freedom from synonymy and nomenclatorial difficulties. Of its members not one appears to have borne any other name than the one here recognized.

RUMFORDIA DC. (ad equitem clarissimum *Benjaminem Thompson* comitem de *Rumford* dedicata). — Capitula mediocria vel majuscula heterogama. Flosculi ♀ 6–20 liguliferi' fertiles; ligulis ellipticis vel oblongis vel linearibus tenuibus et flavis vel aetate indurescentibus et albicantibus nunc simplicibus nunc obscure bilabiatis. Flosculi disci ca. 10 vel multo numerosiores ☿ fertiles, corollis tubulosis flavis, tubo proprio gracili pubescenti quam fauces subcylindrici glabriusculi distincte breviore vel eos subaequanti, dentibus limbi 5 brevibus deltoideis. Achaenia obovoidea modice compressa calva glabra conformia. Involucrum duplex, squamis exterioribus herbaceis ovatis vel ellipticis vel oblongo-lanceolatis laxe patentibus, squamis interioribus multo minoribus ovatis vel lanceolatis paleiformibus erectis cucullatis achaenia flosculorum exteriorum amplectentibus. Receptaculum plano-convexum paleiferum. — Prod. v. 549 (1836); Deless. Ic. Sel. iv. t. 30 (1839); Benth. et Hook. f. Gen. ii. 359 (1873); Hemsl. Biol. Cent.-Am. Bot. ii. (1881); Baill. Hist. Pl. viii. 215 (1886); Hoffm. in Eng. et Prantl, Nat. Pflanzenf. iv. Ab. 5, 230 (1890); Brandegee, Zoe, iii. 241, t. 23 (1892); Greenman, Proc. Am. Acad. xxxix. 99 (1903), xl. 38 (1904), xli. 261 (1905). — Frutices vel rarius herbae elatae perennes, caulibus

circ. 3 mm. longis vel brevioribus; calyce mediam tantum corollam paullulo superante circ. 2.2 cm. longo, flavo et rubello, antice profunde fisso, ad circ. 1/3 altitud. connato, postice minute exciso, rotundato; corolla flava et rubella e calycis fissura longe exserta, 3.5–3.7 cm. longo, tubo circ. 1.5 cm. longo, galea elongata, labii lobis acutis, naviculari-subcorniformibus vix 1 mm. longis.

"Var. *a.* SUBGLABRA Loesen.; ramulis subglabris, foliis glabris. Hab. in GUATEMALA, in dept. Quezaltenango in pratis alpinis supra Totonicapam in 3000 m. altitud.: Sel. n. 2357. — Flor.: Sept.

"Var. *β.* HIRTA Loesen.; ramulis hirtis, foliis pulvereo-puberulis. Hab. in GUATEMALA, in dept. eodem in pratis alpinis ad Zihâ in 2840 m. altitud.: Sel. n. 2933. — Flor.: Jun." Loesen. in Bull. Herb. Boiss. ser. 2, iii. 285 (1903).

54. C. KATAKYPTUSA Loesen., "humilis atque procumbens, tantum circ. 9 cm. alta; ramulis dense hirtis; foliis parvulis, sessilibus, linearibus vel superioribus lineari-lanceolatis, integris, acutiusculis, pulvereo-puberulis, obsolete uni–trinerviis, 8–20 mm. longis, vix 1–4 mm. latis, inferioribus angustioribus brevioribus, superioribus longioribus praecipueque basi latioribus sensim in bracteas transformatis, bracteis summis etiam maioribus, usque 23 mm. longis, et 4 mm. latis, margine utrinque 1–2-fimbriatis, fimbriis ipsis tantum usque 4 mm. longis, linearibus, lamina igitur fimbriis additis tota circ. 10 mm. lata; pedicellis tantum vix 2 mm. longis; calyce circ. 2.5 cm. longo, postice minute atque etiam minus excisulo quam in praecedente, rotundato, corolla circ. 4 cm. longa, tubo circ. 1.7 cm. longo, labii lobis obtusis vel subobtusis, extrinsecus pilosis; cetera ut in praecedente. — Habitat in GUATEMALA: in dept. Huehuetenango in pratis et silvestribus in jugo montium inter Todos los Santos et Chiantla, in 3000 m. altitud.: Sel. n. 2750. — Flor.: Sept." Loesen. in Bull. Herb. Boiss. ser. 2. iii. 286.

ramorum majorum axillaribus solitariis pedunculatis speciosis coccineis; bracteis propriis nullis; calyce tubuloso antice ultra medietatem longitudinaliter fisso, nervoso pubescenti colorato, basi subventricoso, superne compresso; labio superiore longissimo incurvo, apice emarginato dorso pubescenti; inferiore brevissimo trifido, laciniis acutis; sinubus similibus glandulis; stigmate obtuso; capsula ovato-acuminata compressa. — Linn. f. Suppl. 293 (1781); Benth. l. c. 533; Smith, Icon. Ined. t. 40; Hemsl. l. c. 460. — This species can scarcely be considered Mexican, as it has so far been collected only in South and Central America. There are no specimens in hb. Gray from Mexico or Central America.

52. C. IRASUENSIS Oerst., "suffruticosa glabra, foliis linearibus apice trifidis, lacinia intermedia subtrifida, racemo elongato laxo, calyce elongato hinc fisso postice bilobo, lobis retusis, corollae galea tubo subduplo longiore labii lobis brevissimis acuminatis. — Suffrutex erectus, ramosus, 1–2 pedalis. Caulis ramique teretes, glabri, nitiduli. Folia alterna, sessilia, amplexicaulia, linearia, supra medium trifida, glabra, 8–14 lin. longa, lobis linearibus obtusiusculis, intermedio majore sub 3–4-fido. Folia floralia indivisa, cuneata, apice obtusa, 8 lin. longa, trinervia, rubicunda, glabra. Flores pedicellati, 15 lin. longi. Pedicelli 2 lin. longi, villiusculi demum glabriusculi. Calyx elongatus, tubulosus, compressus, tomentosus, fuscus margine flavescente, hinc fissus inde bilobus, 6–7 lin. longus, lobis rotundatis vel retusis. Corolla bilabiata subrecta, calyce tres lineas longior, antice virescens postice rubicunda, labio superiore (galea) apice retuso, inferiore 3-fido, laciniis acuminatis incurvis. Stamina exserta, anticis corolla lineam longioribus, posticis ei aequilongis. Stylus exsertus. Stigma capitatum. Capsula ovato-oblonga, breviter acuminata, fusca, glabra calyce demum tecta, 6 lin. longa. Semina oblonga, numerosa, minutissima, testa laxa, diaphana, reticulata." — Oerst. in Vidensk. Meddel. 1853, p. 27; Hemsl. l. c. 461. — COSTA RICA: alpine region, Volcano Irasu, altitude 2745–3050 m., *Oersted*, part of type material in hb. Gray; same locality, *John Donnell Smith*, no. 4901; Volcan de Turrialba, *Pittier*, no. 13,079 (hb. Nat. Costa Rica, distr. by John Donnell Smith). COLUMBIA: Santa Marta, *H.H. Smith*, no. 1387.

53. C. TEPEINOCLADA Loesen., "humilis atque procumbens, tota planta tantum circ. 6–9 cm. alta; ramulis subglabris vel hirtis; foliis parvis sessilibus linearibus vel lineari-lanceolatis integris, acutis vel subacutis, glabris vel pulvereo-puberulis, uninerviis vel obsolete trinerviis, 6–13 mm. longis, circ. 1–2 mm. latis; bracteis longioribus usque 17 mm. longis et latioribus usque 3 mm. latis, summis ipsis plerumque utrinque uni- vel bifimbriatis, fimbriis lateralibus usque 6 mm. longis; pedicellis

C. G. Pringle, no. 13,659; in hb. U. S. Nat. Mus., near La Providencia, altitude 1982–2440 m., *E. W. Nelson*, no. 4989; Sierra Madre, 45 km. north of Guanacevi, *E. W. Nelson*, no. 4766.

49. C. PURPUSI Brandegee, perennis suffruticosa hirsuta; caulibus multis 1 dm. altis simplicibus ex rhizomatibus longis ramosis; foliis inferioribus lineari-lanceolatis obtusis vel acutis, basi subattenuatis 1.5–2 cm. longis 3–4 mm. latis; foliis superioribus bracteisque trifidis, segmento medio longissimo; calyce antice profunde fisso, postice paulo, segmentis integris vel emarginatis; corolla 3.5 cm. longa exserta; galea tomentosa dorso viridi; labio inferiore brevissimo, dentibus 3 acuminatis, medio breviore. — Zoe, v. 181 (1905). — Mt. Ixtaccihuatl, rocky slopes above timber-line, *C. A. Purpus*, nos. 320 (type), 1711 (both in hb. Univ. Calif., duplicates in hb. Gray). The bracts and calyx are more or less tinged with red, but the entire plant becomes black in drying. The leaves are rather thickly covered with loose spreading long white hairs, and some of the upper leaves are trifid.

50. C. PECTINATA Mart. & Gal., "fruticulosa pilosa; foliis pectinato-subpinnatis, laciniis linearibus distantibus elongatis 2–3-jugis, bracteis laciniato-pectinatis, floribus racemoso-spicatis, pedunculis et calycibus pilosis. — Folia pollicaria pectinato-laciniata, flores rubri similes floribus *Castillejae integrifoliae* L.; sed pedunculati. — Affinis *Castillejae laciniatae* Hook. Dans les forêts de pins de la Cueva del Temascal, au pic d'Orizaba, de 9500 à 12,500 pieds (limites de la végétation phanérogame). Fl. rouge-vermillon. Août." — Bull. Acad. Brux. xii. pt. 2, 27 (1845); Walp. Rep. vi. 651; Hemsl. l. c. 462. — *C. Orizabae* Benth. in DC. Prodr. x. 533, is founded partly on the same number (1074) in Galeotti's collection, also on *Linden*, no. 223. Bentham gives these additional characteristics under *C. Orizabae*, — "canescenti-hispidula, foliis inferioribus integris linearibus sublanceolatisve, superioribus dilatatis incisis, floralibus vix apice coloratis, racemo laxo, calyce elongato amplo hinc fisso postice obtuse 2–4-dentato, corollae galea tubo suo multo longiore, labii lobis brevibus acuminatis. Habitus fere *C. integrifoliae* sed folia pleraque incisa lobis elongatis et flores multo majores. Calyx 15 lin. longus. Corollae galea dorso villosa, calycem lineis 5–6 superans." In hb. Gray the species is represented by a doubtfully identified specimen collected in GUATEMALA: Volcan de Agua, Depart. Zacatepequez, altitude 3670 m., April, 1890, *John Donnell Smith*, no. 2146.

51. C. FISSIFOLIA Linn. f., herbacea quandoque suffruticosa; caulibus erectis parum ramosis foliosis pubescentibus; foliis sessilibus patentissimis, basi ovatis integris, apice pinnatifidis, laciniis patentibus obtusis fere alternis utrinque pubescentibus subtrinerviis; floribus versus apices

bracteis majoribus pedunculisque longioribus differt. Dans les bois de Juquila del Sur (côte pacifique d'Oaxaca) à 5000 pieds, à Talea et dans le Rincon (Cordill. orientale d'Oaxaca), de 3000 à 4000 pieds. Fl. rouges. Septembre." — Bull. Acad. Brux. xii. pt. 2, 28 (1845); Walp. Rep. vi. 651; Hemsl. l.c. 461. The following specimens are in hb. Gray, — Mexico, *Dr. Coulter*, no. 1353. OAXACA: Sierra de San Felipe, altitude 1830 m., *C. G. Pringle*, no. 4817; hills, San Felipe del Agua, altitude 1750 m., *Conzatti*, no. 570. In hb. U. S. Nat. Mus., Valley of Oaxaca, altitude 1830–2287 m., *E. W. Nelson*, no. 1194; 29 km. southwest of City of Oaxaca, altitude 2287–2897 m., *E. W. Nelson*, no. 1459, as to material in hb. Gray.

47. C. INTEGRIFOLIA Linn. f., suffruticosa ramosa glabriuscula vel tenuiter canescenti-hispidula; foliis linearibus integris basi vix dilatatis, floralibus paulo latioribus apice raro coloratis; racemo secundo; calyce elongato hinc fisso postice brevissime 2–4-dentato; corollae galea elongata glabriuscula, labii lobis brevissimis acutis. — Linn. f. Suppl. 293 (1781); Smith, Icon. Ined. 39; Benth. in DC. Prodr. x. 533; Mart. & Gal. l. c. 27; Hemsl. l. c. 461. The species was founded upon specimens collected by *Mutis* in Nova Granata and is also a native of Central America and Mexico. The flowers are conspicuous, falcately spreading, and turning black in drying. The calyx is nearly 2 cm. long and the corolla exserted about 5 cm. — CHIAPAS: *Ghiesbrecht*, nos. 152, 654, and 655. GUATEMALA: *Hyde & Lux*, no. 3099 (distrib. of J. Donnell Smith); *Sutton Hayes;* between Jacaltenango and San Martin, altitude 1670–2135 m., *E. W. Nelson*, no. 3609. OAXACA: between Juquila and Nopala, altitude 1372–2135 m., *E. W. Nelson*, no. 2426. NUEVO LEON: near Monterey, *C. G. Pringle*, no. 1951.

48. C. PATRIOTICA Fernald, simplex vel ramosa, 3–5.5 dm. alta; caulibus piloso-hirsutis vel glabrescentibus; foliis tenuibus hispidulis 3–5 cm. longis lineari-lanceolatis vel pinnatisectis, laciniis 2–6 lineari-lanceolatis; floribus racemosis; pedicellis ca. 1 cm. longis; bracteis foliis similibus minoribus et minus laciniatis; calyce tubuloso 3–4 cm. longo piloso-puberulo rubro viridi et albescenti; corolla 4.5–5.25 cm. longa viridi et albescenti dorso; galea 2.75–3 cm. longa exserta; labio inferiore viridi protuberanti ca. 3 mm., lobis lanceolatis; capsula oblongo-acuminata 1.5 cm. longa. — Fernald in Proc. Am. Acad. xl. 56 (1904). — CHIHUAHUA: near Colonia Garcia, altitude 2310 m., *Townsend & Barber*, no. 156; Cumbre, *E. Palmer*, no. 363, coll. of 1885; Mapula Mts., altitude 2200 m., and cool slopes of the Sierra Madre *C. G. Pringle*, nos. 1154, 1350 (type, in hb. Gray); Colonia Juarez, *E. W. Nelson*, no. 6062. DURANGO: barranca below Sandia Station,

animals. It is related to *C. linariaefolia*, but differs in its shorter tri-nerved leaves, its pubescence, its pedicellate flowers, and the peculiar lower lip of the corolla, which stands out like a small shelf and must be very conspicuous in the living flower, its dark red contrasting strongly with the light red and yellowish green of the other parts of the flower and bracts.

44. C. LAXA Gray, herbacea cinereo-pubescens; caulibus e radice perennis subdiffusis ramosis gracilibus; foliis tenuibus scabridis lineari-lanceolatis integerrimis basi haud dilatatis, floralibus calyce brevioribus rubro-coloratis; floribus paucis confertis breviter pedicellatis; calyce rubello antice profundius postice breviter fisso, dentibus brevibus obtusis; corollae galea magna, lobis labii inferioris brevissimis obtusis. — Gray in Torr. Bot. Mex. Bound. Surv. 119 (1859); Gray, Synop. Fl. N. Am. ii. pt. 1, 296; Hemsl. l. c. 461. — SONORA: mountain sides near Santa Cruz, Wright, no. 1490; Los Pinitos, altitude 1830 m., *C. V. Hartman*, no. 142 (Lumholtz Exped.). DURANGO: San Ramon, *E. Palmer*, no. 59, coll. of 1906. ARIZONA: Santa Catalina Mts., *J. G. Lemmon*, no. 264. There is an abnormal specimen collected at Alamos in 1890, by *E. Palmer*, no. 366.

45. C. STENOPHYLLA Jones, suffruticosa 6 dm. alta ramosissima, ramulis rectis subcinereis; foliis subfiliformibus obtusis 2–5 cm. longis; floribus rectis in spicis capitatis demum elongatis; bracteis oblongis acutis nervatis integris vel laciniatis, laciniis lateralibus paucis linearibus brevibus, media lata; calyce 2 cm. longo, postice in altitudinem 5 mm. fisso, segmentis irregulariter acuminatis vel laciniatis; corolla calycem superanti 3 mm., galea basi 3 mm. lata, apice 1 mm., dorso glandulosa; labio inferiore triplicato, lobis subulatis incurvatis 1 mm. longis; capsula oblongo-ovata 1.5 cm. longa. — Contributions to Western Botany, xii. 67 (1908). The type was collected at Garcia and in San Diego cañon, Sierra Madre, CHIHUAHUA, September, 1903 (hb. Marcus E. Jones). The specimens in hb. Gray are from Colonia Garcia, altitude 2287 m., *Townsend & Barber*, no. 209, also *E. W. Nelson*, no. 6210 a, in part. According to Marcus E. Jones, the flowers have a green back and red face. At almost all the leaf-axils there are small sterile branchlets slender and very leafy.

46. C. LONGIBRACTEATA Mart. & Gal., "caule fruticoso erecto glabriusculo, foliis linearibus acuminatis elongatis 3-nerviis subglabris, floralibus lanceolato-linearibus flore sublongioribus, superioribus vel bracteis obovato-lanceolatis apice fimbriatis, floribus longepedunculatis racemoso-spicatis; calyce tubuloso-inflato glabriusculo, corolla calycem longe excedente apice pilosa. — Flores 1.5 pollicares, pedunculi semipollicares. — A *Castilleja integrifolia* L., cui affinis praesertim

Conzatti & González, no. 1225; San Juan del Estado, 1920 m. alt., *Lucius C. Smith*, no. 407. CHIHUAHUA: near Batopilas, *E. A. Goldman*, no. 195. DURANGO: vicinity of city, *E. Palmer*, nos. 114 and 648, coll. of 1896. VERA CRUZ: Orizaba, *Botteri*, nos. 590 and 431. STATE OF MEXICO: Tacubaya, *W. Schumann*, no. 1013; near Toluca, *G. Andrieux*, no. 156; Vallée de Mexico, *E. Bourgeau*, no. 104. GUANAJUATO: Guanajuato, *A. Dugès*, no. 388. N. W. Mexico, *Seemann*: Mexico without locality, *Dr. J. Gregg*, nos. 434 and 610.

42. **C. xylorrhiza**, n. sp., perennis, investa pilis albis crispis simplicibus vel basi furcatis; radice lignea crassa; caulibus pluribus basi ramosis, supra simplicibus rectis 1.5–2 dm. altis (gracilibus in specimine viso sed anni praecedentis caulibus ligneis grandis); foliis lanceolatis trinerviis 2–3 cm. longis, 2–3 mm. latis, apice acutis, basi obscure auriculatis; floribus breve pedicellatis divaricatis in spicis brevibus confertis, bracteis floribus brevioribus similibus foliis; calyce 2.5 cm. longo, basi ventricoso, laciniis bidentatis; corolla 3.5 cm. longa, galea exserta 5–15 mm., dorso glandulari-pubescenti; labio inferiore protuberanti acuminato-tridentato, sinubus inter dentes angustis crassis, similibus glandibus; stylo filiformi exserto, stigmate integro clavato; capsula oblonga, basi et apice acuminata, compressa, 1.5 cm. longa. — COAHUILA: Sierra Encaruaciore, 28 July, 1896, *E. W. Nelson*, no. 3895 (type, in hb. Gray). This species is related to *C. tenuiflora*, differing in the peculiar pubescence, the habit of growth, and fewer-flowered more capitate spikes.

43. **C. scabridula**, n. sp., suffruticosa scabriduli-puberulens ramosa alta; foliis lineari-lanceolatis apice acutis vel obtusis basi rotundatis vel rarissime auriculatis, trinerviis 2–3 cm. longis, 2–5 mm. latis; foliis floralibus latioribus et brevioribus; floribus breve pedicellatis, junioribus in spicis capitatis, senioribus in racemis; pedicellis filiformibus 3–5 mm. longis; bracteis supremis attenuatis apice coccineis; calyce basi obliquo tubuloso 3 cm. longo, in altitudinem postice 6 mm., antice 2 cm. fisso, segmentis acuminato-laciniatis glandulari-puberulentis, superiore parte coccinea, inferiore psittacina; corollae galea exserta 15 mm., obtusa sed lateraliter emarginata, 2 cm. longa, apice 2 mm. lata, basi 5 mm.; labio inferiore atro-rubro exserto et protuberanti 1 mm., laciniis lineari-acuminatis 1 mm. longis, exterioribus divaricatis, media incurvata, sinubus crassis; stigmata fere exserto clavato; ovario ovato-acuminato. — DURANGO: Tejamen, August, 1906, *E. Palmer*, no. 468 (type, in hb. Gray). Dr. Palmer notes this as one of the showiest of plants, with flowers bright yellow and scarlet. It grows in compact masses on stony hills among other plants, but is not common. The stems are brittle and the plant is not eaten by

indicated: *Hartweg*, no. 191, type; *Coulter*, no. 1354. State of Mexico: Tacubaya (Tokabaya), *Bilimek*, no. 288; Sierra de Ajusco, 2592 m. alt., *C. G. Pringle*, nos. 9476 and 11,063; Chapultepec, *C. G. Pringle*, no. 1472; valley of Mexico, *Bourgeau*, no. 125. Oaxaca: Cerro San Felipe, *E. W. Nelson*, no. 1146; also in hb. U. S. Nat. Mus. nos. 1166 and 1076; west slope of Mount Zempoaltepec, 2300–2440 m. alt., *E. W. Nelson*, no. 559, hb. U. S. Nat. Mus.; near Reyes, *E. W. Nelson*, no. 1735, hb. U. S. Nat. Mus. Coahuila: 9.6 km. east of Saltillo, *E. Palmer*, no. 991, April, 1880; San Lorenzo Cañon, *E. Palmer*, no. 415, coll. of 1904. Hidalgo: Ixmiquilpan, *C. A. Purpus*, no. 1411; Sierra de Pachuca, *Rose & Hay*, no. 5582. Puebla: San Martin, *E. W. Nelson*, no. 8, and on same sheet without separate numbers is a specimen from Mexico and another from Vera Cruz, hb. U. S. Nat. Mus.; in plaza near Calchicomula, *Rose & Hay*, no. 5807; near town of Puebla, *Lucius C. Smith*, no. 905. Jalisco: Guadalajara, *E. Palmer*, no. 265, July, 1886; *C. G. Pringle*, no. 8763. Michoacan: north slope of Mount Patamban, 2897–3355 m. alt., *E. W. Nelson*, no. 6587; *C. & E. Seler*, no. 1281, San Luis Potosi: *E. Palmer*, no. 724, coll. of 1898; no. 88, coll. of 1902; *Parry & Palmer*, no. 692, coll. of 1878. Morelos: Tres Marias Mountains, *C. G. Pringle*, no. 11, 647. Sonora: Huchuerachi, 1220 m. alt., *C. V. Hartman*, no. 299, and *F. E. Lloyd*, no. 436 (Lumholtz Exped.). Vera Cruz: Mount Orizaba, 2745 m. alt., *H. E. Seaton*, no. 160; Boca del Monte, *E. W. Nelson*, no. 194, hb. U. S. Nat. Mus. State of Mexico: Mount Popocatepetl, *Rose & Hay*, no. 6063; foot-hills of Mount Ixtaccihuatl, *Chas. C. Deam*, no. 19; Cholula, *Chas. C. Deam*, no. 85.

41. C. canescens Benth., suffruticosa ramosa canescenti-hispida; foliis linearibus lanceolatisve basi dilatato-amplexicaulibus, floralibus latioribus acutis, summis rarius apice coloratis; spicis confertis; calyce elongato hinc fisso, postice obtuso vel acute 2–4-dentato, corollae galea elongata, labii lobis brevibus obtusis vel acutiusculis. — Benth. in DC. Prodr. x. 533 (1846); Seem. Bot. Voy. Herald, 323; Hemsl. l. c. 460. — It is doubtful if this species can be maintained as distinct from the preceding, though certainly *Hartweg*, no. 191 (*C. tenuiflora*), and *Andrieux*, no. 156 (*C. canescens*), specimens cited by Bentham and represented in hb. Gray, are dissimilar, as is indicated in the above key. The flowers seem alike in the dried specimens, though perhaps those of *C. canescens* spread more widely from the flowering axis. The following are in hb. Gray, — San Luis Potosi: *Parry & Palmer*, no. 688, coll. of 1878; in the mountains of San Miguelito, *Schaffner*, no. 740, also near town of San Luis Potosi, no. 739. Oaxaca: Cerro San Felipe, *C. Conzatti*, no. 689½, April, 1898; Huauchilla, Nochixtlan, alt. 2000 m.,

basi ramoso, foliis, inferioribus suboppositis, horizontalibus deflexisve, e basi amplectente dilatato-auriculata linearibus acuminatis, superioribus latioribus, omnibus trinervibus; bracteis ovato-acuminatis, trinervibus, pallidis, summo apice lateritiis, divergenti-divaricatis; floribus brevissime pedunculatis terminalibus, subracemosis, paucis, calycis tubulosi compressi lobis elongatis, bidentatis (aurantiis), corolla longe exserta, labio superiori attenuato, obtuso, recto (apice rubello), inferiore minuto, bi-, rarius trifido, lobis porrectis, obtusis, stylo parum exserto." — Linnaea, xvi. 312 (1842); Mart. & Gal. l. c. 28; Benth. l. c. 533; Hemsl. l. c. 461. — PUEBLA: near Tehuacan, altitude 1700 m., *C. G. Pringle*, no. 9517, *C. A. Purpus*, no. 1287, *Rose & Hay*, no. 5844 (hb. U. S. Nat. Mus.); also in calcareous soil, altitude 1677 m., *C. G. Pringle*, no. 6250. MICHOACAN: Las Reyes, *E. W. Nelson*, no. 6859; Volcano Jorullo, *E. W. Nelson*, no. 6949. OAXACA: valley of Oaxaca, alt. 1675–2290 m., *E. W. Nelson*, no. 1459 in part (hb. U. S. Nat. Mus.).

39. **C. subalpina,** n. sp., perennis herbacea; rhizomatibus ligneis gracilibus; caulibus 3 dm. altis simplicibus angulatis albo-pilosis; foliis lanceolatis apice acutis basi auriculati-amplexicaulibus 2.5–3.5 cm. longis ca. 5 mm. latis trinerviis sparse pilosis et dense scabrido-puberulis et obscure glandulosis; foliis floralibus quam caulina paulo latioribus apice nunc coloratis nunc viridibus; floribus rectis subsessilibus in spicis demum elongatis; calyce 3 cm. longo piloso antice 2 cm. postice 16 mm. in altitudinem fisso; segmentis pilosis coccineis acute bidentatis; corolla recta 4 mm. longa, galea obtusa 2 mm. longa, dorso barbata, antice rubra membranacea, labio inferiore brevi protuberanti, laciniis incurvis ovato-subulatis brevibus, sinubus similibus glandulis, stylo filiformi exserto 5 mm.; stigmate clavato; capsula ovato-acuminata cauli adpressa, 12 mm. longa. — OAXACA: Sierra de San Felipe, altitude 3140 m., 26 June, 1894, *C. G. Pringle*, no. 4722 in part, distributed under *C. scorzonerifolia* HBK. (type, in hb. Gray). It belongs near *C. longiflora* but has different pubescence, and generally longer and narrower leaves. The flowers are more slender and less crowded than in the other species and generally more erect.

40. C. TENUIFLORA Benth., fruticosa scabrido-pubescens ramosa vel simplex; foliis linearibus vel lanceolatis basi amplexicauli-dilatatis integris; floribus spicatis; bracteis lanceolatis acutis, supremis apice coloratis quam folia latioribus; calyce elongato acute 2–4-dentato; corollae galea elongata; labio inferiore protuberanti, lobis brevibus subulato-acuminatis, primum inflexis demum reflexis. — Pl. Hartweg, 22 (1839); DC. Prodr. x. 533; Hemsl. l. c. 463; Loesen. l. c. 285. — The following Mexican specimens are in hb. Gray unless otherwise

closely clustered, the ribbon-like bracts surpassing the flowers; later the flowers become rather distant on the flowering axis.

36. C. MEXICANA (Hemsl.) Gray, annua vel biennis nana 7–15 cm. alta hirsuta; caulibus dense foliosis; foliis pinnatifidis sessilibus, lobis linearibus utrinque saepius 2; floribus sessilibus 5–6 cm. longis rectis, post anthesim divaricatis; bracteis calyce brevioribus basi latis trinerviis, alte trilobatis, lobis linearibus obtusiusculis, lateralibus paulo brevioribus; calycis lobis viridibus, laciniis anguste linearibus non-numquam idem bifidis; corolla calycem triplo superanti gracili puberula; labiis subaequalibus inferiore tripartito basi obscure saccato. — Gray in Proc. Am. Acad. xxi. 404 (1886). *Orthocarpus mexicanus* Hemsl. Biol. Cent.-Am. Bot. ii. 463, t. 63 A. f. 1–6 (1882). — The type is in hb. Kew and was collected in Zacatecas, North Mexico, by *Coulter*. COAHUILA: Sierra Pata Galana, *C. A. Purpus*, no. 1050; Saltillo, *E. Palmer*, no. 530, coll. of 1905, 992 and 993, coll. of 1880, also no. 13, coll. of 1898; same locality, *C. C. Parry*, no. 20½. NUEVO LEON: near Monterey, altitude 610 m., *C. G. Pringle*, no. 10,156. SAN LUIS POTOSI: San Miguelito Mountains, *Dr. J. G. Schaffner*, no. 82. CHIHUAHUA: on rocky hills near town, *C. G. Pringle*, no. 209; Pueblo de Galleana, no. 657, and Puerto de St. Diego, *C. V. Hartman*, no. 631 (Lumholtz Exped.). *C. sessiliflora* Pursh is excluded as all specimens seen appear to be *C. mexicana*. The two are very closely related.

37. **C. auriculata**, n. sp., suffruticosa canescens pilosa et glandulosa; caulibus ramosis; ramis ascendentibus; foliis imbricatis anguste deltoideis acutis vel apice obtusis, basi auriculate amplexicaulibus, integris 1–3 cm. longis 5–15 mm. latis, palmate trinerviis, nervio medio distinctissimo, cinereis scabridis cum glandulis et pilis glandulosis; floribus imbricate spicatis; bracteis foliis similibus, supremis coloratis; spicis confertis, floribus subsessilibus; calyce 2.5 cm. longo antice in altitudinem 2.5 cm., postice 7 mm. fisso, laciniis integris vel bidentatis, 2–3-nerviis; corolla 3.7 cm. longa, galea paulo tubo longiore, antice membranacea, dorso glandulosa, exserta 5–10 mm.; labii inferioris lobis 3, exterioribus linearibus acutis 3 mm. longis paulo medium excedentibus sinubus intus plicatis; stylo exserto; stigmate clavato apice capitato et obscure emarginato; capsula rhomboideo-orbiculata acuminata compressa 1 cm. longa. — Between Huajuapan, OAXACA, and Retlatzingo, PUEBLA, November 19, 1894, *E. W. Nelson*, no. 1992 (type, in hb. Gray and duplicate in hb. U. S. Nat. Mus.). This species is nearest to *C. longiflora*, differing most noticeably in its broader, conspicuously auriculate, closely imbricated leaves. The flowers are more erect and the corolla in anthesis more in a line with the calyx.

38. C. LONGIFLORA Kunze, "caule suffruticoso, tenui, erecto, imprimis

other species, being much narrower, approaching those of *C. minor.* The lower part of the stem is very leafy, the leaves becoming 1 dm. long, the rhachis and divisions 0.5–2 mm. broad. The type and all specimens are in hb. Univ. Calif. except a small part of a flowering branch in hb. Gray and perhaps also in hb. U. S. Nat. Mus., collected by Lyman Belding no. 4, at Laguna, Lower California, altitude 915 m. The specimens from hb. Univ. Calif. were kindly loaned by T. S. Brandegee and H. M. Hall. The species has been found only in Lower California and at the following localities: San Jorge, San Estaban, Sierra de Laguna, Sierra de San Francisquito, San Jose del Cabo.

34. C. AFFINIS H. & A., perennis herbacea; caule simplici piloso-hispido 3–6 dm. alto; foliis lineari-lanceolatis trinerviis integris raro pinnatisectis; floribus subracemosis, inferioribus pedunculatis, superioribus confertis; bracteis similibus foliis brevioribus; calycis segmentis acute bilobis; corolla calycem superanti et valde divaricate exserta; labio inferiore exserto protuberanti. — Bot. Beech. 154 (1833); Benth. in DC. Prodr. x. 532; Gray in Bot. Cal. i. 573, and Synop. Fl. N. Am. ii. pt. 1, 296; Hemsl. l. c. 460. — This species is distinctively Californian and peculiar to the coast region. It varies extremely in foliage and flowers but can scarcely be divided into varieties. The Mexican specimens in hb. Gray are all from the coast of Lower California, — Todos Santos Island, *A. W. Anthony*, no. 198; San Quentin, *E. Palmer*, no. 642, coll. of 1889.

35. C. MINOR Gray, annua vel perennis glandulosa et sparse pilosa; caulibus simplicibus vel ramosis 1–plurimis 3–6 dm. altis gracilibus foliosis; foliis anguste linearibus apice attenuatis 2–5 cm. longis; floribus racemosis, pedicellis brevibus filiformibus rectis; bracteis terminalibus fasciculatis apice coloratis anguste linearibus et attenuatis; calyce subfalcato in altitudinem 1.5 cm. fisso, laciniis 2 filiformibus 1–5 mm. longis; galea et labio e fissura exsertis, galeae dentibus triangularibus coccineis exsertis; capsula anguste ovoideo-cylindracea acuta. — Gray in Bot. Cal. i. 573 (1876), and Synop. Fl. N. Am. ii. pt. 2, 295. *C. affinis*, var. *minor* Gray in Torr. Bot. Mex. Bound. Surv. 119. *C. affinis* Seemann, Bot. Voy. Herald, 323, not H. & A. — NEW MEXICO: beds of exsiccated streams near the copper mines, *Wright*, no. 1494 (type, in hb. Gray). CHIHUAHUA: *C. V. Hartman*, no. 583 (Lumholtz Exped.); *Bigelow*; *Wright*, no. 1493; Presidio del Norte, *Schott*. SONORA: Los Animos, *Thurber*, no. 330; Tubac, *Parry*; Santa Cruz Mountains, *Captain E. K. Smith*. N. W. Mexico, *Seemann*, distributed as *C. affinis*. This species has more slender flowers than its allies. At the summit of the stem the bracts and flowers are

2287 m., 3 June, 1899, *Townsend & Barber*, no. 449 (type, in hb. Gray), also no. 250; same locality, *E. W. Nelson*, nos. 6227, 6101 (hb. U. S. Nat. Mus.). DURANGO: Otinapa, *E. Palmer*, no. 367 in part, July–August, 1906. The two equal segments of the calyx place this in *Euchroma*, but these parts are sharply cleft as in *Hemichroma*. The lower lip of the corolla resembles that of *C. nervata*.

32. **C. ctenodonta**, n. sp., perennis glandulari-pilosa; rhizoma gracili; caule simplici recto gracili 2–3.5 dm. alto; foliis infimis non rosulatis sed imbricatis lanceolatis integris trinerviis 3 cm. longis 5 mm. latis, apice acuminatis; foliis ceteris imbricatis vel distantibus lanceolato-acuminatis pectinatis cum lobulis filiformibus 1–5 mm. longis distantibus 2–8 mm., saepe pectinato-dentatis, basi cordato-amplexicaulibus et paulo decurrentibus; spicis capitatis non-numquam pedunculatis et demum elongatis; bracteis supremis pectinatis vel anguste laciniatis quam flores brevioribus, apice coccineis; floribus sessilibus paucis subdivaricatis; calyce coccineo nunc paulo longiore nunc corolla breviore, segmentis acuti 1–2 mm. in altitudinem bifidis; corollae galea obtusa exserta 4–7 mm. dorso barbata; labio inferiore non viso; stylo exserto 1–2 mm. gracili; stigmate clavato integro; capsula elliptica acuta. — OAXACA: wet meadows, Sierra de Clavellinas, altitude 2745 m., 16 October, 1894, *C. G. Pringle*, no. 4986 (type, in hb. Gray). — This number was distributed as *C. pectinata*, but cannot be that shrubby plant, nor is it to be classed in the same alliance. It more nearly approaches *C. patriotica*, but differs from that well-marked species in leaves, pubescence, and flowers. It is a more slender plant with simple stems. There are features which ally it to *C. minor*, such as, the narrow segments of the calyx-divisions, the slender red-tipped divisions of the uppermost bracts, and the conspicuously colored lower lip of the corolla. The leaves are typically pectinate with the rhachis lanceolate. It is doubtfully placed in *Euchroma*.

33. C. BRYANTI Brandegee, annua 1.5–3 dm. alta divaricate pilosa ramosa, ramulis gracilibus rectis; foliis inferioribus linearibus integris, ceteris pinnatisectis, laciniis 3–7 linearibus acutis; floribus spicatis apice confertis infra elongatis et interruptis; bracteis similibus foliis superioribus, apice coccineis vel ochroleucis; calycis segmentis 7–9 mm. longis 2–3 mm. in altitudinem bisectis, laciniis lanceolatis; corolla calycem aequanti 15–18 mm. longa, galea brevi, labii inferioris lobis brevibus incurvatis; capsula oblongo-cylindracea vel subellipsoidea 1 cm. longa. — Proc. Cal. Acad. ser. 2, ii. 192 (1889), iii. 157. — In habit of growth and foliage this species resembles *Orthocarpus*, but the flowers are those of *Castilleja*, approaching *C. affinis*, though much smaller and less exserted. The pods are different from those of any

29. C. GUADALUPENSIS Brandegee, frutescens intricate ramosa, 2–3 dm. alta; caulibus senioribus glabris atris, junioribus tomentosis; foliis anguste spatulatis 15–18 mm. longis, 2–4 mm. latis dense tomentosis; calycis segmentis tubum aequantibus; galea calycem paulo superanti tubum aequanti; labio inferiore brevissimo tridentato. — Zoe, v. 166 (1903). — Guadalupe Island off the coast of Lower California, *A. W. Anthony*, 1896 (type, in hb. Univ. Calif.), *Harry Drent*, 1898, *Dr. E. Palmer*, no. 59 (coll. of 1875). This species is related to *C. foliolosa*, but is more intricately and divaricately branched. The stems are harder and more woody, while the leaves are longer and narrower at base. In Dr. Palmer's specimen the longest leaves are 6 cm. long and the broadest almost 1 cm. wide. It is a younger and more vigorous shoot than the type, which has been examined through the kindness of T. S. Brandegee and H. M. Hall.

30. C. INTEGRA Gray, perennis; caulibus tomentosis, basi ramosis 3–7 dm. altis; foliis lineari-lanceolatis tomentulosis integris 3–8 cm. longis 4–8 mm. latis; floribus sessilibus in spicis brevibus demum elongatis; bracteis oblongis obovatis coccineis subpetaloideis floribus paulo brevioribus; calyce 2–3 cm. longo colorato, lobis bifidis lanceolatis obtusiusculis; corolla viridi-coccinea ca. 1.6 cm. longa; labio inferiore brevissimo. — Torr. Bot. Mex. Bound. Surv. 119 (1859); Gray, Synop. Fl. N. Am. ii. pt. 1, 298; Hemsl. l. c. 461. *C. tomentosa* Gray in Torr. Bot. Mex. Bound. Surv. 118 (1859). — CHIHUAHUA: near Colonia Garcia, Sierra Madre, altitude 2287 m., *Townsend & Barber*, no. 448; Santa Eulalia Mts., *C. G. Pringle*, no. 226; hills near Chihuahua, *Palmer*, no. 87, coll. of 1908; Puerto de San Diego, altitude 1982 m., *C. V. Hartman*, no. 598 (Lumholtz Exped.). SONORA: Mabibi, *Geo. Thurber*, no. 438 (type of *C. tomentosa*). In hb. U. S. Nat. Mus. are specimens from Chihuahua, Sierra Madre, *E. W. Nelson*, no. 6495.

31. **C. aspera**, n. sp., suffruticosa; caulibus simplicibus sparse pilosis et scabrido-puberulentibus angulatis rectis 3 dm. altis vel majoribus; foliis oblongis trinerviis scabrido-hispidis 4 cm. longis 5–10 mm. latis, apice obtusis vel acutis; bracteis inferioribus foliis similibus, flores superantibus, superioribus brevioribus apice margineque coloratis quam flores brevioribus; calyce 2.2 cm. longo subaequaliter in altitudinem 8 mm. fisso, segmentis bilobatis, lobulis subulatis 5 mm. longis, tubo nervato; corollae galea calycem superanti 1–2 mm. obtusa, dorso glandulare puberulenti; labio inferiore trisaccato rugoso-tuberculato, laciniis viridibus, media incurva bicarinata acuta, lateralibus latioribus dentatis; stylo exserto curvato, stigmate capitato; ovario oblique acuminato. — CHIHUAHUA: near Colonia Garcia, Sierra Madre, altitude

longiore; galea dorso pubescenti; labio inferiore brevissimo tridentato, dentibus incurvis; stylo exserto, stigmate capitato-emarginato; capsula ovata vix acuta. — HBK. Nov. Gen. et Spec. ii. 331, t. 164 (1817); Benth. l. c. 530; Mart. & Gal. l. c. 28; Hemsl. l. c. 461. *C. angustifolia* Mart. & Gal. l. c. 29 (1845) is considered a synonym of this, but the name is preoccupied. The range of this species, if all that seem to agree with the description and authenticated specimens are correctly identified, is from South America to N. W. Mexico. The type was collected probably in the State of Hidalgo near Real del Monte. Specimens in hb. Gray. — JALISCO: Guadalajara, *C. G. Pringle*, nos. 2565, 9348, 9461. OAXACA: Santa Domingo, *E. W. Nelson*, no. 2679. ORIZABA: San Cristobal, *Bourgeau*, no. 2904; N. W. Mexico, *Seemann*. There is also included no. 4168, collected by *C. G. Pringle* in Michoacan, distributed as *C. angustifolia* Mart. & Gal.

27. **C. Nelsonii,** n. sp., suffrutescens; caulibus simplicibus 3–4 dm. altis striatis glabrescentibus; foliis ovato-acuminatis 3–5-nerviis auriculati-amplexicaulibus apice obtusis integris 5–7 cm. longis 1.5–2 cm. latis desuper glabris, nerviis inferioribus puberulentibus; spicis coccineis investis pilis longis albis, floribus confertis, bracteis apice dilatatis integris et undulatis vel obtuse et breviter lobatis calyces superantibus; calyce 18 mm. longo, 7 mm. lato ad 1 cm. fisso, laciniis inaequale et obtuse bilobatis; galea recta tubum aequanti, calycem superanti, dorso glandulari-puberulenti; labio inferiore triplicato, laciniis 3 rectis acuminatis; stylo curvato exserto; stigmate capitato obscure emarginato; capsula ovata acuminata compressa. — SOUTHWEST CHIHUAHUA: Mount Mohinora, 1 September, 1898, *E. W. Nelson*, no. 4895 (type, in hb. U. S. Nat. Mus. and hb. Gray). This species is related to *C. scorzonerifolia*, but differs in having much larger almost smooth leaves. The corolla is dissimilar, with three rather long acuminate divisions instead of five short teeth. *C. Nelsonii* is a showy plant with a large subcapitate spike of scarlet bracts and flowers terminating the tall stems.

28. C. LANATA Gray, tomentosa floccosa simplici denso undique incana; foliis linearibus integerrimis, floralibus nunc trifidis apice coloratis; spicis demum interruptis; calycis lobis obovato-oblongis integerrimis retusisve. — Gray in Torr. Bot. Mex. Bound. Surv. 118 (1859); Gray, Synop. Fl. N. Am. ii. pt. 1, 298; Hemsl. l. c. 461. — The type (in hb. Gray) was collected along and near the Rio Grande river from Eagle Pass to El Paso. COAHUILA: Saltillo, *E. Palmer*, no. 76, coll. of 1898, and no. 990, coll. of 1880; *C. C. Parry*, no. 20; near Diaz, *C. G. Pringle*, no. 9032, and Carneros Pass, no. 3192. NORTHERN ZACATECAS: Cedros, *F. E. Lloyd*, no. 102. San Luis Potosi to San Antonio, Texas, *C. C. Parry*, no. 689.

HIDALGO: Ixmiquilpan, mountain slopes, *C. A. Purpus*, no. 1411 a; Sierra de Pachuca, altitude 2897 m., *C. G. Pringle*, no. 7618, in part; hills near Julanaciugo, *C. G. Pringle*, no. 13,278. PUEBLA: Mt. Orizaba, *Rose & Hay*, no. 5686. SAN LUIS POTOSI: in montibus San Miguelito, *J. G. Schaffner*, no. 741; *Parry & Palmer*, no. 691. COAHUILA: northeast side of Mt. Colima, *P. Goldsmith*, no. 80. Seemann's plant from northwest Mexico is doubtfully included. These specimens probably represent an aggregate of perhaps two or more species which it seems impossible with the present knowledge to segregate. The line between this species and the preceding is not very clear. It is somewhat doubtful in the light of present investigation how much weight is to be placed on the form of the lower lip of the corolla. The above specimens are all characterized by a lower lip with three teeth incurving in age, separated by a broad infolding sinus, so that when it is spread open the teeth are quite separated. *C. scorzonerifolia* has the teeth of the lower lip rather close and the sinus marked by smaller teeth. The indumentum of *C. glandulosa* is in general pilose, but there is also present a close glandular pubescence or almost puberulence, the glands under a lens appearing shortly and finely stipitate. The leaves are somewhat variable, though the typical specimens in each species have rather long acuminate leaves. Some specimens included among the above have obtuse leaves not at all acuminate.

25. **C. cryptandra**, n. sp., pilosa et hirsuti-scabrida, striata; foliis superioribus ovatis acuminatis integris 3–5-nerviis, apice obtusis, basi cordato-amplexicaulibus, nerviis hispido-scabridis; spicis coccineis, floribus confertis breviter pedicellatis, bracteis obovatis coloratis flores superantibus, 3 cm. longis, 1–1.5 cm. latis integris apice rotundatis; calyce fisso 1 cm., 2.5 cm. longo, laciniis obtuse bilobatis 4 mm. longis, glandulari-pilosis; galea paulo calycem superanti dorso pilosa et glandulosa; stylo curvato exserto, stigmate clavato; capsula compressa ovato-acuminata. — COLIMA: Cuchilla, northeast side of Volcano Colima, 22 July, 1905, *P. Goldsmith*, no. 76 (type, in hb. Gray). — This is a showy species related to *C. scorzonerifolia*, differing in its more veiny leaves, coarse and rough pubescence, and in having the lower lip of the corolla with three instead of five teeth. It is also related to *C. lithospermoides*, but the bracts are much larger, almost completely concealing the flowers.

26. C. LITHOSPERMOIDES HBK., caule recto simplici piloso-hispido; foliis lanceolato-linearibus, apice angustatis et obtusis, integris valde trinerviis piloso-hispidis ca. 5–7 mm. latis 3–6 cm. longis; floribus spicatis sessilibus; bracteis apice dilatatis rubicundis flores excedentibus; calycis segmentis bilobatis, lobis rotundatis; corolla albida vix calyce

altitude 3350 m., *H. E. Seaton*, no. 208. San Luis Potosi: altitude 1830–2440 m., *Parry & Palmer*, no. 690, coll. of 1878; hillsides, Las Canoas, *C. G. Pringle*, no. 3066. State of Mexico: Nevada de Toluca, about timber-line, altitude 4000 m., *C. G. Pringle*, no. 4225; Sierra de Ajusco, *J. W. Harshberger*, no. 123 a. Coahuila: Sierra de Parras, *C. A. Purpus*, no. 1051; Levios, 67 km. east of Saltillo, *E. Palmer*, no. 2026, coll. of 1880. Nuevo Leon: near Monterey, *C. G. Pringle*, no. 2236; north-east side of Volcano Colima, *P. Goldsmith*, no. 80 a. Durango: Otinapa, *E. Palmer*, no. 367, coll. of 1906, in part. Mexico without locality: *Dr. J. Gregg*, no. 407. The following have been examined from hb. U. S. Nat. Mus. — Mount Orizaba, *E. W. Nelson*, no. 282, *Rose & Hay*, no. 5741. Tamaulipas: mountains near Miquihuana, altitude 2135–2745 m., *E. W. Nelson*, no. 4485. This is a showy plant, distinguished from allied species by the pilose pubescence (somewhat glandular only on the inflorescence) and by the five-toothed lower lip of the corolla. The species may prove to be an aggregate when more fully understood. The forms with strictly acuminate capsules do not seem exactly similar to those with capsules subtruncate to acute, but the material has not been sufficient to warrant a division.

24. C. glandulosa Greenman, annua vel perennis basi indurata, pilosa et glandulari-pubescens; caulibus simplicibus rectis 1–3 dm. altis; foliis viridibus vel purpurascentibus sessilibus, paulo basi dilatatis et amplexicaulibus, lanceolato-attenuatis 1.5–5 cm. longis 1–6 mm. latis, acutis integris et saepe crispe undulatis trinerviis; floribus sessilibus et confertis 2–2.8 cm. longis in spicis ornatis 2–18 cm. longis, floribus inferioribus distantibus; bracteis inferioribus lanceolato-acuminatis foliaceis, superioribus oblongis apice rhomboideis coccineis vel flavis saepe flores superantibus; calyce fisso 8 mm., segmentis obtusis vel vix emarginatis; corolla 2–2.7 cm. longa calycem superanti 3–5 mm., galea recta 7–9 mm. longa dorso viridi glandulari-puberulenti, antice alba membranacea; capsula ovoidea acuminata 10–12 mm. longa. — Proc. Am. Acad. xli. 247 (1905). — State of Mexico: hills near Lecheria Station, altitude 2200 m., *C. G. Pringle*, no. 10,000 (type, in hb. Gray); hills above Santa Fé, altitude 2440 m., *C. G. Pringle*, no. 7979; *Schaffner*, no. 322. Durango: Otinapa, *E. Palmer*, no. 367, coll. of 1906, in part; City of Durango, *E. W. Nelson*, no. 4601. Oaxaca: Sierra de San Felipe, altitude 3140 m., *C. G. Pringle*, no. 4722, in part; 10 km. above Dominguillo, altitude 1980 m., *E. W. Nelson*, no. 1644; summit of Mt. Zempoaltepec, altitude 3470 m., *E. W. Nelson*, no. 626 (hb. U. S. Nat. Mus.); Sierra de Tápalo, altitude 2500 m., *González & Conzatti*, no. 759 (doubtful).

dibus, supremis apice coccineis; floribus sessilibus interruptis in spicis elongatis, falcatis bracteas superantibus; calyce fisso 12 mm., tubo anguste cylindrico piloso, segmentis dilatatis 4–5 mm. latis coccineis puberulentibus; galea et labio inferiore calycem superantibus; galea 1 cm. longa, basi 3–4 mm. lata, dorso viridi glandulare pilosa, antice coccinea membranacea; labio inferiore protuberanti trisaccato, dentibus acutis viridibus, sinubus implicatis cum plicaturis interioribus; stylo apice et stigmate subclavato exsertis; capsula ovata oblique-acuminata. —PUEBLA: Mount Orizaba, altitude 3660 m., 14 Aug., 1901, *C. G. Pringle*, no. 8560 (type, in hb. Gray). This is related to *C. Conzattii*, differing in having much longer flowers, with segments of the calyx red instead of green. The falcate flowers spreading outwards resemble those of § *Hemichroma*, but the equally cleft calyx is that of § *Euchroma*. It is a showy species.

22. C. HIRSUTA Mart. & Gal., "caule fruticuloso humili ramoso dense hirsuto-villoso; foliis obovato-spatulatis 3-nerviis apice rotundatis integerrimis villosis scabris, corolla calycem coccineum longe excedente. — Folia ½-pollicaria, flores pollicares. Dans les champs de Zacuapan, à 3000 pieds. Fl. rouge vif. Février-juillet." — Bull. Acad. Brux. xii. pt. 2, 29 (1845); Walp. Rep. vi. 651; Hemsl. l. c. 460; Greenman, Proc. Am. Acad. xli. 460. *C. obovata* Benth. l. c. 528. —HIDALGO: in a barranca below Trinidad Iron Works, altitude 1525 m., *C. G. Pringle*, no. 8935. Through the kindness of Dr. Prain, Director of the Royal Gardens at Kew, specimens under this number were compared with authenticated specimens in hb. Kew and reported as similar. There is a tendency in the specimens in hb. Gray to have incised dentate or laciniate leaves. Bentham placed this in § *Epichroma* on account of the somewhat ampliate calyx-limb. It is entirely unlike the other species in that section in habit, foliage, bracts, and flowers, and has the characteristic equally cleft calyx-divisions of § *Euchroma*, so in this synopsis it is included under the latter section.

23. C. SCORZONERIFOLIA HBK., simplex vel basi ramosa perennis; caulibus piloso-hispidis; foliis linearibus vel lanceolatis hispidulis, apice saepe angustatis; floribus spicatis sessilibus; bracteis oblongis acutis integris pilosis coccineis vel purpurascentibus florem subaequantibus; calycis segmentis coloratis emarginato-bidentatis; corolla calycem vix superanti; galea lineari dorso pilosa; labio inferiori quinquedentato; stylo exserto filiformi; stigmate capitato emarginato-bilobato; capsula oblonga compressa acuminata vel acuta. — HBK. Nov. Gen. et Spec. ii. 331, t. 165 (1817); Mart. & Gal. l. c. 29. *C. scorzoneraefolia* Benth. l. c. 529; Hemsl. l. c. 462. *C. speciosa* Mart. & Gal. l. c. 30 (1845). The following are in hb. Gray: — PUEBLA: Mt. Orizaba,

specimen from the same region, *C. V. Hartman*, no. 150 (Lumholtz Exped.), which may be this species. *C. nervata* resembles *C. aspera* in the tuberculate-rugose sac-like lower lip of the corolla, but differs in having the segments of the calyx quite entire and in the glandular pubescence. The flowers are smaller and the capsules less ovoid. The strongly nerved leaves suggest *C. lithospermoides*, but otherwise it is quite different.

19. C. CONZATTII Fernald, "suffruticosa; caulibus simplicibus erectis glanduloso-puberulis; foliis linearibus vel lineari-lanceolatis, 3–5-nerviis, 2–7 cm. longis dense puberulis, inferioribus integris, superioribus pectinatis, laciniis linearibus patentibus; bracteis oblongis 1.5–2.5 cm. longis, summis coccineis trifidis, lobis lateralibus linearibus vel spatulatis, intermedio majore anguste obovato integro vel obsolete trilobo; pedicellis 1 mm. longis; calyce mediam tantum corollam paululo superante 1.5–1.8 cm. longo viridi et albo, antice et postice aequaliter fisso, lobis oblongis subtruncatis 5–6 mm. longis; corolla viridi et rubella 2.2–2.5 cm. longa, tubo 1.2–1.3 cm. longo, galea elongata, labii lobis obtusis 1 mm. longis." — Proc. Am. Acad. xliii. 67 (1907). — OAXACA: Santa Ines del Monte, Zimatlan, altitude 2700 m. *Conzatti*, no. 1360 (type, in hb. Gray); 25 km. southwest of City of Oaxaca, altitude 2287–2897 m., *E. W. Nelson*, no. 1368.

20. **C. rigida**, n. sp., perennis rigida recte sparseque ramosa 3 dm. alta; caulibus et foliis purpurascentibus albo-pubescentibus; foliis inferioribus oblanceolatis, ceteris lanceolatis apice obtusis basi non dilatatis ca. 3–4 cm. longis 2–5 mm. latis; floribus sessilibus in spicis elongatis; bracteis oblongis apice rotundatis vel acutis coccineis puberulis flores subaequantibus, basi pilosis 2–2.5 cm. longis 5–8 mm. latis; calyce fisso 1 cm., segmentis ca. 5 mm. latis, apice oblique truncatis, 4-nerviis coccineis puberulis; galea exserta 5 mm., dorso puberula viridi, antice membranacea coccinea; labio inferiore obtuso, lobis membranaceis, lateralibus oblique truncatis, medio deltoideo obtuso, 1 mm. longo et lato, sinubus crassis involutis; stylo exserto 3 mm., stigmate bilamellato; capsulis caulibus adpressis oblongo-cylindraceis acuminatis 15 mm. longis. — Hills near Chihuahua, 16 April, 1885, *C. G. Pringle*, no. 188, in part (type, in hb. Gray). As in *C. Conzattii*, to which this species is related, the flower after anthesis has a tendency to curve outward above the capsule.

21. **C. falcata,** n. sp., caule simplici recto 3–3.5 dm. alto glandulare puberulenti et tenuiter piloso rubro angulato; foliis oblongo-lanceolatis integris vel sparse et irregulariter laciniatis 2–2.5 cm. longis, basi 3–10 mm. latis dilatate et auriculate amplexicaulibus 3–5-nerviis glandulare pilosis; bracteis foliis latioribus et longioribus, inferioribus viri-

fisso 7 mm., segmentis oblique emarginatis 4 mm. latis 4-nerviis scabrido-glandulosis; corollae galea calycem superanti 1.5 mm. lata 8 mm. longa, dorso puberulenti-glandulosa; labio inferiore paulo protuberanti triplicato, dentibus acutis; stigmate exserto capitato; capsula elliptica acuta 1 cm. longa, in calyce inclusa. — PUEBLA: dry hills about Chalchicomula, altitude 2592 m., 27 July, 1901, *C. G. Pringle*, no. 8545 (type, in hb. Gray); same locality, *Rose & Hay*, no. 5809.

16. C. TOLUCENSIS HBK., procumbens ramosa; caulibus vel ramis 5–6 cm. altis; foliis lanceolatis obtusis hispidis, inferioribus integris, superioribus apice trifidis, laciniis obtusis ca. 2 cm. longis; bracteis trifidis trinerviis, lobo intermedio oblongo obtuso, lateralibus linearibus intermedium subaequantibus; floribus 2 cm. longis sessilibus; calycis segmentis rotundatis; corollae galea vix exserta, dorso hirta; labio inferiore acute tridentato. — HBK. Nov. Gen. et Spec. ii. 329 (1817); Benth. l. c. 530; Mart. & Gal. l. c. 29; Hemsl. l. c. 463. — High mountains of southern MEXICO: Mt. Ixtaccihuatl, *C. A. Purpus*, no. 230; bare summits of Nevada de Toluca, *C. G. Pringle*, no. 4250; Mt. Orizaba, *Rose & Hay*, no. 5770. In hb. U. S. Nat. Mus. there is also a specimen collected by *E. W. Nelson* on Mt. Toluca.

17. C. MORANENSIS HBK. "caulibus suffruticosis, simplicibus, prostratis, pubescenti-hispidis; foliis lanceolatis, acutis, hispidis, integris, superioribus trifidis; floribus axillaribus, sessilibus; corolla calycem paulo superante; calycis lobis rotundatis emarginatis; corollae labio inferiori brevissimo, dentato." — HBK. Nov. Gen. et Spec. ii. 329 (1817); Benth. l. c. 530; Mart. & Gal. l. c. 30; Hemsl. l. c. 462. — There seem to be no specimens of this in hb. Gray. The type was collected in temperate localities between Pachuca and Moran, State of Hidalgo probably.

18. **C. nervata,** n. sp., herbacea, caulibus 1–5, 1–3 dm. altis divaricate pilosis et glandulare pubescentibus; foliis 3–5-nerviis, inferioribus oblanceolatis apice obtusis rectis integris 3–6 cm. longis, 1 cm. latis, superioribus oblongis apice obtusis basi angustatis et amplexicaulibus, floribus inferioribus sessilibus in axillis foliorum, superioribus confertis et occultis in spicis ornatis, bracteis obovatis 2–3 cm. longis apice coccineis; calyce 2 cm. longo 4 mm. lato vix corollam superanti, fisso 5 mm., segmentis 4-nerviis apice rotundatis; corollae galea 1 cm. longa dorso glandulari-pilosa, labio inferiore protuberanti trisaccato infra lacinias tuberculati-rugoso, laciniis exterioribus 3 triangularibus obtusis, interioribus 2 brevioribus sinus terminantibus; stylo stigmateque exsertis; capsula oblique oblonga compressa 1 cm. longa. — CHIHUAHUA: vicinity of Madera, May to June, 1908, altitude 2250 m., *F. Palmer*, no. 274 (type, in hb. Gray). There is also in hb. Gray a fragmentary

bus purpureis sessilibus in spicis brevibus; calyce 18 mm. longo, fisso 7 mm., laciniis obtuse lobatis vel profunde emarginatis binerviis arachnoideo-pilosis et glandulosis; corollae galea et labio inferiore calycem superantibus, priori 8 mm. longa, basi 3–4 mm. lata, apice acuta, dorso glandulari-puberulenti, antice purpurea membranacea; labio inferiore viridi protuberanti tridentato, dentibus obtusis incurvis infra triplicatis; stigmate exserto bilobato, apice styli curvato; capsula ovato-acuminata 1 cm. longa. — DURANGO: near El Salto, altitude 2440–2600 m., 12 July, 1898, *E. W. Nelson*, no. 4553 (type, in hb. U. S. Nat. Mus., dupl. in hb. Gray). This is related to *C. Schaffneri* and *C. Pringlei*, but differs from all of the same alliance in general habit of growth, pubescence, and most especially in the lower lip of the corolla, which has the divisions separated by a plicate sinus that is often toothed at the top.

14. C. PRINGLEI Fernald, caulibus plurimis decumbentibus 3–6 cm. altis; foliis imis confertis et bracteiformibus ovatis 3–4 mm. longis, superioribus lanceolatis vel oblongo-lanceolatis simplicibus vel apice trilobatis pilosis 1.5–2 cm. longis; bracteis foliis similibus, laciniis angustis coloratis; calyce tubuloso 2.5–3.5 cm. longo, infra ochroleuco piloso, supra rubro puberulenti, segmentis 6–8 mm. longis obtuse bilobatis; corolla vix exserta, galea angusta pilosa, labio inferiore trisaccato, lobis 1 mm. longis. — Proc. Am. Acad. xl. 56 (1904). — HIDALGO: Sierra de Pachuca, *C. G. Pringle*, nos. 9647, 8666 (type, in hb. Gray); *Rose & Hay*, no. 5581. MORELOS: Mount Popocatepetl, *Rose & Hay*, no. 6022. Related to *C. Schaffneri* but with much larger flowers and densely pilose calyx.

15. C. SCHAFFNERI Hemsl., hirsuta scabrida basi ramosa, ramis vel caulibus erectis vel adscendentibus, 2.5–5 cm. altis densissime foliosis; foliis integris anguste lineari-lanceolatis subacutis ca. 2 cm. longis; bracteis trinerviis trifidis, lobis linearibus acutis, medio longiore; calycis lobis rotundatis vel obscure emarginatis; corollae galea paulo exserta, dorso hirsuta; labio inferiore tridentato. — Hemsl. l. c. 462, t. lxiii. B. f. 7–13 (1882). — STATE OF MEXICO: in the valley of Mexico, *Schaffner*, no. 373 (dupl. of type, in hb. Gray); Desierto Viejo, same region, *Bourgeau*, no. 874; Flor de Maria, *C. G. Pringle*, no. 3193; Mount Ixtaccihuatl, altitude 3355–3660 m., *C. A. Purpus*, no. 218. MORELOS: meadows about Tres Marias, altitude 2897 m., *C. G. Pringle*, no. 13,141.

Var. **cinerascens**, n. var., nana pallida foliosa cinerascens; caulibus ramosis caespitosis 1 dm. altis retrorse pilosis; foliis linearibus vel saepissime divaricate pinnatisectis, laciniis 3–5 attenuatis (media elongata) nervatis scabrido-hispidis; bracteis foliis superioribus similibus, apice ochroleucis puberulenti-glandulosis; calyce 1.5 cm. longo

GUERRERO: between Tlapa and Tlaliscatilla, *E. W. Nelson*, no. 2048. JALISCO: vicinity of San Sebastian, *E. W. Nelson*, no. 4070.

11. C. ARVENSIS Schlecht. & Cham., precedenti similis, omnifariam major, bracteis obovatis dilatatis coloratis corollam superantibus. — Linnaea, v. 103 (1830); Benth. l. c. 529; Mart. & Gal. l. c. 31; Hemsl. l. c. 460; Loesen. l. c. 285. — ORIZABA: *Botteri*, nos. 339, 437. MICHOACAN: near Guanajuato, *C. & E. Seler*, no. 1148; corn fields near Patzcuaro, *C. G. Pringle*, no. 3349. AGUAS CALIENTES: *Hartweg*, no. 192. JALISCO: Guadalajara, *C. G. Pringle*, nos. 5348, 11,646; *E. Palmer*, no. 575, coll. of 1886. OAXACA: Sierra de San Felipe, altitude 3050 m., *C. G. Pringle*, no. 5664; same locality, altitude 2000 m., *Conzatti & González*, no. 507; Etla, altitude 1600 m., *Lucius C. Smith*, no. 963. STATE OF MEXICO: Vallée de Mexico, *Schaffner*, no. 375; Atusco, *L. Hahn*, 1865–1866; Salto de Agua, *C. A. Purpus*, no. 1712. VERA CRUZ: Zacuapan and vicinity, dry meadows, *C. A. Purpus*, no. 1925; Cordoba, *Bourgeau*, no. 1893; same locality, altitude 850 m., *Conzatti & González*, no. 1135. S. W. CHIHUAHUA: *E. Palmer*, year 1885, number missing. MEXICO: without locality, *Bilimek*, no. 296; *Uhde*, no. 945.

12. **C. nitricola**, n. sp., herbacea; caule basi ramoso piloso 2 dm. alto; foliis lineari-oblongis apice obtusis basi amplexicaulibus, integris 3–4 cm. longis 2–5 mm. latis, investis pilis basi subpapillosis; foliis superioribus et floralibus flores aequantibus vel floribus brevioribus ovatis vel spatulatis, apice obtusis glandulosis; floribus sessilibus in spicis angustis; calycis segmentis obtusis integris 6 mm. longis puberulente glandulosis; corolla calycem paulo superanti; galea acuta, dorso puberula exserta curvata, labium inferius duplo superanti, 7 mm. longo; labii laciniis membranaceis acuminatis 1.5 mm. longis; stigmate vix exserto capitato-emarginato; capsula ovato-acuminata. — SAN LUIS POTOSI: knolls of alkaline meadows, Hacienda de Angostura, 10 July, 1891, *C. G. Pringle*, no. 3756 (type, in hb. Gray). This was distributed as *C. scorzonerifolia*, "a narrow-bracted form." It seems quite distinct, peculiar in the group in the erect divisions of the lower lip which somewhat resemble those of *C. mexicana*. The plant has a pallid fleshy appearance like many of the *Chenopodiaceae*. The lower part of the stem is marked by bunches of leaf-scales resembling tubercles like those on *C. communis* and *C. arvensis*. The flowers are ochroleucous.

13. **C. saltensis**, n. sp., herbacea sparse arachnoidea 1 dm. alta; caulibus 2–4 simplicibus; foliis radicalibus subrosulatis lineari-lanceolatis 1–1.5 cm. longis; foliis caulinis pinnatisectis, laciniis 3–5 divaricatis linearibus, imis saepe tantum longis quantum mediis; bracteis coloratis similibus foliis superioribus, laciniis latioribus; flori-

C. G. Pringle, no. 4117 (type, in hb. Gray).—This is well distinguished in the group in which it has been placed by the almost lanate pubescence. Often at the base of the stem there is a peculiar thickening due to the old crowded leaf-bases. The leaves are probably present during the wet season.

9. **C. ornata**, n. sp., caulibus 1–2 rectis simplicibus 1.7–2.5 dm. altis glandulari-pilosis striatis; foliis lanceolatis vel oblongis, apice acutis vel obtusis, basi amplexicaulibus, margine integris vel crispi-undulatis, 2–3.5 cm. longis 2–6 mm. latis trinerviis; foliis radicalibus rosulatis, caulinis propinquis, supremis apice coccineis; floribus bracteisque confertis in spicis ornatis; bracteis spatulatis glandulari-puberulentibus, apice rotundatis vel rhomboideis 2–2.5 cm. longis 5–10 mm. latis calycem excedentibus; calyce fisso 1 cm., segmentis undulatis 5 mm. latis; galea exserta 5 mm., apice acuta, dorso viridi puberulenti, antice albo-membranacea; labio inferiore trisaccato membranaceo, laciniis subulati-acuminatis 2 mm. longis; stylo filiformi exserto, stigmate crasso bilobato, in fructu galea stigmateque contortis; capsula oblongo-ovata acuminata compressa 1 cm. longa. — CHIHUAHUA: near Colonia Juarez, Sierra Madre, June–July, 1899, *E. W. Nelson*, no. 6073 (type, in hb. Gray). This approaches more closely to *C. glandulosa* than any other species and resembles it in the trisaccate lower lip with the divisions separated by the folds forming the three sacs below. It has different pubescence and generally obtuse leaves. *C. glandulosa* does not appear ever to have the basal leaves rosulate, but their persistence in this species may be due to a season or locality of greater moisture.

10. C. COMMUNIS Benth., pilosa et hispida ramosa alta; caulis inferiore parte squamulose tuberculata; foliis lanceolatis integris basi angustatis apice acutis vel obtusis; spicis elongatis basi interruptis; bracteis apice coloratis vix dilatatis flores parum aequantibus apice glandulosis viridibus; corolla non exserta; capsula lata obtusa siccitate nigra. — Benth. in DC. Prodr. x. 529; Hemsl. Biol. Cent.-Am. Bot. ii. 460; Schmidt in Mart. Fl. Bras. viii. pt. 1, 323, t. 56, fig. 2; Loesen. l. c. 285. — Southern Mexico, Central America to South America. — GUATEMALA: Alta Verapaz, *H. von Türckheim*, no. II. 1318, also Coban, no. 28; San Miguel Uspantan, *Heyde & Lux*, no. 2878 (both ex hb. John Donnell Smith). NICARAGUA: *Oersted.* COSTA RICA: San Jose, *Tonduz*, no. 7096; Cartago, *Juan J. Cooper*, no. 5873 (both ex hb. John Donnell Smith). YUCATAN: *G. F. Gaumer*, no. 416. VERA CRUZ: Santa Lucrecia, Isthmus Tehuantepec, *Chas. L. Smith*, no. 1102. TEPIC: San Blas, *Frank H. Lamb*, no. 608. Additional specimens in hb. U. S. Nat. Museum. — ORIZABA: Boca del Monte, *E. W. Nelson*, no. 204. This is mounted on sheet no. 257,518 with a specimen of *C. canescens*.

vel 2–3-lobatis pruinosis; calycis segmentis 1 cm. longis 3 mm. latis emarginatis pruinosis tubum aequantibus; galea apice acuta 1 cm. longa purpurea glandulare puberulenti; labio inferiore membranaceo, laciniis linearibus acutis 2 mm. longis; stylo crasso; stigmate globoso 1.25 mm. diametro vix exserto; capsula elliptica acuta compressa. — DURANGO: Otinapa, July–August, 1906, *E. Palmer*, no. 361 (type, in hb. Gray). The peculiar pruinose appearance of the inflorescence is due to the white puberulence closely covering the purplish bracts and flowers.

7. **C. Palmeri,** n. sp., sparse pilosa et glandulari-puberulens; caulibus 1–2 simplicibus rectis 1.5 dm. altis; foliis radicalibus imbricatis caulinis lanceolatis 2–3 cm. longis 1–3 mm. latis trinerviis, basi amplexicaulibus 5–10 mm. latis, apice obtusis vel acutis, margine integris vel undulatis; spicis flavis brevibus compactis, fructiferis elongatis; bracteis flores sessiles superantibus vel aequantibus spatulatis 2–3 cm. longis integris, apice rotundatis; calyce fisso 7 mm., laciniis emarginatis 5 mm. latis; corolla 14 mm. longa, galea acuta, dorso glandulari-puberulenti, calycem superanti; labio inferiore membranaceo protuberanti, lobis subulatis acuminatis 2 mm. longis glandulari-ciliatis; stigmate bilobato crasso paulo exserto; capsula ovato-acuminata compressa; seminibus foveolatis cum testa membranacea. — DURANGO: Otinapa, July–August, 1906, *E. Palmer*, no. 376 (type, in hb. Gray). This species is related to *C. macrostigma*, differing in pubescence, densely flowered spike, and large bracts; from *C. angustata* it differs in pubescence, stigma, foliage, and flowers. There are resemblances to *C. glandulosa* chiefly in the form of the spike, but the bracts in *C. Palmeri* are rounded at apex rather than rhomboid. The corolla is quite dissimilar, the lower lip with three long almost equal divisions, and the body extending outward like a shelf, being very different from the trisaccate lower lip of *C. glandulosa* with its short divisions separated by the folds forming the sacs.

8. **C. angustata** (Robinson & Seaton), n. comb., caulibus 1–2 rectis gracilibus purpurascentibus 1–1.5 dm. altis, basi squamulosis, inferiore parte minute adpresso-pubescenti, superiore parte spicisque albo-tomentosis; foliis integris linearibus 2–4 cm. longis 1–3 mm. latis; bracteis lanceolatis acutis flores subaequantibus, supra viridibus glabrescentibus, subter albo-tomentosis, confertis cum floribus in spicis brevibus; calycis segmentis bidentatis vel crenatis albo-puberulentibus; corollae galea calycem paulo superanti, apice acuta, dorso glandulari-puberulenti; labio inferiore non protuberanti, laciniis 3 lineari-obtusis ciliatis 1.25 mm. longis, sinubus angustis; stigmate crasso bilobato, paulo exserto; capsula ovato-oblonga acuta 5–7 mm. longa. — *C. pallida* Kunth, var.? *angustata* Robinson & Seaton in Proc. Am. Acad. xxviii. 114 (1893). MICHOACAN: grassy slopes near Patzcuaro, 18 July, 1892,

recurvatis; capsula elliptica acuta compressa 8 mm. longa. — Proc. Am. Acad. xxvi. 173 (1891). — STATE OF MEXICO: grassy slopes, Flor de Maria, 28 July, 1890, *C. G. Pringle*, no. 3194 (type, in hb. Gray), also from same locality, altitude 2440 m., no. 9429. To this species, at the time of its original description, was doubtfully referred a specimen collected in Chihuahua by *C. G. Pringle*, no. 1545, which is below made the type of *C. pediaca*. In hb. U. S. Nat. Mus. sheet no. 396,150 contains a specimen of *C. macrostigma* collected at the type locality by *Rose & Hay*, no. 6330, together with a specimen of *C. Schaffneri.*

5. **C. pediaca**, n. sp., annua albo-pilosa et glandulare puberulens; caulibus prope basim recte ramosis, 2 dm. altis; foliis tenuibus lanceolato-acuminatis integris 3–5-nerviis, 2–3 cm. longis 1–2 mm. latis, basi amplexicaulibus 5–10 mm. latis; floribus sessilibus interruptis acclinibus ad axim spicae gracilis; bracteis flores subaequantibus et investientibus, spatulatis 10–12 mm. longis, superiore parte flavo densissime glandulare puberulenti, apice truncato vel obtuso, inferiore parte pilosa nervia; calyce membranacea 12 mm. longo, fisso 6 mm., duobus partibus truncatis vel emarginatis 5 mm. latis; corolla recta 18 mm. longa, galea acuminata apice glandulare pilosa, labio inferiore membranaceo non protuberanti, laciniis linearibus obtusis glandulare ciliatis 1.5 mm. longis, sinubus inter laciniis 1 mm. latis; stigmate bilobo crasso, fere exserto; ovario subcylindrico; capsula compressa oblonga 8 mm. longa 3 mm. diametro, apice acuminata; seminibus rhomboideis 1 mm. diametro, testa membranacea profunde foveolata. — CHIHUAHUA: plains, base of the Sierra Madre, 27 September, 1887, *C. G. Pringle*, no. 1545 (type, in hb. Gray), distributed as *C. lithospermoides*, var. (?) *flava* Watson; also included under *C. macrostigma* Robinson in Proc. Amer. Acad. xxvi. 173 (1891). From this latter species it differs in having a more closely flowered spike, pilose instead of puberulent indumentum and less exserted stigma. The flowers too are dissimilar, but the differences are not so obvious. It is even further removed from *C. lithospermoides*, being a slender-stemmed annual, while that is a robust perennial with somewhat harsh pubescence. The slender spikes of *C. pediaca* have flowers about 1 cm. apart, somewhat distichous and appressed to an axis that is slightly tortuous, and are quite unlike the showy thickly flowered spikes of *C. lithospermoides.*

6. **C. sphaerostigma**, n. sp., caulibus 1–2 simplicibus 1.5–2.5 dm. altis gracilibus adpressi-pubescentibus; foliis integris vel pinnatisectis 1–3-nerviis puberulenti-subscabridis, basi amplexicaulibus, apice obtusis, margine frequente involutis, laciniis 2–6 anguste linearibus; floribus in spicis elongantibus sessilibus, bracteis galeam fere aequantibus vel (sub floribus primis) eam superantibus simplicibus cum margine undulata

et minoribus; spicis vel racemis gracilibus; floribus divaricatis ca. 2–3 cm. longis; calyce tubuloso vel saepissime infundibuliformi 1.5–2.5 cm. longo; galea 5–10 mm. exserta obtusa inferiore labio protuberanti nunc exserto nunc incluso; styli superiore parte et stigmate bilamellato exsertis; capsula oblonga 7 mm. longa apice truncata. — Mart. & Gal. in Bull. Acad. Brux. xii. pt. 2, 30 (1845); Walp. Rep. vi. 651; Hemsl. Biol. Cent.-Am. Bot. ii. 463; Loesen. in Bull. Herb. Boiss. ser. 2, iii. 285. *C. anthemidifolia* Benth. in DC. Prodr. x. 528 (1846).— Southern Mexico in the states of Oaxaca, Michoacan, Guerrero, Morelos. The type was collected in Oaxaca, *Galeotti*, no. 995. OAXACA: Zimatlan, Sta. Ines del Monte, altitude 2800 m., *C. Conzatti*, no. 1358; Sierra de Clavellinas, altitude 2440 m., *C. G. Pringle*, no. 5692. MICHOACAN: Ignatio, *C. & E. Seler*, no. 1209; dry hills near Patzcuaro, *C. G. Pringle*, no. 3348; rock fields near Coru Station, altitude 1830 m., *C. G. Pringle*, no. 13,142. GUERRERO: between Tlapa and Ayusinapa, altitude 1372–1740 m., *E. W. Nelson*, no. 2106. MORELOS: thin soil of the knobs of the Sierra de Tepoxtlan, altitude 2287 m., *C. G. Pringle*, no. 9123.

2. C. AUREA Robinson & Greenman, glabra supra puberula 3 dm. alta graciliter ramosa; laciniis pinnatisectis 2.5–4 cm. longis; laciniis 6–9 lineari-filiformibus; floribus 2–2.5 cm. longis subsecundis in racemis, pedicellis 2–10 mm. longis rectis, saepe in fructu divaricatis; galea obtusa exserta 8 mm.; labio inferiore saepissime exserto; capsula oblongo-acuminata ca. 6 mm. longa. — Proc. Am. Acad. xxxii. 39 (1896). — MORELOS: wet bluffs of barrancas above Cuernavaca, altitude 2135 m., *C. G. Pringle*, no. 6204 (type, in hb. Gray).

3. C. GRACILIS Benth., praecedenti similis; floribus 10–15 mm. longis, saepe sessilibus; galea obtusa 4–6 mm. exserta, calyce non ampliato et viridi-flavo. — Benth. in DC. Prodr. x. 528 (1846); Hemsl. l. c. 460. — OAXACA: Cerro de San Felipe, altitude 1800 m., *Conzatti & González*, no. 490; dry banks in same mountain range, altitude 2287 m., *C. G. Pringle*, no. 4968. FEDERAL DISTRICT: lava fields, valley of Mexico, altitude 2287 m., *C. G. Pringle*, no. 7977. These specimens have not been compared with authenticated ones.

4. C. MACROSTIGMA Robinson, caule basi ramoso et saepe cum ramulis gracilibus brevibus sterilibus in axillis, puberulenti 1–2 dm. alto; foliis viridibus, inferioribus subimbricatis, superioribus integris undulatis vel sparse pinnatisectis lineari-lanceolatis 1–4 cm. longis 2–3 mm. latis 1–5 nerviis; floribus flavis 1.5–2 cm. longis; spicis brevibus demum elongatis; calyce fisso 4–5 mm., segmentis bidentatis; corollae galea obtusa; labio inferiore non protuberanti, laciniis lineari-acuminatis 1–5 cm. longis, media breviore; stigmatibus exsertis, 1–2 mm. longis,

SECTIO IV. HEMICHROMA Benth. l. c. 532 (1846).

Calyx incurvus, antice profunde fissus, postice vix vel paulo fissus integer vel 2–4-dentatis. Folia floralia vulgo caulinis minora et apice vix colorata. Corolla e fissura calycis saepius longiuscule exserta.

a. Flores spicati, *b.*
b. Folia distincte auriculata, auriculis 1–2 mm. longis et latis, *c.*
c. Pubescentia dense canescens et minute glandulosa, caulibus idem pilosis.
Folia deltoidea densissime imbricata. . . . . . 37. *C. auriculata.*
Folia ovato-lanceolata non imbricata. . . . . 38. *C. longiflora.*
c. Pubescentia divaricate pilosa et scabrido-puberula. Folia viridia lanceolata. Flores subrecti.. . . . . 39. *C. subalpina.*
c. Pubescentia plerumque adpressis et scabrido-puberula. Flores divaricati.
Folia lineari-lanceolata saepe 5 cm. longa divaricata, in siccitate atra. 40. *C. tenuiflora.*
Folia eis praecedentis breviora et crassiora. Pubescentia densior. 41. *C. canescens.*
b. Folia obscure auriculata.
Folia lineari-lanceolata viridia, divaricate pilosa. . 42. *C. xylorrhiza.*
Folia linearia viridia scabridula. . . . . 43. *C. scabridula.*
b. Folia basi non dilatata.
Folia lineari-lanceolata saepe deflexa, canescente pubescentia. Flores multo exserti divaricati. . . . . 44. *C. laxa.*
Folia subfiliformia in siccitate atra. Flores recti. . 45. *C. stenophylla.*
a. Flores racemosi, *c.*
c. Folia integra.
Bracteae summae obovatae apice fimbriatae. . 46. *C. longibracteata.*
Bracteae summae lineares. . . . . 47. *C. integrifolia.*
c. Folia pinnatisecta, laciniis elongatis.
Folia scabrido-hispidula tenuia, laciniis linearibus plurimis. 48. *C. patriotica.*
Folia hispida, superiora trifida, lobo medio lateralibus multo longiori. 49. *C. Purpusi.*
Folia pectinato-laciniata, laciniis linearibus distantibus 2–3-jugis. 50. *C. pectinata.*
c. Folia pinnatisecta, laciniis crassis obtusis, saepissime in siccitate atris.
Folia pubescens, laciniis brevibus. . . . . . . 51. *C. fissifolia.*
Folia glabra. . . . . 52. *C. irasuensis.*

C. linearifolia Benth., Sonora, *Geo. Thurber*, no. 981, species hujus sectionis sed valde dubia est.

Locus in clave dubius . . . . . { 53. *C. tapeinclada.*
{ 54. *C. katakyptusa.*

1. C. TENUIFOLIA Mart. & Gal., herbacea glabra vel puberula 1.5–6 dm. alta ramosa; foliis pinnatisectis, segmentis filiformibus vel lineari-subulatis elongatis in siccitate contortis, floralibus simplicioribus

g. Calycis segmenta apice emarginata vel obtuse bilobata, *h.*
  h. Folia basi dilatata.
    Folia bracteis longiora.
      Pilosa. Corollae labium inferius quinquedentatum. 23. *C. scorzonerifolia.*
      Pilosa et glandulosa. Corollae labium inferius tridentatum, sinubus latis involutis. . . . . 24. *C. glandulosa.*
    Folia superiora bracteis breviora. Scabrido-hispida. Folia ovata valde nervata. . . . . 25. *C. crypiandra.*
  h. Folia basi non dilatata, bracteis longiora.
    Scabrido-hispida. Folia lanceolata valde nervata. 26. *C. lithospermoides.*
    Glabrescens. Inflorescentia pilosa. Folia ovato-acuminata longa lataque. . . . . 27. *C. Nelsoni.*
    Lanata densissime. Folia lineari-lanceolata. . . . 28. *C. lanata.*
    Caules in vetustate glabri. Folia lanata anguste longeque spatulata. 29. *C. guadalupensis.*
g. Calycis segmenta acute bilobata, *i.*
  i. Folia integra.
    Tomentosa. Folia lineari-lanceolata. . . . . 30. *C. integra.*
    Scabrido-hispida et glandulari-pilosa. Folia valde nervata lanceolata. 31. *C. aspera.*
  i. Folia saepissime pinnatisecta. Flores subfalcati. (Transitio ad Hemichromam.)
    Folia regulariter pectinata, laciniis brevibus subfiliformibus. 32. *C. ctenodonta.*
    Folia filiformi-pinnatisecta. Corolla breviter exserta. Capsula anguste et oblique cylindracea. . . . . 33. *C. Bryanti.*
    Folia lanceolata saepe pinnatisecta. Corolla exserta 5–15 mm. 34. *C. affinis.*
    Folia anguste linearia apice attenuata. Flores parvi pedicellati. Capsula anguste cylindraceo-ovoideo. . . . . . . 35. *C. minor.*

SECTIO III. CALLICHROMA Benth. l. c. 531 (1846).

Calyx postice breviter, antice profundius fissus, lobis bifidis, laciniis ovatis vel oblongis vel linearibus plerumque acutis vulgo coloratis. Folia floralia (praesertim superiora) caulinis saepius magis incisa, latiora et colorata.

Folia caulinia et floralia pinnatifida, lobis linearibus elongatis. Calycis segmenta linearia bifida. Corollae labii subaequales calycem multo superantes. 36. *C. mexicana.*

C. sessiliflora auct. quoad speciminibus mexicanis est me judice ad *C. mexicanam* referenda.

a. Annua vel biennis, radice brevi, *b*.
  b. Caules recti solitarii vel pauci 1–2 dm. alti. Stigma crassum, *c*.
    c. Folia nunc integra nunc pinnatisecta. Bracteae floribus breviores.
      Puberulens. Folia lanceolata. Stigma exsertum bisectum, partibus recurvatis. . . . . 4. *C. macrostigma.*
      Pilosa et glandulosa. Folia lanceolata. Stigma vix exsertum bilobatum. . . . . 5. *C. pediaca.*
      Albo-puberulens. Folia vulgo pinnatisecta, laciniis linearibus. Stigma globosum vix exsertum. . . . . 6. *C. sphaerostigma.*
    c. Folia et caulinia et floralia integra.
      Glandulare puberulens. Folia lanceolata saepe undulata. Bracteae obovatae flores excedentes. Stigma bilobatum exsertum. 7. *C. Palmeri.*
      Sublanata. Folia linearia. Bracteae lanceolatae flores aequantes vel excedentes. Stigma bilobatum vix exsertum. 8. *C. angustata.*
      Glandulare pilosa. Folia oblonga vel lanceolata. Bracteae spatulatae floribus breviores. Stigma bilobatum. Styli superior pars et galea exsertae. 9. *C. ornata.*
  b. Caules a basi ramosi, infra ramulos squamulose tuberculati, *e*.
    e. Folia oblanceolata basi angustata. Capsula apice truncata.
      Bracteae paulo dilatatae. Stigma bilobatum vix exsertum. 10. *C. communis.*
      Bracteae dilatatae. Stigma bilobatum exsertum. Flores foliaque eis praecedentis majora. . . . . 11. *C. arvensis.*
    e. Folia lineari-lanceolata basi dilatata. Capsula apice acuminata. 12. *C. nitricola.*

a. Perennis nana saepe caespitosa. Alpina vel subalpina. Folia integra vel pinnatisecta, *f*.
  f. Caules recti, non caespitosi.
    Spici breves densique. Galea lata. Labium inferius exsertum. 13. *C. saltensis.*
  f. Caules caespitosi.
    Galea exserta.
      Flores 3–3.8 cm. longi. . . . . 14. *C. Pringlei.*
      Flores 2.5 cm. longi. . . . . 15. *C. Schaffneri.*
    Galea vix exserta.
      Folia apice obtusa. Corolla et calyx subaequantes. 16. *C. tolucensis.*
      Folia acuta. Corolla calycem vix superans. . . 17. *C. moranensis.*

a. Perennis. Caules alti saepissime recti. Bracteae latae coloratae, *g*.
  g. Calycis segmenta integra.
    Folia integra valde nervata, inferiora basi angustata. . 18. *C. nervata.*
    Folia integra vel saepissime pinnatisecta.
      Calyx viridis 1.2–2 cm. longus. . . . . 19. *C. Conzattii.*
      Calyx viridis apice coccineus 2.3 cm. longus. . . . . 20. *C. rigida.*
      Calyx subfalcatus apice coccineus 3–3.2 cm. longus, segmentis vix dilatatis. . . . . 21. *C. falcata.*
      Calyx divaricatus, usque ad ovarium coccineus; segmentis dilatatis. 22. *C. hirsuta.*

form. Great variation and closely related species indicate a recent genus still in process of evolution. The line separating *Orthocarpus* from *Castilleja* is not definitely fixed, and the species on the border may suffer changes in name frequently. At present the tendency is to remove all these doubtful species from *Orthocarpus* and include them in *Castilleja*, thus leaving the former genus represented only by annuals. The two genera are certainly very closely related, for there is scarcely a character of *Orthocarpus* which cannot be found in some species of *Castilleja*. Indeed, it is doubtful if the differences between the two genera are much more pronounced than are the differences between some of the sections of *Castilleja*. The last enumeration of the Mexican and Central-American species of *Castilleja* was in 1881–1882, when Hemsley enumerated 26 species (Biol. Cent.-Am. Bot. ii. 459–463). Since then great activity has prevailed in the biological exploration of Mexico and Central America, and specimens of *Castilleja* have been accumulating in all the large herbaria. The present paper is based upon the specimens in the Gray Herbarium and some from the herbarium of the U. S. National Museum. Besides the key a short diagnosis of each species has been added, sometimes modified from the original description and sometimes quoted.

Sectio I. Epichroma Benth. in DC. Prodr. x. 528 (1846).

Calyx vix fissus, breviter et obtuse sinuato-lobatus. Folia pinnatisecta; rhachi et laciniis filiformibus vel anguste linearibus. Folia floralia caulinis minora et concolora. Flores laxe spicati vel racemosi. Annua.

Flores 2.5 cm. longi. Calyx coccineus infundibuliformis. Galea flava a basi exserta. . . . . 1. *C. tenuifolia.*
Flores 2 cm. longi. Calyx flavus. Galea flava exserta. . . . . 2. *C. aurea.*
Flores 1.5 cm. longi. Calyx viridi-purpureus. Galea viridi-flava paulo exserta. . . . . 3. *C. gracilis.*

Sectio II. Euchroma (Nutt.) Benth. l. c. 529. *Euchroma* Nutt. Gen. ii. 54 (1818).

Calyx in duas partes subaequaliter fissus, segmentis integris vel obtuse bilobatis vel acute bifidis. Folia floralia caulinis latiora, apice dilatata et semper colorata. Flores et bracteae in spicis confertae, demum interruptae.

CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.—NEW SERIES, No. XXXVI.

Presented by B. L. Robinson, March 10, 1909. Received March 12, 1909.

I. SYNOPSIS OF THE MEXICAN AND CENTRAL AMERICAN SPECIES OF CASTILLEJA.

By Alice Eastwood.

The genus *Castilleja* was published by Linnaeus fil. in 1771 (Suppl. 293). It was named by Mutis in honor of Domingo Castillejo of the botanical garden of Cadiz and rested upon the two species collected by Mutis in New Granada, *C. integrifolia* and *C. fissifolia.* At that time *C. pallida* and *C. coccinea* had been described by Linnaeus but under *Bartsia*, so that altogether four species were known. In 1818 Nuttall established the genus *Euchroma* (Gen. ii. 55) founded upon *Bartsia coccinea* and *B. sessiliflora* Pursh. The first satisfactory arrangement, however, came in 1846, when Bentham revised the genus *Castilleja* (DC. Prodr. x. 528–534), establishing four sections. At that time thirty-four species were known, fifteen of which belonged to Mexico and Central America. The subdivisions established by Bentham seem to mark off natural groups, which, however, show connecting characteristics that often render the true position of certain species doubtful. *Epichroma* is probably the most individual subdivision and has, perhaps, the best claim to generic rank; but some species placed in the present synopsis under *Euchroma* have floral characteristics that closely approach those of *Epichroma*, while other species under the same section are difficult to separate from *Hemichroma.* On account of this interrelationship any key must be more or less artificial. Perhaps when the knowledge gained from books and herbarium specimens is supplemented by that of the living plants in their natural environment, an entirely different system of classification may be arranged. Dried specimens often conceal the form of the flower, and when mounted frequently render dissection difficult, so that it is not always possible to obtain accurate knowledge of all of the parts; especially is this true of the lower lip of the corolla, which gives much of the characteristic

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

New Series. — No. XXXVI.

I. Synopsis of the Mexican and Central American Species of *Castilleja*. By A. Eastwood.
II. A Revision of the Genus *Rumfordia*. By B. L. Robinson.
III. A Synopsis of the American Species of *Litsea*. By H. H. Bartlett.
IV. Some Undescribed Species of Mexican Phanerogams. By A. Eastwood.
V. Notes on Mexican and Central American Alders. By H. H. Bartlett.
VI. Diagnoses and Transfers of Tropical American Phanerogams. By B. L. Robinson.
VII. The Purple-flowered *Androcerae* of Mexico and the Southern United States. By H. H. Bartlett.
VIII. Descriptions of Mexican Phanerogams. By H. H. Bartlett.

The time of greatest intimacy between the clergy of Norway and England, as we judge from the English Rolls, was the reign of Hákon Hákonarson (1217–1263), and especially the decade ending in 1230.

Toward the end of the thirteenth century records grow scanty. In the fourteenth century the breach between the English and Norwegian churches became complete.

After 1290 the route between Bergen and Bruges brought Norway into closer contact with France. The popes moved to Avignon. During the fourteenth century France (and Flanders) took the place of England in the eyes of the Norwegian church.

The date 1290 makes a convenient mark of transition. In so far as the Norwegian clergy before that date imported foreign culture, especially foreign literature, we should expect it to come from England; after 1290 from France. When all else is discounted, there remain the actual records of a sufficient number of clergy passing between Norway and England to assure a literary intercourse in the twelfth and thirteenth centuries. For France it is not so. The great body of foreign literature, and notably the Arthurian and Carolingian romances, were translated into Old Norse before 1290. The chief agent of translation was the clergy, and the clergy depended for its foreign relations upon England, to the relative exclusion of the continent. England, then, and not France, was the chief medium of exchange.

Harvard University,
June 1, 1908.

taught Olaf Kyrre (1066–1093) the art of psalmody. Martin was King Sverri's chaplain and favorite. Richard of St. Albans (1234, 1238) served as envoy of King Hákon Hákonarson in England. His position with Hákon may have been like that of Matthew, his colleague. We are sure of only one visit of Matthew to Norway (1248), but before that time he was said to be a "special friend" of Hákon. In Eric's reign (1280–1299) a Yorkshire priest served a long time as secretary at the Norwegian court, and returned to England (1293) bearing letters of recommendation from the king and his brother.

During the twelfth and thirteenth centuries the Norwegian clergy came in large numbers to England. They appear in the English Rolls usually as merchants and envoys, but we must believe many of them came primarily on church business or for study. The archbishops of Nidaros early secured important trading privileges in England, from Henry II, and these were renewed by Richard, John, and Henry III. The Norwegian monasteries, Lyse in particular, and the bishops and cathedral chapters, loaded their ships in English ports with provisions for their houses.

Church dignitaries, lay and secular, served as envoys to the English kings, spending the winter well entertained at London. The same man sometimes served for several succeeding seasons, if, indeed, he did not remain for a term of years in permanent residence abroad. As ambassadors, the abbots and priors of Lyse were most in demand, partly because their ranks were recruited by Englishmen who understood both countries, partly because the association of this abbey with England took its officials thither. In much the same way figure the high officials of the see of Bergen.

The shrine of Becket brought pilgrims; the English monastic schools drew students from Norway.

English establishments in Norway, like Lyse and Hovedö, kept in contact with the mother institution. The first Bishop of Lyse returned in his old age to Fountains. A century later (after 1248) the English Abbot of Hovedö came back to be head of the mother abbey of Kirksted.

Bishops came in person to England or sent their delegates "on affairs of the church." We are sure of three archbishops of Norway who were in England. Eystein spent three years there (1180–1183), nine months of it at St. Edmundsbury. The archbishops were doubtless delayed often in England on their way to and from consecration by the pope, as the English route was preferred over the German alternative.

Papal legates went to Norway via England. England was a stage on the way to the crusades. It was the avenue by which French and Italian influence came to Norway before the fourteenth century.

time for study. John Halldórsson, a Dominican friar in Bergen, who went out to Iceland (in 1332) as Bishop of Skalholt, and died on a visit to Bergen (in 1339), studied in his youth in Paris and Bologna. In Iceland he introduced foreign romantic tales accumulated in student days.[198] In 1307 an Upsala canon, a student at Orleans, made his will; among the witnesses was one Alfinn, a canon of Hamar in Norway.[199] In 1309 two of the twelve canons of Bergen were studying in Paris.[200] Soon after this, Paul Bárdsson, then a canon in Bergen, but later archbishop (1333–1346), studied in Paris and Orleans.[201] In 1317 Olaf Eindrideson went as a student to Paris. In 1346 Síra Einar Haflithason spent "some time" in Paris.[202]

Against this array the records have little to offer in the way of Anglo-Norwegian relations in the fourteenth century.[203] The pendulum has swung to France.

CONCLUSION.

From England Norway received Christianity. Its church was established by English bishops who went thither in the eleventh century. A century and a half later an Englishman reorganized this church and set it apart as an independent province.

Founded by Englishmen, the Norwegian church continued to depend upon England. The Norman Conquest apparently did not break the chain. English clerics continued to go to Norway to teach and reform and make new establishments. The first two Cistercian monasteries in Norway were founded by English monks who went from Fountains to Lyse (1146), and from Kirksted to Hovedö (? 1147). At least one subsequent abbot of Hovedö, Lawrence (*c.* 1246) was an Englishman. In 1247 the Benedictine order in Norway, seeking reform, called upon Matthew of St. Albans, a monk in England. The secular clergy also drew leaders from the English. The first bishop of Stavanger (1135) was an Englishman. So was Bishop Martin of Bergen (1194).

English clerks were also sought by the Norwegian kings for personal service, as teachers, secretaries, or envoys to foreign lands. Turgot

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[198] See introd. to Clári Saga, ed. Cederschiöld in Saga-Bibliothek.

[199] Dipl. Sv., 1557; Munch, IV, 2, 474, note 2.

[200] Dipl. Norv., VI, No. 72; Munch, ibid.

[201] Munch, ibid.

[202] Icelandic Annals.

[203] I have used all my fourteenth-century English material under the thirteenth century. It ends with the murder of the Abbot of Lyse in 1337, and the complaint of the Bishop of Bergen, the following year, that wine no longer came from England.

About 1300 a sea-route between Bergen and Bruges[186] was established,[187] and took the place of the old approach to France and southern Europe via Lynn and Dover. As early as 1258 we read, concerning the retinue which accompanied Princess Christina to Spain, that they returned home in various ways, most of them coming, probably, as they had gone, via England; "but Bishop Peter fared overland into Flanders, and he came somewhat later."[188] In 1285 Bishop Thorfinn of Hamar died in the monastery of Doest near Bruges.[189] In 1301 Archbishop Jörund, returning from Paris to Norway, met in Bruges, John Elk, a refractory cleric, on his way to the pope, and had him arrested.[190] In 1312 envoys from the Council of Vienne returned via Bruges.[191] In 1326 papal messengers came via Flanders;[192] in 1330 another papal nuncius.[193] About 1335 Bruges was a papal subtreasury for the deposit of funds from Norway sent by the bishops of Oslo, Hamar, and Stavanger.[194] Bruges was the route used by Norwegians through the fourteenth century in reaching the papal court at Avignon.[195]

The Bruges route brought Norway into closer contact with France. Shortly before 1295 there came to Norway a learned Fleming who became the archbishop's right-hand man,—"a great clerk," says the saga, "John Fleming; he had stayed long at Paris and in Orleans in study; he was so great a jurist that no one in Norway was his like."[196] In 1301 Archbishop Jörund started for the curia, fell ill in Paris, and returned home via Bruges.[197] Norwegians went to France at this during our period are slight indeed, and do not complicate the Anglo-Norwegian connection. These countries leaned upon Germany and, at times, upon France.

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[186] Not that Bruges was its own seaport.

[187] Our earliest evidence of Norse-Flemish relations is in the reign of Magnus (1280–1299), when Count Guido of Flanders sent his servant William to Norway, Sweden, and Denmark. About 1304 Norwegians were trading between Flanders and Lynn. In 1308 they had their own "street" in Bruges, and in the same year Flanders and Norway made their first recorded treaty (see A. Bugge, Byers Selvstyre, pp. 154 ff.).

[188] *Hákonar S., chap.* 296.

[189] Arna Bisk. S., chap. 54 (Bisk. S., I, 752); Annals, 1285; Munch, IV, 2, 50.

[190] Dipl. Norv., III, No. 48; Munch, IV, 2, 382.

[191] Dipl. Sv., III, 62–64; Munch, IV, 2, 593; Keyser, II, 155–156, 148–149.

[192] Munch, 2. Hovedafd., I, 93.

[193] Ibid., 164.

[194] Dipl. Norv., XVII (publ. 1902), letters, 39 ff. In 1355 (28 November), the pope ordered his legate to pay in Brussels or Bruges moneys collected in Scandinavia (Dipl. Norv., VI, 265).

[195] E. g., papal nuncius via Brügge in 1364 (Munch, 2. Hovedafd., I, 843).

[196] Laurentius Saga, chap. 9 (Bisk. S., I, 799); Munch, IV, 2, 304.

[197] Laurentius S., chap. 13; Annals; Dipl. Norv., III, No. 48; Munch, IV, 2, 381.

After 1290. The French Period.

There must always have been some reaction, however indirect, from France upon the church in Norway. The archbishops went to receive the pallium from the pope. Other Norwegians visited Rome. Messengers came to Norway from the papal court. When we know their route, it is almost always through France and England.[180] In England they tarried long.[181]

An idea prevails that Norsemen flocked to the University of Paris. The list of these students begins and ends with Bishop Thorlak, the Icelander. From 1100 to 1250 I know of only one West Scandinavian who studied in Paris. He is our friend Thorlak, who has been multiplied into a legion. Thorlak stayed in Paris "as long as he thought needful to get the knowledge which he wished to get there." To Lincoln he went to complete his education, and to acquire "great knowledge."[182] This does not prove Lincoln was the better school, but it does show how Icelanders felt about it.

In the second half of the thirteenth century, some extended sojourn in France can be conjectured. In 1254 Einar Gunnarsson, when chosen archbishop, was in Paris, and men were sent out to seek him.[183] In 1274 Archbishop John, Bishop Askatin of Bergen, and Bishop Andrew of Oslo, attended the general council at Lyons.[184]

From such scant evidence we cannot infer any considerable influence from France upon the Norwegian church, except as it came through Anglo-Norman England.

After 1290[185] all is changed. The records of Norwegian clerics in England become meagre, and those for France plentiful.

Hist. Univ. Paris, 1665, esp. II, 385; Fr. Hammerich, En Skolastiker, 1865). In the succeeding century this influence continued, broken, of course, at times by the church in Germany. The clergy of Norway and Denmark do not seem to have been on cordial terms.

[180] Above, pp. 548 ff.

[181] Sabina, for instance, spent four months. England, before 1290, seems to have been the base of papal attack on Norway.

[182] Above, p. 548.

[183] Hákonar Saga, chap. 281.

[184] Árna Biskups S., chap. 14.

[185] There is no charm about this date. It is, on the whole, the most convenient. In this year died Margaret, "the Maid of Norway," who was to unite Scotland and England.

and became Abbot of Abingdon. About 1016 Guthlaug, oldest son of Snorri Gothi, went to England and became a monk.[172] Probably many Icelanders came to England, like Bishops Thorlak and Paul, for travel and study. The sagas claim that Thorlak, after his death and saintship, was reverenced in Scotland and England as well as the Scandinavian countries.[173] From England they record two miracles. One was performed by a likeness of the sainted bishop set up in a church in Kynn (Lynn ?).[174] On the other occasion, merchants in the "English sea" called successfully upon Thorlak to deliver them from a tempest.[175] How many of the travellers who came to England from Norway were Icelanders cannot be determined. Hrafn, as we saw, proceeded to Canterbury after he had spent the winter in Norway.[176] The Icelandic priest Ingimund, who was in Norway at the close of the eighties, came to England to trade, in the spring of 1189, and returned in the autumn with a cargo of wine, honey, wheat, and cloth.[177] About this time (*c.* 1195), an Icelander named Marcus lost his wife, and he went abroad for materials to build a church. "After her death Marcus went away from the land, and in Norway he had good church-wood cut. He went south to Rome; and when he came from the south from Rome, he purchased good bells in England and took them with him to Norway. Afterwards he returned to Iceland with the church-wood and the bells." In Iceland he built a church and gave it the English bells.[178]

The relations of the clergy of Sweden and Denmark [179] to England

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[172] Vigá Styrs S., in Ísl. Sög., II, 307; Dipl. Isl., I, 481.

[173] Bisk. S., I, 124.

[174] Ibid., 357, 810–811.

[175] Ibid., 120, 321.

[176] Above, under "Pilgrims."

[177] Bisk. S., I, 433.

[178] Hrafns S. Sveinbjarnarsonar (in Sturlunga S., ed. Vigfússon, II, 280).

[179] Consult in general the church histories of Maurer, Helveg, and Jörgensen. In the reigns of Cnut the Great and his sons (1016–1042) the ties between England and Denmark must have been fairly intimate. King Erik (1095–1103), at the beginning of his reign, fetched monks from Evesham in England to Odense (J. B. Baugaard, Om de danske Klostre i Middelalderen, Copen., 1830, p. 284). About 1100 Aelnoth, an English priest of St. Albans in Odense, wrote a Latin Martyrology of the Danish St. Cnut (†1086) (H. Olrik, Aelnods Skrift om Knud d. Hellige, Hist. Tidssk., 1893, pp. 205–291; A. D. Jörgensen, Bidrag til Nordens Historie, Copen., 1871, p. 190). Saxo says that Anders Suneson, who became Archbishop of Lund in 1201, "searched through Gaul and Italy, and *Britain* also, in order to gather knowledge of letters and amass them abundantly" (preface to Historia Danica). In the twelfth century, however, the Norwegian church looked to France, whither her clerks went to study. In Paris, as early as 1147, there was a *Collegium Dacicum* (Bulaeus,

Matthew's fourth allusion to his trip occurs in an account of the trouble between pope and emperor. The pope, through his legate, offered Hákon the throne of Emperor Frederick, which Hákon refused, "and this the said king declared to me, Matthew, who wrote these pages, and attested it with a great oath."[169]

Matthew himself, then, accounts for only one visit to Norway, in 1248. The repeated hints of Matthew's friendship with Hákon, especially when the abbot of Holm, in 1247, told the pope "he is a most particular friend to our king," point to previous visits of Matthew to Norway. At least we can be certain that he helped the monks of Holm with their finances before 1248; that he went to Norway at their solicitation and the command of the pope, landing about June 10, 1248; that he bore letters from St. Louis to Hákon, who gave him rich gifts and discussed state secrets with him, and that he stayed in Norway long enough to reform the Benedictine order.[170]

Matthew's narrative gives color and detail to the stiff outlines which I have wrested from the Rolls. No other record shows in clearer light the relation of the Norwegian church to the English, — affection, respect, intimate acquaintance, —than the account which the monks of Holm gave the pope of the Benedictines in England, of St. Albans, and of Matthew Paris.

THE NORSE ISLES, DENMARK, AND SWEDEN.

A whole history could be written about the interests of the Norse clergy of Shetland, the Orkneys, Sodor and Man, in the church in England, and especially in Scotland.[171] Orkney remained nominally under the jurisdiction of Nidaros until *c.* 1475, and Sodor and Man until, in 1458, a papal bull made it subject to York.

What the church in Iceland owed to England was, in general, indirect and via Norway. We have seen how two or three of the English bishops whom Olaf "the saint" took to Norway, carried their work later to Iceland. At least one of them, Rudolph, returned to England, of France, sent by Matthew a letter to Hákon[165] inviting him to share the command of a crusade, and also a letter of protection in France. "When the king of Norway, who was a discreet, modest and learned man, read this letter, he was greatly delighted, and returned thanks to the bearer of it, besides rewarding him with rich and royal presents."[166]

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[169] Chron. Maj., V, 201 (Giles, II, 415).

[170] The next step traceable in his itinerary is Winchester, July, 1251 (see Preface, xx, of Rolls ed. of Hist. Min., vol. III).

[171] For example, St. Magnus, of Orkney, spent some time in England in the reign of Henry I, and was well known there as a saint after his death (Magnûs S.). The Bishops of Man were sometimes consecrated at York to save the voyage to Nidaros (see Keyser, I, 414 f.). With Furness the church of Man had intimate relations (Keyser, I, 414 f.). R. L. C., II, 175, contains a letter from Henry III to Olaf, King of Man, warning him not to interfere with the affairs of Furness Abbey, "que libera elemosina nostra est."

The third mention of Matthew's visit occurs in his account of a terrible fire in Bergen, followed, a day or two later, by a fearful thunderstorm. "A sudden flash of lightning struck a large ship which had arrived from England during the night, killing one man in it, wounding or severely bruising all the others, and, shivering the mast into small pieces, hurled it into the sea; all the ships, too, which were in the harbor, amounting to two hundred in number or more, were injured. The writer of this work had come in the ship whose mast was broken, but at the time of the occurrence he was performing mass in a church near the sea-coast, singing a nautical hymn to return thanks to God after escaping the perils of the sea. When the above-mentioned circumstances were made known to the king, he, out of his regard for the person who had been on board that ship, ordered a larger and better mast to be supplied to it."[167] Fortunately Hákon's Saga enables us to date within a day or two Matthew's arrival in Norway. It too describes the fire, which occurred "fourteen nights before St. John's eve," that is, June 9, and the thunderstorm which followed "a few days later." The saga apparently also describes the accident which happened to the very ship of Matthew Paris, for the lightning, Sturla says, "flew out afterwards into the voe and struck a mast on a ship which floated off the town, and dashed the mast asunder into such small chips that they could scarcely be seen anywhere. One bit of the mast did hurt a man who had got on board the ship from the town to buy finery; but there was no harm done to anyone else who was on board."[168] So Matthew arrived in Bergen about June 10, 1248, and came on a trading ship, or perhaps defrayed the expenses of the voyage by a little incidental bartering, as did Norwegian prelates who went to England.

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[165] That it was the same trip is stated explicitly in Matt. Par., Abbreviatio Chronicorum (Rolls), p. 304: "Et tunc temporis scripsit dominus rex Francorum dicto fratri Matheo in Norwegiam profecturo."

[166] Chron. Maj., IV, 650 ff. (Giles trans., II, 248 f.); Hist. Min., III, 304. The additamenta to the Chronica Majora give a list of hangings presented by Matthew to St. Albans. Among them is an aurifrigium "de dono domini regis Norwagiae Haconis" (p. 391).

[167] Chron. Maj., V, 36 (Giles, II, 278).

[168] Hákonar Saga, chap. 260 (Rolls trans.).

In the following year, 1248, Matthew Paris himself went to Norway on an important church mission. He gives a detailed account of the difficulties which led to his visit.[161] The monastery of St. Benedict of Holm, already in a bad way, was abandoned by the abbot, who got the house into debt and died. "The prior was then sent . . . with one of the brothers accompanying him, and with a sum of three hundred marks, and also bearing letters directed to brother Matthew Paris, begging him to use his diligent endeavors to free them from their debt, and in the end it was happily arranged that the said house should be released on payment of the debt only. After having obtained all writings and instruments by which the convent of Holm was held indebted to the Caursins, who were then at London, he returned safely within a year. But although they breathed freely in temporal matters, they were still languishing in a confused state in spiritual concerns."[162] So the Cardinal of Sabina, then in Norway, advised them to go to the pope for a suitable instructor to reform their order. The abbot and prior accordingly went to the pope, who asked them to choose their adviser, and on deliberation they replied: "Your holiness, we have learnt by experience that the monks of our order are not so well ordered anywhere throughout the whole world, as we believe, as in England; nor is there, as we hear from report, any house so well arranged in the kingdom of England as that of St. Alban, the protomartyr of the English. We therefore ask for a certain monk of that house, named Matthew, whose wisdom and fidelity we have had experience of, to inform and instruct us; besides, he is a most particular friend to our king, who will be able by his means, if he thinks necessary, to subdue any rebels against him." Accordingly, the pope wrote to the abbot asking him to send Matthew to Norway. "The abbot of St. Albans therefore obeyed the pope, as he justly ought; and the said monk obeyed his abbot, the business went on, and was arranged prosperously, so that the abbot of Holm in Norway continued in peace and prosperity, and the monastic order, which was exposed to such peril in that country, now, by the grace of God, recovered breath, as did also some other monasteries there."[163]

I know of no contemporary mention of Matthew's visit to Norway outside the reputed writings of Matthew himself.[164] In three other connections, however, Matthew alludes to his presence in Norway.

When he set out for Norway at the pope's request, Louis IX, king for consecration, as far as their itineraries are preserved, all travelled via England.[149] By this route also came Cardinal Breakspeare, bearing the pallium to the first archbishop (1152).[150] In 1230 Henry III allowed the Abbot de la Dale to depart to Norway "on business of the pope."[151] In 1231 the Cistercian Abbot of Stanley, in England, was appointed with two Norwegians on a papal commission.[152] In 1247 the Bishop of Sabina spent several months in England on his way to crown Hákon.[153]

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[161] Chron. Maj., V, 42 ff.

[162] Giles trans.

[163] Giles trans., II, 283 ff.

[164] Except the indirect confirmation in Hâkonar S. (cited below).

The visit of William, Cardinal Bishop of Sabina, to Norway in 1247, invested with all the powers of the pope, his coronation of Hákon and the attending festivities, constitute perhaps the most spectacular event in Norway in the thirteenth century. Sturla, the Icelandic historian, devotes chapter upon chapter of his Hákonar Saga to a glowing account,[154] and Matthew Paris, of St. Albans, the great Anglo-Latin historian, who was a personal friend of King Hákon, refers to it in several connections.[155]

In 1240 Hákon's rival, Duke Skuli, was overthrown and slain, and Hákon's rule became undisputed. He desired, however, church sanction and coronation.[156] Accordingly he opened negotiations with the pope,[157] culminating in 1245 with the embassage of Lawrence, the English Abbot of Hovedö. At his solicitation,[158] the pope replied that he was sending William, Cardinal of Sabina, to perform the ceremony. So "King Hacon sent ships west to England and to other lands . . . to gather those stores which seemed to him to be most lacking in Norway, to welcome the cardinal as he wished."[159] About this time, according to Matthew Paris, the cardinal arrived in England on his way to Norway. He assured the English, who thought he had come to rob them, that he wished merely to proceed from Dover to Lynn. At Lynn, however, he stayed three months, secretly enriching himself, and departed in a veritable Noah's Ark, laden with all the good things of England.[160]

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[149] Above, under "Nidaros."

[150] Saxo Grammaticus (Müller's ed.), p. 697.

[151] Close Rolls, 1227–1231, p. 358.

[152] Dipl. Norv., I, 10.

[153] Below.

[154] 246 ff.

[155] Rolls Series, Chron. Maj., IV, 612, 626, 650; V, 195, 201, 222, 230; Hist. Min., III, 23, 31, 95; Abbrev., pp. 300, 304.

[156] He was an illegitimate son, and, as such, according to the church agreement of 1164, had no real title to the crown.

[157] Hákonar Saga, chaps. 246 ff.

[158] Chron. Maj., V, 222.

[159] Hákonar Saga, chap. 248 (Dasent trans. in Rolls Series).

[160] Chron. Maj., IV, 626.

the Martyr."[140] As late as 1309 one of the canons of Bergen Cathedral was setting out to perform his vow to "Saint Thomas in England."[141, 142]

Clerks and laymen also came through England on their way to the Holy Land. We have seen how Sigurd and his host spent a winter in England as guests of Henry I. In 1215–1216 King Inge sent ships crusading, and in 1217 other Norwegians joined the fleet which assembled off the Netherlands and touched at Dartmouth on the way to Acre.[143] One crusader of this year — called in the saga "Hroar, the king's kinsman," and in the English Rolls "Roherus, relative of the King of Norway" — secured safe-conduct in the name of Henry III while waiting over in England. Presumably he spent the winter there.[144] In the thirties, Duke Skuli was intending to pass through England, for (July 29, 1233) Henry III issued letters of "safe-conduct for Sverri, Duke of Norway, going on pilgrimage to the land of Jerusalem, until his return;"[145] and again (June 22, 1235) "safe-conduct until Michaelmas, 20 Henry III, for the Duke of Norway passing through England on pilgrimage to the Holy Land.[146]

Papal Legates and messengers passing between Norway and the pope, sometimes tarried weeks and months in England. Before 1290 there were two principal routes from Norway to Rome, — one through Germany,[147] which was often impracticable, the other via England and France.[148] The archbishops of Nidaros who went south before 1290

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[140] Rymer, I, 205; Pat. Rolls, 1225–1232, p. 485.

[141] Dipl. Norv., VI, No. 72.

[142] I suspect that some royal letters to the sheriffs of Canterbury concern pilgrims. In 1155 the sheriff of Canterbury paid 3*s*. to "envoys of the king of Norway" (Great Rolls of the Pipe, 1155–1158, p. 15); in 1223 Henry III ordered the sheriff of Canterbury to pay 20*s*. etc., to Norwegian envoys (R. L. C., I, 562 a).

[143] Munch, III, 569, 594.

[144] Hákonar Saga, chap. 30; Pat. Rolls, 1216–1225, p. 103.

[145] Pat. Rolls, 1232–1247, p. 21.

[146] Rymer, I, 218; Pat. Rolls, 1232–1247, p. 109.

[147] Forty-six days from Aalvorg in Denmark to Rome, according to the Icelandic Itinerary of Abbot Nicholas (*c.* 1194) (Werlauff's Symbolae ad Geogr. Medii Aevi, pp. 15–22). This route involved Danish jealousies, Saxon robbers, and the passion of German princes for locking up strangers found in their woods. Some Norwegians, in 1251, learned this to their sorrow (Hákonar Saga, chap. 275).

[148] I have yet to find a ship between 1150 and 1350 which went direct from Norway to France, or vice versa, without stopping in England. The traveller from France sailed to one of the Cinque Ports (e. g., Rouen to Dover), and travelled overland to some eastern port like Lynn, which communicated with Norway (e. g., Cardinal of Sabina, below).

1309, we learn, by chance from a Bergen church letter, that one of the canons was at that time in England for study.[133]

Pilgrimage also was a link between England and Norway. If we may believe the legendary St. Olaf's Saga, Englishmen visited the shrine of St. Olaf at Trondhjem.[134] Certainly there were so many foreign pilgrims that, in 1297, King Eric issued orders to all officers of the realm to protect foreigners who came as pilgrims to Olaf's shrine.[135]

The death of Thomas à Becket made a profound impression in Norway and Iceland, and is frequently alluded to in the sagas. In Iceland the legendary history of his life was translated, soon after his canonization, into the so-called Thomas Archbishop's Saga. It was widely popular in Iceland and Norway, to judge from the large number of extant manuscripts. One of the earliest representations (about 1220) of the murder of St. Thomas is a little brass shrine, once used as a reliquary, and still preserved in the church of Hedal in Valders.[136] Becket's shrine brought Norwegian pilgrims to Canterbury. The Saga of Hrafn Sveinbjarnarson, the Icelander (1190–1213), tells an amusing tale of how he was fishing and caught a narwhale which he could not land, and promised the narwhale's tusks to St. Thomas if he would help. His prayer was answered. Hrafn went to Norway and stayed there through the winter.[137] In the spring, true to his vow, he voyaged to Canterbury and deposited the tusks on Becket's shrine. In 1225, according to Hákonar Saga, John Steel was met by King Hákon, sailing home from England, where "he had gone for a vigil to Saint Thomas the Archbishop."[138] In 1229 the bailiffs of Ipswich were ordered to allow a Norwegian ship held there to go freely, and the passengers who came to England on a pilgrimage freely to perform their vow.[139] In 1332 Duke Skuli was given letters of safe-conduct from June 25 until Easter of the following year, "and those whom he shall bring with him into England to visit as a pilgrimage the shrine of Blessed Thomas

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[133] Dipl. Norv., VI, No. 72..

[134] Heilagra Manna Sögur, II, 182 (miracle of an English knight who obtained relief at Nidaros after other European shrines had failed).

[135] Norges Gamle Löve, II, 31.

[136] T. B. Willson, History of the Church and State in Norway, Westminster, 1903, p. 246, note and photograph. A church in Norway dedicated to St. Thomas of Canterbury was destroyed in 1808.

[137] Hrafns S., chap. 4, printed in Sturlunga S., II, 277.

[138] Hákonar Saga, chap. 130.

[139] Close Rolls, 1227–1231, p. 216: "Permittentes similiter homines ejusdem navis, qui causa peregrinationis venerunt in terram regis, libere et sine inpedimento exequi votum suum."

Students, Pilgrims, Papal Messengers.

Norwegian clerks are named in the English Rolls because they figure as merchants or diplomats; church business and private affairs demanded no royal writ. So we must assume that these traders and envoys often had ulterior ends. For instance, John Steel, a Norwegian noble, in 1225 secured a license to come to England as a merchant,[128] while, according to the saga, he went on a pilgrimage to Canterbury, and had dealings with the newly elected Archbishop of Nidaros and other Norwegian priests in England.[129] A prelate who commanded his own ship naturally defrayed expenses by taking a load of fish to Lynn or Yarmouth, to be replaced in wheat, wine, or cloth. At the same time the king entrusted him with a despatch. Accordingly, his name is recorded in the Rolls, but not his church mission, — and this in addition to the great silent majority to whose number we have no index.

Had we no evidence, it would still be safe to assume that Norwegians came to England for *study*. Bishop Sigurd learned the Cistercian rules at Fountains; Archbishop Eystein may have done some reading in his nine months at Bury.[130] The Rolls naturally are silent upon Norwegian students; what little confirmation we find must be from Scandinavian sources. About 1160 Thorlak, an Icelander who became Bishop of Skalholt, studied at Lincoln. He went abroad, says the saga, and "came to Paris, and was there at school as long as he thought needful to get the knowledge which he wished to get there. Thence he came to England, and was at Lincoln, and there he gat, moreover, great knowledge, and fraught with blessings both to himself and others."[131] The next bishop of Skalholt, Paul (d. 1211), a nephew of Thorlak, likewise studied in England in his youth. "He went south to England, and was there at school, and got great learning there, so that there was scarce any example of any man's having got so deep and so much knowledge in the like time. And so when he came back to Iceland, he surpassed all other men in his courtliness and his learning, and in making of verse, and in book-lore."[132] These two accounts show the respect in which English schools were held in the North. Again, in

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[128] "Johannes Stel, mercator de Norwegia," Pat. Rolls, 1216–1225, p. 542.

[129] Hákonar Saga, chap. 130.

[130] Above, p. 536.

[131] Bisk. Sögur, I, 92, Thorláks Saga, chap. 4 (Powell and Vigfusson trans., in Orig. Islan.).

[132] Póls Saga, chap. 1 (Powell and Vigfusson trans., in Orig. Islan.).

"frater Benedictus canonicus et Radulfus clericus" in 1228;[116] "friar Ivoer of the order of Minors" in 1297.[117]

English clerics, in their turn, served as diplomats in Norway, sometimes as servants of the Norwegian as well as the English crown. In 1234 Henry III ordered the bailiffs of Lynn to permit "Richard of St. Albans, envoy of the King of Norway," to have one of four ships detained in their port on account of a contention between subjects of the King of England and those of the King of Norway to return to Norway in order to treat with the king about the difficulty.[118] Again, four years later, this same "Richard of St. Albans, envoy of the King of Norway," was given by letter of Henry III protection without term.[119] Richard seems to have been on the same confidential footing with Hákon as his brother monk at St. Albans, Matthew Paris. In 1247 Norwegian monks told the pope that Matthew of St. Albans was "a most particular friend to our king,"[120] and in the following year he bore letters even from the king of France, St. Louis, to king Hákon,[121] who gave him rich presents[122] and confided state secrets to him.[123]

Disputes over the Hebrides were occasions for sending church dignitaries from England and Scotland to Norway. In 1244, for instance, Alexander II sent two bishops.[124] About 1290, when Margaret, "the Maid of Norway," was coming to rule Scotland, the clergy played important rôles.[125]

English clerks were employed also as secretaries to the Norwegian crown. In Sverri's time the chaplain occupied much the position of chancellor, and Sverri's chaplain, Martin, was an Englishman.[126] In 1293 one Geoffrey, formerly a clerk in Yorkshire, brought letters to King Edward, one from Duke Hákon, another from King Eric, his brother, highly recommending the bearer to Edward. Geoffrey had long served King Eric and Duke Hákon in the capacity of secretary.[127]

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[116] Close Rolls, 1227–1231, p. 80.

[117] Pat. Rolls, 1292–1301, p. 255.

[118] Close Rolls, 1231–1234, p. 532, "Quod permittant Ricardum de Sancto Albano, nuntium regis Norwegie."

[119] Rymer, I, 236; Pat. Rolls, 1232–1247, p. 226.

[120] Matt. Paris, Chron. Maj. (Rolls), V, 44.

[121] Chron. Maj., IV, 650 f.; Hist. Min., III, 304.

[122] Chron. Maj., Addit., VI, 391.

[123] Chron. Maj., V, 201.

[124] Hákonar Saga, chap. 245.

[125] Pat. Rolls, 1281–1292, p. 350.

[126] Sverris Saga (ed. Unger), chap. 119. Cf. above, under "Bergen."

[127] Rymer, I, 787, 788.

In another letter he gave Askeldus himself his protection for three years.[106]

In 1269 Chancellor Askatin became Bishop of Bergen.[107] In that year he helped draw up at Winchester a trade treaty between England and Norway.[108] In previous years, also, he had been sent as an envoy to England and Scotland.[109]

In 1309 one of the canons of Bergen Cathedral was studying in England, and another was just setting out for the shrine of Becket.[110] In 1322 Bishop Audfin was sending a ship and two representatives to England "on affairs of our court and our own."[111] But the good old days of English affiliations were over. In 1338 Bishop Hákon wrote out to Iceland, to his friend Bishop John of Skalholt, bewailing the fact that wine no longer came from Flanders and England, but from Germany only.[112]

Envoys of State.

"The King's Mirror," a book of courtesy and instruction, written in Norwegian, apparently at the court of Hákon Hákonarson (1217–1263), shows us that church dignitaries were much in demand as ambassadors. "And if the king orders a clerk or an abbot or a bishop of his realm on an embassy to foreign kings or to the pope, if the king insists, he who is called is obliged to go, unless he wishes to incur the king's displeasure and be driven from his realm."[113]

We have noted the state errand of the Abbot of Lyse to England in 1217, of Richard the Cistercian in 1218 and 1223, of Archdeacon Andrew of Bergen in 1223, of Archdeacon Askeldus of Bergen in 1225, of Prior Andrew of Lyse in 1229, of Lawrence in 1233, of Canon Adam of Stavanger in 1264, of Askatin in 1265 and 1269, of Abbot Richard of Lyse in 1280, and of Canon Hugh of Stavanger in 1299 and 1309. In 1215 "the nephew of the King of Norway" brought his chaplain.[114] Other priestly ambassadors were Skuli's chaplain, John, in 1222,[115]

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[106] August 31, Pat. Rolls, 1216–1225, p. 548.

[107] For Askatin's career see Hákonar S., chaps. 86, 275, 305, 319; Munch (index); Lange, pp. 117, 404–405.

[108] Rymer, I, 480.

[109] Magnûs Saga Hákonarsonar, chap. 4.

[110] Dipl. Nor., VI, No. 72.

[111] Ibid., IV, No. 153.

[112] Ibid., VII, No. 155.

[113] Translated from Speculum Regale, Christiania ed., p. 62.

[114] R. L. C., I, 231 a.

[115] Ibid., I, 508 b.

Henry III a letter from King Magnus, announcing the death of Hákon Hákonarson.[92] In 1299 Master Hugh, Canon of Stavanger, served as envoy for King Eric in England, and secured letters of safe-conduct from King Edward, on his return.[93] About 1303 the Bishop of Stavanger had goods on ships coming to England.[94] In 1309 Canon Hugh came again to renew old treaties between England and Norway.[95]

HAMAR. — The inland bishopric of Hamar probably sent fewer men to England. In 1265 Bishop Gilbert went as peace commissioner to Scotland, via England, with Chancellor Askatin.[96]

BERGEN. — The Bishop and Chapter of Bergen, like the monks of Lyse, were close to the Norwegian Court, and close to England.

Bishop Sigurd of Bergen, while visiting in England, arranged to found, in 1146, a Cistercian abbey at home.[97]

At least one bishop was an Englishman. In 1194 King Sverri had his chaplain, Martin, consecrated Bishop of Bergen. This man, the saga says, was "English in all his kin."[98] Martin remained bishop until his death in 1216.[99] In 1208 King John of England gave him letters of protection, for self, property, and men.[100] We need not suppose this was his only visit to England.

An Archdeacon of Bergen, Andrew, went to England as royal envoy in 1223,[101] apparently spending the winter there.[102] In the autumn of the next year the English regents sent by Andrew a gift of corn and malt for King Hákon.[103] In the following year, 1225, another Archdeacon of Bergen, Askeldus, served as diplomat,[104] and performed his mission so successfully that "Henry" wrote the bailiffs of Lynn to receive in a friendly way all subjects and merchants of "his friend," the King of Norway, and allow them free export for three years.[105]

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in 1214 and 1222 ordered the Bishop of Winchester to send presents to Norway (R. L. C., I, 168 a, 508 b).

[92] Calendar of Documents Relating to Scotland, I, No. 2355.

[93] Pat. Rolls, 1292–1301, p. 420.

[94] A. Bugge, Byers Selvstyre, pp. 135 ff., 200 ff.

[95] Rymer, II, 81; Close Rolls, 1307–1313, p. 224.

[96] Magnús Saga Hákonarsonar, chap. 4.

[97] See above, under "Lyse."

[98] Sverris Saga (ed. Unger), chap. 119.

[99] Cf. Keyser, I, 291, 302, 304 f., 314, 327, 331, 337.

[100] R. L. P., I, i, 85 b.

[101] Royal Letters of Henry Third, I, 216–217.

[102] R. L. C., I, 584 a.

[103] Ibid., I, 622 b.

[104] Letter of August 30, R. L. C., II, 60 a.

[105] Another letter of August 30, ibid.; also August 31, Pat. Rolls, 1216–1225, p. 548.

carefully insisted by the English crown that the Rolls preserve at least five letters to the sheriff of Norfolk regarding their conviction.[80] As late as 1303 clerks from Nidaros traded at the Lynn market,[81] and in 1316, after the commercial rupture,[82] the men of Elanus, Archbishop of Nidaros, obtained royal leave to trade in England for one year.[83]

Archbishops of Nidaros came to England in person, and on business other than trade. Eystein did not spend his three years peddling dried fish or filling his hold with corn. In all probability he studied ecclesiastical institutions, engaged in church politics for the advantage of his see, and secured English clerks to accompany him to Norway, and English artisans and materials for the construction of his cathedral. Again, England, until 1290, seems to have been the favorite route to Rome,[84] and every archbishop had to go to the pope to receive his pallium. In the thirteenth century ten archbishops were consecrated.[85] Archbishop Guttorm chose the English route in 1215, securing from King John a safe-conduct for himself and his men.[86] Peter of Housesteads, the next archbishop, returned via England, and tarried there during the summer of 1225.[87]

Oslo. — Bishop Nicholas of Oslo sent, in 1213, an envoy with presents of hawks and gerfalcons to King John,[88] who in return sent several casks of wine to the bishop.[89]

About 1303 the Bishop of Oslo was exporting to England.[90]

Stavanger. — The first bishop of Stavanger was an Englishman, and the cathedral was dedicated to St. Swithun, patron of Winchester in England.[91] In 1264 Master Adam, Canon of Stavanger, brought to was Lawrence who brought the mission to a successful issue. He was an Englishman by birth, and later returned to England, and became abbot of Hovedö's mother abbey, Kirksted.[70] In this case, certainly, an English abbey continued intimate relations with her offspring in Norway for at least one hundred years.

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[80] R. L. C., II, 156 b, 158 b, 162 b, 167 b, 174 a.

[81] A. Bugge, Byers Selvstyre, pp. 135 ff., 200 ff.

[82] In 1312. Cf. A. Bugge, Handelen, pp. 68 ff.

[83] Rymer, II, 285.

[84] See below, under "Papal Messengers."

[85] I do not know of a single archbishop before 1290 of whom it can be shown that he did *not* go via England.

[86] R. L. P., p. 180 a, dated May 12. Archbishop Thorer died August 8, 1214. Munch (III, 558, 567; so Keyser, I, 336), who does not know this letter, shows that Guttorm probably attended the council at Rome, November 11–30, 1215. In that case, this letter was secured by Guttorm on his way to Rome, and made to read so that he could use it on his return, "in eundo per totam potestatem domini Regis et transfretando ad partes suas." There is no time limit.

[87] Pat. Rolls, 1216–1225, p. 542; Hákonar Saga, chaps. 100, 130.

[88] R. L. C., I, 156 b.

[89] Ibid., I, 138 b.

[90] A. Bugge, Byers Selvstyre, pp. 135 ff., 200 ff.

[91] See above, p. 534. Munch (II, 615 ff.) argues for a connection between Stavanger and Winchester; the first bishop, he believes, was a member of the Winchester chapter-house. In this connection it is of interest that the crown

The Five Norwegian Bishops and Cathedral Chapters. Their Interests in England.

The archbishops of Nidaros (Trondhjem) and the bishops of the four dioceses of Oslo, Stavanger, Hamar, and Bergen kept in frequent contact with England, either in person or through their cathedral chapters.

The archbishops of NIDAROS enjoyed extraordinary trade rights in England. Henry II, Richard I, John, and Henry III each gave a license to the church of Nidaros, the archbishop and his successors, every year, whether fertile or not, to load one ship in England with corn and provisions, without challenge or exaction, and to take it to Norway to the church.[71] This privilege was perhaps first gained by Archbishop Eystein, during his visit in England (1180–1183). It was renewed in 1203,[72] 1222,[73] and 1241.[74] Ships belonging to the Archbishop of Nidaros are mentioned in the English Rolls in 1223,[75] 1225,[76] 1226,[77] 1233,[78] and 1236,[79] — presumably in addition to the "one a year" allowed by the license. The punishment of Englishmen who, in 1226, robbed a ship at Hull belonging to the Archbishop of Nidaros, was so ings of the same in 1280 to Edward,[60] who highly recommended the abbot in a letter to King Eric.[61] The following year Edward gave a safe-conduct to "Richard of Norway," whom he was sending to Norway on his affairs.[62]

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[70] Matt. Paris, Chronica Majora (Rolls Series), V, 222: "Per manum domini Laurentii, abbatis postea de Kirkestude in Lindeseia, qui totum illud negotium Romam pergens effectui mancipavit, Anglicus natione et ordinem professus Cisterciensem."

[71] "Rex justiciario, vicecomitibus, et omnibus baillivis suis Anglie et portuum maris, salutem. Sciatis nos, pro amore Dei et ad peticionem G. Niderosiensis archiepiscopi, concessisse Niderosiensi ecclesie et ipsi G. archiepiscopo, et successoribus suis, ut singulis annis usque ad etatem nostram, sive fuerit tempus fertilitatis vel non fuerit, unam navem faciant honerari blado et victualibus in Anglia sine omni occasione et exactione et duci in Norwegiam ad ecclesiam suam, et prohibemus ne inde disturbentur." Pat. Rolls, 1216–1225, p. 338.

[72] Rotuli Chartarum, p. 110 b.

[73] Pat. Rolls, 1216–1225, p. 338

[74] Ibid., 1232–1247, p. 259.

[75] R. L. C., I, 559 a.

[76] Pat. Rolls, 1216–1225, p. 542.

[77] R. L. C., II, 139 a.

[78] Two ships, Close Rolls, 1231–1234, p. 242.

[79] Pat. Rolls, 1232–1247, p. 144.

The following century furnishes only one record, — and that of a deed of violence. In 1336 or 1337 Abbot Arne of Lyse was seized off England by pirates, and beheaded with all his crew.[63]

The frequent employment of abbots and priors of Lyse, in the thirteenth century, as ambassadors between England and Norway, may be explained by the probability that Lyse kept in close contact with Fountains, and continued recruiting from England. Monks of English birth, who knew the languages and life of both countries, would be much in demand as diplomats.

Hovedö.—Soon after the foundation of Lyse, English monks from Kirksted Abbey in Lincolnshire founded the second Cistercian monastery in Norway, St. Mary's of Hovedö (*Caput Insula*), in the diocese of Oslo (Christiania).[64] Hovedö, like Lyse, traded in England, though fewer records remain.[65] In 1224 a ship belonging to the Abbot of Hovedö was allowed to embark from Lynn.[66] In 1237 Henry III wrote the Governor of Norwich to exempt all the goods belonging to the Abbot of Hovedö, on board his ship, which had been detained, but to sell all other goods in it and six other Norwegian ships, to settle the King of Norway's debt to an English merchant.[67] This, indeed, shows marked discrimination toward Hovedö on the part of the English crown. About this time the abbot was an Englishman, one Lawrence. In 1233 a Lawrence, probably the same, appears in the Rolls, when King Henry orders forty shillings to be given to "Brother Lawrence, a messenger from the King of Norway" for his expenses.[68] In 1246 Hákon Hákonarson sent the Abbot of Hovedö, with a canon of Nidaros, to the pope to arrange for his coronation.[69] According to Matthew Paris, it traders, sometimes as state envoys, their names appear in the English Rolls.

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[60] Rymer, I, 577 (two letters), 579.

[61] Ibid., I, 587.

[62] Pat. Rolls, 1272–1281, p. 456.

[63] Icelandic Annals (Copen., 1847; Christ., 1888), A. D., 1336 and 1337.

[64] Langebek, Scriptores Rerum Danicarum, IV, 417.

[65] Kirksted, too, may have sent ships to Hovedö. In 1224 the bailiffs of Lynn were ordered to allow the Abbot of Kirksted to export wool "to foreign parts" (R. L. C., I, 609 b, 634 a).

[66] R. L. C., I, 606 b.

[67] Not yet printed, but a Norwegian summary is in Regesta Norvegica, No. 452.

[68] Issues of the Exchequer, ed. Devon, I, 513.

[69] Diplomatarium Norvegicum, I, No. 30 (Potthast, No. 12330).

King John, in 1212, ordered the bailiffs of all ports to allow a ship of the Abbot of Lyse to export from England duty-free.[48] In 1217 the Abbot of Lyse concluded a treaty of trade and friendship between England and Norway, and remained in England some time after Henry III (or the regency) sent favorable answers to King Hákon and Earl Skuli.[49] Richard, a "Cistercian monk," spent the winter of 1218–1219 in London as Hákon's ambassador, receiving presents of money and clothing by order of Henry III, November 8[50] and February 1.[51] On November 9, 1218, Henry ordered the bailiffs of Yarmouth to protect the monks and men of Lyse Abbey, "according to the letters of King John."[52] Richard was serving again in 1221.[53] In 1223 a ship of the Abbot of Lyse secured two years' leave to export free from any English port.[54] In 1225 the king ordered the bailiffs of Lynn, "despite the export prohibition," to allow Brother William "de Luse in Norwegia" to buy in Lynn fifty quarters of corn to take home.[55] In 1229 Henry III ordered £20 for a present to be sent King Hákon by Prior Andrew of Lyse, *nuncius* of that king;[56] and late in the year he requested the bailiffs of Yarmouth to deliver to the same prior a ship detained in their port which had brought new year's presents from Hákon to Henry, so that the prior might return home in her.[57] In 1233 the sheriff of Norfolk was directed to release two ships detained at Lynn, to Brother Ernisius, Cellarer of Lyse, and Brother Nicholas, "canon of Teseberia in Norway," provided they could prove ownership.[58] About 1275 one Richard was Abbot of Lyse.[59] He served Edward I on intimate state business, securing the arrest in Norway of a man supposed to be the fugitive Guy de Montfort, and brought tid-

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[48] Rotuli Litterarum Patentium, p. 95, col. a.

[49] Rymer's Foedera, 1816 ed., I, 149; Rotuli Litterarum Clausarum, I, 336 b.

[50] Rot. Litt. Claus., I, 382 a (two letters).

[51] Ibid., I, 387 a.

[52] Ibid., I, 382 a.

[53] January 23, the king ordered clothing for him (R. L. C., I, 446 a); April 23, money for journey home (Ibid., 454 b).

[54] Patent Rolls, 1216–1225, p. 384.

[55] R. L. C., II, 61 a.

[56] Close Rolls, 1227–1231, pp. 218, 219.

[57] *Calendar of Documents relating to Scotland*, I, No. 1058; Close Rolls, 1227–1231, p. 277.

[58] Close Rolls, 1231–1234, p. 247.

[59] Lange, p. 350; Munch, IV, 2, 86

The impression Eystein made upon Englishmen is expressed by William of Newburgh in the words "vir magnus."[45]

THE THIRTEENTH CENTURY.

The records of clerical visits between England and Norway accelerate considerably during the century after Eystein, especially in the reign of Norway's great patron of culture, Hákon Hákonarson (1217–1263). We can best group these records under the various forms of church and secular business which drew clerks from one country to the other, such as the interests of related abbeys, trade, embassage, pilgrimage, study.

Related Foundations.

Various churches and monasteries in Norway were dedicated to English saints, such as St. Edmund, St. Alban, and St. Swithun. Doubtless many were connected with parent foundations in England. There is certain evidence for two abbeys, Lyse and Hovedö.

LYSE. — In 1146 English monks from Fountains founded the oldest Cistercian monastery in Norway, St. Mary's at Lyse, south of Bergen (*Coenobium Vallis Lucidae*). The account is preserved in the Memorials of Fountains.[46]

Bishop Sigurd of Bergen, during a stay in England, learned at Fountains Abbey the rules of the Cistercian order, and determined to establish an abbey at home. Abbot Henry of Fountains sent with him to Norway a convent of his own monks, among them Runulf or Ranulf, under whose direction Lyse was established. Ranulf was first abbot, serving until, "released at last from his charge by the Abbot of Fountains, he returned to his own, full of days."

For sixty-seven years the abbey remained under the immediate direction of Fountains; in 1213 the Abbot of Alvestro in Sweden became supervisor.[47]

Even after this date the monastery probably continued connections with the English mother abbey. Certainly its abbot and monks came often to England, where they enjoyed special privileges. Sometimes as allowed Eystein ceased. At that time, then, we may suppose he left the abbey. About that date, "one year and three months having elapsed since the death of Abbot Hugh (November 15, 1180), the king commanded by his letters that one prior and twelve of the convent, in whose mouth the judgment of our body might agree, should appear on a certain day before him to make choice of an abbot."[39] Two days later the thirteen set forth. Now, Eystein is not mentioned by Jocelin as a member of the cavalcade, but Jocelin does say that Eystein "was of considerable assistance in obtaining for us our free election, bearing witness of what was well, and publicly declaring before the king what he had seen and heard."[40] It seems, then, likely that Eystein left the abbey on the 15th, after the receipt of letters from the king, and proceeded to Waltham, where he interceded with Henry on behalf of the abbey-convent. Partly as the result of Eystein's intercession, Henry, instead of appointing an objectionable stranger, gave to the delegates their free choice of Samson, the sub-sacrist, for their new abbot.

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[45] Note 25, (3).

[46] Printed in (1) Dugdale's Monasticon Anglicanum (new ed., 1817–1830), V, 301; (2) Langebek's Scriptores Rerum Danicarum, Copen., 1776, IV, 406 ff.; (3) Memorials of Fountains Abbey, I, Surtees Soc., No. 42, p. 89.

[47] See a general order of the Cistercians, Martene, Thesaurus Nov. Anecdot., vol. IV, col. 1313: "Quoniam abbas de de Fontanis in Anglia abbatiam de Lysa in Norvegia secundum formam ordinis nostri competenter non potest visitare, eadem domus de Lysa domui de Alvestro committitur in filiam."

Where Eystein stayed during the remainder of his English visit, a year and four months, we have no inkling. King Henry did not delay long in England. The day after Samson's election he made his will, and on March 10–11 embarked again for France, not returning until June, 1184, a year after Eystein reached Norway. While he was with the king, it is probable that Eystein gained that privilege from Henry II for the Archbishop of Nidaros to export each year from England a shipload of grain free of duty, a license which was renewed by Richard, John, and Henry III.[41] At any rate, Eystein influenced English ideas of Norwegian politics. Although King John in 1201 sent troops to aid Sverri,[42] the chronicler William of Newburgh heaps abuse upon Eystein's enemy — "*sacro ordine abjurato*," "*caput execrabile*."[43]

Early in the summer of 1183, then, according to Sverris Saga, Eystein returned to Norway, made his peace with Sverri, and retired north to his see at Trondhjem. The Icelandic annals barely record that "Eystein the archbishop came to Norway from England."[44] After his return he lived in retirement from politics until his death in 1188. His last years were spent in revising the old laws of the land. He also began the cathedral of Trondhjem, probably on Anglo-Norman models, — not completed for fifty years.

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[39] Clarke trans., p. 24.

[40] Note 25 (4) (trans. Clarke, p. 23).

[41] See below, p. 543.

[42] Sverris Saga (ed. Unger), chap. 194; Rotulus Cancellarii, p. 322.

[43] Note 25 (3).

[44] Note 25 (7).

death the humbled king was likely to be very gracious to archbishops. Probably the Archbishop of Norway, also, after paying his devotions at Canterbury, met Henry on his arrival. The strong resemblance between Becket's troubles and Eystein's present situation must have affected Henry. At that time the abbot's house at St. Edmunds was vacant, Abbot Hugh having died on November 15th of the preceding year. The king had taken over the government of the abbey, which was in a bad state financially, until the new abbot should be appointed.[34] We may suppose the king thought the abbot's house a good place to lodge the nation's guest. At any rate, on August 9th, twelve days after Henry landed, Eystein took up his residence in the vacant mansion, receiving ten shillings a day by Henry's order.

The house itself, we may gather from Jocelin, was ill-furnished. Before the last abbot was dead, "everything was snatched away by his servants, so that nothing at all remained in the abbot's house except the stools and the tables, which could not be carried away. There was hardly left for the abbot his coverlet, two quilts, old and torn, which some, who had taken away the good ones, had placed in their stead." [35]

A very pretty story might be written about the Norse archbishop's stay at Edmundsbury. For Jocelin mentions Eystein in the same breath in which he chats about the gossip of the monks during the vacancy.

Carlyle's imagination [36] would reconstruct Eystein's life at Old Bury, how he talked with the prior over a bottle of wine about the latter's prospects for election to the abbacy; how he nodded in passing to "Bozzy" Jocelin or Samson the sub-sacrist; how he spent long hours in the abbey library, and weeks at his own desk writing his Miracles of St. Olaf, of which a copy was for centuries preserved at Fountains.[37]

Certainly Carlyle is correct in saying, "At Waltham, 'on the second Sunday of Quadragesima,' *which Dryasdust declares to mean the 22d day of February, year 1182*, thirteen St. Edmundsbury Monks are, at last, seen processioning towards the Winchester Manor-house; and in some high Presence-chamber and Hall of State, get access to Henry II, in all his glory." [38] Just two weeks earlier (December 14) the corrodies

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[34] Jocelin, chaps. i and ii (cf. trans. by E. Clarke, London, 1903, pp. 262, 263).

[35] Clarke's trans., pp. 10–11.

[36] Cf. Past and Present.

[37] See Metcalfe, Passio Olaui.

[38] Past and Present, Book ii, chap. viii; cf. Jocelin, trans. Clarke, pp. 31, 263.

Eystein "by letter of the king" amounted to £94 10*s.* and covered 189 days, from the 9th of August, 1181, to the 14th of February, 1182. By computation it will readily be seen that the daily allotment amounted exactly to the ten shillings mentioned by Jocelin.[26]

In view of existing evidence we may safely construct Eystein's itinerary somewhat as follows.

In Sverris Saga we hear of Eystein in the spring of 1180 as sailing north with Magnus to Trondhjem.[27] The saga does not mention him again until 1183, when it relates that "Archbishop Eystein had arrived from England early in the summer, having been there for three years, absent from his see. He now made peace with King Sverri and sailed north in the summer."[28] In 1180, then, Eystein went to England. The English chroniclers, Roger of Hoveden[29] and Benedict of Peterborough,[30] relate that in that year, "unwilling to subject himself to Sverri the Priest," he left his see, came to England, and excommunicated Sverri. William of Newburgh also asserts that Sverri, "having abjured the sacred order, and taken in marriage the daughter of the Gaut-king, wished to be solemnly crowned by the archbishop. But he, since he was a great man and not to be induced by prayers or threats to pour sacred ointment on an execrable head, was driven by Sverri from his fatherland."[31] Hence we infer that Eystein left Norway after the Battle of Íluvellir, and arrived in England early in the summer, "breathing anathemas" upon Sverri.

Where did Eystein go when he reached England? Probably he visited friends among the prelates; possibly he crossed the Channel to seek King Henry II in Normandy, whither he had sailed on April 15th.[32] Henry did not return to England until July 28th, 1181, when he landed at Portsmouth. He then moved about England for seven months, devoting much of his time to bishoprics and abbeys and church appointments. On September 12th another foreign prelate, the Archbishop of Rheims, who had visited Becket's shrine early in the month, found the king at Winchester.[33] In those years after Becket's of Trondhjem, with the same kindness with which your grace has been wont to receive us and ours."[24]

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[26] The same amount per day the wardens paid for Abbot Hugh's expenses during the last six weeks of his life, — £21 (Chronica Jocel., Camden Soc., 1840, pp. 109–110).

[27] Chap. 44.

[28] See note 25 (6); trans. J. Sephton, p. 99.

[29] Note 25 (1).

[30] Note 25 (2).

[31] Note 25 (3).

[32] R. W. Eyton, Court, Household, and Itinerary of K. Henry II, London, 1878, p. 231.

[33] Ibid., p. 243.

During the three years spent by Archbishop Eystein in England,[25] we can follow him only nine months, which he spent in the abbot's house of Bury St. Edmunds, just before the election of Abbot Samson. "While the abbey was vacant," says Jocelin de Brakelond, "Augustine, Archbishop of Norway, tarried with us, residing in the abbot's house, and received by command of the king ten shillings each day from the funds of the abbey." Jocelin is corroborated in the accounts rendered by the wardens of the abbey to the king, who took over the abbot's revenues during the vacancy. According to these, the corrodies allowed to conciliate the Norse, the Englishman highest in the church. "There never came a foreigner to Norway," says Snorri Sturlason, "whom all men respected so highly, or who could govern the people so well as he did. After some time he returned to the South with many friendly presents, and declared ever afterwards that he was the greatest friend of the people of Norway." [19, 20]

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[24] Epistolae Sancti Thomae, ed. J. A. Giles, Oxon., 1845, I, 301; Migne, CXC, 612–614.

[25] The sources for the residence of Eystein in England are as follows: (1) Roger of Hoveden (Rolls Series), II, 214–215 (for 1180 A. D.): "Et Augustinus Nidrosiensis archiepiscopus, nolens aliquam facere subjectionem Swerre presbytero, archiepiscopatum suum reliquit, et venit in Angliam, et excommunicavit Swerre presbyterum. Est autem sciendum quod iste Magnus rex primus fuit rex coronatus de regno Norweiae." (2) Benedict of Peterborough (Rolls), I, 268–269 (for 1180 A. D.): "Eodem anno, scilicet M°C°LXXX°, Augustinus Nidrosiensis archiepiscopus, nolens aliquam subjectionem facere Suero presbytero, sedem archiepiscopatus sui reliquit, et venit in Angliam, et tulit sententiam excommunicationis in Suerum presbyterum." (3) William of Newburgh (Rolls), I, 231–232: "Qui, sacro ordine abjurato, et accepta in conjugem filia regis Gotorum, ab archiepiscopo terrae illius sollemniter coronari voluit. Verum ille cum esset vir magnus, et neque precibus neque minarum terroribus flecteretur ut caput execrabile sacra unctione perfunderet, ab eodem patria pulsus est." (4) Jocelin of Brakelond, in Memorials of St. Edmund's Abbey (Rolls), I, 222–223; same in Chronica Jocelini de Brakelonda (Camden Soc., London, 1840), p. 12: "Vacante abbatia perhendinavit Augustinus archiepiscopus Norweie apud nos in domibus abbatis, habens per praeceptum regis singulis diebus x. solidos de denariis abbatiæ; qui multum voluit nobis ad habendam liberam electionem nostram, testimonium perhibens de bono, et publice protestans coram regie quod viderat et audierat." (5) Pipe Roll, 27 Hen. II, Norfolk and Suffolk: "Abbatia de S' Aedmundo . . . in corredio Archiepiscopi Norwegiae xxxv. li., a vigilia S. Laurencii [August 9] usque ad diem S. Luce Evangeliste [October 18], scilicet de lxx diebus per breve Regis." Same, 28 Hen. II: "Et in liberatione Archiepiscopi de Norweia lix li. & x s. de xvii., septimanis per breve Regis." (Printed in Chron. Jocelini, Camden Soc., 1840, pp. 109–110.) (6) Sverris Saga (Fornmanna Sögur, VIII, 193), chap. 78 (1183 A. D.): "Eysteinn erkibiskup hafđi þat sumar komit vestan af Englandi snemma, ok hafđi verit III vetr á Englandi frá stóli sínum; ok đa saettist erkibiskup viđ Sverri konung, ok fór hann um sumarit norđr til stóls sîns." (7) Skálholts-Annaler, in Isl. Annaler, ed. Storm, Christ., 1888, p. 180, in the events for the year 1183 A. D.: "Eysteinn erkibyskup kom af Englandi." The other annals give 1182 as the date (Storm, pp. 118, 323). But Skalholt is confirmed by the saga. Some MSS. insert "til Noregs" after "kom."

In 1157 the new archbishop, whom Breakspeare consecrated, died, and the great Eystein succeeded him.[21]

Eystein is of especial interest in our study. He had communications with his great contemporary, Thomas of Canterbury, and himself spent three years of exile in England, after Becket's martyrdom. Moreover, Eystein was an author, and, in his case, we have certain evidence of literary connection between Norway and England. The oldest Latin account of the martyrdom and miracles of St. Olaf is by Eystein, and the fullest manuscript of this work was preserved in England, at Fountains Abbey.[22] We also have letters and laws attributed to Eystein. To him is dedicated perhaps the earliest existing history written in Norway, the Latin work of Tjodrek the monk.

The political career of Eystein cannot detain us here. He made a king and lost him. He made the crown of Norway subject to his own see, and won many other triumphs for the church, and lost most of them. He fought beside Magnus, the king of his creation, against the "Birchshanks" and their great leader, King Sverri the Priest, until Sverri's decisive victory at Iluvellir (May 27, 1180).[23] Then Eystein fled to England.

Already, more than ten years before, Eystein was in communication with Thomas à Becket. In an undated letter from Thomas to the Bishop of Meaux (near Paris), written apparently in France about 1168–1169, in which he complains of his exile, he adds, "Welcome, if it please you, besides, the bearers of these presents, Master Godfrey and Master Walter, messengers of our reverend brother, the Archbishop

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[19] Heimskringla, Saga of Sigurd, Inge, and Eystein, chap. 23 (Laing trans.).

[20] Two years later he became pope, under the title Adrian IV (1154–1159), being the only Englishman who has achieved that eminence. For Breakspeare's visit to Norway see Keyser, I, 219 ff., Munch, II, 865. I have nothing new to offer. The best mediæval accounts of his life are by Matthew Paris, William of Newburgh, and John of Salisbury.

[21] A good brief life of Eystein is that by L. Daae in the Trondhjem Jubilee Book (Festskrift udgivet i Anledning af Trondhjems 900 Aars Jubilæum, pp. 11–23, Trond., 1897).

[22] See F. Metcalfe, Passio et Miracula Beati Olaui, Oxford, 1881.

[23] Cf. Munch, III, 116.

mody."[11] Geoffrey of Durham, in his Life of Bartholomew the Anchorite of Farne, states that, when a youth, Bartholomew,[12] "fastidiosus novitatum amator," visited Norway, where he became a priest, refused an offer of marriage, and, after three years, returned to England.[13]

In 1107 King Sigurd, with sixty ships and about 10,000 men, by permission of Henry I, spent the winter in England, on his way to the Crusades.[14] "The sons of the last Magnus, Hasten and Siward," says William of Malmesbury, "yet rule conjointly, having divided the empire : the latter, a seemly and spirited youth, shortly since went to Jerusalem, by the route of England, performing many famous exploits against the Saracens."[15]

In 1135 the first bishop of Stavanger, in Norway, an Englishman, was executed by King Harald Gilli. According to the sagas, "Bishop Reinald of Stavanger, who was an Englishman, was considered as very greedy of money. He was a great friend of King Magnus, and it was thought likely that a great treasure and valuables had been given into his keeping." Harald tried to make him surrender his funds, but "the bishop declared he would not thus impoverish his bishop's see, but would rather offer his life. On this they hanged the bishop on the holm."[16]

About 1146 English monks founded two Cistercian abbeys in Norway.[17]

In 1152 an Englishman, Nicholas Breakspeare, reorganized the Norwegian church under its own metropolitan see at Nidaros (Trondhjem).[18] Breakspeare was at that time Cardinal Archbishop of Albano ; so the pope chose for this Scandinavian mission the man most likely

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[11] Symeon of Durham (Rolls ed.), II, 202–204. Turgot returned to England, became a monk in Durham, and later Bishop of St. Andrews (see index to above ed. of Symeon).

[12] He lived in the twelfth century; his dates are uncertain. See Symeon of Durham (Rolls), I, 295.

[13] Symeon of Durham (Rolls), I, 298.

[14] William of Malmesbury, Gesta Regum Anglorum (Rolls), II, 318–319; Heimskringla, Sig. Eyst. Ól., chap. 3; Fagrskinna (Munch-Unger, ed.), chap. 242.

[15] John Sharpe's trans., London, 1815.

[16] Heimskringla, Saga of Magnus the Blind and Harald Gille, chap. 8 (Laing trans.).

[17] Below, pp. 540, 542.

[18] Before this, since 1103, Lund had been the archbishop's seat for all Scandinavia. Under Nidaros were ten bishoprics, four in Norway, two in Iceland, and one each for Greenland, Sodor and Man, the Orkneys, and the Færöes.

In view of Taranger's results, only the briefest outline is necessary for the period preceding 1066. King Hákon the Good (reigned 935–961) was educated in England at Athelstan's court. After he became king he sent to England for a bishop and other teachers and made several ineffectual attempts to convert Norway from heathendom. The work was left for Olaf Tryggvason (995–1000), and he accomplished it with the aid of the sword. He was converted in England, and had with him in Norway, Sigurd, an English bishop. Iceland, too, was christianized in Olaf's reign, largely through Thangbrand, a missionary from England.[8] Olaf Haraldsson (*c.* 1016–1030), afterwards "St. Olaf," also received his Christian education in England. He continued Tryggvason's labors and organized the church in Norway. "He had with him," as Adam of Bremen says,[9] "many bishops and priests from England, by whose admonition and doctrine he himself prepared his heart for God, and to whose guidance he committed the people subject to him; among those famous for teaching and virtues were Sigafrid, Grimkil, Rudolf, and Bernard." Bernhard later worked in Iceland; so did Rudolph, who returned eventually to England, and became Abbot of Abingdon. Bishop Grimkell, with King Olaf, drew up a Christian law for Norway, in the vernacular. After Olaf's death he disinterred his body and pronounced him a saint.[10] Because of its dependence on England, the church in Norway stood in ill favor with its overlord, the Archbishop of Bremen. He forbade Harald Hárdráde (1047–1066) to have bishops consecrated in England, but Harald persisted. Among the Englishmen who came over to Norway in Harald's reign were Asgaut, nephew of Grimkell and third bishop of Trondhjem, and Osmund, who returned and died, at an advanced age, in the monastery at Ely.

The Period after the Norman Conquest.

Although our records are slight for the half-century after 1066, they indicate that the intimate relations between the Norwegian and the parent church remained unbroken. Symeon of Durham tells of a monk Turgot, who was imprisoned in Lincoln, and, escaping, hid as a stowaway on a ship sailing from Grimsby to Norway (*c.* 1069). King Olaf Kyrre (1066–1093) received him well. "Having heard that a clerk had come from England, he took him for his master in psalwhere in the middle ages, men in holy orders were the scholars and collectors of the old, and took a large part in creating new, literature. One of the greatest living authorities on Old Norse literature, Finnur Jónsson, is convinced that "the sagas in an overwhelming number are composed by Icelandic priests and ecclesiastics."[3] The two Sturlas (lawmen) — great exceptions indeed — are almost the only non-clerical saga writers whose names stand out of the blank of anonymity. Of clerical writers in Iceland we have Abbot Karl Jónsson (author of Sverris Saga), the monks Gunnlaug and Odd, each of whom wrote a life of Olaf Tryggvason; in Norway, Theodoric the monk (author of a twelfth-century Latin History of Norway), Archbishop Eystein, his contemporary (who wrote in Latin upon the martyrdom of St. Olaf), Abbot Robert (who translated the *Tristan* of Thomas and *Elye de St. Gilles* into the vernacular), and many others.[4] Finnur Jónsson thinks that most of the sagas were written down in the abbeys.[5] In the libraries of the monasteries and cathedrals curious scholars collected works from abroad, and Norwegian monks, returning from visits in England, deposited the illuminated vellums which they brought with them. There, we may believe, English clerks visiting in Norway left books from their native land; similarly manuscripts made in Norway came to English abbey libraries.

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[8] A Fleming in origin. The Althing of Iceland adopted Christianity 1000 A. D. Shetland, the Orkneys, and the Færöes yielded about this time.

[9] II, 55.

[10] For the cult of St. Olaf in England, cf. F. Metcalfe, Passio Olaui, Oxford, 1881, pp. 33 f.

In this investigation it will best serve our purposes not to examine comparative institutions so much as the actual visits of the clergy of one country to the other.[6]

Norway received its Christianity and its Christian Church from England. This has been demonstrated by Taranger.[7] The terminology and the peculiar institutions of the Norwegian Church were borrowed from the Anglo-Saxon. The church in Norway was established by kings educated in England, and by Anglo-Saxon bishops.

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[3] Litt. Hist., II, 1, 289.

[4] Ibid., II, 1, 10 ff.

[5] Ibid., II, 1, 289, etc.

[6] The best authorities on the Norwegian Church are still P. A. Munch, Det Norske Folks Historie, 8 vols., Christiania, 1852–1863; C. C. A. Lange, De Norske Klostres Historie, Copenhagen, 1847, revised 1856; R. Keyser, Den Norske Kirkes Historie under Katholicismen, 2 vols., Christ., 1856–1858. A list of principal authorities may be found on pages xi and xii of History of the Church and State in Norway, by T. B. Willson, Westminster, 1903. To this list add K. Maurer, Über Altnordische Kirchenverfassung und Eherecht, Leipzig, 1908. In the following essay I rely upon Munch, Lange, and Keyser for the general background. Therefore I need not give detailed references for well-attested statements which are not concerned directly with Anglo-Norwegian relations.

[7] A. Taranger, Den Angelsaksiske Kirkes Indflydelse paa den Norske, Christ., 1890.

THE RELATIONS OF THE NORWEGIAN WITH THE ENGLISH CHURCH, 1066–1399, AND THEIR IMPORTANCE TO COMPARATIVE LITERATURE.[1]

By Henry Goddard Leach.

Presented by G. L. Kittredge, March 10, 1909. Received March 10, 1909.

The relations of England with the Scandinavian countries after the Norman Conquest are obscure and little understood.

Scandinavia, especially Norway and Iceland, borrowed, translated, and redacted a large body of the common European literature. From whence did it come? Some critics have assumed an English literary counting-house for the romances translated in Norway during the reign of Hákon Hákonarson (1217–1263). Finnur Jónsson, writing in 1901, favored England. And yet Rudolf Meissner, one of the most recent and voluminous writers on these romances, takes it for granted that not only the romances but foreign culture and "courtesy" in general were imported by Norwegian students from France.[2]

As the translations themselves seem not to reveal the country from which their originals were borrowed, it is pertinent to ask, With what foreign land did Norway at that time stand in intimate relations? Also, with what foreign country were the producers of literature in Norway in such relations? As far as we know, the two classes in Norway who produced literature in the middle ages were the patron aristocracy and the clergy. It is my purpose here to examine the foreign relations of the latter with England.

The history of the Church in Norway and Iceland is closely identified with that of the literature. For in the North, no less than else-

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[1] The following essay is part of a dissertation entitled "The Relations between England and Scandinavia, from 1066 until 1399, in History and Literature," presented to the Faculty of Harvard University, 1908, in part fulfilment of the requirements for the degree of Doctor of Philosophy.

[2] Die Strengleikar, Halle, 1902, p. 132: "Bekanntschaft mit der französischen Dichtung vermittelten vor allem die sorgfältiger gebildeten norwegischen Geistlichen, die in Frankreich studiert hatten. Sie brachten die Ideen des Rittertums, der höfischen Bildung (*kurteisi*) nach dem Norden." Cf. p. 317, note 1.

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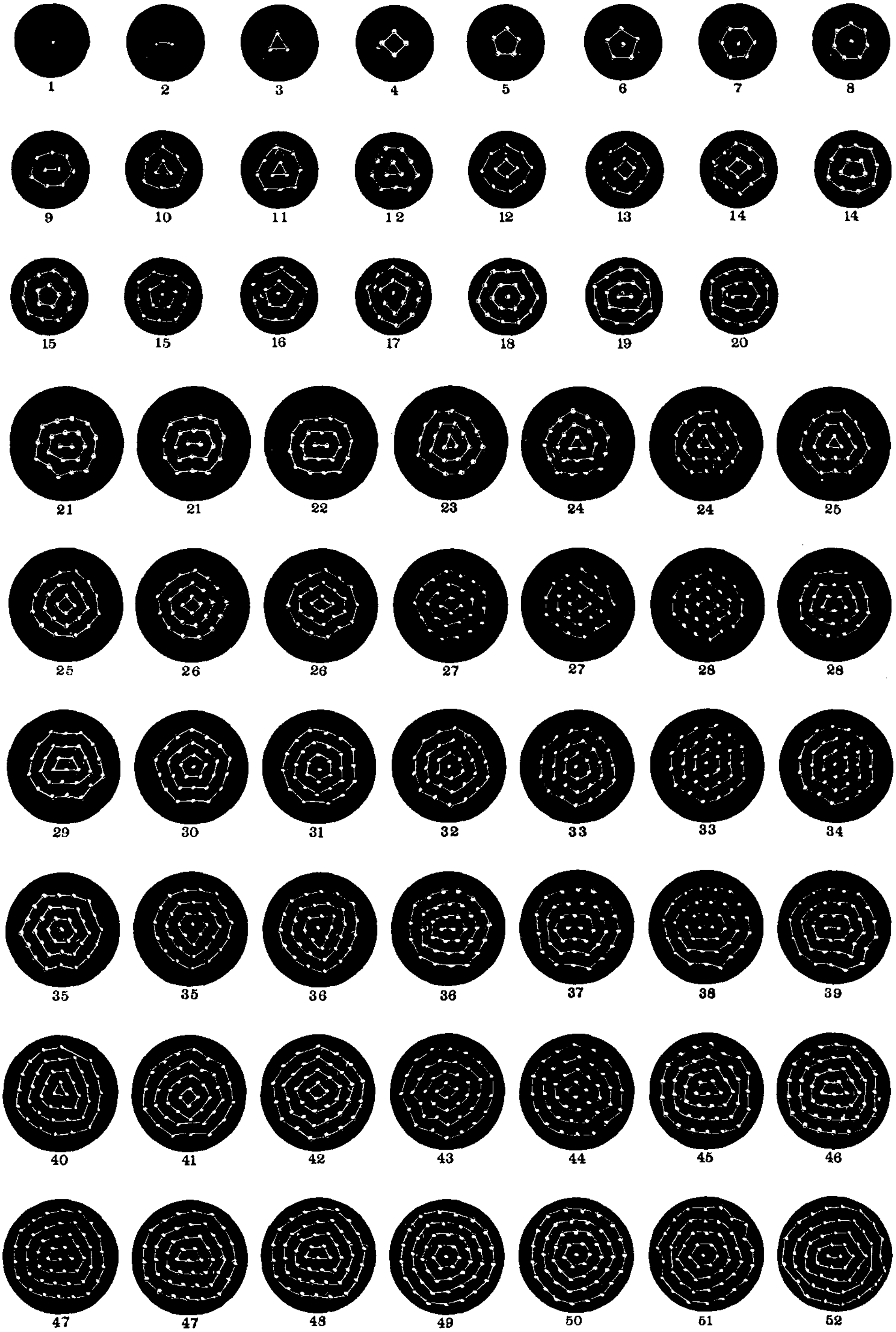
By Henry Goddard Leach.

DERR. — FLOATING MAGNETS. PLATE.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and larger rings. With fifty-two balls the central nucleus changes again from one to two, and the series continues much as before; but the figures are much crowded, and unless the balls are perfectly uniform it is often difficult to decide just which ones go to form particular rings.

The preceding table presents the results as calculated by the Thomson method and as photographed. It is curious that so many of the calculated minimum numbers should be obtainable with ease; with care in manipulation of the balls I have obtained a number of others, but not of sufficient stability to be photographed on the vibrating floor where the experiments were carried out.

CONFIGURATIONS AS CALCULATED AND OBSERVED.

Number of Magnets.	Calculated Rings.	Calculated Rings photographed.	Other Forms photographed.
1 to 5	1 to 5	1 to 5	. . .
6	1–5	1–5	. . .
7	1–6	1–6	. . .
8	1–7	1–7	. . .
9	1–8	. . .	2–7
10	2–8	. . .	3–7
11	3–8	3–8	. . .
12	3–9	. . .	4–8
13	3–10	. . .	4–9
14	4–10	4–10	5–9
15	5–10	5–10	1–5–9
16	5–11	. . .	1–5–10
17	1–5–11	. . .	1–6–10
18	1–6–11	1–6–11	. . .
19	1–6–12	. . .	2–7–10
20	1–7–12	. . .	2–7–11
21	1–8–12	. . .	2–7–12 and 2–8–11
22	1–8–13	. . .	2–8–12
23	2–8–13	. . .	3–8–12
24	3–8–13	3–8–13	3–9–12
25	3–9–13	3–9–13	4–9–12
26	3–9–14	. . .	4–9–13 and 4–10–12
27	3–10–14	. . .	4–9–14 and 4–10–13
28	4–10–14	4–10–14	. . .
29	5–10–14	5–10–14	. . .
30	5–10–15	. . .	1–5–10–14
31	5–11–15	. . .	1–6–11–13
32	1–5–11–15	. . .	1–6–11–14
33	1–6–11–15	1–6–11–15	1–6–12–14
34	1–6–12–15	1–6–12–15	. . .
35	1–6–12–16	1–6–12–16	1–7–12–15
36	1–7–12–16	. . .	1–8–12–15 and 2–7–12–15
37	1–8–12–16	. . .	2–8–12–15
38	1–8–13–16	. . .	2–8–12–16
39	2–8–13–16	2–8–13–16	. . .
40	3–8–13–16	. . .	3–9–13–15
41	3–8–13–17	. . .	4–9–13–15
42	3–9–13–17	. . .	4–9–14–15
43	3–9–14–17	. . .	4–9–14–16
44	3–10–14–17	. . .	4–10–14–16
45	4–10–14–17	. . .	5–10–13–17
46	5–10–14–17	5–10–14–17	. . .
47	5–10–15–17	5–10–15–17	5–10–14–18
48	5–10–15–18	5–10–15–18	. . .
49	5–11–15–18	. . .	1–6–11–14–17
50	1–5–11–15–18	. . .	1–6–11–14–18
51	1–6–11–15–18	1–6–11–15–18	. . .
52	1–6–12–15–18	. . .	2–7–12–14–17

the unmagnetized ones are apt to draw into contact and spoil the figure unless kept several diameters apart. The balls are much more convenient than needles; and as they give very nearly the same figures, the law of force cannot be very different in the two cases.

Professor J. J. Thomson[5] has discussed the stability conditions of a ring of negatively electrified corpuscles within a sphere of positive electricity, and has given a method of calculating the minimum number of such corpuscles required to hold an outer ring of a given number in stable equilibrium. It is interesting to compare the figures actually obtained with the results of the calculation. Complete agreement can hardly be expected, partly because the calculated numbers are minimum values and may represent in some cases forms of such slight stability that they might be difficult to reproduce, but chiefly because the law of force in the concrete case is quite different from the simple law of electric attraction. With the floating balls only the horizontal component of the central attraction is available in producing motion toward the centre of the figure; and as this is an increasing fraction of the entire force as the distance from the centre increases, the pull on a large outer ring is virtually increased and a larger number of balls will be required to hold it in equilibrium. This is exactly what takes place, as may be seen from the accompanying table, where for a considerable number of balls the number inside the outer ring is almost always larger than the calculated minimum.

The configurations shown are those obtainable without much difficulty, no special effort having been made to secure forms of very slight stability. In fact, with perfectly clean balls and mercury it is not easy to obtain many "isomers" unless the apparatus is very free from vibration, a figure which is quite stable enough to be photographed sometimes working itself over into quite another form after five or ten minutes. The effects of surface tension modify the results greatly, as shown by A. W. Porter,[6] who was able to obtain a ring of fifteen magnets without a central nucleus, in a dish of water filled to overflowing. Lack of perfect equality in the balls will distort figures otherwise symmetrical, and if the mercury surface is even slightly dirty the inner balls arrange themselves with nearly uniform spacing, without much reference to the number in the outer ring. The white lines in the figures have been drawn upon the negatives to mark the contours, and are not a part of the experiment. The figures clearly show the periodic nature of the structure, as has been noted from the first; the larger figures are obtained from the smaller by the simple addition of more

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[5] Phil. Mag., Ser. 6, **7**, 237.

[6] Nature, 64, 563.

CONTRIBUTIONS FROM THE ROGERS LABORATORY
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LII.—A PHOTOGRAPHIC STUDY OF MAYER'S FLOATING MAGNETS.

By Louis Derr.

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Though Professor A. M. Mayer's beautiful experiment of floating magnetized needles over a magnetic pole has been variously modified in details, it has for many years been regarded chiefly as an interesting study of a rather special set of forces; but the recent investigations into the structure and possible electrical nature of the atom have lent a new interest to the equilibrium figures formed by the floating magnetic poles, and have suggested that they may illustrate the arrangement of sub-atomic corpuscles, at least in the limited degree possible in two dimensions. Mayer's original paper[1] gives drawings of 8 arrangements of 3 to 7 needles, and a later one[2] gives all the configurations of 2 to 8 needles. A fuller discussion[3] gives a list of all the configurations up to 51, with drawings up to 20 needles. Professor R. W. Wood[4] showed that bicycle balls could be used, and gives 20 symmetrical figures. I have therefore thought it might be of interest to assemble pictures of an entire series, in order to show the progression from one form to another more clearly than can be done by tables; and the accompanying Plate is a reproduction from photographs of the more stable forms assumed by the magnets when their number is varied from 1 to 52.

The magnets were clean quarter-inch steel balls, floated on freshly-filtered mercury as described by Wood, but initially magnetized by placing them one by one between the jaws of a powerful electromagnet. Equilibrium figures may be obtained with unmagnetized balls, both hard and soft; but the magnetized balls are more easily managed, as

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[1] American Journal of Science, **95**, 276.

[2] Ibid., p. 477.

[3] Ibid., **96**, 247.

[4] Phil. Mag., Ser. 5, **46**, 162.

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With a Plate.